

## ABSTRACT

Title of dissertation: Thesis for  
 $G_E^n$  at  
moderate  $Q^2$

Jonathan Miller, Doctor of Philosophy, 2009

Dissertation directed by: Professor Elizabeth Beise  
Department of Physics

The electric form factor of the neutron,  $G_E^n$ , is a parameter of interest among nuclear physicists, both in providing greater understanding of the nucleon structure, and for providing values with which to analyze other experiments. There has been a renewed interest in this quantity as the result of the improved quality of data provided by double polarization experiments, which have matured in the last decade. A survey of these double polarization experiments is presented in this paper, as are the techniques of recoil polarization and target polarization used to make the measurements. Selected models used to interpret the data are also presented. Double polarization measurements have provided values for the kinematic range of 0.4 to 1.5 GeV/c<sup>2</sup>, with a current measurement reaching to 3.4 GeV/c<sup>2</sup>.

Thesis for  $G_E^n$   
at moderate  $Q^2$

by

Jonathan Miller

Dissertation submitted to the Faculty of the Graduate School of the  
University of Maryland, College Park in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy  
2009

Advisory Committee:  
Professor Elizabeth Beise, Chair/Advisor  
Dr. Bogdan W.  
Professor Carter Hall  
Professor Michael Coplan  
Professor Steve Wallace

© Copyright by  
Jonathan Miller  
2009

# Table of Contents

List of Tables	v
List of Figures	viii
List of Abbreviations	xvii
 1 Physics of the Nucleon	 1
1.1 Introduction to the Nucleon . . . . .	1
1.2 QCD . . . . .	2
1.2.1 Perturbative QCD . . . . .	4
1.3 Nucleon Structure . . . . .	5
1.3.1 Partons . . . . .	6
1.3.2 The Constituent Quark Model . . . . .	8
1.3.3 Generalized Parton Distributions . . . . .	9
1.3.4 Vector meson dominance models . . . . .	10
1.3.5 Chiral effective field theory . . . . .	11
1.3.6 Dyson-Schwinger Equations . . . . .	12
1.4 Experimental Investigation . . . . .	13
 2 Physics of Electron and Nucleon Scattering	 14
2.1 Introduction to electron scattering . . . . .	14
2.2 Definition of kinematic variables . . . . .	15
2.2.1 Current operator . . . . .	16
2.2.2 Dirac and Pauli form factors . . . . .	16
2.3 Breit frame . . . . .	17
2.4 Quasi-elastic scattering . . . . .	19
2.4.1 Impulse approximation . . . . .	20
2.4.2 PWIA . . . . .	21
2.5 Polarization observables . . . . .	21
2.5.1 Recoil polarization . . . . .	23
2.5.2 Target polarization . . . . .	25
2.6 Cross section with unpolarized observables . . . . .	25
2.7 Cross section with polarized observables . . . . .	26
 3 Form Factors	 28
3.1 Models and fits . . . . .	28
3.1.1 Fits . . . . .	28
3.1.2 Vector meson dominance model . . . . .	29
3.1.3 Pion cloud and CQM . . . . .	29
3.1.4 Model of Generalized Parton Distributions . . . . .	30
3.1.5 pQCD predictions . . . . .	30
3.2 Measuring G <sub>En</sub> via the Rosenbluth technique . . . . .	31
3.2.1 Elastic e-D scattering . . . . .	33

3.2.2	Quasi-elastic scattering and the Rosenbluth Technique . . . .	34
3.2.3	Difficulties with Rosenbluth method . . . . .	35
3.3	Double polarization techniques . . . . .	35
3.3.1	Quasi-elastic scattering and (e,e'n) . . . . .	37
3.3.2	Recoil polarimetry . . . . .	40
3.3.3	Polarized target measurement . . . . .	42
3.3.4	Experiments with polarized deuterium . . . . .	43
3.3.5	Experiments with polarized helium-3 . . . . .	44
3.4	Summary of past measurements . . . . .	47
3.5	E02-013 at Jefferson Lab . . . . .	49
4	$G_E^n$ experiment E02-013 . . . . .	51
4.1	Experimental Overview . . . . .	51
4.2	Principle and Experiment setup . . . . .	52
4.3	The Electron Beam . . . . .	57
4.4	The Helium-3 Target . . . . .	60
4.4.1	Target System . . . . .	63
4.4.2	Polarization . . . . .	64
4.4.3	Polarization Measurement . . . . .	68
4.5	Big Bite . . . . .	73
4.5.1	The Big Bite Apparatus . . . . .	73
4.5.2	The Big Bite Scintillators and Shower . . . . .	75
4.5.3	The Wire Chambers and Tracking . . . . .	76
4.5.4	Big Bite Electronics . . . . .	79
4.6	Big Hand . . . . .	81
4.6.1	Neutron Geometry . . . . .	82
4.6.2	Veto and Marker Bars . . . . .	83
4.6.3	Neutron Arm Electronics . . . . .	84
4.7	Data Acquisition . . . . .	84
4.7.1	Encoding and Decoding . . . . .	88
5	Analysis . . . . .	92
5.1	Event filtering and scintillator timing . . . . .	93
5.2	Electron parameters . . . . .	99
5.3	Hadron identification . . . . .	103
5.4	Identification of the hadron charge . . . . .	112
5.5	Dilution of the neutron sample by protons . . . . .	118
5.5.1	Single Track Analysis and Background Correction . . . . .	129
5.5.2	Nitrogen Dilution . . . . .	134
5.5.3	RF correction . . . . .	135
5.6	Asymmetry calculation . . . . .	135
5.7	Corrections . . . . .	142
5.7.1	FSI Corrections . . . . .	142
5.7.2	Inelastic Contribution . . . . .	145

6	Results and Discussion	148
6.1	Framework Formalism . . . . .	148
6.2	Results . . . . .	150
6.3	Pauli and Dirac Form Factors . . . . .	150
6.3.1	Flavor Form Factors . . . . .	152
6.4	GPD Models . . . . .	154
6.4.1	Impact Parameter Space . . . . .	154
6.4.2	Fit to our data . . . . .	156
A	Neutron Arm Analysis	159
A.1	A Neutron Arm Analysis . . . . .	159
A.2	Simulations . . . . .	160
A.3	Calibrations . . . . .	161
A.3.1	HV Tune . . . . .	163
A.3.2	Time Offsets . . . . .	165
A.3.3	Initial Calibration . . . . .	168
A.3.4	Amplitude Correction . . . . .	178
A.3.5	Energy Calibration . . . . .	179
A.3.6	Veto Calibration . . . . .	179
A.4	Neutron Software . . . . .	180
A.4.1	Cluster formation . . . . .	180
A.5	Veto Analysis . . . . .	185
B	Error Analysis	191
C	Calculations	203
C.1	Asymmetry Best Value . . . . .	203
	Bibliography	205

## List of Tables

1.1	The three light quarks and their respective charges. Other quarks are the c (charm), b (beauty), and t (top) quark, but they are too heavy to play a large role in the experiments described here. These three light quarks form the SU(3) group. . . . .	4
3.1	Double polarization experiments to measure the electric form factor of the neutron that have been carried out to date[64]. . . . .	38
4.1	A table showing the running conditions, time, accumulated charge, and kinematics of the different measurements referred to here and elsewhere as kinematics or for short kin. Kinematic one includes both the commissioning time and the first measurement. The measurement of $Q^2 = 1.3$ and $1.7 \text{ GeV}^2$ provide a comparison of the polarized $^3\text{He}$ technique with previous recoil polarization data while the other two measurements extend the measured range of $Q^2$ . Not shown is the periods for commissioning and kinematic 1 which won't be covered in this thesis. . . . .	54
4.2	A table of the beam polarization as measured by different means at different times. The Møller measurements were done in the Hall A Møller. The Mott measurements were done using the injector Mott measurement. Only selected Compton data is presented, the average is the averaged of all data for Compton. . . . .	91
5.1	A table of the initial event selection for analysis. Sanity cut 1 is the requirement that Big Bite has a track, Sanity cut 2 is the requirement that at least one scintillator paddle in Big Bite has an event, Sanity cut 3 is that the coincidence time (L1A) was recorded. Sanity cut 4 is the requirement that there is a coincidence event. Negative is a requirement that only negative particles in the drift chambers can be considered as electrons. . . . .	103
5.2	A table of selection criteria used to determine which veto hits to consider when doing charge identification. The spatial cuts of Y1, Y2, and X determine which bars are to be included, events were only differentiated by time and amplitude within a veto detector which was within range of the hadron hit. The subscript $V$ signifies veto, while $C$ signifies the cluster in the neutron arm. . . . .	118

5.3	A table of the ratios for different criteria. Over the course of a kinematic setting, runs varied in rate. This had a small effect on the purity factor, the results presented in this table is for a normal run. .	125
5.4	The effective ratio of $\frac{Z}{N}$ for the target and cuts. . . . .	125
5.5	A table for the Nitrogen Dilution factor determination. Uncertainties are discussed in Appendix B. Densities are determined from measurements of the target [51][46][68]. . . . .	134
5.6	Table of quasi-elastic event selection cuts. The Single QE cut is that there is one and only one hit in the associated region of time and $q_{\perp} < 0.15$ . This is in addition to the region of time and $q_{\perp}$ which are selected as quasi-elastic event candidates. The vertical cut and horizontal cut are neutron arm fiducial cuts. . . . .	137
5.7	Determination of the physical asymmetry from the measured experimental asymmetry. A full discussion and analysis of errors is presented in Appendix B. . . . .	139
5.8	Expansion Coefficients in the determination of $\lambda$ from the physical asymmetry. Uncertainty in $\lambda$ calculated as discussed in Appendix B. .	141
5.9	Displayed is the calculated values for $\lambda$ and $Q^2$ , and the value of $G_M^n$ as arrived at from a parameterization [48]. From this the value of $G_E^n$ can be determined. Determination of uncertainty is discussed in Appendix B. . . . .	142
5.10	Shown are preliminary inelastic asymmetries and dilutions for the various kinematic [69]. The * row relates to an expanded selection of quasi-elastic events, with a maximum invariant mass of 1.2 GeV rather than the usual maximum of 1.05 (see Table 5.6). . . . .	146
A.1	Table of the list of high voltage changes (HV) for the neutron arm in the E02-013 experiment. . . . .	189
A.2	A table of the correlation between hits in Veto plane 1 and Veto plane 2. The is for all events within a single run in kinematic 2A (3190). . . . .	190
B.1	A table showing the measurements and error calculations for the nitrogen dilution of the uncharged sample, for all three kinematics. $N_{^3He}$ is the number of background corrected counts for the selected $^3He$ runs, $\rho_{N_2}$ is the nitrogen density, $Q_{^3He}$ is the total charge of the selected $^3He$ runs. This calculation is as described in section 5.5.2. .	197



B.2	A table of the errors for the calculation of $A_{proton}$ . The calculation is calculated according to equation 5.54. The uncertainty in $G_M^p$ and $G_E^p$ is determined using a parameterization [48] and the propagation of the error is according to equation B.1. . . . .	198
B.3	A table for all three kinematics showing the measurements for the charged and uncharged background and the uncertainty in the background ratio. . . . .	199
B.4	A table of the errors for the calculation of $G_E^n$ . Uncertainty in the ratio of form factors was calculated from the physical asymmetry as expressed in section 5.6 and Table 5.9 and Table 5.8. The uncertainty in $G_M^n$ is determined from a parameterization [48] and propagation of the error is according to equation B.1. . . . .	199
B.5	A table of the uncertainties that contribute to the physical asymmetry ( $A_{phys}$ ). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation ???. This table shows the errors for $Q^2 = 2.5 \text{ GeV}^2$ . . . . .	200
B.6	A table of the uncertainties that contribute to the physical asymmetry ( $A_{phys}$ ). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation ???. This table shows the errors for $Q^2 = 3.5 \text{ GeV}^2$ . . . . .	201
B.7	A table of the uncertainties that contribute to the physical asymmetry ( $A_{phys}$ ). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation ???. This table shows the errors for kinematic 4. . . . .	202

## List of Figures

1.1	Shown is a schematic of the DVCS process, in the framework of generalized parton distributions (GPDs). Here $\xi$ is the skewness, $x$ is the longitudinal momentum fraction, $N$ is the nucleon, $\gamma^*$ is the virtual photon, and $\gamma$ is the detected photon. The skewness gives the longitudinal momentum asymmetry, as shown in the figure. . . . .	8
2.1	Feynman diagram for one interaction between an electron and a nucleon. The electron has four-momentum $k^\mu$ and the nucleon has four-momentum $p^\mu$ . . . . .	15
2.2	Feynman diagram for interaction between an electron and the nucleus. The electron has four-momentum $k^\mu$ and the ejected nucleon has momentum $p_f^\mu$ . . . . .	19
2.3	Polarization exchange in the one photon exchange approximation for polarized-electron, polarized nucleon scattering. $\theta$ and $\phi$ define the polarization of the recoiling nucleon. . . . .	22
3.1	Various fits discussed in the text. The points plotted are from [55], which was used to fit the pQCD curve. These data are only representative, see figure 3.2 for the full set. The Bijker curve is a representative Vector Meson Dominance model, the Guidal curve is a representative GPD based model, the Miller curves are from a CQM. . . . .	32
3.2	Previous $G_E^n$ data from experiments involving polarization techniques. The curve is the Galster parametrization [33]. Herberg provided new calculations of the Ostrick data, while Golak provided a FSI corrected analysis of the Becker data. For references see Table 3.3. . . . .	36
3.3	Double polarization data and selected models in the region where polarized data is available. Models are described in section 3.1 while data is referenced in Table 3.3. . . . .	48
4.1	Image of Jefferson Laboratory. Shown is the accelerator and the mounds over the three experimental halls. The mound to the left is Hall A. . . . .	53

4.2	General layout, not to scale, of the experiment. Shown is the beam-line, the target, the Big Bite detector stack, and the Neutron Arm detector. Also shown are the two polarimeters in the hall before the target, and the approximate distances to the detectors. The components of the Big Bite (BB) detector stack are also shown and consist of the Big Bite dipole magnet, multi-wire drift chambers (MWDCs), pre-shower, shower, and timing plane. . . . .	55
4.3	Coordinate systems for Big Hand, Target, and Lab. The lab $z$ is along the beam line while the target $z$ is toward Big Bite. . . . .	56
4.4	Schematic of the accelerator as it existed in 2006. Shown are the three halls, injector, and IINACs. . . . .	57
4.5	Diagram of the helium-3 nucleus. Shown is the most common state with the proton spins aligned anti-parallel. The neutron carries $\approx 86\%$ of the polarization of the nucleus while the protons carry $\approx 3\%$ . . . . .	61
4.6	Picture of an empty $^3\text{He}$ cell. On top is the spherical pumping chamber, and below is the cylindrical target chamber. The beam passes through the the 40cm of target material[34]. . . . .	62
4.7	Measurement of the holding field as a function of position along the beamline in the target. The origin is the target center. . . . .	65
4.8	Magnet box for Target [46]. Shown is the coils that create the magnetic field. . . . .	66
4.9	Figure showing the polarization process [46]. Optical pumping excites the electron to an excited state, which can exchange (50%) to a state that decays to a spin state that is not optically pumped by the lasers. . . . .	67
4.10	In a traditionally polarized Helium cell, one that contains only Rb and He, the Rb exchanges spin in one of three ways. Either by changing the spin of the He nucleus, or by just rotating the atoms it interacts with. This rotation is not useful for polarizing the He, so by adding K, which is likely to engage in 'spin rotation' with the Rb, but likely to engage in 'spin exchange' with He, the polarization efficiency of the cell is increased. Only the spin-exchange collision process causes polarization of the $^3\text{He}$ [78] [46]. . . . .	69

4.11	The coils used for the NMR measurement of target polarization. Shown is the holding field coils (iron box), the RF coils (which can be adjusted), and the pick up coils. Two sets of pick up coils are down at the target chamber, one more set is up above the pumping chamber. One of the RF coils could be adjusted, as could the pick up coils, to maintain the transverse relationships needed for an NMR measurement. . . . .	70
4.12	Measured polarization in the $^3\text{He}$ cells during the experiment [45]. Shown is all cells, including Dolly, Edna, and Barbara used during Kinematics 2, 3, and 4. . . . .	73
4.13	Schematic of the Big Bite detector. Shown is the large dipole which provides a magnetic field integral of 1.2 T·m, the 15 planes in three chambers that made up the wire chamber, and the lead glass shower and preshower with the layer of plastic scintillator between. . . . .	74
4.14	Not to scale diagram showing Big Bite, the location of the dipole, target, drift chambers, and shower, and the quantities known as the deflection angle ( $\theta_{def}$ ), $\theta_{tgt}$ , and the back and front tracks. Shown on the upper left is the coordinate system for the target coordinates. The shower is shown providing a fourth location for the particle, to anchor the track reconstruction. Big Bite detector coordinates have their origin at the center of the first plane. The $z$ direction is perpendicular to the first chamber, and $x$ is the magnetic dispersion direction. . . . .	78
4.15	Momentum resolution achieved with the latest optics model. Shown is the momentum resolution for all four kinematics. . . . .	80
4.16	Resolution of the carbon foils after vertex corrections. The foils should be 6.7 cm apart with the BeO foil at 0 cm. This histogram is of data from a foil target run in kinematic 3. . . . .	80
4.17	The electronics schematic for Big Bite. Shown are the modules and the wire lengths. . . . .	81
4.18	Neutron Arm electronics schematics. Included in the schematic is the length of the cables. From the PMTs of the various tubes, the signal goes into both a TDC and a sum module with the non-summed output going to an ADC. The different summed signals are added together and sent to both an ADC and TDC, with an OR of the TDCs forming the trigger. . . . .	85
4.19	. . . . .	86

4.20	Plot showing the reference time and coincidence counts in relation to the Big Bite trigger time. Best coincident events are in the circled region. . . . .	89
5.1	The flowchart for the analysis. This is after the preliminary replay of events that is described in section 4.7. The resulting ROOT files contain the subset of events necessary to undertake the remaining analysis, also assigns charge, and identifies quasi-elastic event candidates. Two separate scripts are ran as part of this process for each run before this post processing. This is to determine the RF correction (see section 5.5.3) and the hadron time-of-flight. . . . .	94
5.2	Flowchart for the event processing. Events in the analysis are ignored if no tracks or Big Bite scintillator hits exist, or if the event does not have a well defined trigger time as demonstrated by the existence of the Level 1 Accept (L1A) Time. The optics can be updated here if necessary, and the kinematics of the event are calculated to ease post processing. All event cuts are detailed in Table 5.1. . . . .	95
5.3	Flowchart associating clusters, both with each other and as quasi-elastic event candidates. Some clusters should be associated together, where some neutron detector did not fire to make the cluster continuous. Charged candidates are identified, and momentum is calculated for the hit. . . . .	96
5.4	Amplitude distribution for all events showing pions and electrons in Big Bite. In blue is the distribution after cuts have been applied to remove events which are lacking the information needed for analysis, shaded is the region selected. See Table 5.1 for cuts, in black is all events and in blue is with just the “sanity“ cuts applied. This distribution is for $Q^2 = 3.5 \text{ GeV}^2$ . . . . .	97
5.5	Electron momentum for all tracks in Big Bite. In blue are just those tracks for which the preshower energy is greater than 450 channels, shaded is the selected momentum. See Table 5.1 for cuts, in black is all events and in blue is with the “sanity” and pre-shower energy cuts applied. This distribution is for $Q^2 = 3.5 \text{ GeV}^2$ . . . . .	100
5.6	X position (m) within the Big Bite scintillator detector. This is for before cuts (black), after electron and sanity cuts (blue), with the selected region of X shaded. Variation is due to the lack of calibration for the total shower in the Big Bite detector. See Table 5.1 for cuts, in black is all events and in blue is with the “sanity”, pre-shower energy, and electron momentum cuts applied. This distribution is for $Q^2 = 3.5 \text{ GeV}^2$ . . . . .	101

5.7	Vertex, this shows the target $z$ position reconstructed by the Big Bite wire chambers for all events with tracks (black). In blue is after cuts on particle ID. Shaded is the region selected as being within the target. See Table 5.1 for cuts, in black is all events and in blue is with the “sanity“, pre-shower energy, Big Bite fiducial, and electron momentum cuts applied. This distribution is for $Q^2 = 3.5 \text{ GeV}^2$ . . . .	102
5.8	$q_\perp$ versus $W$ for neutral particles in the various kinematics. Shown is the selection of quasi-elastic events in the red square. As shown, at higher $Q^2$ the inelastic tail becomes stronger. . . . .	104
5.9	$q_\perp$ versus $W$ for charged particles in the various kinematics. In the red box is the selection of quasi-elastic events. . . . .	105
5.10	$q_\parallel$ versus $W$ for charged particles in kinematic 3. ? - ? In the conversion section or the ND section ? - ? . . . . .	106
5.11	Distribution of $q_\perp$ in the various kinematics. Shown in blue is the neutral spectrum, in black is the total spectrum (That passed the event based selection). The majority of neutral quasi-elastic events fall under 0.15 GeV. . . . .	107
5.12	$\Delta T$ (ns) for the various kinematics. In blue is the neutral time-of-flight spectrum, in black is the total time-of-flight spectrum. The photon peak is shown at $\approx -3\text{ns}$ . Quasi-elastic events are selected with time between -1 and 1 ns. . . . .	109
5.13	Missing mass (GeV) for the various kinematics. In blue is the neutral scaled up by a factor of 20 for $Q^2 = 3.5 \text{ GeV}^2$ and 10 for $Q^2 = 2.5 \text{ GeV}^2$ , in black is the total spectrum. . . . .	110
5.14	$\Delta T$ (ns) for the various kinematics. In blue is the charged time-of-flight spectrum, in blue is the charged time-of-flight spectrum. The charged spectrum is scaled down to the neutral spectrum. . . . .	111
5.15	Shown in these figures is the time versus $q_\perp$ for various kinematics. Also shown is the box showing which cuts were used for identifying quasi-elastic clusters within the neutron arm. As is shown, the ability to select quasi-elastic clusters is much better for the lower kinematics. . . . .	113
5.16	Shown in these figures is the time versus invariant mass for various kinematics. Also shown is the box showing which cuts were used for identifying quasi-elastic clusters within the neutron arm. As is shown, the ability to select quasi-elastic clusters is much better for the lower kinematics. . . . .	114

5.17	Shown in these figures is the time versus missing mass for various kinematics. Also shown is the box showing which cuts were used for identifying quasi-elastic clusters within the neutron arm. As is shown, the ability to select quasi-elastic clusters is much better for the lower kinematics. . . . .	115
5.18	Shown in these figures is the invariant mass versus missing mass for various kinematics. Also shown is the box showing which cuts were used for identifying quasi-elastic clusters within the neutron arm. As is shown, the ability to select quasi-elastic clusters is much better for the lower kinematics. . . . .	116
5.19	Vertical position (m) within the Big Hand detector. This is for before cuts (black), after event selection cuts (blue), with the selected region of $x$ shaded. In the vertical direction the neutron arm was segmented, giving the segmented structure observed. GLA bars are removed, as seen in the most negative region. See Table 5.6 for cuts. This distribution is for $Q^2 = 3.5 \text{ GeV}^2$ . . . . .	119
5.20	Horizontal position (m) within the Big Hand detector. This is for before cuts (black), after event selection cuts (blue), with the selected region of $y$ shaded. In the horizontal direction the neutron arm was not entirely covered by the veto planes, giving the peaks on the two edges. See Table 5.6 for cuts. This distribution is for $Q^2 = 3.5 \text{ GeV}^2$ . . . . .	119
5.21	Presented is the ratio of protons to neutrons within the target for nitrogen. Shown are two models, one a simple extension to the plane wave impulse approximation (in blue), and the other is a model by Udias (in black). Shown is the ratio for various cuts of $p_\perp$ . A cut of 150 MeV was used in this experiment. This is for $Q^2 = 3.5 \text{ GeV}^2$ . . . . .	121
5.22	Presented is the ratio of protons to neutrons within the target for $^3\text{He}$ . Shown are from cuts on $p_\perp$ . The applied cut on $p_\perp$ was 0.15 GeV with $P_\parallel$ cuts of 400, 250, and 200 MeV for kinematics of $Q^2 = 3.5, 2.5$ , and $1.7 \text{ GeV}^2$ . Shown is the ratio for various cuts of inverse distance. This study was done by Aidan Kelleher. . . . .	122
5.23	Shown in these figures is the measured uncharged to charged ratio, the uncharged to charged ratio within the model, and the fit line of the model in nitrogen. These are plotted against the natural dependence $\frac{\xi}{T}$ . . . . .	126
5.24	Shown in these figures is the measured uncharged to charged ratio, the uncharged to charged ratio within the model, and the fit line of the model in hydrogen. These are plotted against the natural dependence $\frac{\xi}{T}$ . . . . .	127

5.25	Shown in these figures is the measured uncharged to charged ratio, the uncharged to charged ratio within the model, and the fit line of the model in $^3\text{He}$ . These are plotted against the natural dependence $\frac{\xi}{T}$ .	128
5.26	Plot $q_\perp$ for a region in time. This is 5 ns removed from the peak region.	130
5.27	Plot $q_\perp$ for a region in time. This is at the quasi-elastic region. . . . .	130
5.28	Plot of the ratio of uncharged to charged for the region of $q_\perp$ between 0.55 and 0.60 $\text{GeV}/c$ . The region near 3 ns is removed due to the photon peak being there. Add axis information time (ns), update description (changed plot) . . . . .	131
5.29	Plot of $q_\perp$ versus time-of-flight for events with only a single hit in the $ t  < 7$ and $q_\perp < 0.15$ region. The area in the spectrum of events coming from $k$ photons via $\pi_0$ production was removed (between 2 and 4 ns). This is for runs 4090-4099. . . . .	131
5.30	Spectrum used to determine the RF correction. This is for run 3888. The peak location is determined from the Mid parameter, which is $t_{rfshift}$ . . . . .	136
5.31	RF correction versus run number for $Q^2 = 2.5 \text{ GeV}^2$ . Note that since the plotted function is <b>mod</b> $(\frac{1}{0.499})$ that the values of 2 ns and 0 ns are almost equivalent. . . . .	136
5.32	Shown is the run number versus raw asymmetry for $Q^2 = 2.5 \text{ GeV}^2$ . .	138
5.33	The four dominant lowest order diagrams in single nucleon scattering. Shown is (a) impulse approximation, (b) meson exchange currents, (c) isobar contributions, and (d) FSI. [75] . . . . .	143
5.34	In nuclei with more than a single nucleon, the plane wave impulse approximation can be used to give the description of the interaction. Shown are the leading order diagrams for $^3\text{He}$ which is the PWIA diagram, single re-scattering diagram, and double re-scattering diagram. [75] . . . . .	143
5.35	The asymmetry as a function of invariant mass. This shows that the invariant mass doesn't over the region selected (see Table 5.6). . . . .	146
6.1	Pauli form factor ( $F_1$ ) dependent on $G_E$ and the dipole approximation for $G_M$ . . . . .	151
6.2	Dirac form factor ( $F_2$ ) dependent on $G_E$ and the dipole approximation for $G_M$ . . . . .	151



6.3	Ratio of the down and up quark flavor Pauli form factors. The form factors not dependent on $G_E^n$ are just dipole approximations. . . . .	153
6.4	Ratio of the down and up quark flavor Dirac form factors. The form factors not dependent on $G_E^n$ are just dipole approximations. . . . .	153
6.5	Diagram showing the transverse charge density of a simple model where the nucleon is polarized along the x-axis. Lighter is greater density. . . . .	158
A.1	Calibration of second plane to first plane using Rob's script. . . . .	169
A.2	Horizontal calibration of second plane to first plane using Rob's script.	169
A.3	Horizontal calibration of the first plane using Rob's script. . . . .	170
A.4	Vertical calibration of the first plane using Rob's script. . . . .	170
A.5	Calibration tof plane 1 bar 20. This shows the time of flight peak for quasi-elastic protons between the BigBite scintillators and the front plane of the neutron arm. . . . .	172
A.6	Calibration lr plane 1 bar 20. This histogram shows the coincidence peak in the horizontal bars time, with coincidence with the marker counters. . . . .	173
A.7	Calibration diff plane 2 bar 12. This is the difference between neighboring bars in different planes. In particular, this is the difference between plane 1 bar 12 and plane 2 bar 12. . . . .	175
A.8	Calibration lr plane 2 bar 12. This is the horizontal position (transformed to time) of the hits in plane 1 bar 12 and plane 2 bar 12 for events with well timed hits within the two bars. . . . .	176
A.9	Calibration summary plot for run 3975 after pass 3 calibration. In this plot, X is the result, + is the before calibration. On the x axis, 0 is for plane 1 and 1-6 show the differences between planes 1 and planes 2-7). At 7 on the x axis is the horizontal absolute position calibration for plane 1. . . . .	177
A.10	Calibration veto tof left 1 bar 16 . . . . .	181
A.11	Calibration of one side of the veto detector. . . . .	182

A.12	Plot of X position in the veto subtracted by the X position in the neutron arm versus $\Delta t$ . These are for quasi-elastic hits in the neutron arm. Plotted on the x axis is the time difference between the neutron arm and the veto arm. This identifies a region of 0.3-0.5 m to consider in the veto detector to identify charge hits. . . . .	186
A.13	Plotted is the time difference between the veto hit and the neutron arm hit for a single veto plane versus the amplitude in the veto bar. This figure shows all bars in the plane. The gate is seen to start around 40 ns before the coincidence hit in the neutron arm. The deadtime of the various veto detectors is between 40 and 110 ns. During this dead time, the hits are identified as charged if the amplitude is above 200. For a region of $\pm 10$ ns around 0, all events are identified as charged. . . . .	186
A.14	Plot showing the rate both with and without a 200 channel ADC cut for the veto detector L1-14. The ADC cut rate is in blue. The window between 300 and 350 ns was used to calculate the rate. . . .	187
A.15	Plot of hits within the Veto detector. Colors are given to show coincidence between the two separate halves of the V1 veto detector. . . .	188
B.1	The physical asymmetry as the accidental background ratio of charged to uncharged used in the calculation is changed. This figure shows that the dependence on this ratio is very small, with an estimated fractional uncertainty on the accidental background charge ratio of 20%. This esimated fractional uncertainty corresponds to a fractional uncertainty of less than .2%. Plotted is a linear fit, showing the small dependence. This is for an analysis of Kinematic 2(b). . . . .	193

## List of Abbreviations

$\alpha$	alpha
$\beta$	beta
$\gamma$	gamma
QED	quantum electro-dynamics
QCD	quantum chromo-dynamics
SRF	superconducting radio frequency
RF	radio frequency
RTD	resistive temperature device
CEBAF	continuous electron beam facility
EPR	electron paramagnetic resonance
BB	Big Bite
BH	Big Hand
NA	Neutron Arm
Rb	Rubidium
He	Helium
K	Potassium
SEOP	Spin Exchange Optical Pumping
NMR	nuclear magnetic resonance

## Chapter 1

### Physics of the Nucleon

#### 1.1 Introduction to the Nucleon

In 1933 with the discovery that the magnetic moment of the proton (having mass  $M_N$  and charge  $e$ ) was not simply that of a point particle,  $\frac{e}{2M_N}$ , nucleons were known to have internal structure [42]. This structure has been the object of interest to physicists for over 50 years. Nucleon structure may be examined with scattering experiments utilizing electromagnetic, hadronic or “weak” probes. The electromagnetic interaction, by scattering real or virtual photons off the nucleon, can be used to parametrize the unknown electromagnetic structure into effective form factors. One set of parametrizations for the electromagnetic form factors are the Sachs form factors,  $G_E$  and  $G_M$ , which relate to the charge and magnetism distributions of the nucleons. These form factors only depend on the square of the four-momentum transfer carried by the photon,  $Q^2$ .

However, the electromagnetic form factors provide more than just a relation to the charge and magnetization distributions of the nucleon. Any theory of the nucleon must explain the nucleon’s structure, and the form factors provide an experimental test of this. The form factors are also important for the analysis of many other experiments, so having precise measurements can enable better determination of other quantities.

This thesis reports on a measurement of the neutron's electric (otherwise known as charge) form factor extracted from an asymmetry using polarized electron scattering from a polarized  $^3\text{He}$  target. The structure of this thesis is as follows. In Chapter 1, the nucleon structure is introduced and discussed within the framework of various models. In Chapter 2, the formalism required for describing the form factors and the discussion of the measurement is developed. In Chapter 3, previous measurements of the electric form factor are presented in addition to selected previous fits within the framework of the models discussed in Chapter 1. In Chapter 4, the experiment is introduced, in addition to a description of the experimental apparatus. In Chapter 5, the analysis is presented along with the experimental values for  $G_E^n$ . Finally, in Chapter 6 new results are discussed in the context of the presented models, the flavor form factors are decoupled, and the quark orbital angular momentum is presented. The remainder of this chapter will have a short introduction to quantum chromo-dynamics (QCD) presented in section 1.2 and an introduction to the various models of nucleon structure in section 1.3.

## 1.2 QCD

The ground state structure of the nucleon can be investigated using momentum transfer in an elastic scattering experiment, where the initial state of the nucleon is not changed. In electron scattering, the electromagnetic properties of the nucleon are probed, and they are characterized by the aforementioned form factors. These form factors express the difference between scattering from an object with internal

structure and scattering from a point particle.

In the very successful theory of the strong interaction, quantum chromodynamics (QCD), the nucleon is the lowest energy three-quark bound state. QCD is similar in structure to quantum electro-dynamics (QED), but where in QED the strength of interaction is governed by electric charge and the exchange particle is photon, in QCD the exchange particles are gluons and the interaction strength is due to color charge. The nucleons are then made up of these gluons and quarks, with the individual quarks being resolved in the Bjorken limit. The three lightest quarks are identified in Table 1.1.

The notion of valence versus “sea” quarks, where it is the valence quarks that define the electromagnetic attributes of the hadron, is useful. For the proton, the valence quarks are two up quarks and one down quark, for the neutron the valence quarks are two down quarks and one up quark.

Protons and neutrons, as has been implied here, share many similar properties. Models and theories have been developed to explain both proton and neutron properties together, in terms of their shared properties as nucleons. Similar properties are expected because both nucleons are made up of the same constituents, and have the same types of valence quarks. Both also have the same spin, are very close in mass, and are long lived. This allows one to discuss the two nucleons as two states of a single object, differentiated by the additional quantum number “isospin”. When the next lightest quark, the strange quark (with strangeness charge  $-1$ ), is included an  $SU(3)$  symmetry group can be formed, although this symmetry is only approximate due to the larger mass of the strange quark.

The charge in QCD is color, with three colors instead of the single electromagnetic charge which is familiar from classical physics. A significant distinction of QCD is that, unlike the photon in QED, the exchange particles (gluons) hold the property of color themselves. As a result, the gluons can interact with each other, and a quark of a particular color will be “anti-screened”, causing the force to increase as distance increases. This phenomenon results in confinement. The three color charges of the quark form a  $SU(3)$  group as well. This confinement and the creation of the QCD vacuum implies the existence of “sea quarks” ( $\bar{u}u, \bar{d}d, \bar{s}s$ ) and gluons within the nucleon [38].

name	electric charge	isospin	strangeness	mass
u (up)	$+\frac{2}{3}$	$\frac{1}{2}$	0	1.5 to 4 MeV
d (down)	$-\frac{1}{3}$	$-\frac{1}{2}$	0	4 to 8 MeV
s (strange)	$-\frac{1}{3}$	0	-1	80 to 130 MeV

Table 1.1: The three light quarks and their respective charges. Other quarks are the c (charm), b (beauty), and t (top) quark, but they are too heavy to play a large role in the experiments described here. These three light quarks form the  $SU(3)$  group.

### 1.2.1 Perturbative QCD

Quantum electrodynamics is a highly successful and calculable theory, in part because it is perturbative. This is not the case for QCD. At large distances from the bare charge, the QED coupling is the familiar  $\alpha(Q^2) = \frac{1}{137}$  while the QCD coupling,  $\alpha_s(Q^2)$ , is  $\approx 1$  at low momentum transfer making an expansion in  $\alpha_s(Q^2)$  impossible. However, the coupling decreases with increasing  $Q^2$ , the theory is said

to be “asymptotically free”. The renormalization scale,  $\Lambda$ , defines the energy at which the effective coupling becomes large. When  $Q^2$  is well below this, quarks and gluons are bound into hadrons [38].

At sufficiently high momentum transfers ( $Q^2 > \Lambda^2$ ), we would expect to be able to understand the nucleon structure using perturbative QCD. A virtual photon of high enough transferred momentum will see the nucleon as consisting of three massless quarks moving collinearly. In (quasi-)elastic scattering the momentum of the virtual photon is transferred among the three quarks through two hard gluon exchanges, with each gluon being proportional to  $Q^2$ . This gives the dominant scaling of  $1/Q^4$  for the helicity conserving form factor (known as  $F_1(Q^2)$ ). This power counting is justified by QCD factorization theorems[12].

### 1.3 Nucleon Structure

When the transferred momentum is below the scale  $\Lambda$ , the QCD interaction is strong and quarks are confined. In this parameter space there is no clear way to calculate quantities using QCD analytically. There is much promise for future calculations using direct computational techniques such as Lattice QCD. In the meanwhile, various models have been developed to describe the dominant features of nucleon structure.



### 1.3.1 Partons

In deeply inelastic scattering (DIS), where the proton breaks up, the final state of the interaction cannot be described by a particle in a single final state. Because of this, a tensor  $W^{\mu\nu}$  is used to parametrize the unknown final state. In the Bjorken limit ( $Q^2 \rightarrow$ ), apparent point particles known as quarks are resolved inside of the nucleon, each of which is carrying some fraction of the nucleon's four-momentum. The nature of the strong interaction is such that individual quarks are never seen; instead these “partons” exit the nucleon in jets of colorless hadrons; either  $qqq$  baryons or  $q\bar{q}$  pairs (mesons). At lower momentum transfers where deep inelastic scattering is not applicable, mesons can also be produced through production and then decay of a nucleon excited state.

In the infinite momentum frame, where the momentum of the hadron is large and all masses can be neglected, and in the case that the invariant mass of the system is large, the tensors  $W^{\mu\nu}$  can be represented in terms of dimensionless structure functions. It is useful for these to be expressed in terms of Bjorken  $x$ , a dimensionless kinematic variable of the virtual photon. This variable gives the component  $(xE)$  of the initial energy/momentum carried by the resulting “parton”. In the limit  $x \rightarrow 1$ , all of the momentum is being carried by the parton, and so the valence quarks dominate the interaction. As  $x$  approaches 0, the sea quarks dominate. The momentum which is not carried by the quarks and antiquarks is carried by the gluons [38].

The analysis of partons and the constituents of the nucleon is made more

complicated because many energetic terms must be included. This means that relativistic effects should be considered. However, this makes the interpretation of the wave function unclear, because a Lorentz boost mixes the momentum states. If QCD is quantized at a fixed light-cone time,  $\tau$ , where  $\tau = t + \frac{z}{c}$ , the challenges of mixing due to Lorentz boosts is removed along with the complexity of the vacuum in relativistic quantum field theory. The following models and interpretations of nucleon structure are mostly considered in such a quantization. Further discussion on this is in Section 6.1 [53][18][17][19].

In elastic scattering, where the nucleon neither breaks up nor is excited, there is a single final state. The tensor  $W_{\mu\nu}$  then can be reduced to a form factor. Obviously, a complete model of the internal structure of the nucleon must include a description of form factors. In the case of a spin  $\frac{1}{2}$  object like a nucleon, there are two independent form factors, which can be expressed in the form of Sachs electric and magnetic form factors ( $G_E$  and  $G_M$ ) or as the Pauli and Dirac form factors,  $F_1$  (helicity conserving) and  $F_2$  (helicity-flip) (The relationship between these will be described in Chapter 2). In addition to elastic and deeply inelastic scattering other hard exclusive processes such as deeply virtual Compton scattering (DVCS), can also be used to investigate nucleon structure (for a pictorial description see figure 1.1)[38] [20].

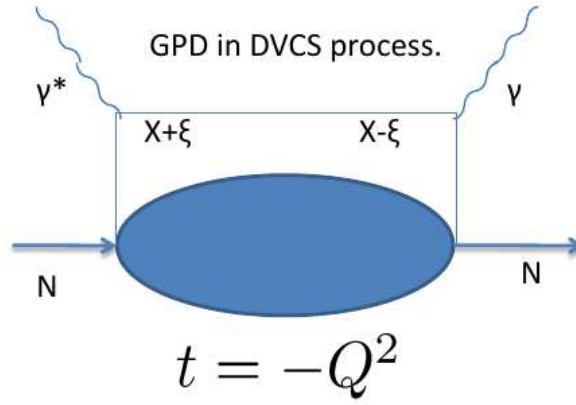


Figure 1.1: Shown is a schematic of the DVCS process, in the framework of generalized parton distributions (GPDs). Here  $\xi$  is the skewness,  $x$  is the longitudinal momentum fraction,  $N$  is the nucleon,  $\gamma^*$  is the virtual photon, and  $\gamma$  is the detected photon. The skewness gives the longitudinal momentum asymmetry, as shown in the figure.

### 1.3.2 The Constituent Quark Model

An early model of nucleon structure is the constituent quark model (CQM). In this case, the nucleon is a ground state of three massive quarks in a confining potential. The masses of these quarks are determined by  $SU(3)$  flavor symmetry and it is assumed that the mass of the hadron is held by just the valence quarks. While relativistic modifications of this model have had some descriptive success, the model does not satisfy the chiral symmetry of the QCD Lagrangian. Some modifications, such as goldstone-boson-exchange (GBE) where there is an additional quark-quark interaction, and the one-gluon exchange (OGE), provide a general fit to the data for the form factors and the radii of the nucleons.

The long distance behavior of hadronic wave functions should be able to be described by the exchange of the lightest of the  $q\bar{q}$  states, the pions. As a result, pion signatures should be seen in the low momentum behavior of hadrons, and the

addition of a this pion cloud to the constituent quark model is used to form a better model of the nucleon. An early model of this type is the cloudy bag model. Here the pion cloud interacts with the confined quarks such that chiral symmetry is restored. This cloudy bag model provides a good fit of low  $Q^2$  nucleon form factors [63][57].

### 1.3.3 Generalized Parton Distributions

In a number of reactions, the scattering amplitudes factorize such that they can be described by a simple diagram known as the hand bag diagram. Here there is one valence quark interacting with the virtual photon, with the rest of the structure contained within a generalized parton distribution (GPD). An appealing feature of GPDs is that, depending on the kinematic limit selected, they can be related to a wide variety of scattering processes, such as deeply inelastic scattering, deeply virtual Compton scattering, wide angle compton scattering, and elastic scattering.

Currently, GPDs cannot be measured explicitly, and are instead expressed by models which are generally simple parametrizations constrained by the GPD relationship to the form factors, properties of the nucleon, and by the fact that the GPDs become parton densities in vanishing momentum transfer.

In this framework, hard exclusive reactions (like DVCS) result in an interaction between the probe and a single quark within the nucleon (see Figure 1.1). At leading order, the interaction that described by the traditional form factors can also be described by four Generalized Parton Distributions (GPDs), these GPDs depend on the longitudinal momentum fractions (light-cone and skewness), and the squared

four-momentum transfer ( $Q^2$ ) to the nucleon. The four GPDs are  $H^v$  and  $E^v$ , where  $v$  indicates either an up quark or a down quark.<sup>1</sup>

These GPDs depend on Bjorken  $x$ , momentum transfer  $Q^2$ , scewness ( $\xi$ ), and a scale parameter  $\mu$ . The following sum rules, which are independent of  $\xi$ , relate the GPDs and the form factors:

$$\int_{-1}^1 dx H_v(x, \xi, Q^2) = F_1^v(Q^2) , \quad (1.1)$$

$$\int_{-1}^1 dx E_v(x, \xi, Q^2) = F_2^v(Q^2) . \quad (1.2)$$

$F_{1,2}^v(Q^2)$  are the flavor form factors, which describe the distribution of that flavor of quark within the nucleon. These can be expressed in terms of the electromagnetic form factors as  $F_1^u = 2F_1^p + F_1^n + F_1^s$  and  $F_1^d = 2F_1^n + F_1^p + F_1^s$ , where  $F_1^s$  is the strangeness form factor of the nucleon. It is assumed, here as in most analyses, that the strange form factor is 0. Similar relationships exist for the Pauli form factor,  $F_2$ . The GPDs must still be modeled in order to predict form factors, and are usually done so using the sum rules [25][37][24].

### 1.3.4 Vector meson dominance models

Some of the earliest models of the nucleon were of a virtual photon coupling to the nucleon both through its internal structure and through exchange of intermediate vector mesons with a meson cloud. These vector mesons (the  $\rho$ ,  $\omega$ , and  $\phi$ ) have the same quantum numbers as the virtual photon and are the lightest hadrons in the

---

<sup>1</sup>There are other GPDs which aren't related to the electromagnetic form factors such as one to describe the polarized distribution.

time-like region ( $Q^2 < 0$  in the standard definition), and so could potentially play a major role in nucleon structure at low  $Q^2$ . The form factors are commonly described by a dipole form, resulting from two nearby vector meson resonances (poles) that have opposite residue. This structure can be written in terms of a photon-meson coupling strength ( $C_{\gamma V_i}$ ) and a meson-nucleon vertex ( $F_{V_i N}$ ):

$$F^{iv,is} = \sum \frac{m_i C_{\gamma V_i}}{Q^2 + m_i^2} F_{V_i N}(Q^2). \quad (1.3)$$

These isovector and isoscalar form factors ( $F^{iv}$  and  $F^{is}$ ) are then written as linear combinations of the electromagnetic form factors[63][64].

The internal structure coupling is sometimes identified with the three valence quarks. The initial models had only  $F_1$  coupling with the internal structure, but some of those considered in this thesis include  $F_2$  as well[15][54].

### 1.3.5 Chiral effective field theory

In chiral effective field theory, the electromagnetic form factors are computed using a chiral Lagrangian with pion, nucleon, and  $\Delta$  fields. The short distance physics is determined by low-energy-constants, which are fit using the measured nucleon charge radius and moments. While this technique has had some success at low  $Q^2$ , by about  $0.4 \text{ GeV}^2$  the predictions break down, due to an increasing role of vector mesons. Since this thesis deals with  $G_E^n$  at intermediate  $Q^2$ , such models can not reliably provide a description of the discussed data. Thus, the focus will be on models that are more appropriate to the energy regime investigated in this experiment [63].

### 1.3.6 Dyson-Schwinger Equations

As has been expressed earlier, there is a significant difference between the bare quark and the dressed (or “anti-screened”) quark. All models of nucleon structure at intermediate  $Q^2$  try to describe this difference. In a framework that uses Dyson-Schwinger equations, the primary cause of this parton behavior is a dense cloud of gluons which surround the low momentum quark, at low momentum transfer ( $Q^2 < M_N^2$ ). This is a manifestation of dynamical chiral symmetry breaking (DCSB), and becomes a source for the nucleon’s mass[22].

Describing the relativistic region in a framework that is consistent with this low energy description is difficult. In terms of these DSE, the nucleon appears as a pole in a six-point Green’s function. Here, the three dressed quarks are described in terms of a bystander quark and a quark-quark correlation (diquark) pseudoparticle in a single color- $\bar{3}$  channel. The binding depends on the exchange of the bystander quark and quarks in the diquark pseudoparticle. This framework combines the descriptions of mesons and nucleons. It is the observation of this diquark description of the nucleon which is used to truncate the DSE and allow calculations using DSE.

An interesting prediction of models in this framework is that the radius of the “dressed“  $u$ -quark in the neutron is greater than that of the  $d$ -quark [14] [70] [22]. Another is that since the up and down current quark masses are thought to be small, the dominant mechanism for the helicity flip behavior in QCD comes from quark orbital angular momentum and an additional polarized gluon[22].

## 1.4 Experimental Investigation

These various models described above provide a description of the nucleon which is more complete than the simple constituent quark model or dynamical chiral symmetry breaking. Most of these models share in common as part of their description a non-zero current quark orbital angular momentum. In this regime, the constituents of the nucleus are dominated by the up and down quarks (with possibly a small strange quark component). The simplest probe available of this structure is the electron. The common experimental techniques used to study this behavior and to investigate the models are deep inelastic scattering (DIS), deeply virtual Compton scattering (DVCS), and (quasi-)elastic scattering. By expanding our knowledge of the nucleon form factors, the pieces needed to complete our picture of the neutron are brought together. The measurement described in this thesis is one important piece: the physics of electron scattering will be developed in the next section.



## Chapter 2

### Physics of Electron and Nucleon Scattering

#### 2.1 Introduction to electron scattering

Scattering is the process in which a particle is deflected during an interaction. There are two main categories of scattering, elastic, where the particles are left intact and only the momentum changes, and inelastic, where the scattered particles are modified, excited, or destroyed. The differential scattering cross section is often written as the product of a cross section for scattering from a spin-0 point particle and a “form factor” which characterizes the structure of the target

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{Mott} f_{rec} |F(q^2, \theta_e)|^2 . \quad (2.1)$$

Here  $F$  is the form factor,  $q$  is the momentum transfer, the Mott cross section is that of a spin- $\frac{1}{2}$  electron scattered from a point-like particle, and  $f_{rec}$  is a kinematic factor (to be defined later). For a spin  $\frac{1}{2}$  particle,  $F(q^2, \theta_e)$  can be expressed in terms of the form factors  $G_E$  and  $G_M$ , which will be developed later[38].

The neutron is known to have no net charge. With high energy electrons, distances smaller than the size of a nucleon can be probed, allowing the structure to be described at different scales. Nonetheless, the neutron electric form factor is small compared to the other nucleon form factors. Various models exist that provide detailed descriptions of the nucleon over different kinematic ranges; specific

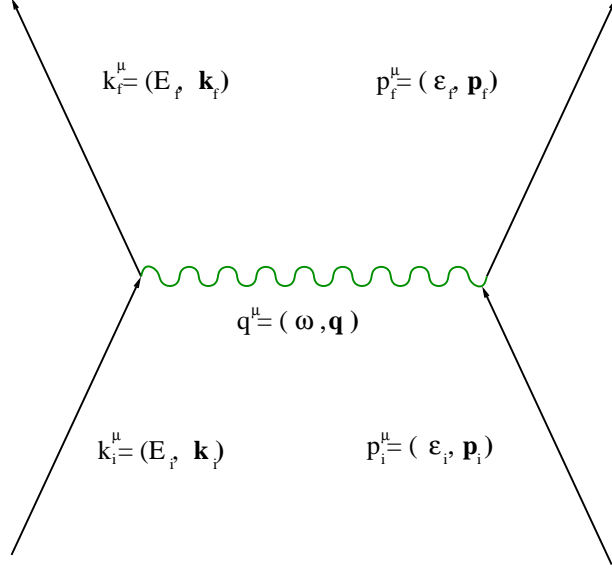


Figure 2.1: Feynman diagram for one interaction between an electron and a nucleon. The electron has four-momentum  $k^\mu$  and the nucleon has four-momentum  $p^\mu$ .

examples of models giving  $G_E^n$  at intermediate  $Q^2$  will be given in more detail in section 3.1.

## 2.2 Definition of kinematic variables

The interactions of electrons and photons are well understood within the theory of quantum electrodynamics (QED). In this interaction, the properties of the incident and scattered electron and recoiling nucleon can be measured.

In the Feynman diagram in figure 2.1, the following kinematic variables are defined. The nucleon four-momentum is  $p^\mu = (\epsilon, \mathbf{p})$ ,  $k^\mu = (E, \mathbf{k})$  is the electron four-momentum, and  $q^\mu = (\omega, \mathbf{q})$  is the virtual photon four-momentum. The four-momentum transferred to the nucleon is  $q^2 = q_\mu q^\mu = \omega^2 - \mathbf{q}^2$ , and is carried by the virtual photon. If evaluated in the laboratory frame,  $\mathbf{p}_i = 0$  and  $\epsilon_i = M_N$ , where

$M_N$  is the recoil nucleon mass. Since  $q^2$  in electron scattering is space-like,  $Q^2$  is best defined as  $Q^2 = -q^2 = 4E_i E_f \sin^2 \frac{\theta_e}{2}$  (in the approximation  $m_e = 0$ ), where  $\theta_e$  is the angle the electron deviates from its initial direction.  $Q^2$  is typically the quantity used to parametrize the form factors.

### 2.2.1 Current operator

The current of the electron in Figure 2.1 is arrived at from QED [38]

$$j^\mu(k_f, k_i) = \bar{u}(k_f)(-ie\gamma^\mu)u(k_i) . \quad (2.2)$$

Here  $\gamma^\mu$  is the standard Dirac matrix and  $e$  is the electric charge. In this expression  $u$  is the Dirac spin  $\frac{1}{2}$  spinor. For the spinor the following completeness relation is true

$$\sum_{s=1,2} u^{(s)}(p)\bar{u}^{(s)}(p) = \gamma^\mu p_\mu + m . \quad (2.3)$$

Similarly the current for a structureless, spin  $\frac{1}{2}$  charged particle can be expressed as

$$J^\mu(p_f, p_i) = \bar{u}(p_f)(-ie\gamma^\mu)u(p_i) . \quad (2.4)$$

These provide the invariant amplitude  $\mathcal{M}$  using the rules for Feynman diagrams.

Following standard conventions, repeated indices are summed over:

$$i\mathcal{M} = J^\mu(p_f, p_i) \frac{-ig_{\mu\nu}}{Q^2} j^\nu(k_f, k_i) . \quad (2.5)$$

### 2.2.2 Dirac and Pauli form factors

The vertex factor for the nucleon-photon interaction is not exactly known, but rather parametrized in terms of form factors. These form factors must be

functions of  $q^2$  since it is the only independent Lorentz scalar at the nucleon vertex in elastic scattering. Generally, the form factors can be constructed with any Lorentz-invariant constituent;  $\gamma^\mu$ ,  $\gamma^5$ , or  $\sigma^{\mu\nu}$ . However, the choices of  $i\sigma^{\mu\nu}q_\nu$  and  $\gamma^\mu$  are made to conserve parity. Other possible forms are restricted due to current and parity conservation. The two terms,  $\gamma^\mu$  and  $i\sigma^{\mu\nu}q_\nu$ , give a current of

$$J^\mu(p_f, p_i) = \bar{u}(p_f)[-ie(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2(q^2))]u(p_i) , \quad (2.6)$$

where  $F_1$  and  $F_2$  are known as the Dirac and Pauli form factors. They are normalized to give the proper charge and magnetic moment at  $q^2 = 0$ , so that

$$\begin{aligned} F_1^p(0) &= 1, \quad F_2^p(0) = \kappa_p \\ F_1^n(0) &= 0, \quad F_2^n(0) = \kappa_n . \end{aligned} \quad (2.7)$$

where  $\kappa$  is anomalous contribution to the nucleon's magnetic moment.

The Dirac and Pauli form factors are a parametrization of the electromagnetic structure in a different basis than the Sachs form factors. The Sachs form factors can be expressed in terms of  $F_1$  and  $F_2$  as [47]

$$G_E = F_1 - \tau F_2 , \quad G_M = F_1 + F_2 , \quad (2.8)$$

where  $\tau = \frac{Q^2}{4M_N^2}$ .

### 2.3 Breit frame

As mentioned previously, the Sachs form factors can be related to the charge and magnetic moment distribution, in a particular reference frame called the Breit frame. No energy is transferred, only momentum ( $\mathbf{p}_i = -\mathbf{p}_f$ ), therefore

$$Q^2 = \mathbf{q}_B^2 . \quad (2.9)$$

An important aspect of the Breit frame is that the Fourier transform of the electric form factor gives the charge density distribution, and that of the magnetic form factor gives the magnetic current density distribution.

The four components of the hadronic current in this frame are [38] [63]:

$$J^0 = ie2M_N\bar{u}_f(F_1 - \tau F_2)u_i = ie2M_N\bar{u}_f G_E u_i , \quad (2.10)$$

$$\vec{J} = -e\bar{u}_f(\vec{\sigma} \times \mathbf{q}_B)(F_1 + F_2)u_i = -e(\vec{\sigma} \times \mathbf{q}_B)\bar{u}_f G_M u_i . \quad (2.11)$$

Using this, Sachs[74] showed that  $G_E$  and  $G_M$  can be expressed as the Fourier transforms of the nucleon charge and magnetization densities:

$$G_E(Q^2) = \frac{4\pi}{Q} \int r dr \rho_{ch}(r) \sin Qr \quad (2.12)$$

$$G_M(Q^2) = \frac{4\pi}{Q} \int r dr \mu \rho_{mag}(r) \sin Qr. \quad (2.13)$$

At low  $Q$ , equation 2.12 can be expanded such that

$$G_E(Q^2) = 1 - \frac{1}{6}Q^2\langle r_E^2 \rangle + \frac{Q^4}{120}\langle r_E^4 \rangle + \dots \quad (2.14)$$

From this we can see a simple formula for the charge radius of the neutron. At very low momentum transfer, the slope of the form factor with respect to  $Q^2$  defines the mean square radius of the distribution[49],

$$\langle r_n^2 \rangle = -6 \frac{d(G_E^n)}{dQ^2} \Big|_{Q^2 \rightarrow 0}. \quad (2.15)$$

This is obviously only true in the non-relativistic limit. Recently, Miller *et al.* have developed a formalism that allows a representation of the transverse charge and magnetization densities that are not reference frame dependent [58].

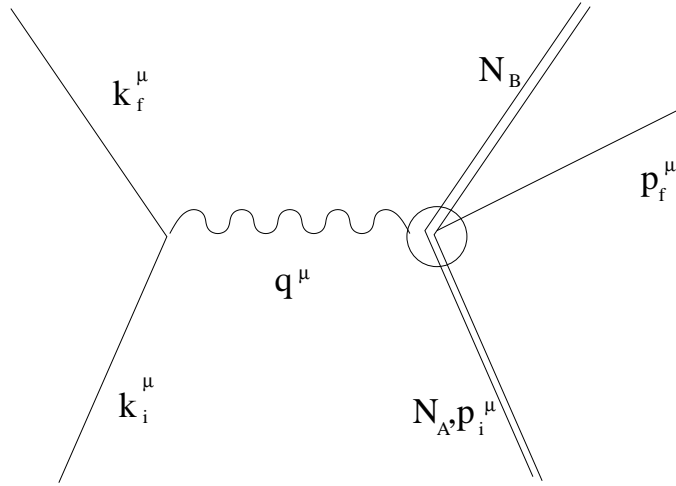


Figure 2.2: Feynman diagram for interaction between an electron and the nucleus. The electron has four-momentum  $k^\mu$  and the ejected nucleon has momentum  $p_f^\mu$ .

## 2.4 Quasi-elastic scattering

Rarely in scattering experiments is there an opportunity to scatter from a single nucleon. This is especially true of the neutron, which is short lived in its free state. There are many different approximations that can be used to put the nucleus in terms of elements that are better understood. Some of these are referenced in sections 5.5 and 5.7.1. While discussing all of them is out of the scope of this thesis, two which allow the final state nucleus to be understood in terms of a remainder nucleus and a scattered nucleon merit development. These are the impulse approximation (IA), and the plane wave impulse approximation (PWIA). Here and throughout much of this thesis, discussion will be formulated in terms of the one-photon exchange approximation.

### 2.4.1 Impulse approximation

In the impulse approximation the virtual (off-shell) photon interacts with a single nucleon. Afterward, the struck nucleon leaves the nucleus without any further interaction with the remaining “spectator” nucleons. This approximation can also be described as the assumption that the current of the nucleus is given by the sum of the currents of the individual nucleons, where the nucleons are treated as free particles[23].

The simplest nucleus that can be discussed in such terms is the deuteron. In the IA, the differential cross section for scattering from a neutron or proton embedded in the deuteron can be factorized. This allows the contributions of the neutron and proton to be separated. For an electron to scatter from a nucleon inside the deuteron, the contributions from the proton and the neutron can be separated

$$\frac{d\sigma_{ed}}{d\Omega} \approx \frac{d\sigma_{ep}}{d\Omega} + \frac{d\sigma_{en}}{d\Omega} . \quad (2.16)$$

This approximation, however, completely neglects nuclear binding and the inherent Fermi motion of the bound nucleons. It also neglects that the nucleon might re-scatter, the virtual photon might couple to a virtual meson exchanged between the nucleons, or the virtual photon might couple to an excited state of a nucleon (known as Isobar Configuration or IC). The quasi-elastically scattered nucleon interactions with the other “spectator” nucleons are known as final state interactions (FSI).

### 2.4.2 PWIA

A more detailed simple approximation is the Plane Wave Impulse Approximation (PWIA). This approximation is a quasi-free one, and is important in discussions of neutron double polarization experiments, where either the target or the recoiling nucleon is polarized. In PWIA the polarization of the recoiling nucleon or of the target is restricted to the scattering plane. The longitudinal component is parallel to the recoiling nucleon's momentum vector, and the transverse component is perpendicular[55]. In this approximation the initial and final state of the target nucleus are represented as products of plane waves for a nucleus and nucleon, where the nucleon is bound (with momentum  $\mathbf{p}_i$ ). The nucleon absorbs the virtual photon, while the nucleus remains as a spectator. The nucleon then leaves the nucleus, which is assumed to be free after the absorption of the virtual photon, without any further interaction.

### 2.5 Polarization observables

A convenient coordinate system to work with observables when the beam, target, or recoiling nucleon are polarized is defined by[64]

$$\begin{aligned}\hat{z} &\parallel \mathbf{q}, \\ \hat{y} &\parallel \mathbf{k}_i \times \mathbf{k}_f, \\ \hat{x} &= \hat{y} \times \hat{z}.\end{aligned}\tag{2.17}$$

It is useful to refer to the polarization vector  $\vec{P}$ , which can serve as either the polarization of the nucleon inside the target in beam-target asymmetry mea-



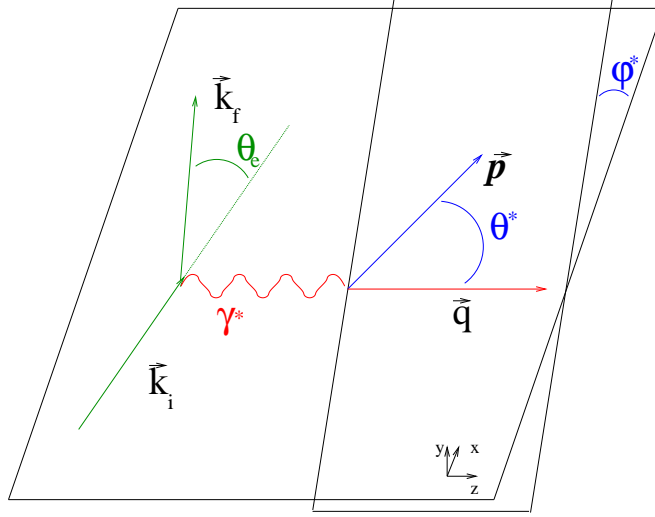


Figure 2.3: Polarization exchange in the one photon exchange approximation for polarized-electron, polarized nucleon scattering.  $\theta$  and  $\phi$  define the polarization of the recoiling nucleon.

surements, or the polarization of the ejected nucleon in beam-recoil polarization experiments.

$$\vec{P} = P(\sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}) \quad (2.18)$$

In the general case of parity conserving polarized scattering there are 18 independent response functions. Under the approximations of one photon exchange, PWIA, and parallel kinematics, only four terms survive. The cross sections of both a recoiling polarized nucleon and a polarized target share similar forms, and can be written in terms of products of kinematic quantities and response functions[47]. The kinematic variables (following the development in [47]) for these reduced terms are

$$V'_{TL} = (1 - \tau)^{-1} \tan \frac{\theta_e}{2} , \quad (2.19)$$

$$V_T = \frac{1}{2(1 - \tau)} , \quad (2.20)$$

$$V_L = \frac{1}{(1 - \tau)^2} , \quad (2.21)$$

$$V'_{TT} = \sqrt{(1 - \tau)^{-1} + \tan^2 \frac{\theta_e}{2}} \tan \frac{\theta_e}{2} . \quad (2.22)$$

The response functions depend on the polarization of the recoiling nucleon and the target nucleus. These kinematic variables arise from the electron tensor in an alternate basis, where the coefficients refer to the nuclear response function with which the kinematic variable is associated ( $_L$ ,  $_{TL}$ ,  $_T$  or  $_{TT}$ ). The response functions arise from the nuclear response tensor ( $W^{\mu\nu}$ ). The nuclear response tensor serves to parametrize the increased generality of the nucleus, where the same current doesn't exist in the initial and final states [38]. The four reduced response functions are  $R'_{TT}(\hat{z})$ ,  $R'_{LT}(\hat{x})$ ,  $R_L$ , and  $R_T$ , the first two of which reflect components associated with the polarized nature of the cross section. These subscripts  $_L$  and  $_T$  correspond to “longitudinal” and “transverse”.

Two additional response functions exist for nuclei with more than one nucleon. These are  $R_{TT}$  and  $R_{TL}$  and only appear when the change in total angular momentum is greater than two units [26] [67].

### 2.5.1 Recoil polarization

From PWIA it can be shown that the unpolarized nuclear response tensor is symmetric,  $W^{\mu\nu} = W^{\nu\mu}$ . This means that the response tensor for recoil polarization is

$$W^{\mu\nu}(\hat{a}) = -W^{\nu\mu}(\hat{a}) = W^{\nu\mu}(-\hat{a}) . \quad (2.23)$$

In double-polarization experiments parallel kinematics are often used; under

these kinematics the nucleon's final momentum is parallel to the momentum transfer. Also, at the electron beam energies in the experiments described here, only the longitudinal component of the electron polarization is relevant.

In the laboratory reference frame there are two response functions that describe the polarization (in PWIA),  $R'_{TT}(\hat{z})$  and  $R'_{LT}(\hat{x})$ . These are the longitudinal and transverse response functions from which the transverse ( $P_x$ ) and longitudinal ( $P_z$ ) components of recoil polarization arise[47].

The general form of the nuclear structure tensor, with a polarized target or recoiling nucleon, can be written as[63]:

$$W_{\mu\nu} = J_\mu J_\nu^* = W_{\mu\nu}^0 + W_{\mu\nu}(\vec{P}_f) + W_{\mu\nu}(\vec{P}_i) + W_{\mu\nu}(\vec{P}_i, \vec{P}_f) . \quad (2.24)$$

In recoil polarization experiments, typically only the final nucleon polarization is measured, and  $W_{\mu\nu}$  reduces to (in the Briet frame):

$$W_{\mu\nu} = \frac{1}{2} \text{Tr} F_\mu F_\nu^\dagger \vec{\sigma} \cdot \vec{P} . \quad (2.25)$$

Here  $F_0 = 2mG_E$  and  $F_j = imG_M \vec{\sigma} \times \mathbf{q}_B \cdot \hat{j}$ , where  $j = x, y, z$ . This gives  $W_{\mu\nu}(P_x) \propto 2mG_E G_M$  and  $W_{\mu\nu}(P_z) \propto G_M^2$ .

As an example, this leads, after transformation back to the lab frame, to the following components of the recoil polarization of a free neutron [77][39]:

$$P_x = -P_e \frac{\sqrt{2\tau\epsilon(1-\epsilon)} G_E^n G_M^n}{\epsilon(G_E^n)^2 + \tau(G_M^n)^2} , \quad (2.26)$$

$$P_y = 0 , \quad (2.27)$$

$$P_z = P_e \frac{\tau\sqrt{1-\epsilon^2}(G_M^n)^2}{\epsilon(G_E^n)^2 + \tau(G_M^n)^2} , \quad (2.28)$$

where  $\epsilon = [1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2}]^{-1}$ , and  $P_e$  is the polarization of the electron beam.

### 2.5.2 Target polarization

The arguments and results are similar for a polarized target, where the detected nucleon is either unpolarized or not detected. Once more, in parallel kinematics, the one photon interaction approximation, and PWIA, only the  $\hat{x}$  and  $\hat{z}$  components of the target polarization vector remain.

In the response function formulation, and with spin- $\frac{1}{2}$  nucleons, to within a multiplicative constant, are a function of the components  $F_L = (1 + \tau)G_E$  and  $F_T = \sqrt{2\tau(1 + \tau)}G_M$ . The general expression for a response function is

$$\sum_{j,j'} A_{j,j'}(k) F_j F_{j'} \quad (2.29)$$

where the  $A_{j,j'}(k)$  has only two terms left for the polarized response, the  $F_T^2$  and  $F_L F_T$  terms. This formulation is developed in general for more complicated systems than elastic scattering with a polarized nucleon by Donnelly and Raskin [26] [67].

The response functions for the target are then[47]:

$$R_{LT}^T(\hat{x}) = 2\sqrt{\tau(1 + \tau)}G_E G_M , \quad (2.30)$$

$$R_{TT}^T(\hat{z}) = 2\tau G_M^2 . \quad (2.31)$$

## 2.6 Cross section with unpolarized observables

The unpolarized differential cross section is dependent on the square of the scattering amplitude (spin averaged since it is unpolarized),

$$\frac{d\sigma}{d\Omega} = \frac{m_e^2}{4\pi^2} \frac{E_f}{E_i} f_{rec} \frac{1}{4} \sum |\mathcal{M}|^2, \quad (2.32)$$

where  $f_{rec} = [1 + \frac{2E_i}{M} \sin^2 \frac{\theta_e}{2}]^{-1}$ . It is useful to define the Mott cross section, which corresponds to a relativistic electron scattering from a structured target

$$\frac{d\sigma_{Mott}}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta_e}{2}}{4E_i^2 \sin^4 \frac{\theta_e}{2}}. \quad (2.33)$$

Here  $\alpha$  is the fine structure constant.

Using the equations 2.2, 2.6, 2.32, and the Feynman rules the following relationship between the four-momenta, angles, and other elements of the electron and interacting fermion give the differential cross section. This relationship between the form factors and the cross section is also known as the Rosenbluth formula.

$$\frac{d\sigma}{d\Omega} = \left( \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta_e}{2}} \right) \frac{E_f}{E_i} \left[ \left( F_1^2 - \frac{q^2}{4M^2} F_2^2 \right) \cos^2 \frac{\theta_e}{2} - \frac{q^2}{2M^2} (F_1 - F_2)^2 \sin^2 \frac{\theta_e}{2} \right] \quad (2.34)$$

Putting this in terms of the Sachs form factors gives[47]:

$$\frac{d\sigma}{d\Omega} = f_{rec} \left( \frac{d\sigma}{d\Omega} \right)_{Mott} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta_e}{2} \right]. \quad (2.35)$$

This is much simpler than equation 2.34 and readily suggests a method of measurement of the form factors.

## 2.7 Cross section with polarized observables

The general form for the cross section when the beam of electrons is polarized and the polarization of the ejected particle is detected is:

$$\frac{d^6\sigma_{h,s}}{dE_f d\Omega_{k_f} d\epsilon_f d\Omega_{p_f}} = \sigma_{unpol} \frac{1}{2} (1 + \vec{P}_i \cdot \vec{\sigma} + h(A + \vec{P}_T \cdot \vec{\sigma})). \quad (2.36)$$

Here  $P_i$  is the induced polarization in the proton,  $h$  is the helicity of the incident electron,  $A$  is the beam analyzing power,  $s$  is the nucleon spin projection on  $\sigma$ ,  $\sigma^{unpol}$

is the unpolarized cross section, and  $P_T$  is the polarization transfer coefficient[47]. In coplanar kinematics  $A = 0$ ,  $\mathbf{P}_i \cdot \vec{\sigma} = \mathbf{P}_i \cdot \hat{N}$ , and  $\mathbf{P}_T \cdot \hat{N} = 0$ . Since  $\mathbf{P}_i \cdot \hat{N} = 0$  in PWIA, two terms survive to provide the nucleon polarization, and from equations (??) and (??) an elegant measurement of  $G_E^n$  is possible.

This is similar in form to the cross section of polarized electrons on a polarized target where the recoil polarization is not measured:

$$\frac{d^6\sigma}{dE_f d\Omega_{k_f} d\epsilon_f d\Omega_{p_f}} = \sigma_{unpol} \frac{1}{2} (1 + \vec{P}_t \cdot \vec{A}_T + h(A + \vec{P}_t \cdot \vec{A}'_T)). \quad (2.37)$$

The new variables here are  $P_t$  which is the target polarization vector,  $A_T$  which is the target analyzing power, and  $A'_T$  which is the correlation vector[47].

Using a longitudinally polarized electron beam and a polarized target, the cross section can be written as:

$$\sigma^{pol} = \Sigma + h\Delta, \quad (2.38)$$

where  $\Sigma$  is the unpolarized cross section from equation 2.35, and  $\Delta$  is the helicity dependent part[26]. Under the approximations of PWIA and one photon exchange, and in the case of parallel kinematics and relativistic electron beam, the cross section simplifies to[47]:

$$\sigma^{pol} = \sigma_{Mott} f_{rec} [V_L R_L + V_T R_T + h P_{target} (V'_T R'_{TT}(\hat{z}) + V'_{TL} R'_{TL}(\hat{x}))]. \quad (2.39)$$

Using equations 2.19, 2.22, 2.31, 2.30,  $\Delta$  is given by

$$\Delta = -2\sigma_{Mott} f_{rec} \tan \frac{\theta_e}{2} \sqrt{\frac{\tau}{1+\tau}} \left( \sqrt{\tau(1 + (1+\tau) \tan^2 \frac{\theta_e}{2})} \cos \theta G_M^2 + G_E G_M \sin \theta \cos \phi \right). \quad (2.40)$$

## Chapter 3

### Form Factors

#### 3.1 Models and fits

While the theory for strong interactions, QCD, is well known, because of confinement it is impossible to use perturbative techniques to do calculations at low  $Q^2$ . Since it is low  $Q^2$  that is pertinent to the structure of hadrons, approximate models must be used to understand this physics. Understanding the form factors, especially  $G_E^n$  since it is relatively unknown, provides assistance in understanding nuclear structure. Models have been developed to describe the structure of hadrons, and data is needed to determine which models provide a better description.

There are a large number of models of nuclear form factors, a few classes of which will be discussed here. Although lattice QCD has the most long term promise for understanding the physics of strong interactions, since the comparison of the results with data will show the current limitation in computing power, they will not be included.

##### 3.1.1 Fits

In the early studies of nucleon form factors, a phenomenological fit was performed to determine the behavior of  $G_E^n$ , based upon unpolarized data and the Rosenbluth technique. This fit of the data is known as the Galster parameteriza-

tion, and is

$$G_E^n(Q^2) = -\frac{\mu_n}{1 + 5.6\tau} G_D(Q^2), \quad (3.1)$$

where  $G_D$  is the dipole form factor

$$G_D = \frac{1}{(1 + \frac{Q^2}{0.71})^2}. \quad (3.2)$$

The other three form factors ( $G_E^p, \frac{G_M^p}{\mu_p}, \frac{G_M^n}{\mu_p}$ ) all show good agreement  $G_D$  using data acquired from the Rosenbluth method. As new data have become available, this simple fit has been updated. See for example the fit by Kelly [48].

### 3.1.2 Vector meson dominance model

A representative vector dominance model which is in reasonable agreement with the low  $Q^2$  data is one by Bijker *et al.*[15] This model has pQCD scaling relations built into the fit, and a phenomenological contribution attributed to the nucleon's  $qqq$  structure. The added term for  $F_2^V$  is  $\frac{g(Q^2)}{1+\gamma Q^2}$  introduced to reflect pQCD. The intrinsic form factor used was  $g(Q^2) = (1 + \gamma Q^2)^{-2}$ .

Another representative model, by Lomon *et al.*, also included meson pole terms and a term with pQCD behavior. This model also includes the additional vector mesons of  $\rho'$  and  $\omega'$  compared to the previously described one, requiring a total of 14 free parameters [54].

### 3.1.3 Pion cloud and CQM

Miller, *et al.*, expanded the constituent quark model into the light-front cloudy bag model [57]. Here the three relativistic constituent quarks are surrounded by a



nonrelativistic cloud of pions. Poincaré invariance provides an additional constraint needed to fit the  $G_E^n$  data. This model has recently been expanded into the light front cloudy bag model, with the pion cloud contributing through a relativistic  $\pi$ -nucleon form factor. The corrected form factors are in terms of the form factors without relativistic effects, and the virtual pion four momentum. An additional parameter is introduced in the  $\pi$ -nucleon form factor, for the relativistic correction[57].

### 3.1.4 Model of Generalized Parton Distributions

The interest in GPDs recently have led to several models being developed. Among these are models by Guidal *et al.*[37][63] and Diehl *et al.* [25]. In these models, the GPD framework is used to describe observables which are independent of skewness  $\xi$  (see Figure 1.1). Guidal *et al.* parameterize the GPDs in the following way

$$H_{R2}^q(x, 0, Q^2) = q_v(x)x^{\alpha'(1-x)Q^2} , \quad (3.3)$$

$$E_{R2}^q(x, 0, Q^2) = \frac{\kappa^q}{N^q}(1-x)^{\eta^q}q_v(x)x^{\alpha'(1-x)Q^2} . \quad (3.4)$$

Here the  $\alpha'$  is the universal Regge slope, while the  $\eta^q$  govern the behavior for the helicity-flip GPDs as  $x$  approaches 1. This was added to produce a faster falloff at large  $Q^2$  in the  $x \rightarrow 1$  limit. The model by Diehl *et al.* is described in Section 6.4.2.

### 3.1.5 pQCD predictions

As stated in section 1.2.1, perturbative QCD gives the prediction that [12]

$$F_1 \propto \frac{1}{Q^4} . \quad (3.5)$$

This relationship is also expected to be relevant at moderate  $Q^2$  when higher order QCD terms are included, which treat small- $x$  partons better. The small quark masses mean that the dominant mechanism for helicity flip in QCD comes from the quark orbital angular momentum. Generalized power counting including this orbital angular momentum gives a scaling of

$$F_2 \propto \frac{1}{Q^6} . \quad (3.6)$$

This behavior depends on the leading order and next to leading order light-cone wave functions, the latter dominated by the probability amplitude for one quark to carry one unit of orbital angular momentum. It is suggested that the higher order resummation suppresses the low  $x$  contribution, providing an effective cut off for the integrals is at  $x \approx \ln^2 (\Lambda^2/Q^2)$  [12].

$$\frac{F_2}{F_1} \propto \frac{\ln^2 \frac{Q^2}{\Lambda^2}}{Q^2} . \quad (3.7)$$

Here  $\Lambda$  is the QCD renormalization scale. While this scaling relation is accurate for the proton, a rough calculation of  $Q^6 F_2(Q^2)$  gives about  $\frac{1}{3}$  the value of experimental data for  $Q^2 < 5 \text{ GeV}^2$  [12].

### 3.2 Measuring GEn via the Rosenbluth technique

As mentioned earlier, equation 2.35 suggests one technique to determine the electric and magnetic form factors. By carrying out multiple measurements at different angles for a given  $Q^2$  the two components of the cross section may be independently determined, allowing a separation of the electric and magnetic form

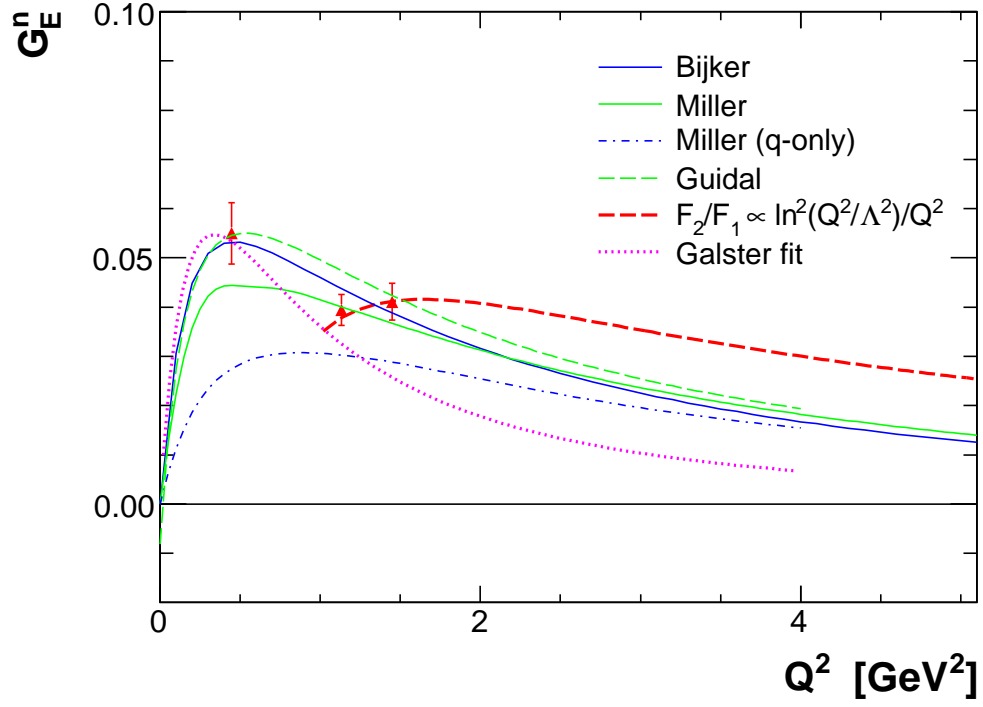


Figure 3.1: Various fits discussed in the text. The points plotted are from [55], which was used to fit the pQCD curve. These data are only representative, see figure 3.2 for the full set. The Bijker curve is a representative Vector Meson Dominance model, the Guidal curve is a representative GPD based model, the Miller curves are from a CQM.

factors. This technique is known as Rosenbluth separation. Experiments to measure the electric form factor of the neutron that use this Rosenbluth method fall into two categories: those in which the impulse approximation for quasi-elastic electron deuteron scattering with the Rosenbluth method is applied directly to the neutron component, and those where the Rosenbluth method is applied to the deuteron through elastic scattering, and an NN model is used to extract a value for  $G_E^n$  [47]. The latter is described first.

### 3.2.1 Elastic e-D scattering

The deuteron, containing two nucleons, is spin-1. Because of this, the equation for the elastic cross section has three form factors, the magnetic  $G_M$ , the charged  $G_C$ , and the quadrupole term  $G_Q$  [64]:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{Mott}}{d\Omega} \left[ G_C^2 + \frac{8}{9} \tau^2 G_Q^2 + \frac{2}{3} \tau(1 + \tau) G_M^2 + \frac{4}{3} \tau(1 + \tau)^2 G_M^2 \tan^2 \frac{\theta_e}{2} \right]. \quad (3.8)$$

While the  $\theta_e$  dependence allows  $G_M$  to be separated out, the charged and quadrupole terms cannot be separated using the Rosenbluth technique. In addition, it is the coherent sum of proton and neutron electric form factors which is extracted ( $G_E^p + G_E^n$ ). Both of these complications add to the uncertainty in the determination of  $G_E^n$ . The electric form factors must be “unfolded” from  $G_C$  and  $G_Q$  using a model, which makes the results of this technique dependent on the model of the deuteron’s wave function [63] [71]. The most recent extraction uses high precision data for  $G_C$ , and the Arenhovel model of the deuteron [6] [29].

### 3.2.2 Quasi-elastic scattering and the Rosenbluth Technique

Recalling equation 2.16, a technique using quasi-elastic scattering off a nucleon rather than elastic scattering off a deuteron and the Rosenbluth formula (equation 2.35) directly presents itself. By subtracting the cross section of the proton from that of the deuteron, or by measuring the neutron in coincidence with the electron, at the quasi-elastic peak, the cross section of the neutron remains. Then the Rosenbluth formula can be used to arrive at a value for  $G_E^n$ .

Most commonly the neutron is detected in coincidence with the scattered electron, to identify that the interaction was with the neutron. However, this technique has also been performed with anti-coincidence measurements, where “no proton” is required in coincidence with the electron [64]. An additional technique used is to measure the ratio of the neutron to proton production cross sections in the electrodisintegration of the deuteron. This method has worked very well for the neutron magnetic form factor.

There have been some serious difficulties with these types of experiments. Among these are a heavy dependence on the proton form factors and the aforementioned problems with the Impulse Approximation (section 2.4.1). Additionally, coincidence and anti-coincidence experiments depend on the absolute detection efficiency, which is difficult to determine for neutrons [64].

### 3.2.3 Difficulties with Rosenbluth method

The uncertainties for these measurements based upon unpolarized quasi-elastic scattering from deuterium are too large to give a definitive nonzero measurement of  $G_E^n$ . The dependence on the proton form factors and the uncertainty directly from IA dominate the measurement.

The Rosenbluth technique has another inherent problem: the magnetic form factor is much larger than the electric form factor for the neutron, and the ratio  $(\frac{G_E}{\tau G_M})^2$  becomes smaller as the transferred momentum increases. This means that the electric form factor is difficult to decouple from the magnetic form factor, since both the tangential and constant (relative to angle) portions of the cross section are proportional to  $G_M^2$  [76].

### 3.3 Double polarization techniques

With the difficulties in measuring the electric form factor of the neutron using the Rosenbluth method, new techniques have been sought. Following the suggestion of Arnold, Carlson, and Gross [10] a technique using a longitudinally polarized beam and a recoil polarized neutron has been investigated. The most obvious reaction for such an experiment would be  $^2H(\vec{e}, e'\vec{n})p$ .

The most important advantage is that the measured term is proportional to  $G_E^n G_M^n$  instead of  $(G_E^n)^2$ . Additionally, only one measurement is needed for a given  $Q^2$  if all interesting components can be measured.

As mentioned earlier, polarized targets can allow the same physics to be ac-

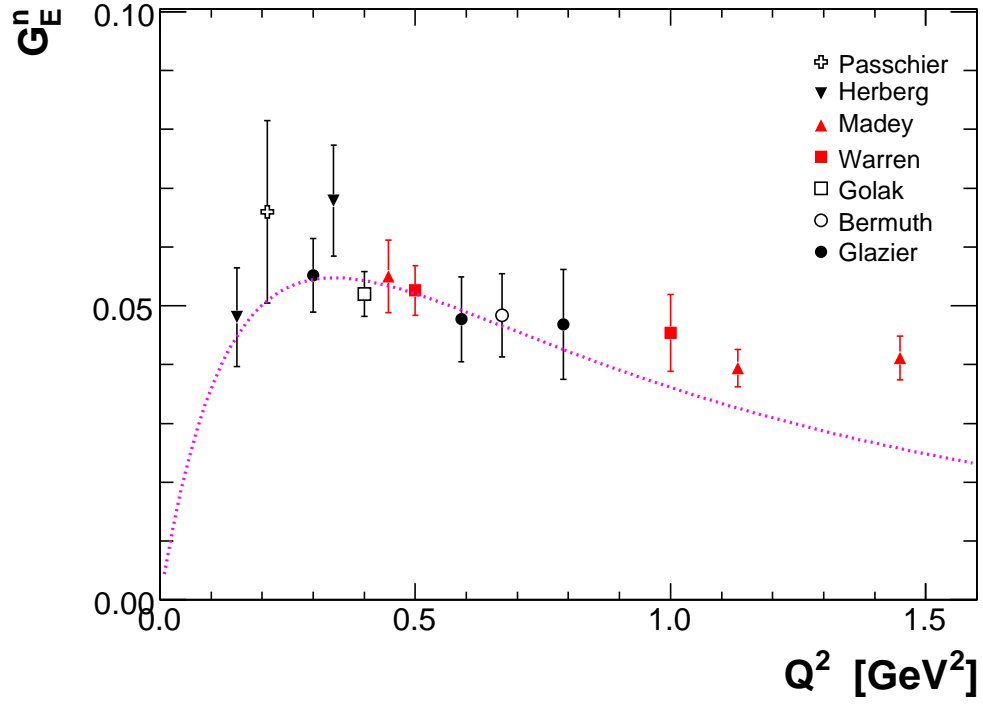


Figure 3.2: Previous  $G_E^n$  data from experiments involving polarization techniques. The curve is the Galster parametrization [33]. Herberg provided new calculations of the Ostrick data, while Golak provided a FSI corrected analysis of the Becker data. For references see Table 3.3.

cessed as detecting a polarized recoil neutron. Experiments have been carried out with either recoil polarization, where the polarized neutron is detected in a polarimeter, or with a polarized target, either polarized deuterium or polarized  $^3\text{He}$ , with a detected unpolarized neutron[55]. Table 3.3 contains a summary of recent experiments all of these techniques.

All experiments using the double polarization technique are dependent on a helicity dependent experimental asymmetry,

$$A_{exp} = \frac{N_+ - N_-}{N_+ + N_-} , \quad (3.9)$$

which can then be related to  $G_E^n$ . Here  $N_+$  gives the number of events with helicity aligned with the nucleon polarization or with the beam direction and  $N_-$  gives the number of events with helicity with the opposite sign compared to nucleon polarization or against the beam direction.

### 3.3.1 Quasi-elastic scattering and (e,e'n)

For PWIA to be applicable, quasi-elastic events must be selected. Even though the cross section is not being measured, the process of selection of these events is important, as double polarization techniques depend on asymmetries, which are diluted by events which are not part of the desired asymmetry. Two useful quantities for selecting quasi-elastic events in coincidence experiments are the missing energy (defined by  $E_m = m_f + m_B - m_{A+i}$  with  $A$ ,  $B$ ,  $i$ , and  $f$  as shown in figure 2.2) and the missing momentum (defined by  $\mathbf{p}_m = \mathbf{p}_i - \mathbf{q}$ ).

In the experiments that have been carried out to date, the experimental trig-



$Q^2$	<i>Date</i>	<i>Reference</i>	<i>target</i>
0.16	1991	Jones-Woodward[44]	$^3\overrightarrow{\text{He}}$
0.2	1992	Thompson[80]	$^3\text{He}$
0.31	1994	Meyerhoff[56]	$^3\overrightarrow{\text{He}}$
0.255	1994	Eden[28]	$^2\text{H}$
0.15, .34	1999	Ostrick[61]	$^2\text{H}$
0.4	1999	Becker[11]	$^3\overrightarrow{\text{He}}$
0.21	1999	Passchier[62]	$^2\overrightarrow{\text{H}}$
0.67	1999	Rohe[71]	$^3\overrightarrow{\text{He}}$
0.45, 1.13, 1.45	2003	Madey[55]	$^2\text{H}$
0.5, 1	2003	Warren[82]	$^2\overrightarrow{\text{H}}$
0.3,0.6,0.8	2003	Glazier[35]	$^2\text{H}$
0.67	2003	Bermuth [13]	$^3\overrightarrow{\text{He}}$

Table 3.1: Double polarization experiments to measure the electric form factor of the neutron that have been carried out to date[64].

ger was typically formed by coincidence between the scattered electron and recoiling neutron. The electrons were detected in a spectrometer or granulated calorimeter. The measured electron kinematics, in concert with large solid angle neutron detectors, allow for event-by-event reconstruction of the particle tracks. For the neutron detector, in higher  $Q^2$  experiments a multi-plane neutron detector was used, with a layer or two of veto detectors in front. In other experiments, two separated layers of scintillator were used, with each layer having veto detectors in front.

Recoil polarimetry experiments required a different detection scheme in order to detect the polarization of the neutron, as it leaves the target. Polarimeters were set up to measure the “up-down” scattering asymmetry due to the transverse component of the recoil polarization. The polarimeters included a plastic or mineral oil scintillator which determined the neutron time of flight (TOF), and also served as a scatterer, with the polarized scattering exhibiting an asymmetry that was measured in a second set of scintillators. Since the spin vector of the neutron could have any combination of longitudinal and transverse polarizations, a dipole magnet was sometimes used to precess the neutron’s spin vector.

These detectors provided scattering angles for both neutrons and electrons. The momentum of the particle(s) provides the needed fifth quantity to select quasi-free  $(e, n)$  scattering, and is determined by considering the time of flight (TOF) for the particle(s). A missing signal in the veto detectors along with the determination of the flight time of the particle from the target to the detector provided neutron detection. Additionally, pulse heights in the neutron detector scintillators can be used to filter out accidental and inelastic events. Quasi-elastic events can also be

selected by comparing the expected time of flight (from the electron energy and the angles) to the measured time of flight.

Other events, which dilute the quasi-elastic neutron events, were one of the main sources of uncertainty in these measurements. A major source of background is proton to neutron conversion in the shielding in front of the neutron detectors. This can be minimized with a tight time of flight cut at low hadron momentum. Good time resolution also allows removal of pion events, another source of dilution. Monte Carlo simulations have been used to determine the proper dilution factor from the uncertainties and to average out the theoretical asymmetries and the effects of finite acceptance.

### 3.3.2 Recoil polarimetry

Four experiments used recoil polarization techniques to measure  $G_E^n$ . These experiments were Madey *et al.*[55] in Hall C at Jefferson Lab, Glazier *et al.*[35] at the Mainz Microtron, Ostrick *et al.*[61] at the Mainz Microtron, and Eden *et al.*[28] at the MIT-Bates laboratory. The first three named experiments (Eden *et al.* was a proof of concept experiment) produced published data.

In these experiments, neutrons were produced through electrodisintegration of deuterons in an unpolarized liquid deuterium target using longitudinally polarized electrons. Contamination from hydrogen within such targets was found to be small[28] [55].

From equations 2.26 and 2.28 we can develop a relationship between the ratio

of the components of polarization, and the ratio of the form factors

$$\frac{G_E}{G_M} = -\frac{P_x}{P_z} \frac{E_i + E_f}{2M} \tan \frac{\theta_e}{2} . \quad (3.10)$$

Here  $P_x$  and  $P_z$  are the  $x$  and  $z$  components of polarization. In order to determine the up-down asymmetry  $A_{meas}$  must be observed. It follows a sinusoidal dependence

$$A_{meas} = A_y |\vec{P}| \sin(\chi + \chi_0) . \quad (3.11)$$

Here  $A_y$  is the beam analyzing power, and  $\chi$  is the spin precision angle, of the detected particle as it travels through a magnetic field.

$$\tan \chi_0 = \frac{P_x}{P_z} = -\frac{G_E}{G_M} \frac{\cos \frac{\theta_e}{2}}{\sqrt{\tau + \tau^2 \sin^2 \frac{\theta_e}{2}}} . \quad (3.12)$$

Using this spin precision technique removes the need for absolute calibration of the electron beam polarization and effective analyzing power [65].

Arenhovel[8][7] showed that for a deuteron target, corrections due to meson exchange currents and isobar configurations are small, and these have little dependence on  $Q^2$ . It was found, however, that especially at lower values of  $Q^2$ , effects from final state interactions (FSI) can play a large role [61].

Ostrick *et al.* found at  $Q^2 = 0.35 \text{ GeV}^2$  that the neutron polarization was reduced by less than 4% due to FSI effects, but at  $Q^2 = 0.12 \text{ GeV}^2$  the transverse polarization is reduced by 50%. These final state interactions were calculated to mostly arise from  $p - n$  charge exchange via pion exchange. A model was used to calculate the change to the polarization by FSI, and a correction was applied [61].

Glazier *et al.* found that at neutron energies of a few hundred MeV the scattering in the polarimeter was dominated by quasi-elastic scattering, which added

to the effective analyzing power of the polarimeter. This means that  $A_{meas}$ , the effective asymmetry, could not be computed accurately, and the statistical precision was not exactly known[35].

Madey *et al.* also used a cross ratio technique, which makes the neutron polarimeter results independent of any luminosity change based on beam helicity, as well as of the efficiencies and acceptances of the two halves of the polarimeter. This experiment used a different technique to account for FSI effects. By using possible  $G_E^n$  values in a simulation, the neutron polarization was calculated which was then compared to experiment[55].

### 3.3.3 Polarized target measurement

To measure  $G_E^n$  using a polarized target, we measure the asymmetry  $A$

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{\Delta}{\Sigma} , \quad (3.13)$$

where  $\sigma_{\pm}$  are the cross sections (Equation 2.38) with electron helicity  $\pm 1$ . If the polarization direction of the target is flipped, this changes the sign of the asymmetry. This asymmetry is not explicitly measured, but rather

$$A_{meas} = P_e P_t A , \quad (3.14)$$

where  $P_e$  is the polarization of the electron,  $P_t$  is the polarization of the target, and

$$A = - \frac{2\sqrt{\tau(1+\tau)} \left[ \tan \frac{\theta_e}{2} \sin \theta \cos \phi G_E^n G_M^n + \sqrt{\tau[1 + (1+\tau) \tan^2 \frac{\theta_e}{2}]} \cos \theta (G_M^n)^2 \right]}{(G_E^n)^2 + \tau[1 + 2(1+\tau) \tan^2 \frac{\theta_e}{2}](G_M^n)^2} . \quad (3.15)$$

Here  $\theta$  and  $\phi$  are the angles defined in Figure 2.3. Setting the angles properly ( $\theta = 90^\circ$  and  $\phi = 0^\circ$ ) greatly simplifies things, giving the perpendicular asymmetry:

$$A_\perp = \frac{-2\sqrt{\tau(1+\tau)} \tan \frac{\theta_e}{2} \frac{G_E}{G_M}}{(\frac{G_E}{G_M})^2 + \tau[1 + 2(1+\tau) \tan^2 \frac{\theta_e}{2}]}. \quad (3.16)$$

Since the ratio of  $\frac{G_E}{G_M}$  is small, the perpendicular asymmetry is roughly proportional to the ratio  $\frac{G_E}{G_M}$ . The parallel asymmetry can be used to normalize the value, so that:

$$G_E^n = \frac{a}{b} G_M^n \frac{(P_e P_t V)_\parallel A_\perp}{(P_e P_t V)_\perp A_\parallel}, \quad (3.17)$$

where  $a = 2\sqrt{\tau(1+\tau)}\sqrt{\tau[1 + (1+\tau) \tan^2 \frac{\theta_e}{2}]}$  and  $b = 2\sqrt{\tau(1-\tau)} \tan \frac{\theta_e}{2}$ .

### 3.3.4 Experiments with polarized deuterium

Two experiments used polarized deuterium targets to measure the value of  $G_E^n$ , one in Hall C at Jefferson Lab (Warren *et al.*[82]) and the other at NIKHEF in Amsterdam, the Netherlands (Passchier *et al.* [62]).

Under PWIA, for deuteron targets with polarization in the scattering plane and after proper averaging of the asymmetry (symmetrically around  $\mathbf{q}$ ), the measured asymmetry is

$$A_{meas} = V \frac{P_e P_1^d A_{ed}^V}{1 + P_2^d A_d^T}, \quad (3.18)$$

where  $V$  is the dilution factor,  $P_1^d$  is the vector polarization,  $P_2^d$  is the tensor polarization,  $A_{ed}^V$  is the deuteron tensor beam-target asymmetry, and  $A_d^T$  is the vector deuteron target asymmetry. For most targets the tensor polarization is small, and that term can be ignored.

In these experiments data were compared to predictions by Arenhovel [7] [8] [9], which accounted for nuclear and FSI effects. This physics model is non-relativistic, and includes meson exchange, isobar configuration currents, and relativistic corrections. The comparison was done by simulating different observables, like  $\mathbf{p}_m$ , and plotting them versus the asymmetry, all using different values of  $G_E^n$  within the confines of the model[62] [82].

In the experiment performed by Warren *et al.*, polarized deuterated ammonia was used as the target. The ammonia granules were submerged in liquid helium and aligned with 5 T magnetic field resulting in a typical polarization of 24%. Good agreement with MC predictions showed that quasi-elastic scattering dominated the scattering reaction. This experiment had a narrow range of acceptance, so it was fairly insensitive to  $Q^2$  dependence [82].

In the experiment performed by Passchier *et al.*, polarized electrons were injected into a recirculating storage ring, allowing for large beam currents. An atomic beam source injected a flux of polarized deuterium atoms into a cell in the storage ring with an electromagnet used to orient the polarization axis. This was created by deuteron atoms in two hyperfine states, with an electromagnet to orient the polarization axis [62].

### 3.3.5 Experiments with polarized helium-3

Another class of experiments to measure  $G_E^n$  have used  ${}^3\overrightarrow{\text{He}}$  as the target. Various experiments were carried out at the Mainz Microtron (MAMI) [11][71][13][56]

and at the MIT-Bates laboratory[44][80]. The results presented in [13] include those from the experiment reported in [71]. Additionally, experiment E02-013 is of this type and in early 2006 ran at Jefferson Lab in Hall A. The analysis of this experiment, with data up to  $Q^2 = 3.5 \text{ GeV}^2$ , is being presented in this thesis.

Optical pumping techniques are used to polarize the  $^3\text{He}$  either through metastability exchange where the  $^3\text{He}$  is pumped directly, or through spin exchange. In the latter, the valence electron in rubidium is optically pumped and the polarization is transferred to the  $^3\text{He}$  nucleus through collisions. The polarization process for  $^3\text{He}$  generally does not take place in the target chamber, rather a pumping chamber is used and the polarized helium-3 diffuses into the target cell or is forced in with a compressor. The polarization vector inside of the  $^3\text{He}$  target can then be measured with NMR.

One of the advantages of a polarized  $^3\text{He}$  target is that in the ground state the spins of the protons are to a large extent aligned antiparallel to each other. This means that most of the spin of the helium nucleus is carried by the neutron; the polarization of the neutron is  $\approx 86\%$  of the polarization of the nucleus [16]. Further details about polarized  $^3\text{He}$  targets will be presented in section 4.4.

At low  $Q^2$ , the complications due to the nuclear wave function can be accounted for with calculations. However, at  $Q^2 > 0.6 \text{ GeV}^2$  this proves difficult, and extrapolation was used in these earlier experiments. Data for  $G_M^n$  is used, or a parametrization, in order to arrive at a value of  $G_E^n$  from the experimental asymmetry (equation 3.16). A Monte Carlo simulation was used to determine radiative loss and neutron energy loss before the detector, as well as other corrections.



It is possible to select the kinematics such that the measured asymmetry is expressed as:

$$A_{exp} = P_{beam} P_{target} D_{neutron} V_{other} A_{\perp}. \quad (3.19)$$

Here  $D$  accounts for dilution from sources related to the target (such as it not being a free neutron), and  $V$  is the dilution caused from other sources and from reactions other than quasi-elastic scattering. This formula assumes that the angles in equation 3.15 are exactly such that  $A_{\parallel} = 0$ ; in practice this isn't possible.

The experiment performed by Becker *et al.* used circulated  $^3\text{He}$  which was compressed into the target cell by a Toepler compressor. An experiment using deuterium provided a parallel measurement, using the same equipment, pointed to a significant role for FSI that decreases as  $Q^2$  increases. The work by Golak *et al.* expanded upon the analysis of Becker *et al.* by summing the cross sections before forming asymmetries using Faddeev calculations. It was discovered that even at the quasi-elastic peak FSI effects played a significant role. Using theoretical ratios determined from models and comparing to experimental values,  $G_E^n$  was extracted from the FSI corrected results [11][36].

In the experiments performed by Rohe *et al.* and Bermuth *et al.*, the data were accumulated by rotating the target spin so that both  $A_{\perp}$  and  $A_{\parallel}$  could be measured. The measurement of both parallel and perpendicular asymmetries allowed the measurement of the target analyzing power ( $A_y$ ) which provided a check on the understanding of FSI effects. These were compared to the calculation by Golak *et al.* and FSI effects caused a 3.4% decrease in the reported value of  $G_E^n$ . These FSI

effects are primarily caused by the photon coupling to one of the protons followed by charge exchange [71][13].

### 3.4 Summary of past measurements

These three different double polarization techniques have a variety of advantages over the Rosenbluth method, as described in this chapter. While models are used to make corrections, and for polarized deuterium for the measurement itself, the general method is model independent. The three different, independent, techniques used in double polarization measurements provides a check on the understanding of the different types of corrections required.

Because  ${}^3\overrightarrow{\text{He}}$  behaves similarly to a polarized neutron, and because of the low detection efficiency neutron polarimeters, polarized  ${}^3\overrightarrow{\text{He}}$  experiments provide the best statistical precision for measuring  $G_E^n$ . FSI effects are taken into account by models for polarized deuterium and calculations for  ${}^3\overrightarrow{\text{He}}$ . Because of the robustness of the deuterium model, FSI effects have traditionally played less of a role in deuterium experiments compared to those using  ${}^3\overrightarrow{\text{He}}$ . For the discussion and calculation fo FSI effects in polarized  ${}^3\text{He}$  for this experiment, at intermediate  $Q^2$ , please see the section 5.7.1.

The completed experiments provided precise enough data to clearly identify a small enhancement in  $G_E^n$  at  $Q^2 = 0.4 \text{ GeV}^2$ . This enhancement is larger in value than the measurements from the Rosenbluth data. At the highest measured values of  $G_E^n$  a noticeable difference between the Rosenbluth and polarized data is seen.

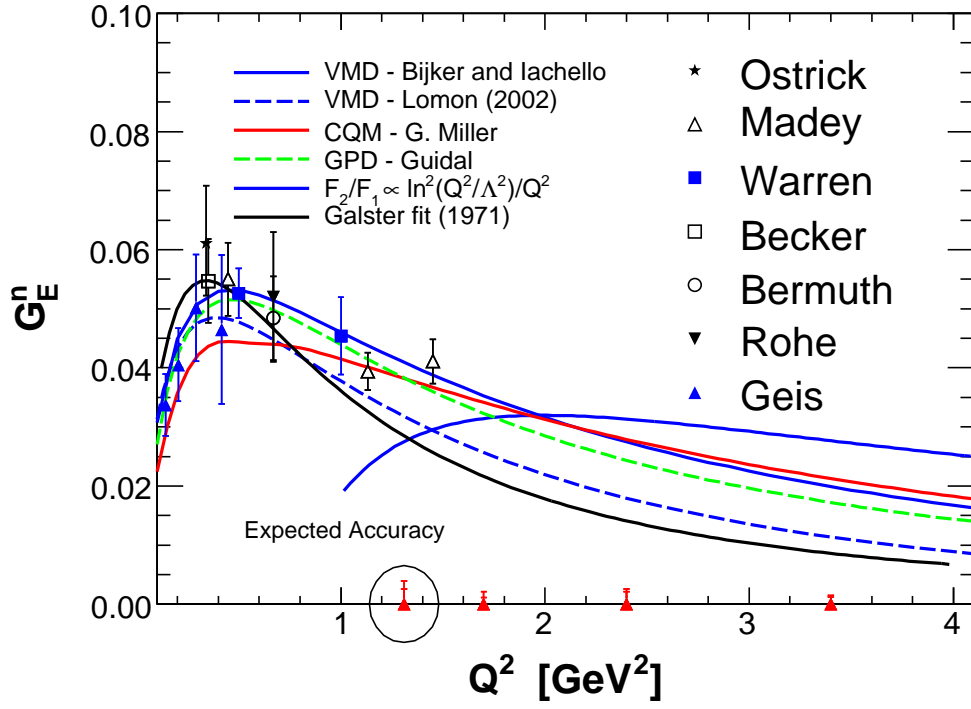


Figure 3.3: Double polarization data and selected models in the region where polarized data is available. Models are described in section 3.1 while data is referenced in Table 3.3.

This is speculated to be due to incomplete radiative corrections for the Rosenbluth data.

In the past decade, precise measurements of the neutron electric form factor have become possible and have provided precise knowledge of the structure of nucleons for  $Q^2$  below 1.5 GeV<sup>2</sup>. In this region, several models provide a good agreement with the data. The majority of the recent measurements have used the double polarization method, which has the advantage of having greater precision and less model dependence compared to the historic Rosenbluth method.

### 3.5 E02-013 at Jefferson Lab

As introduced, the physics of nucleons move from the bound behavior described by massive partons, into the behavior described by bare quarks in QCD. There is a complicated transitive region, in the models presented in Section 1.3, that starts at less than 1 GeV and continues to many GeV. The actual point where this behavior changes, and the description of the change, is unknown. In the experiment presented here, the key quantity  $G_E^n$  needed to understand nucleon structure, and behavior such as quark orbital angular momentum, is measured at three separate points in the  $Q^2$  region between 1 and 4 GeV<sup>2</sup>.

In this experiment, E02-013, numerous improvements were made to increase the figure of merit (effective statistics) for a measurement in this region. Advances in both instrumentation and in the polarized <sup>3</sup>He target technology have been accomplished that enable this measurement. By injecting a small amount of potas-

sium along with the rubidium, the polarization transfer efficiency from the optically pumped rubidium was improved to provide polarizations as high as 50% in the target chamber. The experimental apparatus and target will be explained in detail in Chapter 4. The value of the asymmetry will be analyzed as a function of  $\mathbf{p}_{m,\perp}$ , the perpendicular component of the missing momentum, to study FSI effects. A tight cut on  $\mathbf{p}_{m,\perp}$  is needed to select low nucleon momenta in the  $^3\text{He}$  wave function, but also will suppress dilution from proton polarization and FSI interactions. The high  $Q^2$  at which this measurement is undertaken should suppress meson exchange currents. Additionally, meson exchange currents, delta isobar contributions, and finite acceptance effects should be suppressed by a tight cut on  $\mathbf{p}_{m,\perp}$ [21]. This will be described in detail in Section 5.7.1.

## Chapter 4

### $G_E^n$ experiment E02-013

Experiment E02-013 was carried out at Jefferson Lab from February 28 to May 11, 2006 to measure the electric form factor of the neutron. In this chapter the experiment setup and instrumentation needed to measure the interaction  ${}^3\overline{\text{He}}(\vec{e}, e'n)$  are described. First the experimental overview and relevant coordinate systems will be presented. Then a short summary of the beam and accelerator will be presented in section 4.3. The target and method of polarization is described in section 4.4. In section ?? the spectrometer used to detect the electron and measure its momentum is portrayed. The neutron arm, used to detect the coincident hadron, is described in section ?. Finally, the data acquisition, software used for decoding, and initial physics analysis is presented in section 4.7.

#### 4.1 Experimental Overview

During the running period of E02-013, four kinematic settings were taken in six intervals, not including the time spent on commissioning the apparatus. Three separate polarized  ${}^3\text{He}$  target cells were used for the measured asymmetry. For calibration purposes, a foil target (containing six carbon foils and one BeO foils), a reference cell (providing an empty cell target, hydrogen target, nitrogen target), and a setting with no target cell were used.

Big Bite optics calibrations and beam spot check were achieved using the foil target. The nitrogen target is used to determine the contamination in scattering events from  $N_2$  in the  $^3\text{He}$  cells. The hydrogen target was used to calibrate the momentum and yields of the elastic events for Big Bite and Big Hand. Additionally, the nitrogen and hydrogen targets are used to determine the dilution of the neutral sample by protons empirically, which is an important correction for the physics asymmetry.

Data taking took place in this experiment for several different settings dependent on target, beam energy, neutron arm location, and electron scattering angle. These are described in Table 4.1. Kinematic 1 will not be presented in this thesis. Kinematic 2(a) used two different settings for the neutron arm threshold, this change in threshold creates a difference in the neutron and proton detection efficiencies which are a key component in the empirical measurement of the dilution of the neutral sample by protons. This difference in efficiencies is calculated using a Monte Carlo.

## 4.2 Principle and Experiment setup

This experiment took place in Hall A of Jefferson Laboratory in Newport News, Virginia, USA. Jefferson Laboratory is the home of CEBAF, a continuous electron accelerator that can provide beam energies of 0.6 - 6 GeV [52] [3]. The three experimental halls, A, B, and C, can all receive beam simultaneously. Each hall contains various standard equipment detector systems that facilitate the types of experiments



Figure 4.1: Image of Jefferson Laboratory. Shown is the accelerator and the mounds over the three experimental halls. The mound to the left is Hall A.



name	kin 2 (a)	kin 3 (a)	kin 2 (b)	kin 3 (b)	kin 4
Target Cell	Dolly	Edna	Edna	Edna	Edna
Beam E. (MeV)	2637	3291	2641	3290	2079
$Q^2$ GeV <sup>2</sup>	2.5	3.5	2.5	3.5	1.5
BB Angle (°)	51.59	51.59	51.59	51.59	51.59
NA Distance (m)	10	10	10	10	8
Time Frame	3/09-3/24	3/24-4/17	4/17-4/24	4/24-5/02	5/02-5/09

Table 4.1: A table showing the running conditions, time, accumulated charge, and kinematics of the different measurements referred to here and elsewhere as kinematics or for short kin. Kinematic one includes both the commissioning time and the first measurement. The measurement of  $Q^2 = 1.3$  and  $1.7$  GeV<sup>2</sup> provide a comparison of the polarized <sup>3</sup>He technique with previous recoil polarization data while the other two measurements extend the measured range of  $Q^2$ . Not shown is the periods for commissioning and kinematic 1 which won't be covered in this thesis.

the halls were designed to accomplish. While the beam was provided similarly to other experiments in Hall A, the equipment used to make this measurement was a custom installation that included a large momentum acceptance spectrometer, Big Bite, a high efficiency segmented neutron detector, Big Hand, and a polarized <sup>3</sup>He target, which serves as a source of highly polarized neutrons. A general layout of the apparatus is shown in Figure 4.2.

Using simulations, the requirements for Big Bite and Big Hand were developed. To match Big Hand to Big Bite, the required acceptance for elastics was simulated, and then expanded to insure the acceptance of all events with at least  $p_{m,\perp} = 150$  MeV. Because at  $Q^2 = 3.5$  GeV<sup>2</sup> the desired hadron momentum is 2.6 GeV, the thresholds, shielding, and depth of the neutron arm were developed to maximize

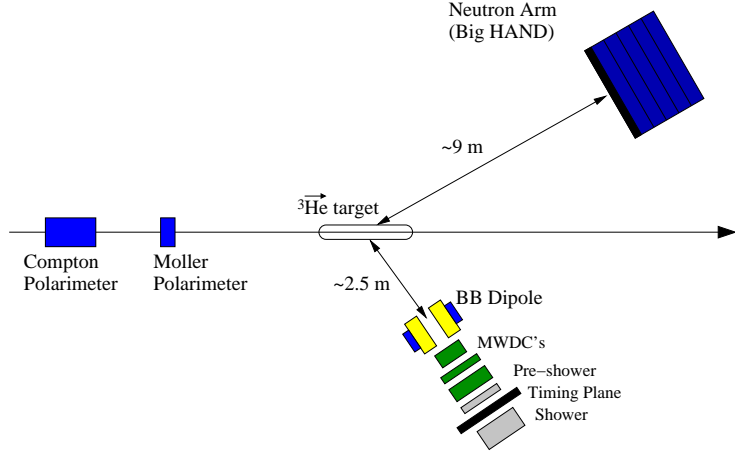
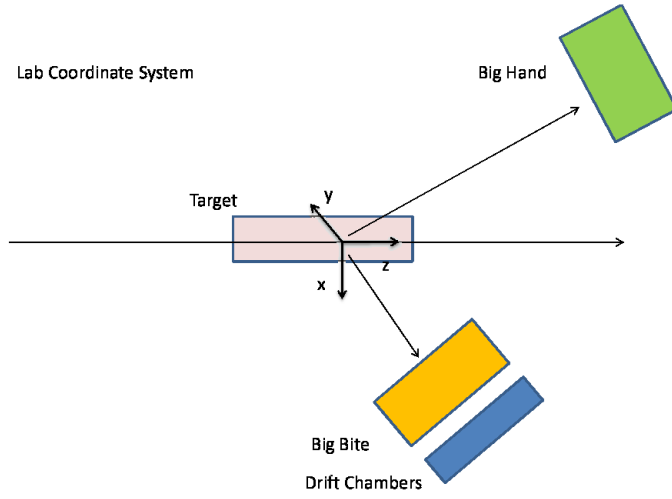


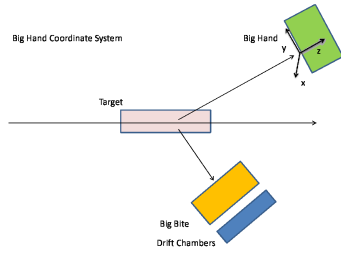
Figure 4.2: General layout, not to scale, of the experiment. Shown is the beamline, the target, the Big Bite detector stack, and the Neutron Arm detector. Also shown are the two polarimeters in the hall before the target, and the approximate distances to the detectors. The components of the Big Bite (BB) detector stack are also shown and consist of the Big Bite dipole magnet, multi-wire drift chambers (MWDCs), pre-shower, shower, and timing plane.

statistics. The desired Big Bite momentum resolution was 1-1.5% and the desired vertex resolution was 6 mm[21].

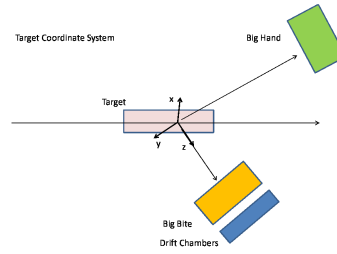
There were four major coordinate systems in this experiment: one for the lab, one for Big Bite, one for the target, and one for the Neutron Apparatus. In the hall system, the origin is at the center of the target,  $y$  is vertical with + as “up”,  $z$  is nominally along the direction of the beam, and  $x$  forms a right handed coordinate system. For the target coordinate system, the origin coincides with the hall origin,  $x$  is vertical with + as “down”,  $z$  is parallel to hall floor along the Big Bite central ray, and  $y$  completes the right-handed coordinate system. For the Big Bite detectors the origin is at the center of the first plane of the drift chambers. The  $+z$  direction is defined in respect to the direction of particles perpendicular to the first drift chamber (and so is at a  $\approx 10^\circ$  angle with the  $x - z$  plane in the



(a) Lab Coordinate System



(b) Neutron Arm (NA) Coordinate System



(c) Target Coordinate System

Figure 4.3: Coordinate systems for Big Hand, Target, and Lab. The lab  $z$  is along the beam line while the target  $z$  is toward Big Bite.

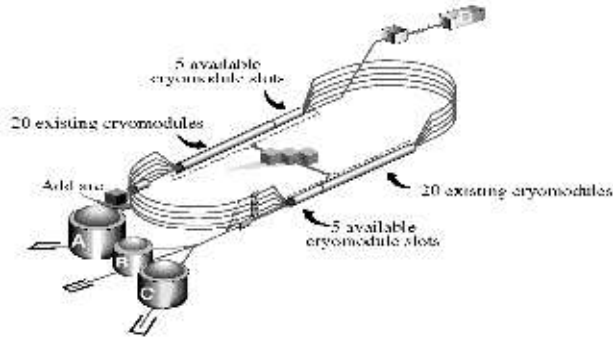


Figure 4.4: Schematic of the accelerator as it existed in 2006. Shown are the three halls, injector, and LINACs.

lab frame), while  $+x$  is in the magnetic dispersion direction, and  $y$  completes the right-handed coordinate system. In Big Hand, the depth is  $z$ , the height is  $x$ , and the horizontal position is  $y$ . In this coordinate system positive  $x$  is associated with “down”, positive  $z$  is associated with deeper into the Neutron Apparatus (NA), and positive  $y$  is away from the beam line. The origin was located in the center of the active area at the front of the shielding. Figure 4.3(c) shows the target, lab, and Neutron Arm coordinate systems and their relation to each other.

### 4.3 The Electron Beam

During 2006, the accelerator was capable of delivering beams with energy up to 5.7 GeV at currents of up to  $150 \mu\text{A}$  semi-simultaneously to all three halls. The two main parts of the accelerator are the injector and the two recirculated linear accelerators (LINACs). These delivered beams are not quite continuous, rather the beam was pulsed with 2 ns wide bunches at a rate of 499 MHz. Inside the injector, the lasers for each hall are 120 degrees out of phase compared to each

other. This allows for the energy and current to be determined for each hall semi-independently. After the initial acceleration, a prebuncher and chopper make sure that the three individual beams are separated in time and longitudinal spread. After traveling through 18 superconducting radio frequency (SRF) cavities, the electrons are injected into the main accelerator at 23-68 MeV. See Figure 4.4 for a schematic of the accelerator.

Polarized electrons are produced from a strained GaAs photo-cathode via the photo-electric effect. A maximum polarizations of approximately 85% can be achieved. A Mott polarimeter at the injector can be used to measure the polarization using the asymmetry in elastic Mott scattering.

The polarization of the high energy beam as it enters Hall A can be measured with a Compton polarimeter. This is done by scattering electrons from polarized photons, and measuring the asymmetry in the cross section due to the beam helicity change. The scattering takes place when electron beam interacts with a photon beam in a Fabry-Perot cavity (which enhances the yields). This method of measurement was used at the same time as data collection during kinematics two and three. On four separate occasions the beam polarization was measured by a Møller polarimeter in Hall A. This was done using magnetized foils which provide a target of polarized atomic electrons. The cross section was measured for two different orientations of the foil, in order to separate the longitudinal and transverse components of the beam polarization. At various times during the experiment both of these two methods along with a Mott polarimeter near the injector, were used to measure the polarization. This allowed for a good understanding of systematic errors for the

beam polarization. An abridged summary of the beam polarization measurements is presented in Table 4.2.

The sign of the electron's helicity was put into the CODA data stream with a copy sent to the E02-013 trigger supervisor. The algorithm responsible produces helicity states (that last for a duration of 33.3 ms) in sets of four (+ − − + or − + + −). A 105 kHz clock was used to reconstruct the helicity if there was a problem with a missing helicity. After every transition there is a short unknown time during which the helicity is unknown (given by 0) in the data.

In the Hall A beamline before the target, a set of field coils are located to move the beam to create a uniform pattern across a section of the cell. This is done so that a small portion of the target is not overheated and the target cell is not compromised. The beam sweeps through its pattern at a rate of 17 to 24 kHz in a process known as rastering. The current used to do this is read into the data stream.

Beam position monitors (BPMs) are located upstream of the target (at 7.52 m and 1.29 m). The average position of the BPM is recorded at a rate of 1 Hz in EPICS. The fast rastering system is located 23 m upstream of the BPMs. The BPMs were connected to an ADC for readout, such data could then be converted to a position for the beam. Harp scans in which a thin set of wires are swept through the beam, were carried out to provide absolute position information which could be used to calibrate the positions. The BPM and raster calibrations were carried out by Brandon Craver and discussed in greater detail in another thesis [68].

During each pass, the two LINACs each accelerate the beam by up to 600

MeV during each pass. The beam can be bent and returned in RF cavities for up to five passes through the LINACs. The beam is accelerated in the east and west and bent 180 degrees in the arcs. See Figure 4.4 for details of the accelerator setup and beam cavities. Because different electron energies need to travel through different magnetic fields to have the proper bend angle, the beam is split in a spreader and then recombined after traveling through the arc. Each cavity is also at 499 MHz to keep the beam properly bunched. The beam can be sent into the switch yard after any pass, where it is sent to hall A, B, or C. Beam energies were determined using the Tiefenbach method and recorded in EPICs. This method relates the beam energy to the current applied to one of the arc magnets [43].

#### 4.4 The Helium-3 Target

Polarized targets and polarized  $^3\text{He}$  have been used in previous experiments, as described in Section 3.3.3. To review and expand upon this description, polarized  $^3\text{He}$  targets are one of the preferred methods of realizing a polarized neutron target. As described, nuclear targets are used rather than free neutrons as the latter only survives with a halflife of  $885.7 \pm 0.8$  s [4]. For polarization experiments  $^3\text{He}$  is ideal, because the polarization in the helium-3 is almost entirely carried by the neutron[4]. Additionally, the relatively simple nature of the  $^3\text{He}$  nucleus allows final state interactions to be understood. These are thought to be small at high momentum transfer (see section 5.7.1). In polarized  $^3\text{He}$ , the neutron carries  $\approx 86\%$  of the spin of the nucleus (Figure 4.5)[4]. Most of the time, the spins of the protons

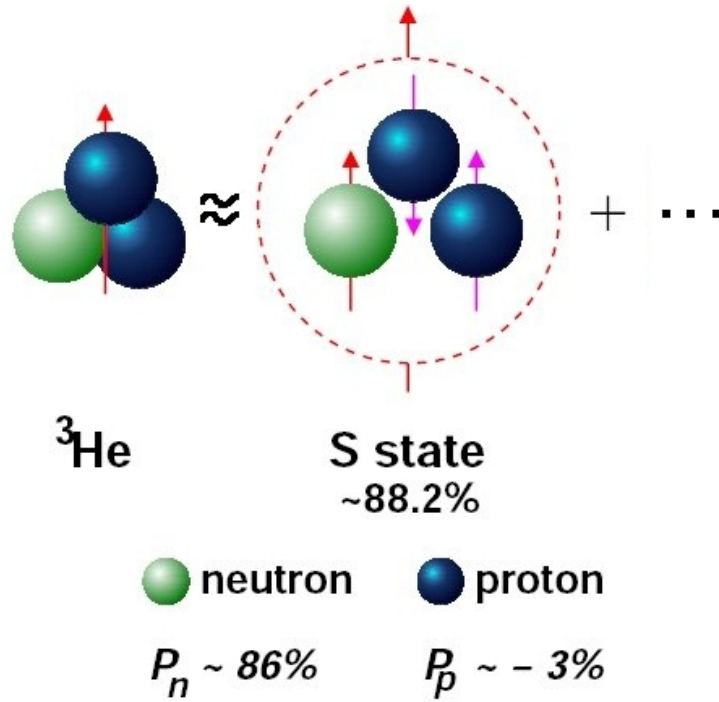


Figure 4.5: Diagram of the helium-3 nucleus. Shown is the most common state with the proton spins aligned anti-parallel. The neutron carries  $\approx 86\%$  of the polarization of the nucleus while the protons carry  $\approx 3\%$ .

are in opposite directions. About 3% of the time, the protons are polarized, and this proton asymmetry is corrected for when considering proton contamination of the neutral hadron candidates (see Section 5.5 and 5.6).

The process of spin-exchange optical pumping (SEOP) was used to polarize the  $^3\text{He}$  nucleus. In this process, alkali atoms are optically pumped and, through spin exchange, polarize the  $^3\text{He}$  nucleus. An image of the target cell is provided in Figure 4.6. A unique feature of this target was the use of potassium (K) and rubidium (Rb) for the optical pumping, rather than the more traditional rubidium only. The process of SEOP will be described in Section 4.4.2.



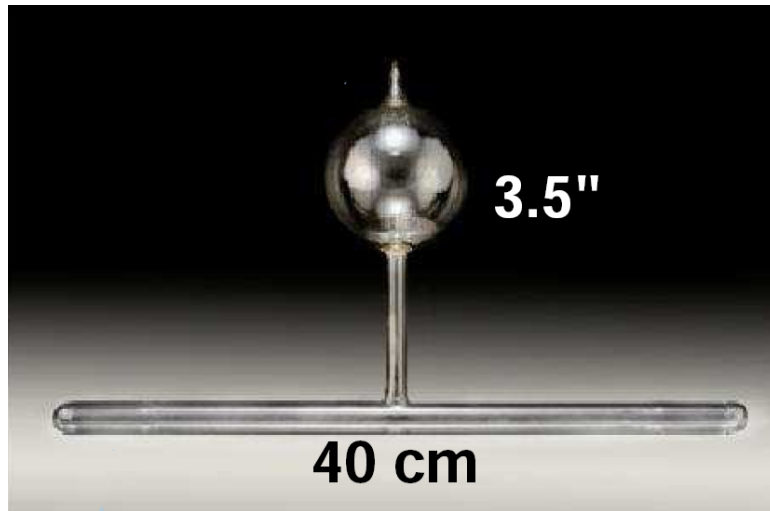


Figure 4.6: Picture of an empty  $^3\text{He}$  cell. On top is the spherical pumping chamber, and below is the cylindrical target chamber. The beam passes through the the 40cm of target material[34].

#### 4.4.1 Target System

The goal of the target system was to realize a polarized  $^3\text{He}$  target. To have a polarized target, a magnetic field is required to provide an axis with which to orient the target spin. To do this, the target system consists of a ladder holding the targets including a cell to hold the  $^3\text{He}$  gas, a laser system to polarize the gas, and a system to create the uniform magnetic field.

The target ladder contained four different targets, and was enclosed in an iron box to provide a “constant” magnetic field in the target that is immune from interference due to the Big Bite dipole. The targets used were the  $^3\vec{\text{He}}$ , a set of optics (carbon and beryllium oxide) foils for calibration, and a reference cell (containing  $\text{N}_2$  or  $\text{H}_2$ ) targets. The reference cell was similar to the helium-3 cell and approximately 40 cm in length.

An oven was used to keep the pumping chamber at a constant temperature of  $240^\circ\text{C}$  in order to maintain a sufficient density of potassium vapor for the spin exchange process.

The target cell was made from hand-blown glass, with two chambers, a pumping chamber and a target chamber, and a transfer tube between the two. The polarized gas in the heated top chamber would diffuse into the bottom target chamber where the polarized  $^3\text{He}$  served as the target. The cell was constructed of alumine-silicate glass and was filled with 8 atm at room temperature of  $^3\text{He}$  in addition to  $\text{N}_2$  and sealed.

To optically pump the Rb, five 30 W lasers were used (totalling 150 W). In

order to decrease radiation damage and ease safety concerns, the lasers were not kept in the hall, rather they were kept in a shed with 75 m of optical fiber carrying the laser light to the target where it was combined in the pumping chamber.

All of the target system was held in an iron box, except the lasers. Around the target, there were coils to drive the RF field for the polarization measurements, and sets of pickup coils to measure the NMR signals, and a coil to provide the excitation for the EPR measurement (see Figure 4.11). The magnitude of the field provided by the iron box was limited to 25 G in order to keep the beam from bending away from the beam dump. The iron box provided shielding from Big Bite, with eight coils arranged to create a "uniform" field in the target region (Figure 4.8). The holding field wasn't completely uniform, and since the experiment was very sensitive to polarization direction, this was measured using a custom built compass[50]. Figure 4.7 shows the relationship between position along the target and direction of the magnetic field.

#### 4.4.2 Polarization

The  $^3\text{He}$  target used in E02-013 was the first implementation of a hybrid target at Jefferson laboratory. The target's hybrid nature is due to containing two alkali metal vapors to reach a higher sustained polarization in less time than the previous mono-alkali targets. This target system was developed by Alan Gavalya and the polarized  $^3\text{He}$  groups at the University of Virginia, the College of William and Mary, the University of Kentucky, and the Hall A staff. By using a Potassium

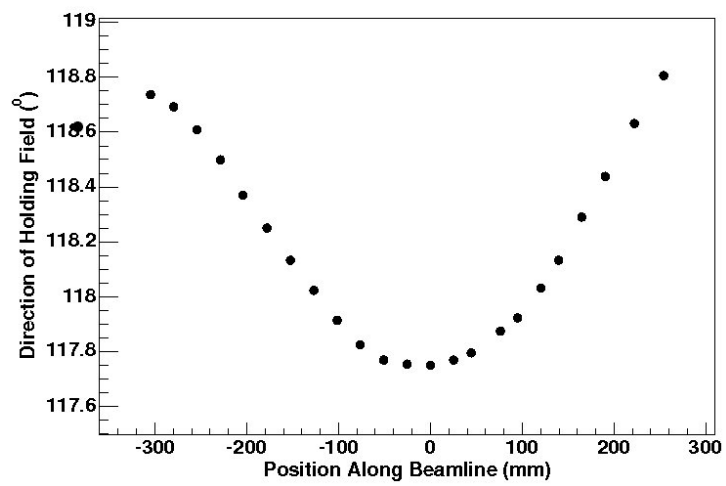
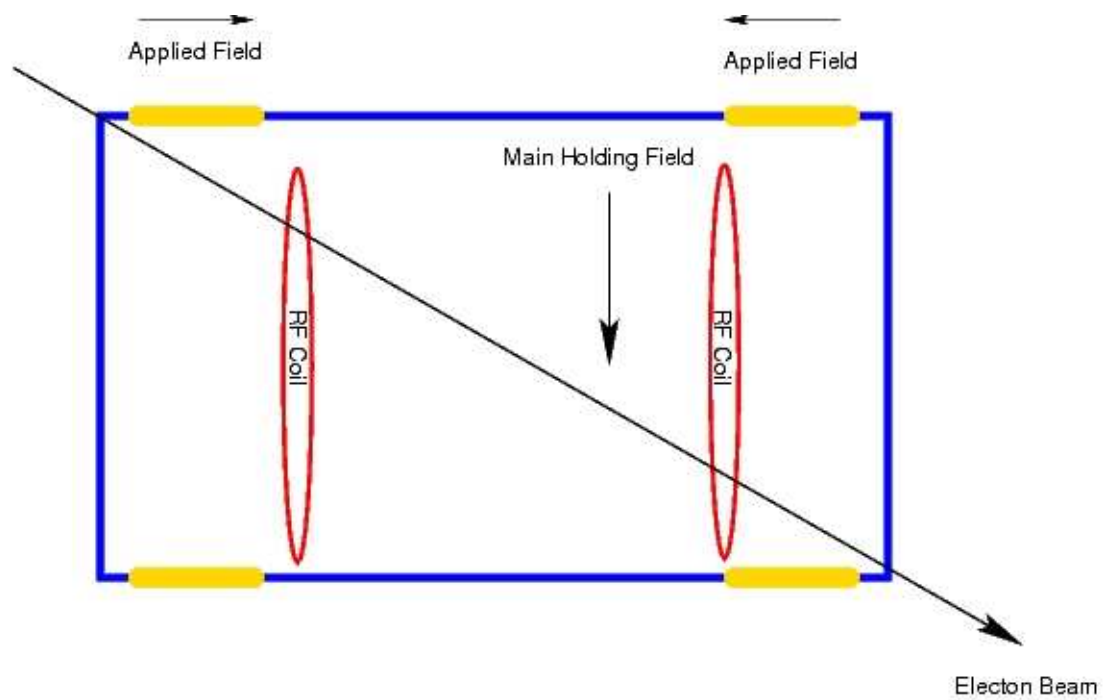


Figure 4.7: Measurement of the holding field as a function of position along the beamline in the target. The origin is the target center.



### Coil Orientation (Top View)

Figure 4.8: Magnet box for Target [46]. Shown is the coils that create the magnetic field.

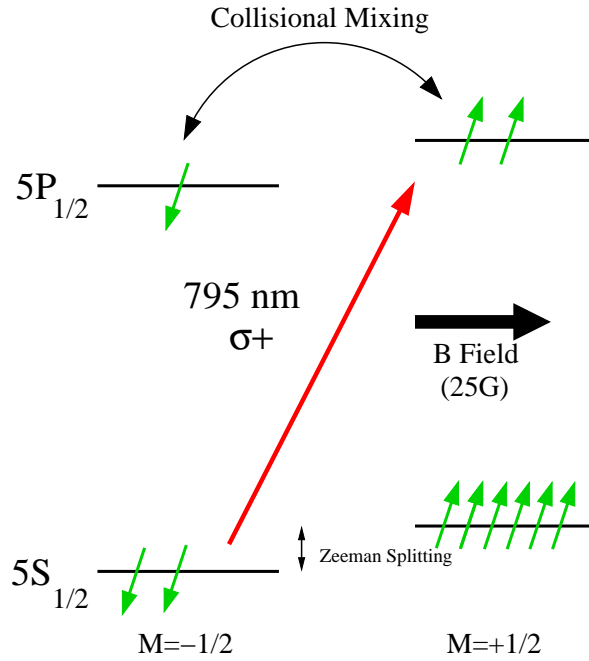


Figure 4.9: Figure showing the polarization process [46]. Optical pumping excites the electron to an excited state, which can exchange (50%) to a state that decays to a spin state that is not optically pumped by the lasers.

and Rubidium mixture in the cells, a polarization higher than 50% was achieved (see Figure 4.12).

In the process of spin exchange optical pumping (SEOP), Rubidium is put in a magnetic field and exposed to the circularly polarized light, this pumps up the valence electron to a spin up sublevel in a P state. This excited state will either directly decay, or exchange spin through collision and then decay (see Figure 4.10). The photon released during decays can depolarize the nitrogen in the cells (roughly 2% of  $^3\text{He}$  volume) serves to quench this, allowing the transition back to a bound state through kinetic collisions rather than radiation. After decay there is a 50% chance to end in a spin state. This spin state is not excited by light from the lasers and so becomes densely populated (see Figure 4.9). Angular momentum can

be transferred from the polarized alkali valence electron to  $^3\text{He}$  via the hyperfine interaction. A description of these processes is presented in Figure 4.10.

The spin exchange efficiency for K- $^3\text{He}$  interaction is 10 times more efficient than Rb- $^3\text{He}$ . Commercial lasers for optically pumping Rb are more readily available than those for Potassium. Potassium (K) and other alkali metals have a high spin exchange cross section with Rubidium and so the polarization of two such gases in a cell will be the same. This means that including the Potassium into the cell enhances the transfer of polarization into the helium-3, decreasing the time to polarize the cell by a factor of two and causing a corresponding increase in sustained polarization in beam.

#### 4.4.3 Polarization Measurement

The polarization of the target was measured by a combined use of EPR (electron paramagnetic resonance) and NMR (nuclear magnetic resonance) techniques. EPR provides an absolute measurement of  $^3\text{He}$  polarization, while NMR provides a relative measurement. Both techniques cause depolarization of the target, but NMR disrupts the experiment less due to causing less depolarization. Because of this and because it could be carried out directly in the scattering chamber, NMR was done more frequently. These two techniques used two separate locations in the target cell. Due to using the alkali gases, EPR measured the polarization in the pumping chamber while NMR could be used to measure polarization in the target chamber. The diffusion of polarized gas in the cell was modeled to relate the two techniques

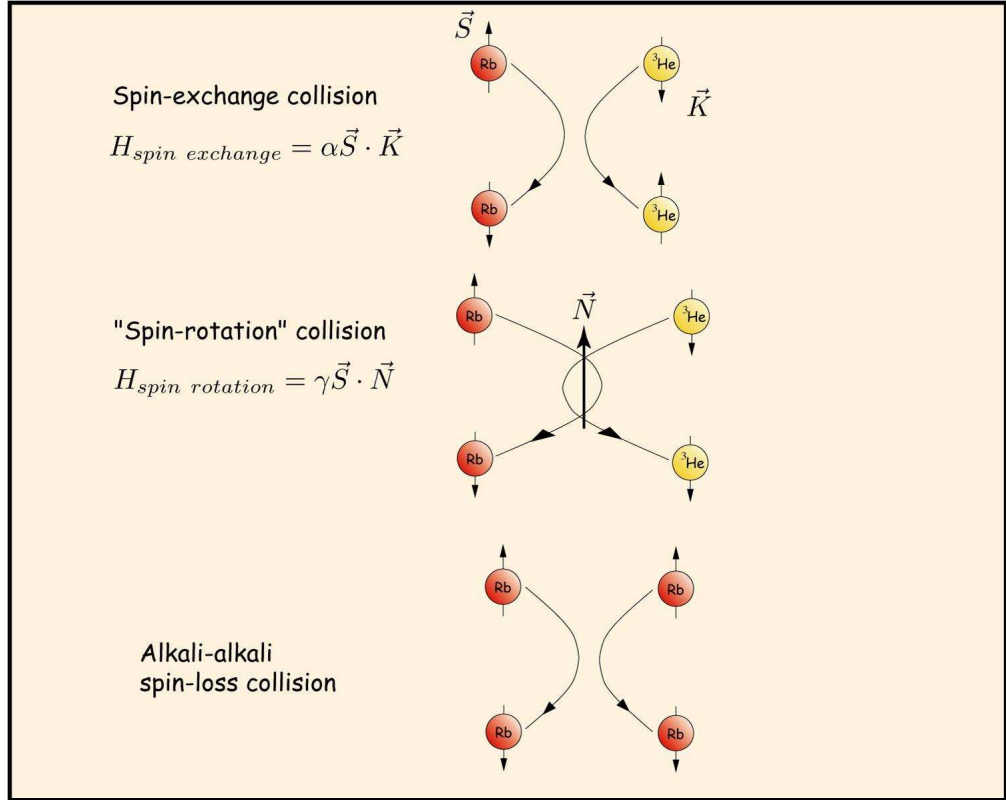


Figure 4.10: In a traditionally polarized Helium cell, one that contains only Rb and He, the Rb exchanges spin in one of three ways. Either by changing the spin of the He nucleus, or by just rotating the atoms it interacts with. This rotation is not useful for polarizing the He, so by adding K, which is likely to engage in 'spin rotation' with the Rb, but likely to engage in 'spin exchange' with He, the polarization efficiency of the cell is increased. Only the spin-exchange collision process causes polarization of the <sup>3</sup>He [78] [46].



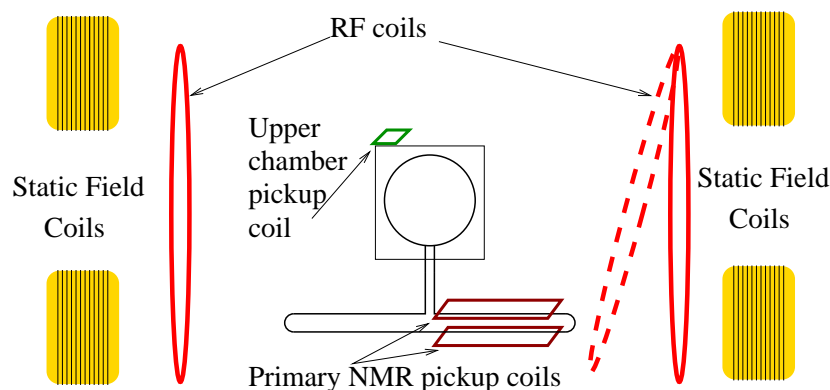


Figure 4.11: The coils used for the NMR measurement of target polarization. Shown is the holding field coils (iron box), the RF coils (which can be adjusted), and the pick up coils. Two sets of pick up coils are down at the target chamber, one more set is up above the pumping chamber. One of the RF coils could be adjusted, as could the pick up coils, to maintain the transverse relationships needed for an NMR measurement.

and properly calibrate the NMR signal.

To perform a NMR measurement, control of the magnetic field of the target is necessary. After an application of a radio frequency (RF) field, when resonance conditions are met, a signal is measured that is proportional to the polarization of the target.

The resonance was found using adiabatic fast passage (AFP). In adiabatic fast passage the magnetic field is changed with a perpendicular radio frequency field (91 kHz) held constant to find the resonance. At resonance,  $^3\text{He}$  undergoes spin reversal. This spin reversal produces an EMF signal that was detected in a separate set of coils known as pick up coils. See Figure 4.11 for a description of the coils involved in the NMR measurement.

This measurement technique switches the direction of the spins, so the magnetic holding field was swept back to give the original target polarization direction

during each measurement. The depolarization caused by this process was found to be on the order of 1%. This measurement technique is relative due to the EMF signal depending on the magnetic flux through the coils, the amplification of the electronics, and the density of  $^3\text{He}$ . The calibration of this measurement technique is presented in another thesis [45].

In an EPR measurement of the polarization of the  $^3\text{He}$ , it is light from the polarization of the alkali metals in the cell that is measured. This is due to the alkali atoms being very sensitive magnetometers. This shift is due to the Zeeman effect. Obviously this means that there are two shifts in the Zeeman responses of the K and Rb, one due to spin exchange and another due to the magnetic field experienced by the metal including that of the polarized  $^3\text{He}$ .

There are numerous magnetic fields which cause this shift, such as the holding field and interactions with the other atoms. However, these other shifts are not dependent on the polarization of the target, and by flipping the polarization the effect due to the polarized  $^3\text{He}$  on the magnetic field creating slight differences in the K energy levels can be determined. For a spherical sample, combining shifts due to collision and the classical magnetic field, the following relationship for the signal shift is obtained [41]

$$\Delta\nu_{EPR} = \frac{8\pi}{3} \frac{d\nu_{EPR}(F, M)}{dB} \kappa_0 \mu_{He} P_{He} . \quad (4.1)$$

Here  $B$  is the magnet field,  $P_{He}$  is the polarization of the  $^3\text{He}$ , and  $\kappa_0$  is a parameter. To flip the target polarization, the holding field is kept constant and the applied RF field is swept (which is an resonance with the  $^3\text{He}$ ). The EPR resonance causes

depolarization and florescence caused by that repolarization can be tracked as a function of RF frequency. This decay is caused by a shift in electron energy levels in the potassium atoms.

To understand the two measurement techniques relative to one another, NMR was done both before and after each EPR measurement. The EPR measurements had the disadvantage of causing significant target depolarization. This is because the EPR transition causes the  $^3\text{He}$  spins to be anti-aligned with that of the spin state the alkali is being pumped into. To minimize the depolarization, this transition was done twice for every EPR polarization measurement. The EPR measurement depends on  $\kappa_0$ , which is different for every alkali and noble gas combination and has temperature dependence [72] [27].

The measurements required to relate the absolute EPR measurements and the relative NMR measurements and the model used to relate the polarization at the different locations of the two measurements is presented in detail in another thesis [45]. The corrected polarization is shown in Figure 4.12. From equation 4.1 it is obvious that the target density needs to be known well. Resistive temperature devices (RTDs) were placed at eight locations on the cell to measure the temperature, from which the density was computed using the ideal gas law. These RTDs did not measure the internal temperature, and so a series of NMR measurements were carried out under various conditions to determine the true internal temperature [45].

Many corrections are needed to provide the needed polarization and direction of polarization to properly determine the physical asymmetry. The material surrounding the helium-3 had to be understood to build a proper Monte Carlo, while

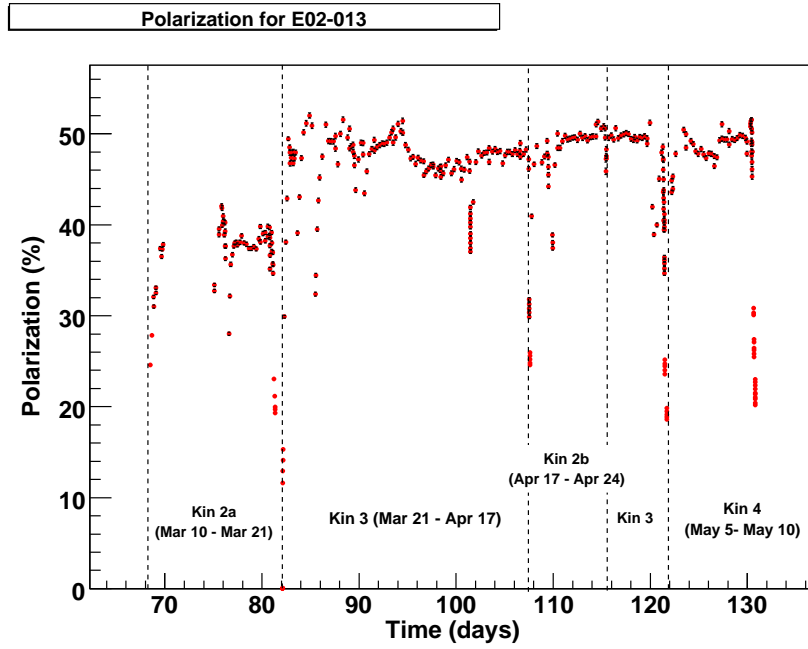


Figure 4.12: Measured polarization in the  $^3\text{He}$  cells during the experiment [45]. Shown is all cells, including Dolly, Edna, and Barbara used during Kinematics 2, 3, and 4.

the unpolarized nitrogen within the cell caused a dilution in the asymmetry. Details of these corrections are presented in section 5.5.2.

## 4.5 Big Bite

### 4.5.1 The Big Bite Apparatus

Big Bite is the name of the electron spectrometer, and of the large 1.2 T dipole magnet that provides the magnetic field for the spectrometer. It consists of the large magnet, a plane of 13 scintillators, a calorimeter, and three multi-wire drift chambers (MWDC) containing a total of 15 wire planes. The calorimeter is split into a shower and preshower, and is constructed of 250 lead glass blocks. A

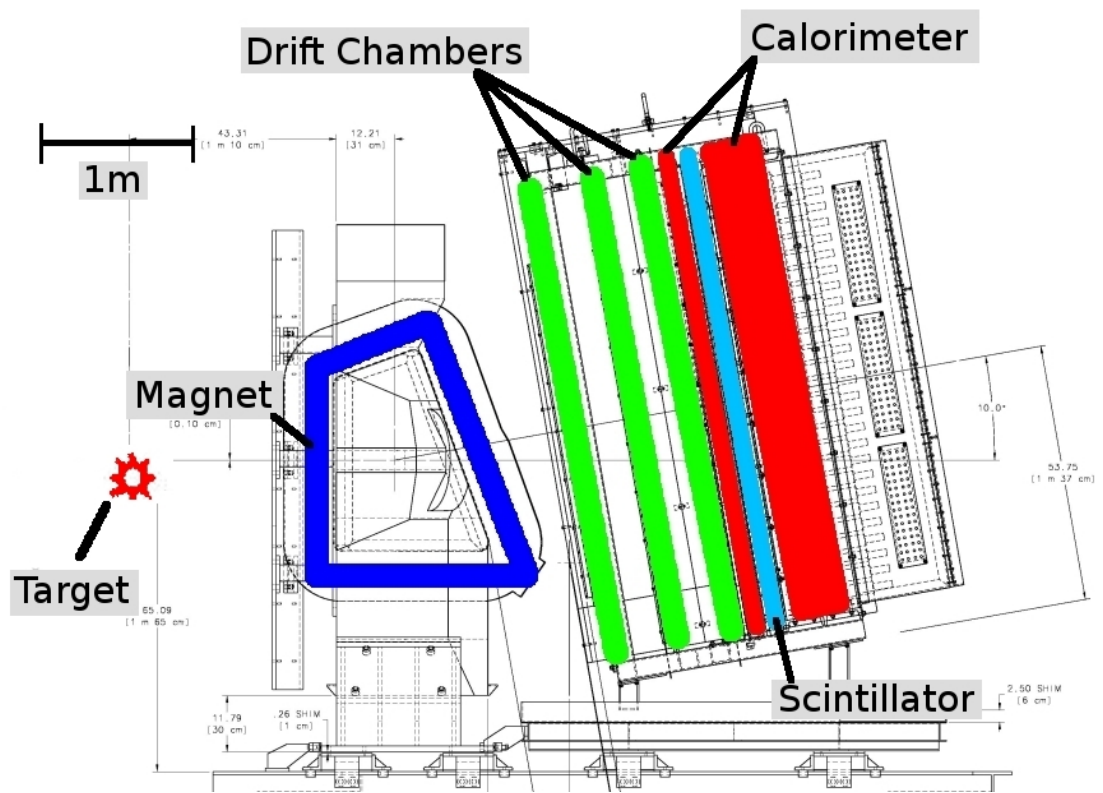


Figure 4.13: Schematic of the Big Bite detector. Shown is the large dipole which provides a magnetic field integral of 1.2 T·m, the 15 planes in three chambers that made up the wire chamber, and the lead glass shower and preshower with the layer of plastic scintillator between.

schematic of the Big Bite detector is shown in Figure 4.13.

#### 4.5.2 The Big Bite Scintillators and Shower

The Big Bite scintillators are located between the preshower and shower and provide the time of the event within Big Bite tracker. Each of the thirteen paddles has a photomultiplier tube on each end, with the signal split going to both a TDC via a discriminator and an ADC. The scintillators are used to calculate the time of the particle at the drift chambers, which are about 1 m away. The scintillator time, with resolution  $\sigma = 300$  ps, is required for reconstructing the hadron time of flight due to its use in providing the reference time. These scintillators are 64 cm by 220 cm.

The front plane of the total shower, known as the pre-shower, is located 1 m behind the drift chambers. The preshower consists of 54 lead glass blocks in two columns of 27 rows. Behind the preshower, the shower consists of seven columns and 27 rows. Its lead glass blocks are 8.5 cm by 8.5 cm, while the preshower has lead glass blocks that are 35 cm by 8.5 cm. The total height of the structure was 230 cm. Each block is connected to a single photomultiplier tube, which collects the Cerenkov light. The signal is then sent to both an ADC and summation module, with the summed signals going both to an ADC and a TDC. The sum of the shower and preshower gives a signal roughly proportional to the energy of the particle. The total shower provides the trigger for Big Bite. A final set of information provided by the total shower is the rough location of the particle. This aids track reconstruction

in the multi-wire drift chambers and reduces the the search space by a factor of 10. The energy resolution provided by the total shower is approximately  $\sigma_{\frac{dE}{E}} = 10\%$ .

### 4.5.3 The Wire Chambers and Tracking

The drift chambers were constructed by the University of Virginia for this experiment in order to reconstruct the trajectory of the electron as it travels through the detector. To do this, the planes which make up the drift chambers were constructed with three different orientations, known as  $U$ ,  $X$ , and  $V$ . The chambers were roughly 35 cm apart. In the Big Bite detector coordinate system, the  $X$  plane has wires running parallel to the  $y$  axis, while the  $U$  and  $V$  wires are rotated by  $\pm 30^\circ$  with respect to that axis. The sense wires were 1 cm apart from each other within the plane. Cathode planes were placed 3 mm above and below each wire plane and field shaping wires were placed between each pair of sense wires to create a roughly symmetric potential around the sensing wires.

Charged particles which pass through the chamber release electrons by ionizing the gas within the chamber. This gas is a 50% argon and 50% ethane mixture that had been bubbled through ethyl alcohol and is kept slightly above atmospheric pressure. Since a voltage difference exists between the sense wires, the field shaping wires, and the cathode planes, the charges are attracted to the detector wires and interact with them forming a signal which is then detected in a TDC (after amplification and discrimination). The time it takes to drift to the wire can be determined and used to calculate the distance between the wire and the particle track. Each

plane was set at a different voltage to create the desired field surrounding the sensing wires.

The detector determines more than just the track that the particle traveled, it also determines the particle momentum and from where along the target the particle originated. An effective bend plane model was used to determine this, where the interaction of the magnet is treated as occurring at the magnetic mid-plane. If we assume that there is only dispersion along Big Bite detector  $x$ , then the complete track between the target and the total shower can be reconstructed. By finding the point in the magnetic mid-plane that the observed track (back track) in the drift chambers points to, and assuming that dispersion happens only in the  $x$  (Big Bite) direction, the origin of the track along the beam (forward track) can be determined ( $V_0$ ). All coordinates in this section are in the Big Bite detector coordinate system unless indicated.

In this model, the vertex is (after corrections  $c_{x'}$ ,  $c_y$ ,  $c_{y'}$ , and  $c_x$ )

$$V_{LAB} = c_0 V_0 + c_{x_0} x_0 + c_{x'_0} x'_0 + c_{y_0} y_0 + c_{y'_0} y'_0 + f(x_{bend}, y_{bend}) \quad (4.2)$$

Here the  $V$  is the  $z$  vertex location in the lab coordinate system, and  $f(x_{bend}, y_{bend})$  is a parameter to determine deviations outside of this model. The  $x_0$  and  $y_0$  describe location of the intersection of the track with the plane  $z = 0$  in the detector coordinate system. The variables  $\Theta_{tgt} = x'_0$  and  $\Phi_{tgt} = y'_0$  describe the track between the bend plane and the target and are found by  $x'_0 = \frac{dx}{dz}$  and  $y'_0 = \frac{dy}{dz}$  in the detector coordinate system. The bend coordinates  $(x_{bend}, y_{bend})$  are the detector coordinates where the track intersects the bend plane. A diagram of the effective bend plane



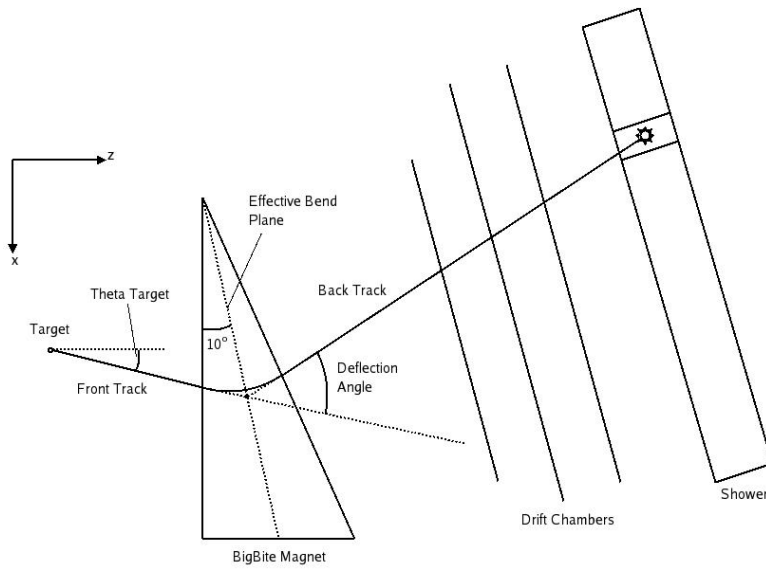


Figure 4.14: Not to scale diagram showing Big Bite, the location of the dipole, target, drift chambers, and shower, and the quantities known as the deflection angle ( $\theta_{def}$ ),  $\theta_{tgt}$ , and the back and front tracks. Shown on the upper left is the coordinate system for the target coordinates. The shower is shown providing a fourth location for the particle, to anchor the track reconstruction. Big Bite detector coordinates have their origin at the center of the first plane. The  $z$  direction is perpendicular to the first chamber, and  $x$  is the magnetic dispersion direction.

model is in Figure 4.14. A histogram showing the foil target vertex reconstruction is shown in Figure 4.16. This shows that the position along the target is reconstructed properly.

The momentum can also be determined, as

$$p_{BB} = \frac{c_0(x_{bend}, y_{bend}) + c_x x_{bend}}{\theta_{def}} + c_\theta \theta_{tgt} + c_y y + c_{y'} y' + f(x_{bend}, y_{bend}) . \quad (4.3)$$

This  $\theta_{def}$  is the deflection angle, and defined using the vectors  $\vec{x}_f$  and  $\vec{x}_b$  which describe the tracks the particle takes between the target and the magnetic mid-plane and between the magnetic mid-plane and the wire chambers, and is defined to be

$$\theta_{def} = \cos^{-1} \left( \frac{\vec{x}_f \cdot \vec{x}_b}{|\vec{x}_f| |\vec{x}_b|} \right) \quad (4.4)$$

The energy for the electron in elastic events can also be determined as

$$E_{elastic} = \frac{m_p E_e}{m_p + E_e (1 - \cos \theta_e)} , \quad (4.5)$$

where  $m_p$  is the mass of the proton and  $E_e$  is the energy of the electron, and  $\theta_e$  is the electron scattering angle. These momentum calibration was done using hydrogen data. The momentum resolution is demonstrated in Figure 4.15.

#### 4.5.4 Big Bite Electronics

Signals from each wire in the multi-wire drift chamber travel through an amplifier and discriminator before terminating at a LeCroy 1877 multi-hit TDC running in common-stop mode. Information is read out from these after each trigger. The scintillators have their signal split between a LeCroy 1881 ADC and a discriminator,

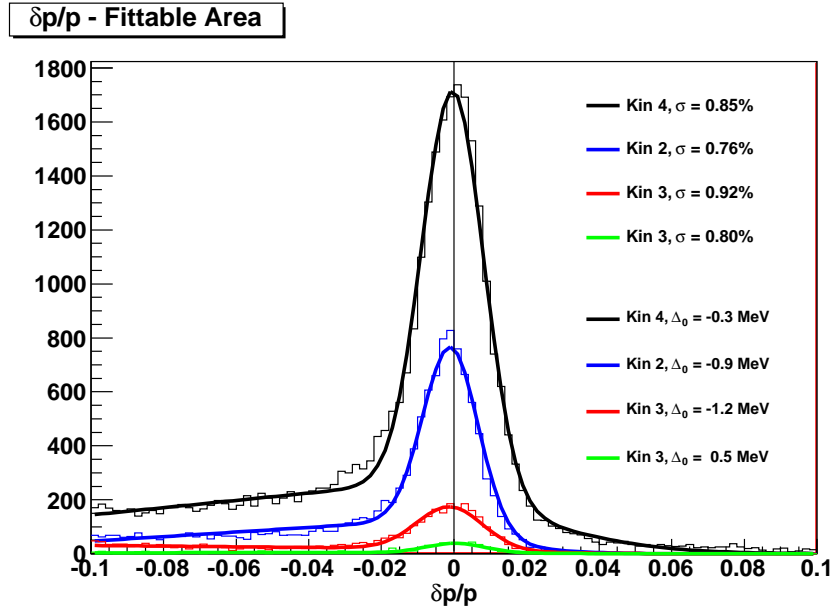


Figure 4.15: Momentum resolution achieved with the latest optics model. Shown is the momentum resolution for all four kinematics.

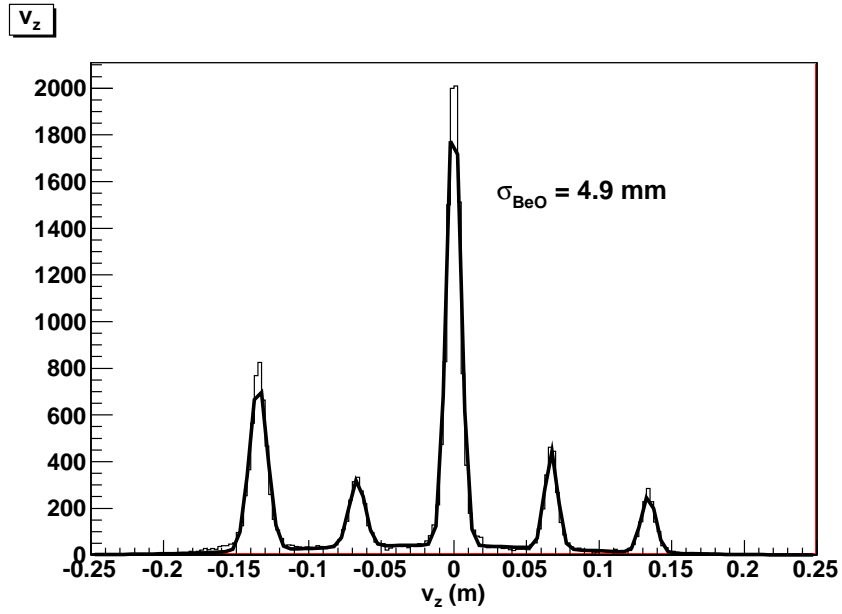


Figure 4.16: Resolution of the carbon foils after vertex corrections. The foils should be 6.7 cm apart with the BeO foil at 0 cm. This histogram is of data from a foil target run in kinematic 3.

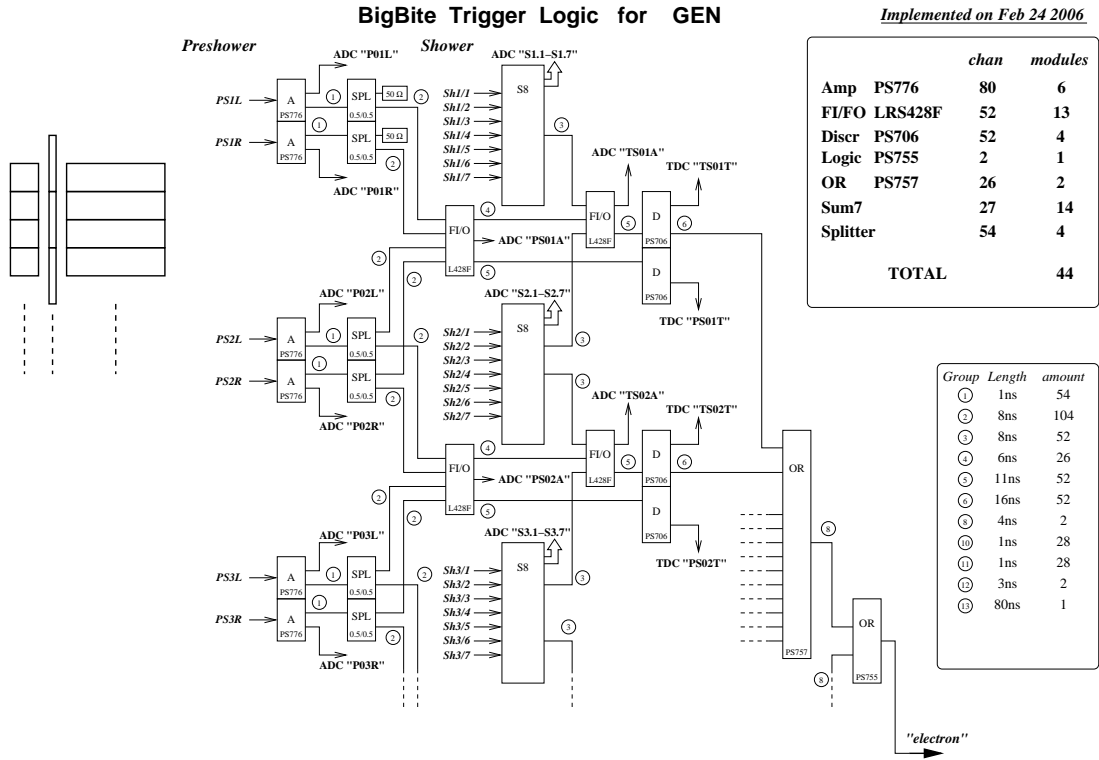


Figure 4.17: The electronics schematic for Big Bite. Shown are the modules and the wire lengths.

with the discriminated signal terminating on a CAEN 775 TDC. The shower and preshower have all their signals (individual and summed) put into a LeCroy 1881 ADC, with the sums also having the signal discriminated and put into a LeCroy 1877 TDC.

The sums of the preshower and shower that are over some predefined threshold provide the Big Bite trigger.

## 4.6 Big Hand

While the subject of Big Hand is a major component of this thesis, the calibration and detailed analysis of the neutron arm's behavior is not needed directly

for the analysis. As a result, the geometry was described in detail in a document put together by Tim Ngo [60] and the software, detector description, analysis, and calibration are presented in Appendix A.

#### 4.6.1 Neutron Geometry

The neutron apparatus (NA), sometimes referred to as Big Hand, is a large hadron detector, designed to match Big Bite's acceptance at the highest kinematic with  $Q^2 = 3.5 \text{ GeV}^2$ . The dimensions of Big Hand are  $4.2 \times 2.0 \times 6.2 \text{ m}^3$  and it has a 100 msr solid angle at a distance of 8 meters. The neutron detector is made up of 244 neutron bars in seven planes and 192 segmented, single sided veto bars in two planes. These counters were provided by University of Virginia (UVA), University of Glasgow (GLA), and Carnegie Mellon University (CMU). For the UVA counters PMT type XP 2282B were used, for the CMU counters PMT type XP 2262B, while for the GLA EMI 5" were used [81]. In order to properly identify quasi-elastic events at the highest kinematic, a time resolution of 0.3 ns was desired, which corresponds to neutron momentum resolution of 250 MeV/c at a hadron momentum of 2.58 GeV[21]. The scintillation material in the counters was the standard organic plastic scintillation material[1]. To increase the detection efficiency of neutrons, a thin (one inch) layer of iron was placed in front of each counter to cause some portion of the incident neutrons to undergo hadronic interactions. A high degree of segmentation was important so that the vertical ( $X$ ) position of the incident particle could be measured with the required resolution, this allows the selection of quasi-elastic events

for which it is necessary to have a good measurement of momentum perpendicular to the virtual photon. Four "marker" counters were included for calibration purposes.

#### 4.6.2 Veto and Marker Bars

The veto counters were divided into two so that they would provide the needed time resolution without problems with attenuation. The two veto planes were offset from each other both horizontally and vertically, for complete coverage of the active area of the neutron bars. This, in addition to the height of veto bars (11 cm) compared to the neutron bars (15cm for CMU bars), meant that multiple veto bars could possibly fire to define the charge for an event within the neutron bars.

The marker counters were included late in the construction to calibrate the horizontal position reconstruction of the neutron detectors. These are long vertical scintillators placed before the first plane of the neutron detector, but after the shielding and veto detectors. They were placed within two constructed channels, with the same amount of space between the top marker bars and the bottom marker bars for each channel[59]. Even with two marker bars to cover the whole height of the neutron arm, most coincident hits between a marker bar and a neutron bar were only measured in a single PMT in the marker bar. Using these marker bars assisted in the time offset calibration of the neutron arm.

### 4.6.3 Neutron Arm Electronics

The initial discriminators, amplifiers, summing modules, were all placed in an electronics hut behind the neutron detector. The ADCs and TDCs were put behind shielding some 100 meters away, where the signals were discriminated again to provide a good signal. Signals from the detectors were combined into sums. This was done to increase the neutron detection efficiency at  $Q^2 = 2, 5 \text{ GeV}^2$ . Figure 4.19 shows the neutron bars and veto bars, a sum is made up of the left or right PMTs for two neighboring color coded sections.

The ADCs used for the detectors within the neutron arm were all Lecroy 1881, while the TDCs used for the veto detectors and sums were LeCroy 1877 TDCs. The F1 TDCs were specially developed electronics for Jefferson Lab and were used for the neutron counters. These TDCs were used in a common-stop mode and provided a resolution of 118 ps. The F1 TDC required that a reference signal be used, this was a delayed signal from the trigger and the F1 TDC signals could be reconstructed relative to it. Summing modules, amplifiers, fan in/out modules, discriminators, ADCs, TDCs are all presented in Figure 4.18 (and logic). This gives the NA side of the trigger as arising from any sum channel being past threshold on the left or right.

## 4.7 Data Acquisition

Six triggers were used in this experiment. These were the neutron arm trigger (1) known as T1, the Big Bite trigger (2) known as T2, the coincidence trigger (3)

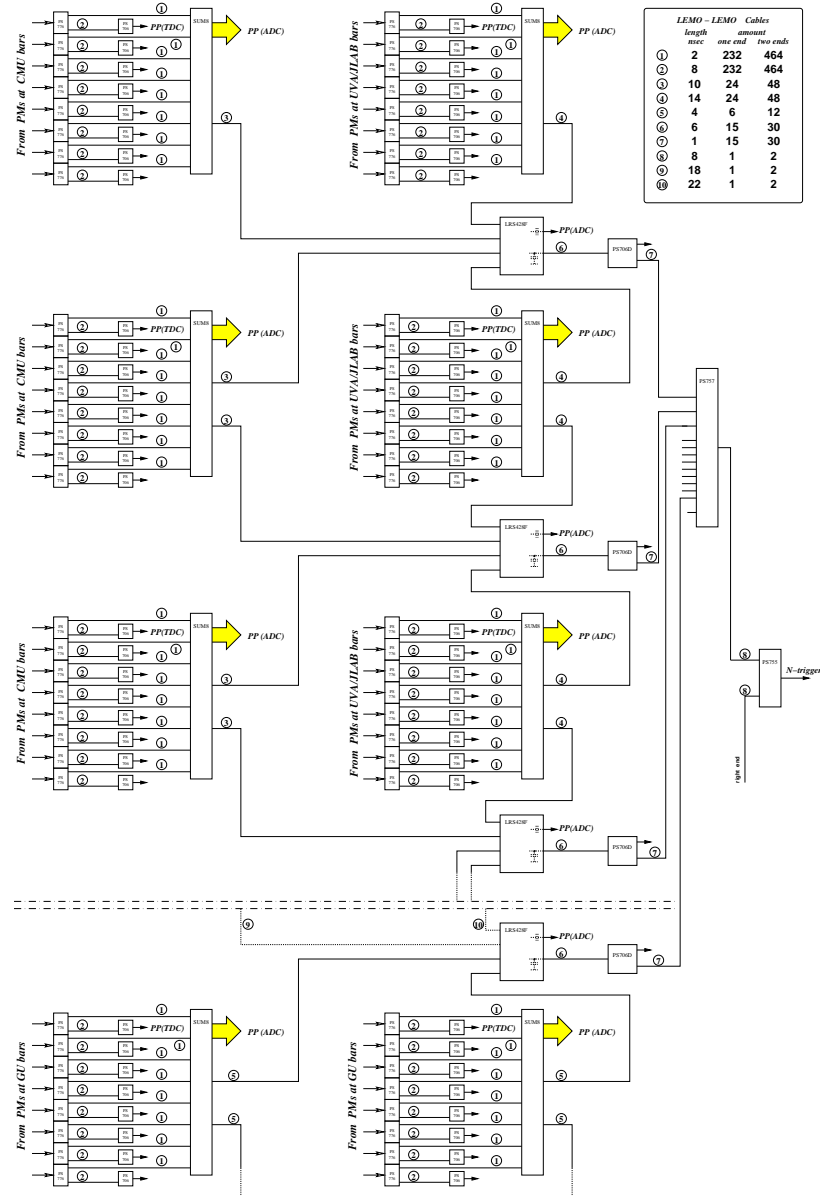


Figure 4.18: Neutron Arm electronics schematics. Included in the schematic is the length of the cables. From the PMTs of the various tubes, the signal goes into both a TDC and a sum module with the non-summed output going to an ADC. The different summed signals are added together and sent to both an ADC and TDC, with an OR of the TDCs forming the trigger.





known as T3, the 8.5 Hz pulser (7), the 105 kHz helicity synchronization signal (8), and the 30 Hz helicity quad duration signal (9).

The T3 trigger was formed by coincidence between the neutron arm and Big Bite. This coincidence was formed between any single sum channel on the neutron arm (left or right) and detection beyond a certain threshold in the preshower and shower. In Figure 4.2 the grouping of neutron bars is presented. Two neighboring colors would be linked together to form one sum, with the left and right PMTs being considered separately. This was done to increase neutron detection efficiency at a higher threshold[66].

The time of the BigBite Shower signal was used for the time of the T2 trigger. The Big Bite scintillator time associated with this was then sent to the neutron arm to provide both the coincidence for the trigger and the reference time of the neutron arm events.

The neutron detector real time was recorded relative to a readout time or reference time. Thus the neutron arm recorded time was given as  $t_{recorded} = t_{real} - t_{readout}$ . The Big Bite trigger time relative to this same read out time was also recorded as  $t_{L1A} = t_{readout} - t_{trig}$ . The neutron arm time relative to the Big Bite trigger time is then  $t_{real} = t_{recorded} + t_{L1A}$ .

A trigger from both detectors, Big Hand and Big Bite, created a T3, the coincidence trigger. The  $t_{readout}$  time was based on this T3 time. The T1 came 35 ns later than the T2 trigger due to delays added in. If the T1 came too early,  $t_{readout}$  and T3 were based on on the T2 trigger. If the T2 trigger came very early, then the Big Bite readout was determined by  $t_{readout}$  and not the T2 trigger. This is known

as forced retiming. A plot showing the data with the different types of events is shown in Figure 4.20.

Both detectors provided information at each coincidence event. Single trigger events, were also taken but prescaled to provide a minimum number of events in the experiment. All three multi-wire drift chambers provide TDC information, the scintillators provide both ADC and TDC, and the preshower and shower provided ADC information (with the preshower and shower sums providing TDC in addition to ADC information). All the neutron arm detectors provided both TDC and ADC information, with the neutron detectors also grouped into sums which provided ADC and TDC information as well. The neutron arm reference time was recorded in both the Neutron Arm TDCs and the Big Bite TDCs.

#### 4.7.1 Encoding and Decoding

There are two main data acquisition systems. One is EPICS, which provides updates occasionally (no more often than every few ms) and provides information about the target, accelerator, electronics, and scalars. This is included in the data stream with the events from the detectors. The events from the detectors are acquired using CODA, which can monitor, acquire, record, and decode data taken during experiments[40]. There was some target information which was stored separately.

To do the analysis and turn the raw data into usable information, a modification of the ROOT software package called the Hall A Analyzer was used[5]. This

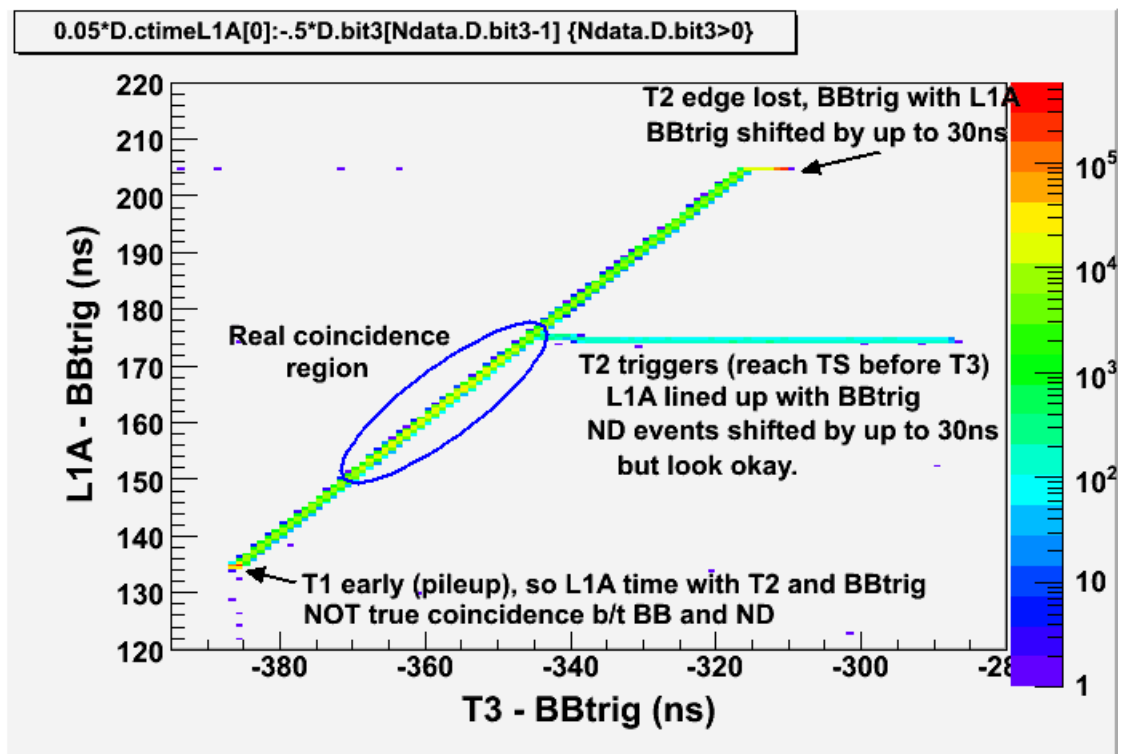


Figure 4.20: Plot showing the reference time and coincidence counts in relation to the Big Bite trigger time. Best coincident events are in the circled region.

package has features which make it easier to deal with large amounts of data and to make histograms and graphs. Special classes for the analysis of the different detectors have been developed for the analysis of the neutron arm data and for tracking and momentum reconstruction in Big Bite and are discussed in other sections. The library which contains these classes is the AGen library[?]. The E02-013 experiment used a modification of this Hall A Analyzer. This was done so that in the decoding and initial calculation of physics quantities, the Big Bite tracking code could take advantage of other detectors. The scalar data were put in separate trees in this initial decoding and analysis of the data. During the initial decoding and analysis steps, the first 1000 events of a run were discarded (due to being helicity diagnostic events) as were non-coincidence events. During the decoding and processing, the data is put into trees, which is a specific data format common for ROOT and are referred to as root files[73].

These root files were then processed an additional time and then the quasi-elastic events were selected using the Big Bite preshower, the reconstructed momentum, the reconstructed vertex hit location, and the hit locations and times of the coincident hadron and electron. This post processing and analysis is described in Chapter 5, with the final value of  $G_E^n$  calculated for the kinematics  $Q^2 = 3.5, 2.5,$  and  $1.5 \text{ GeV}^2$ .

Date	Method	Polarization	Statistical	Systematic
March 30, 2006	Mott	83.32 %	1.45	1.00
March 30, 2006	Mott	81.62 %	1.45	0.98
April 13, 2006	Mott	84.12 %	1.11	1.01
April 13, 2006	Mott	83.25 %	1.11	1.00
Average	Mott	83.08 %	0.65	1.00
February 28, 2006	Møller	88.8 %	0.2	3.0
February 28, 2006	Møller	86.8 %	0.2	3.0
March 4, 2006	Møller	88.2 %	0.14	3.0
March 9, 2006	Møller	86.5 %	0.15	3.0
March 25, 2006	Møller	82.2 %	0.3	3.0
Average	Møller	86.5 %	0.09	3.0
April 18, 2006	Compton	82.92 %	2.31	2.40
April 20, 2006	Compton	85.63 %	3.87	1.55
April 22, 2006	Compton	86.47 %	1.63	3.55
April 24, 2006	Compton	82.65 %	1.64	3.20
Average	Compton	83.5 %	0	1.1 %

Table 4.2: A table of the beam polarization as measured by different means at different times. The Møller measurements were done in the Hall A Møller. The Mott measurements were done using the injector Mott measurement. Only selected Compton data is presented, the average is the averaged of all data for Compton.

## Chapter 5

### Analysis

In experiment E02-013 the electric form factor of the neutron was measured at four kinematic points. Three of these kinematic points are presented in this thesis. The analysis procedure, excluding the initial decoding and analysis steps, is shown in flow charts in Figures 5.1, 5.2, and 5.3. The initial decoding and analysis passes provide the electron arm physics information, as described in section 4.7, in addition to neutron arm cluster information. The first step of this procedure is the event based filtering, as described in Table 5.1, which uses the time-of-flight of the electron, momentum, invariant mass of the recoiling hadron, and transferred momentum vectors which were already calculated from the electron arm (in section 5.1 and 5.2) information to select hadron event candidates. Next, hadrons are identified using the transferred momentum vectors and the constructed time-of-flight of the hadron, which is discussed in more detail in section 5.3. A charge is assigned to these hadrons as described in section 5.4. Next, various dilutions of the asymmetry (section 3.3), and corrections to it, are described and calculated in section 5.5. The neutron physical asymmetry determined from the neutral quasi-elastic events is then calculated in section 5.6, using the Table 5.6. In this section preliminary results for the electric form factor are also presented with corrections. The final state interaction corrections which will be included in the final value of  $G_E^n$  are discussed

in section 5.7. Discussion of error propagation can be found in Appendix B.

Electron event selection in the Big Bite spectrometer was achieved by means of the shower detector and scattering angle. Selection of events from within the target and by a coarse cut on transferred momentum were made using the multi-wire drift chamber (MWDC). Hadron identification was achieved using the location and time of the hit in the neutron detectors, and by using the veto detectors to differentiate the protons from the neutrons. Finally missing mass, missing momentum, and invariant mass were used to select the quasi-elastic process.

## 5.1 Event filtering and scintillator timing

The total shower detector, as described in section 4.5.2, is divided into two parts, called the shower and the pre-shower, separated by a plane of scintillator detectors. Using only the pre-shower amplitude, pions and electrons could be identified and clearly differentiated, (as shown in Figure 5.4). Obviously, for any selection based on preshower energy which doesn't remove an unreasonable portion of electrons, some portion of pions will still exist in the selected sample. These pions will create a dilution of the  $(e,e'h)$  interaction. An empirical study was done to determine the contamination of the neutral hadron candidate sample by  $(ep, e'\pi^0n)$  events. This contamination was shown to be less than 2% [2].

While the total shower was the hardware trigger for the electron arm, the time based on the calorimeter alone would not provide the needed resolution for the event. This function is provided by the plane of scintillators directly behind the



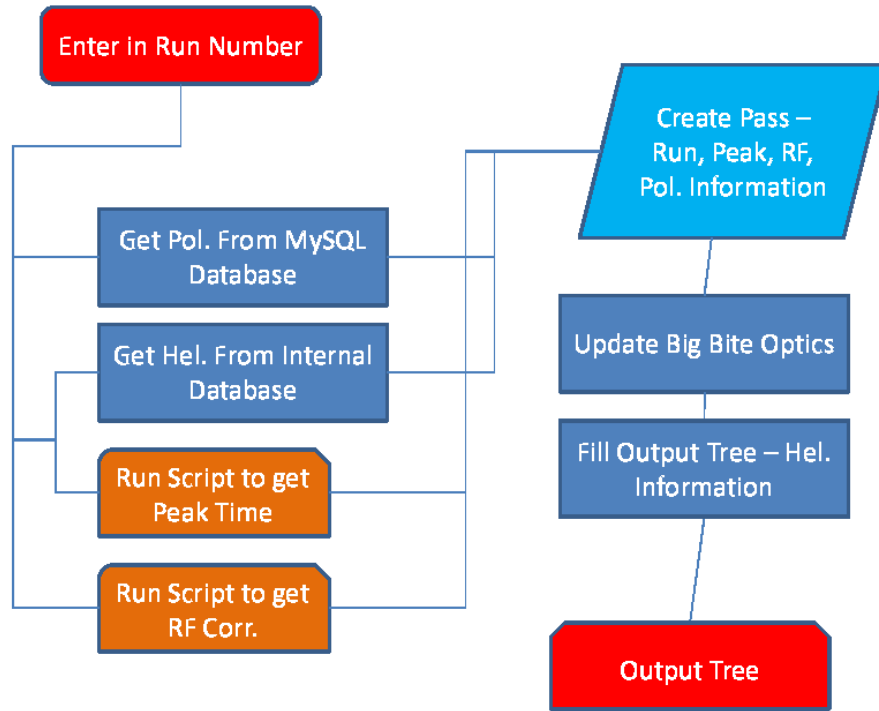


Figure 5.1: The flowchart for the analysis. This is after the preliminary replay of events that is described in section 4.7. The resulting ROOT files contain the subset of events necessary to undertake the remaining analysis, also assigns charge, and identifies quasi-elastic event candidates. Two separate scripts are ran as part of this process for each run before this post processing. This is to determine the RF correction (see section 5.5.3) and the hadron time-of-flight.

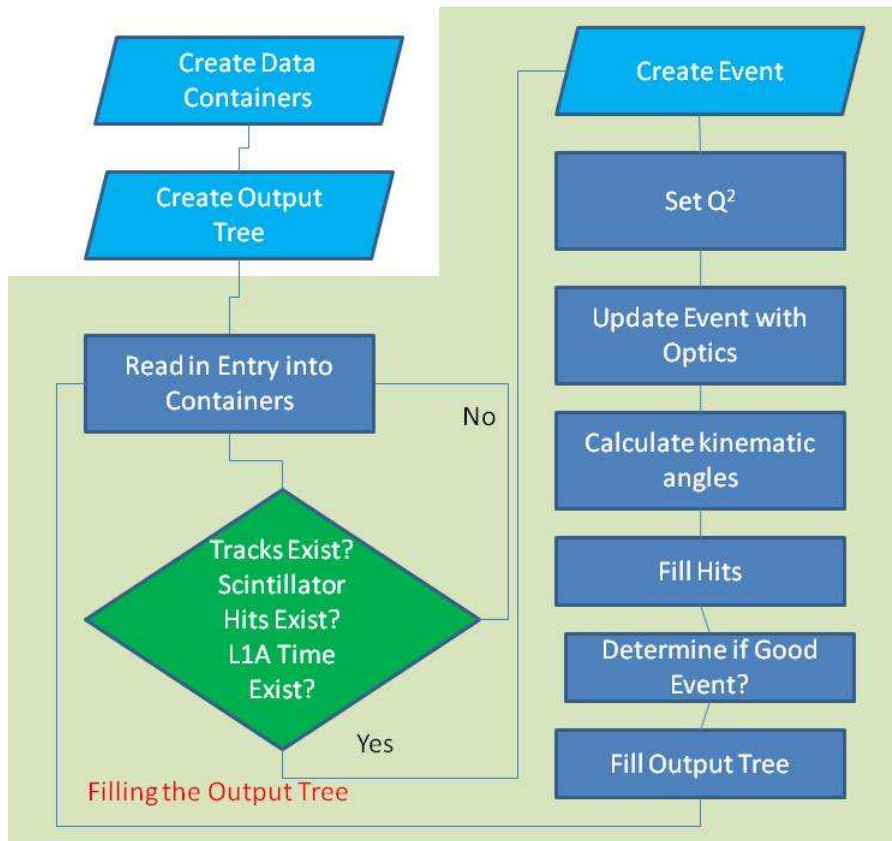


Figure 5.2: Flowchart for the event processing. Events in the analysis are ignored if no tracks or Big Bite scintillator hits exist, or if the event does not have a well defined trigger time as demonstrated by the existence of the Level 1 Accept (L1A) Time. The optics can be updated here if necessary, and the kinematics of the event are calculated to ease post processing. All event cuts are detailed in Table 5.1.

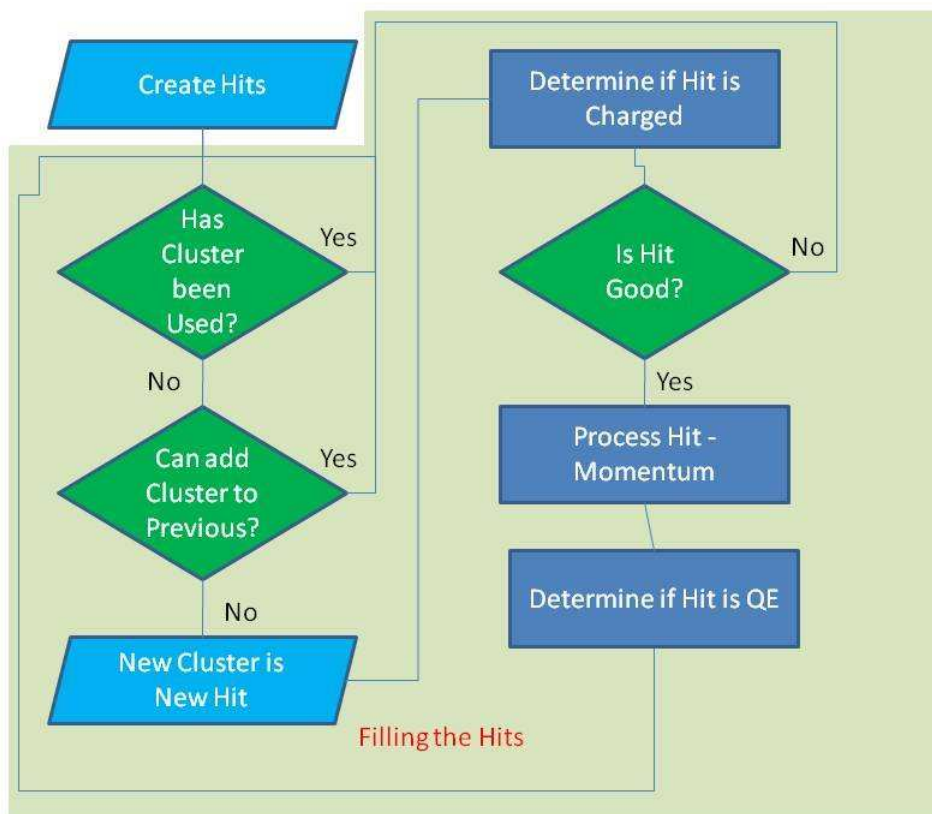


Figure 5.3: Flowchart associating clusters, both with each other and as quasi-elastic event candidates. Some clusters should be associated together, where some neutron detector did not fire to make the cluster continuous. Charged candidates are identified, and momentum is calculated for the hit.

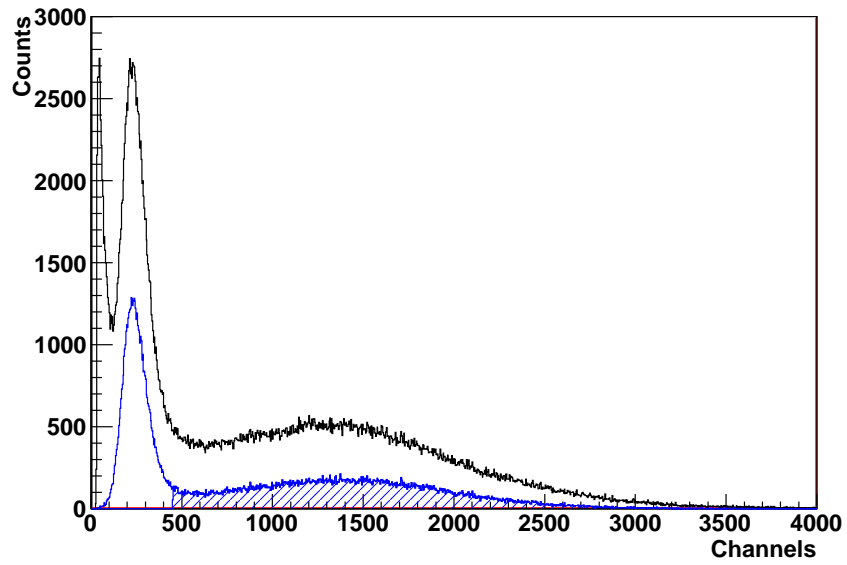


Figure 5.4: Amplitude distribution for all events showing pions and electrons in Big Bite. In blue is the distribution after cuts have been applied to remove events which are lacking the information needed for analysis, shaded is the region selected. See Table 5.1 for cuts, in black is all events and in blue is with just the “sanity” cuts applied. This distribution is for  $Q^2 = 3.5 \text{ GeV}^2$ .

pre-shower, which served as the primary time reference for both the electron and neutron arms. The path length for the electron was empirically determined using the optics with respect to this plane. This reference time was corrected for the variations in path length coming from differences in vertex location and corrections for vertical and horizontal position in the scintillator stack.

The time of the event within the electron arm,  $t_e$ , can be presented as the time in the scintillator  $t_s$ , with corrections for the vertical path length  $t_x$ , the horizontal path length  $t_y$ , and the difference between the scintillator plane and the wire chamber  $t_c$ .

$$t_e = t_s - t_x - t_y - t_c \quad (5.1)$$

$$t_x = \frac{1}{c} 1.2 |P_{x,BB} + 0.25| \quad (5.2)$$

$$t_y = \frac{1}{c} \frac{[2.2532 \sin(-\theta_{BB}) + Y_{BB} \cos(-\theta_{BB})]}{\sin[\tan^{-1}(P_{y,BB}) - \theta_{BB}]} - 2.2 \quad (5.3)$$

$$t_c = \frac{1}{c} (L_{D2S}(1 + P_{x,BB}^2 + P_{y,BB}^2))^2 \quad (5.4)$$

Here,  $L_{D2S} = 0.95$  m is the time from the detector to the scintillator,  $P_{x,BB}$  is the track slope  $\tan \Theta = \frac{dx}{dz}$  in Big Bite detector coordinates,  $P_{y,BB}$  is the track slope  $\tan \Phi = \frac{dy}{dz}$  in Big Bite detector coordinates,  $Y_{BB}$  is the reconstructed horizontal position vector  $y$  in Big Bite detector coordinates, and  $\theta_{BB} = 51.59$  degrees is the central angle at which the detector was placed for the  $Q^2$  of 1.7, 2.5, and 3.5 GeV<sup>2</sup>. The other constants were empirically determined using the optics data.

## 5.2 Electron parameters

Quasi-elastic (from  $^3\text{He}$ ) and elastic events (from hydrogen) were selected using the momentum of the electron as measured by the Big Bite spectrometer in addition to deposited energy in the preshower (see Figure 5.5). Additional event selection was performed by removing events that were reconstructed to start from outside of the target (see Figure 5.7), or that travelled through a region of Big Bite where the magnetic field is not well understood (see Figure 5.6). Among the quantities computed from the identified electron tracks are the invariant mass of the hadron, the transferred momentum of the virtual photon, the electron momentum, and the electron scattering angle.

The transferred momentum,  $\vec{q}$ , calculated from the beam energy and scattered electron momentum, was used in the identification of quasi-elastic hits within the neutron arm. The transferred momentum is entirely independent of the neutron arm time-of-flight. The parallel and perpendicular components of transferred momentum with respect to the hadron momentum vector can be defined as

$$q_{\parallel} = \vec{q} \cdot \hat{p}_h , \quad (5.5)$$

$$q_{\perp} = \vec{q} \times \hat{p}_h . \quad (5.6)$$

Here  $\hat{p}_h$  is the unit vector of the hadron momentum as determined by the neutron arm. This is determined purely from the hit location ( $x$ ,  $y$ , and  $z$ ) in the neutron arm, and is independent of time-of-flight.

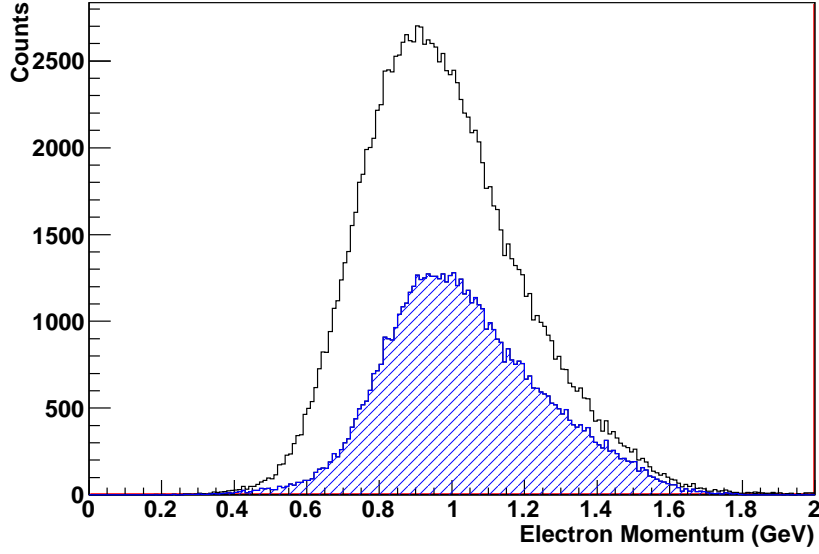


Figure 5.5: Electron momentum for all tracks in Big Bite. In blue are just those tracks for which the preshower energy is greater than 450 channels, shaded is the selected momentum. See Table 5.1 for cuts, in black is all events and in blue is with the “sanity” and pre-shower energy cuts applied. This distribution is for  $Q^2 = 3.5 \text{ GeV}^2$ .

The invariant mass of the hadron is

$$W^2 = M_N^2 - Q^2 + \frac{2\vec{p}_e \cdot \vec{q}}{M_N} . \quad (5.7)$$

Here  $M_N$  is the nucleon mass,  $Q^2$  is the transferred four-momentum squared (see section 2.2), and  $\vec{p}_e$  is the electron momentum.

The invariant mass and perpendicular transferred momentum ( $q_\perp$ ) are the two time-of-flight independent variables which are used for quasi-elastic event selection. The selection of quasi-elastic events are shown for  $Q^2 = 2.5 \text{ GeV}^2$  in Figure 5.8(a) and 5.9(a). Both of these figures include fiducial cuts in the neutron arm. The final quasi-elastic neutral candidates selection for kinematic 2 as well as the overall spectrum is presented in perpendicular transferred momentum is presented in Figure

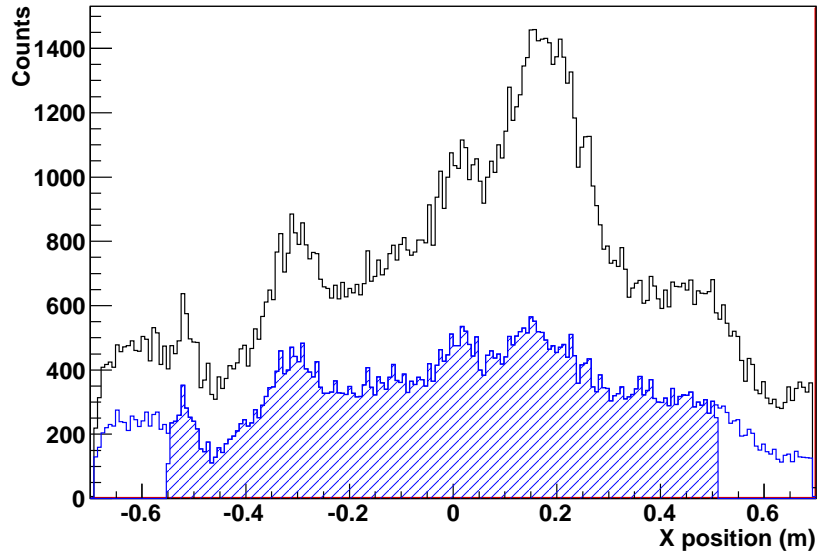


Figure 5.6: X position (m) within the Big Bite scintillator detector. This is for before cuts (black), after electron and sanity cuts (blue), with the selected region of  $X$  shaded. Variation is due to the lack of calibration for the total shower in the Big Bite detector. See Table 5.1 for cuts, in black is all events and in blue is with the “sanity”, pre-shower energy, and electron momentum cuts applied. This distribution is for  $Q^2 = 3.5 \text{ GeV}^2$ .



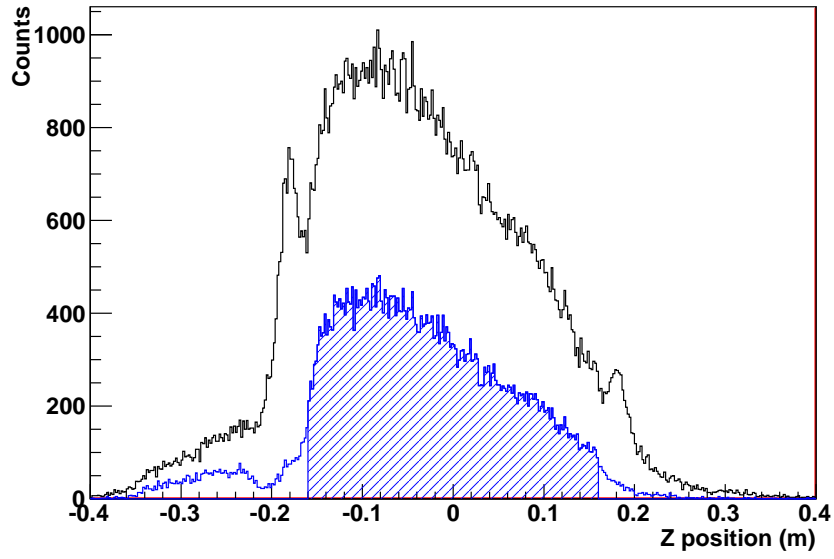


Figure 5.7: Vertex, this shows the target  $z$  position reconstructed by the Big Bite wire chambers for all events with tracks (black). In blue is after cuts on particle ID. Shaded is the region selected as being within the target. See Table 5.1 for cuts, in black is all events and in blue is with the “sanity“, pre-shower energy, Big Bite fiducial, and electron momentum cuts applied. This distribution is for  $Q^2 = 3.5 \text{ GeV}^2$ .

name	$Q^2 = 2.5 \text{ GeV}^2$	$Q^2 = 3.5 \text{ GeV}^2$	$Q^2 = 1.7 \text{ GeV}^2$
W (GeV)	$ W - 0.95  < 0.6$	$ W - 0.95  < 0.6$	$ W - 0.95  < 0.6$
E. E. (MeV)	$E_{ps} > 450$	$E_{ps} > 450$	$E_{ps} > 450$
E. Mom.(GeV)	$ p_e - 1.1  < 0.9$	$ p_e - 1.1  < 0.9$	$ p_e - 1.1  < 0.9$
Vertex (m)	$ V_{tgt}  < 0.16$	$ V_{tgt}  < 0.16$	$ V_{tgt}  < 0.16$
X Fid. (m)	$-0.4 < X_{BB} < 0.45$	$-0.4 < X_{BB} < 0.45$	$-0.4 < X_{BB} < 0.45$
Sanity 1	$N_{Tracks} > 0$	$N_{Tracks} > 0$	$N_{Tracks} > 0$
Sanity 2	$N_{Paddles} \geq 0$	$N_{Paddles} \geq 0$	$N_{Paddles} \geq 0$
Sanity 3 (Ch)	$t_{L1A} < 4000$	$t_{L1A} < 4000$	$t_{L1A} < 4000$
Sanity 4	$N_{Type3} > 0$	$N_{Type3} > 0$	$N_{Type3} > 0$
Negative	$P_{x,BB} - 0.3X_{BB} < 0$	$P_{x,BB} - 0.3X_{BB} < 0$	$P_{x,BB} - 0.3X_{BB} < 0$

Table 5.1: A table of the initial event selection for analysis. Sanity cut 1 is the requirement that Big Bite has a track, Sanity cut 2 is the requirement that at least one scintillator paddle in Big Bite has an event, Sanity cut 3 is that the coincidence time (L1A) was recorded. Sanity cut 4 is the requirement that there is a coincidence event. Negative is a requirement that only negative particles in the drift chambers can be considered as electrons.

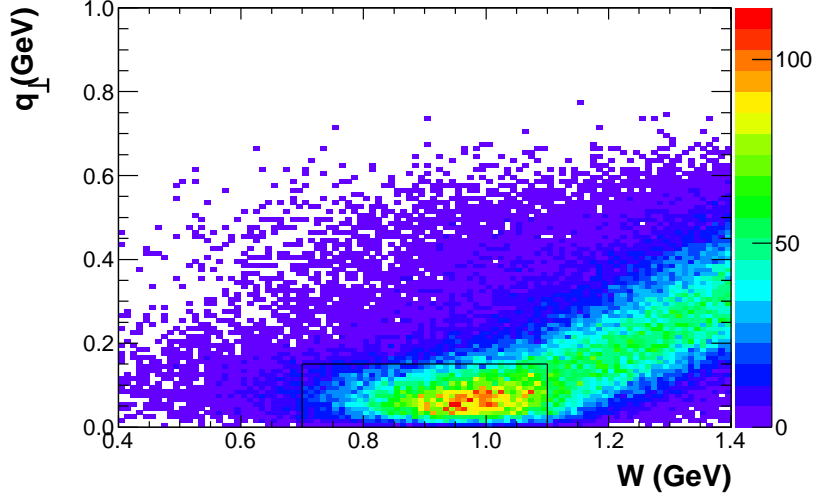
5.11(a). The expected invariant mass is about 0.94 GeV, the mass of a nucleon.

### 5.3 Hadron identification

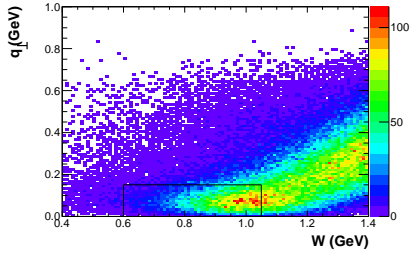
Once electron events are selected and the invariant mass and the direction and magnitude of  $\vec{q}$  are determined from information from the Big Bite spectrometer, the neutron arm is used to determine the three-momentum and charge of the hadron. Hits in the neutron arm were joined together into clusters as described in Appendix A.4.1. Typically, multiple bars recorded hits in the same event in the neutron apparatus due to particles with high energy traveling deep into the neutron apparatus, and to the segmented nature of the neutron apparatus.

The time of the counter with the earliest hit (of those nearby in time and space) was used as the time of the cluster. This time, the time of the electron

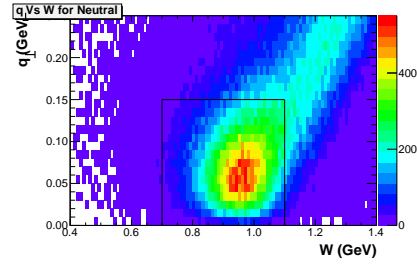
in the Big Bite scintillator, and the relevant path lengths allow the calculation of the momentum of the particle, assuming it is a hadron. Only those hits where the calculated momentum is less than  $c$  are included in the analysis.



(a)  $Q^2 = 2.5 \text{ GeV}^2$   $q_\perp$



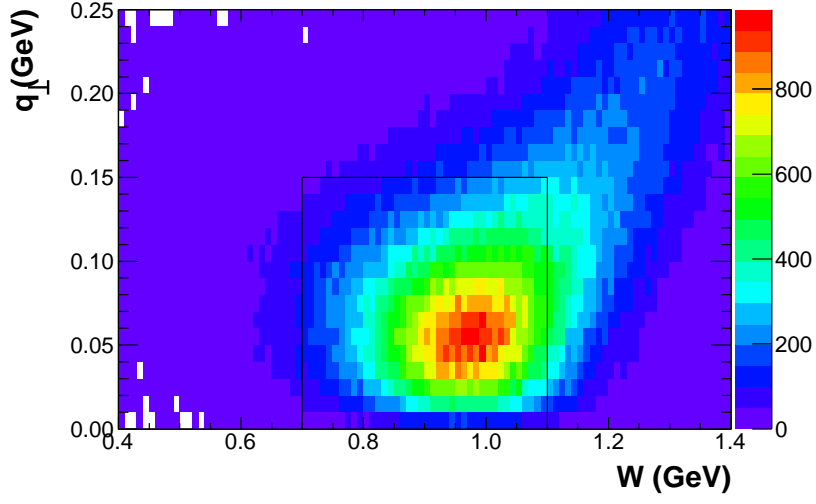
(b)  $Q^2 = 3.5 \text{ GeV}^2$   $q_\perp$



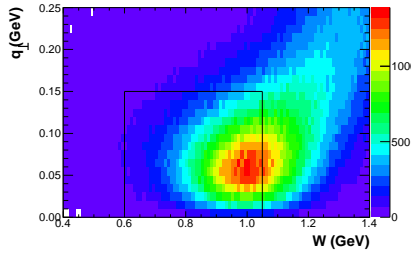
(c)  $Q^2 = 1.7 \text{ GeV}^2$   $q_\perp$

Figure 5.8:  $q_\perp$  versus  $W$  for neutral particles in the various kinematics. Shown is the selection of quasi-elastic events in the red square. As shown, at higher  $Q^2$  the inelastic tail becomes stronger.

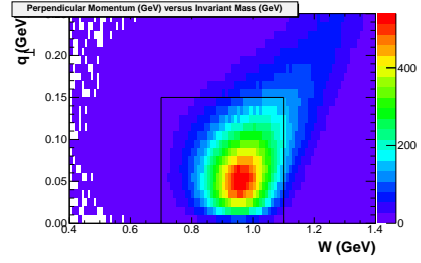
The clusters were combined if they were nearby in time and space. The time and position of only the first hits were used to determine whether the later cluster(s) should be considered part of the earlier cluster. Multiple clusters in a single event were considered as a single cluster if  $\delta X$  (vertical position) and  $\delta Z$  (depth) were within 0.4 m, and  $\delta Y$  (horizontal position) was within 0.2 m. Additionally, the time



(a)  $Q^2 = 2.5 \text{ GeV}^2$   $q_{\perp}$

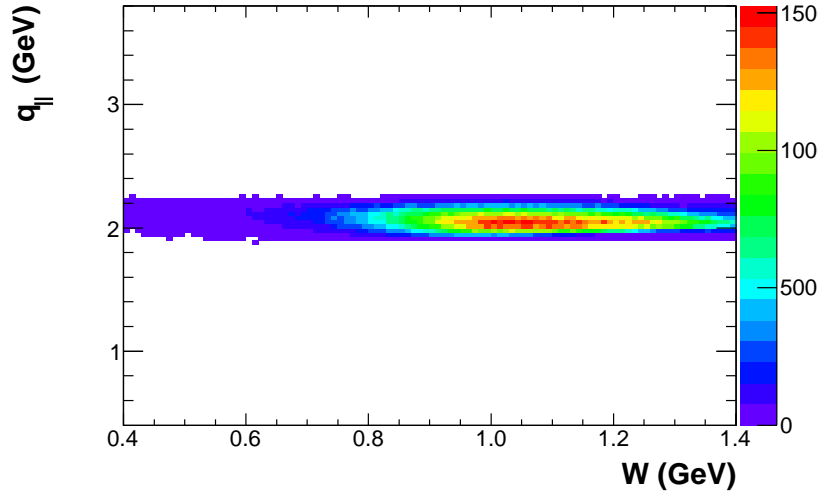


(b)  $Q^2 = 3.5 \text{ GeV}^2$   $q_{\perp}$

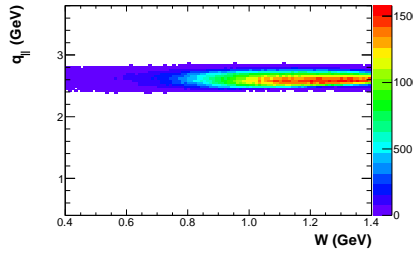


(c)  $Q^2 = 1.7 \text{ GeV}^2$   $q_{\perp}$

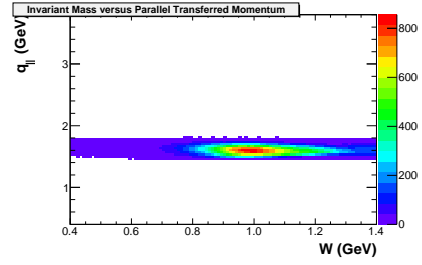
Figure 5.9:  $q_{\perp}$  versus  $W$  for charged particles in the various kinematics. In the red box is the selection of quasi-elastic events.



(a)  $Q^2 = 2.5 \text{ GeV}^2$   $q_{\parallel}$

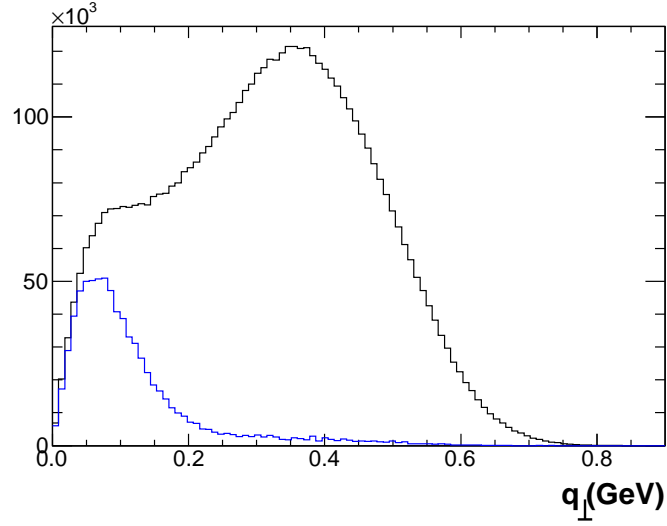


(b)  $Q^2 = 3.5 \text{ GeV}^2$   $q_{\parallel}$

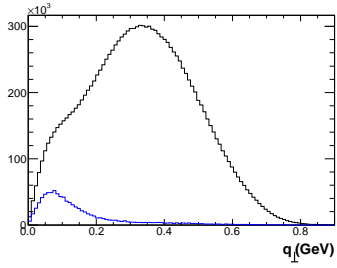


(c)  $Q^2 = 1.7 \text{ GeV}^2$   $q_{\parallel}$

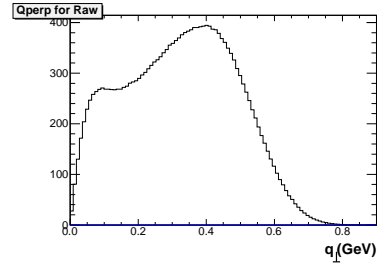
Figure 5.10:  $q_{\parallel}$  versus  $W$  for charged particles in kinematic 3. ? - ? In the conversion section or the ND section ? - ?



(a)  $Q^2 = 2.5 \text{ GeV}^2$   $q_{\perp}$



(b)  $Q^2 = 3.5 \text{ GeV}^2$   $q_{\perp}$



(c)  $Q^2 = 1.7 \text{ GeV}^2$   $q_{\perp}$

Figure 5.11: Distribution of  $q_{\perp}$  in the various kinematics. Shown in blue is the neutral spectrum, in black is the total spectrum (That passed the event based selection). The majority of neutral quasi-elastic events fall under 0.15 GeV.

difference had to be within 10 ns. See Appendix A for more discussion of clusters and neutron arm software.

The expected time for hadron event within the neutron arm  $T_h$  is given by

$$T_h = \frac{\sqrt{L_{NA}^2 + Z_{tgt}^2 - 2L_{NA}Z_{tgt}\hat{p}_h \cdot \hat{z}}}{\frac{q}{\sqrt{m_h^2 + q^2}}} . \quad (5.8)$$

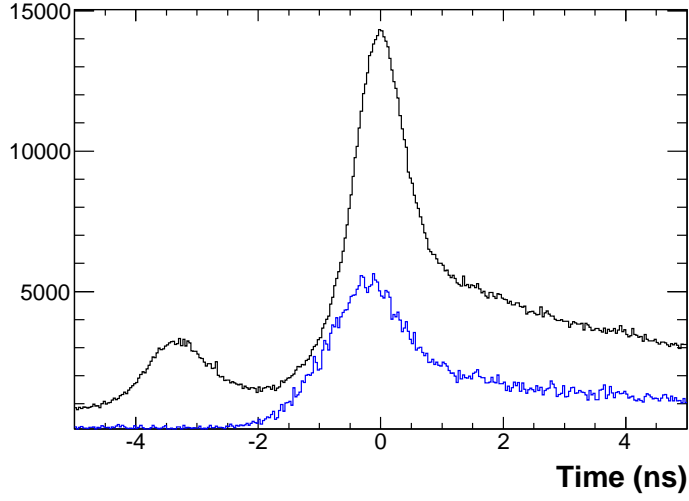
Here  $L_{NA}$  is the path measured from the center of the target to the neutron arm hit,  $Z_{tgt}$  is position of the interaction in the target as reconstructed by Big Bite,  $\vec{p}_h$  is the hadron momentum vector (in the laboratory coordinate system). Since at this point the charge of the hadron is unknown, its mass is nominally set to the nucleon mass  $m_N = 0.939$  GeV.

This is compared to the actual time-of-flight which is constructed from the measured location of the time-of-flight peak measured using the neutron arm and Big Bite scintillators. This actual time is constructed as

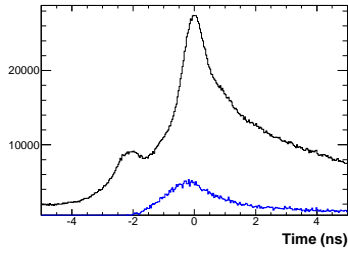
$$T_{TOF} = t_{NA} - t_e + t_{rf} + t_{L1A} - t_{peak} . \quad (5.9)$$

Here, the  $t_{NA}$  is the time measured in the neutron arm,  $t_{rf}$  is the correction that can be done using the beam information (to be discussed in section 5.5.3),  $t_{L1A}$  is the level 1 accept time, and  $t_{peak}$  is the peak location in the total time-of-flight spectrum. To select quasi-elastic events, the expected time-of-flight is compared to the actual time-of-flight, giving  $\Delta T$ . The time-of-flight spectrum for total events and quasi-elastic neutral candidates in kinematic 2 is shown in Figure 5.12(a).

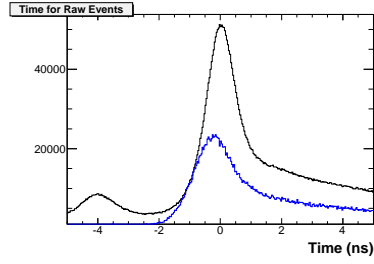
From the time-of-flight, the hadron velocity,  $\beta$ , and momentum,  $p_h$ , can be



(a)  $Q^2 = 2.5 \text{ GeV}^2$  time



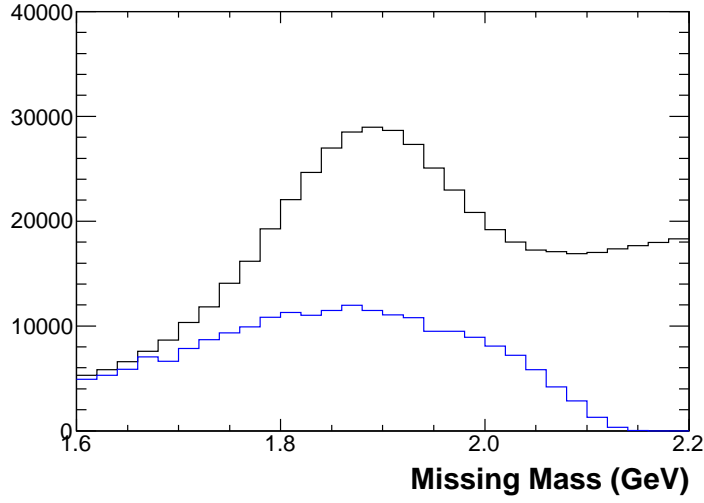
(b)  $Q^2 = 3.5 \text{ GeV}^2$  time



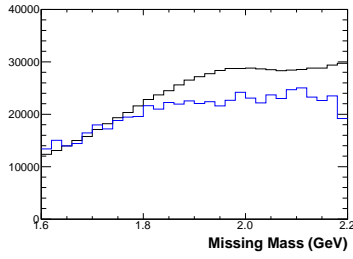
(c)  $Q^2 = 1.7 \text{ GeV}^2$  time

Figure 5.12:  $\Delta T$  (ns) for the various kinematics. In blue is the neutral time-of-flight spectrum, in black is the total time-of-flight spectrum. The photon peak is shown at  $\approx -3\text{ns}$ . Quasi-elastic events are selected with time between -1 and 1 ns.

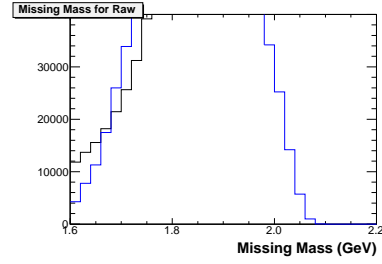




(a)  $Q^2 = 2.5 \text{ GeV}^2$  missing mass

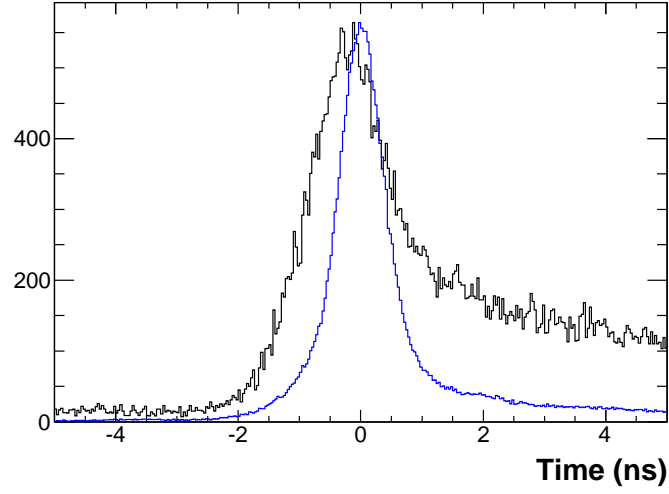


(b)  $Q^2 = 3.5 \text{ GeV}^2$  missing mass

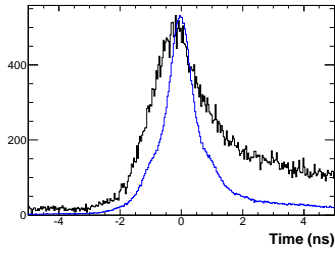


(c)  $Q^2 = 1.7 \text{ GeV}^2$  missing mass

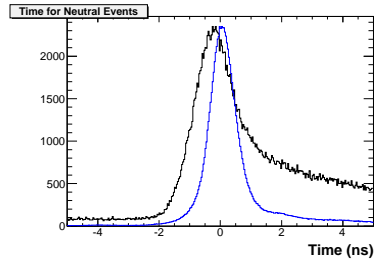
Figure 5.13: Missing mass (GeV) for the various kinematics. In blue is the neutral scaled up by a factor of 20 for  $Q^2 = 3.5 \text{ GeV}^2$  and 10 for  $Q^2 = 2.5 \text{ GeV}^2$ , in black is the total spectrum.



(a)  $Q^2 = 2.5 \text{ GeV}^2$  time



(b)  $Q^2 = 3.5 \text{ GeV}^2$  time



(c)  $Q^2 = 1.7 \text{ GeV}^2$  time

Figure 5.14:  $\Delta T$  (ns) for the various kinematics. In black is the neutral time-of-flight spectrum, in blue is the charged time-of-flight spectrum. The charged spectrum is scaled down to the neutral spectrum.

constructed (all in laboratory coordinates):

$$\beta = \frac{L_{NA}^2 + V_{tgt}^2 - 2L_{NA}V_{tgt}\hat{p}_h \cdot \hat{z}}{cT_{TOF}} , \quad (5.10)$$

$$p_h = \frac{m_h}{\sqrt{1 - \beta^2}} . \quad (5.11)$$

Also calculated is the unit vector relating this hit and the central location in the target. Additionally, the perpendicular and parallel momenta can be calculated:

$$p_{h,\parallel} = \vec{p}_h \cdot \hat{q} \quad (5.12)$$

$$p_{h,\perp} = \vec{p}_h \times \hat{q} . \quad (5.13)$$

An additional quantity used to select quasi-elastic events which is dependent on hadron identification is the missing mass. This is defined as

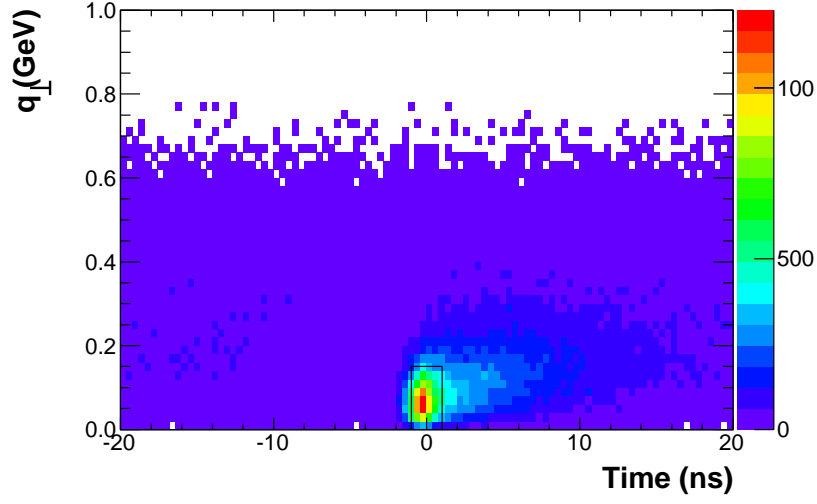
$$M_{miss}^2 = (\mathbf{P}_i + \mathbf{q} - \mathbf{p}_h)^2 = (E_{He} + E_q - E_p)^2 - (\vec{q} - \vec{p}_h)^2 , \quad (5.14)$$

where  $\mathbf{P}_i$  is the initial  $^3\text{He}$  momentum four-vector,  $\mathbf{q}$  is the transferred momentum four-vector and  $\mathbf{p}_h$  is the nucleon four-vector.

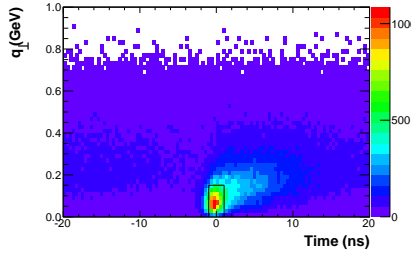
The selection of quasi-elastic neutral events using time, invariant mass, and  $q_\perp$  is demonstrated in Figures 5.15(a) and 5.16(a). The inelastic contribution becomes larger at higher  $Q^2$  and so more aggressive cuts in invariant mass must be used.

## 5.4 Identification of the hadron charge

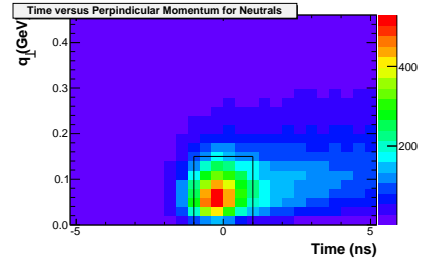
While a particle can be determined to be a recoil hadron candidate by hit location and time-of-flight, it is not easy to differentiate between protons and neutrons using only this information from within the neutron counters. For the purpose



(a)  $Q^2 = 2.5 \text{ GeV}^2$  time versus  $q_\perp$

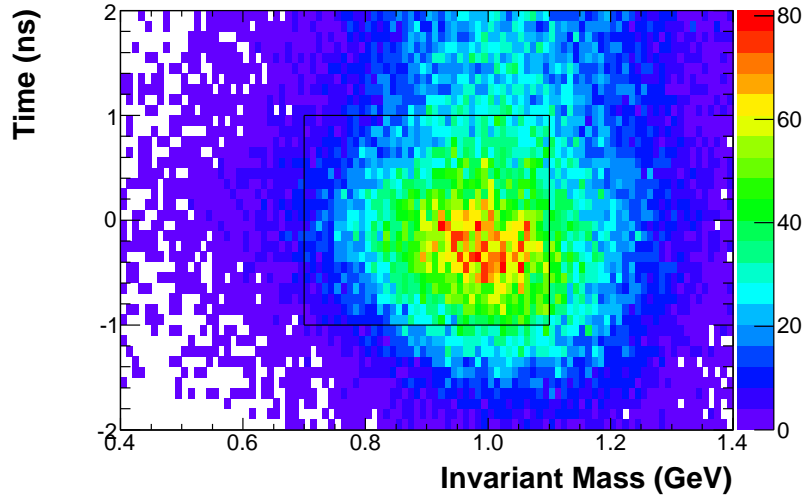


(b)  $Q^2 = 3.5 \text{ GeV}^2$  time versus  $q_\perp$

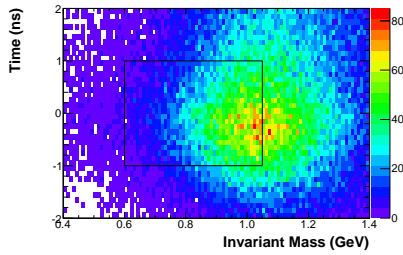


(c)  $Q^2 = 1.7 \text{ GeV}^2$  time versus  $q_\perp$

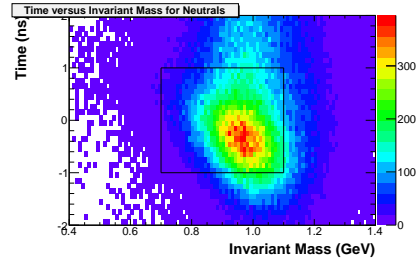
Figure 5.15: Shown in these figures is the time versus  $q_\perp$  for various kinematics. Also shown is the box showing which cuts were used for identifying quasi-elastic clusters within the neutron arm. As is shown, the ability to select quasi-elastic clusters is much better for the lower kinematics.



(a)  $Q^2 = 2.5 \text{ GeV}^2$  time versus invariant mass

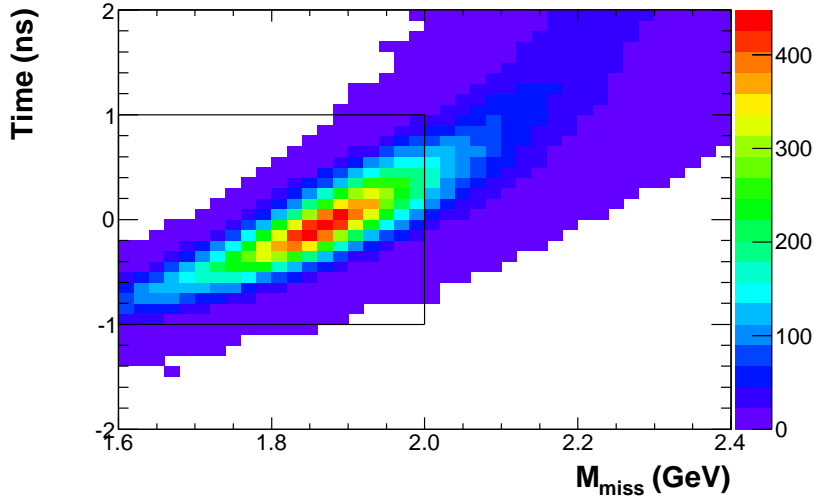


(b)  $Q^2 = 3.5 \text{ GeV}^2$  time versus invariant mass

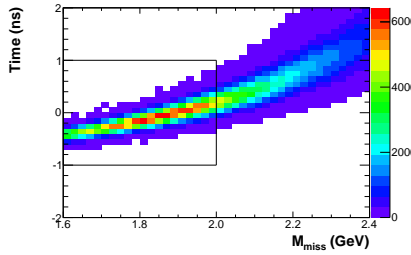


(c)  $Q^2 = 1.7 \text{ GeV}^2$  time versus invariant mass

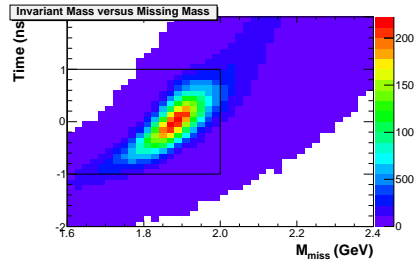
Figure 5.16: Shown in these figures is the time versus invariant mass for various kinematics. Also shown is the box showing which cuts were used for identifying quasi-elastic clusters within the neutron arm. As is shown, the ability to select quasi-elastic clusters is much better for the lower kinematics.



(a)  $Q^2 = 2.5 \text{ GeV}^2$  invariant mass versus missing mass

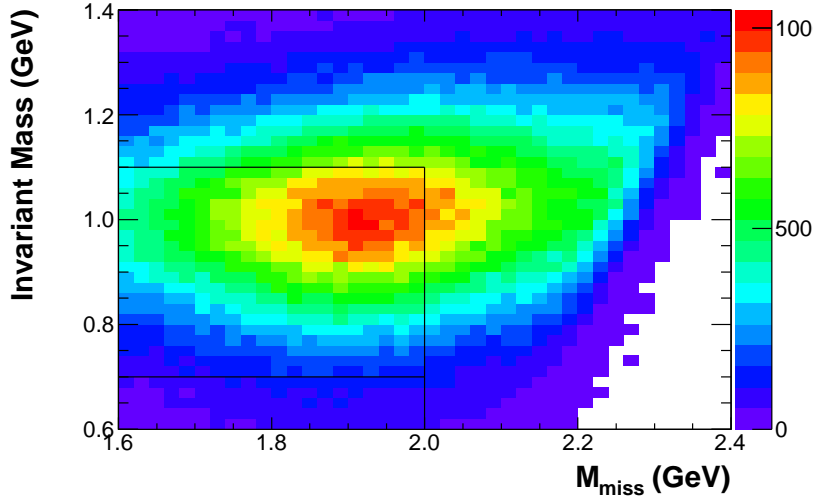


(b)  $Q^2 = 3.5 \text{ GeV}^2$  invariant mass versus missing mass

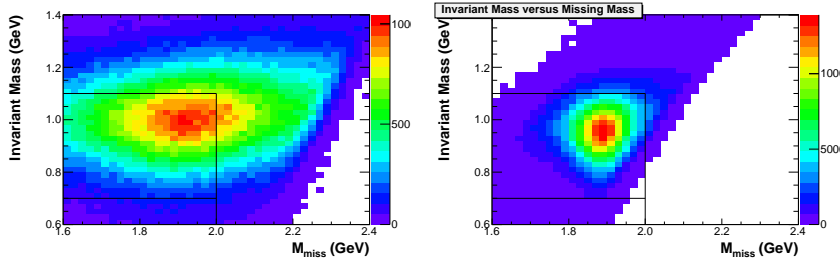


(c)  $Q^2 = 1.7 \text{ GeV}^2$  invariant mass versus missing mass

Figure 5.17: Shown in these figures is the time versus missing mass for various kinematics. Also shown is the box showing which cuts were used for identifying quasi-elastic clusters within the neutron arm. As is shown, the ability to select quasi-elastic clusters is much better for the lower kinematics.



(a)  $Q^2 = 2.5 \text{ GeV}^2$  invariant mass versus missing mass



(b)  $Q^2 = 3.5 \text{ GeV}^2$  invariant mass versus missing mass (c)  $Q^2 = 1.7 \text{ GeV}^2$  invariant mass versus missing mass

Figure 5.18: Shown in these figures is the invariant mass versus missing mass for various kinematics. Also shown is the box showing which cuts were used for identifying quasi-elastic clusters within the neutron arm. As is shown, the ability to select quasi-elastic clusters is much better for the lower kinematics.

of charge identification, two layers of veto counters were positioned in front of the neutron counters but behind some of the lead shielding (see section 4.6.1).

The charge of these hadrons was determined by an analysis of hits in the veto in a neighborhood around the hadron hit position in the neutron counters. The veto detectors were segmented vertically similarly to the neutron detectors. Due to scattering in the shielding and detectors, the relevant veto hit position could be greater than a few centimeters away from the identified hadron hit location. The factors used to select relevant veto detectors are listed in Table 5.4.

Determination of whether the hadron is charged or not was accomplished using the time between the neutron bar hit and the veto hit and the segmentation of the detectors. This segmentation defined the  $x$  and  $y$  position of each veto hit. The individual veto detectors had high rates (1 MHz) and high dead-time (up to 110 ns). This indicates that there exists a large number of possible hits where an earlier accidental hit masked the hit from the quasi-elastic event candidate. For coincident events, the hit in the veto bars varied in veto amplitude in addition to time, the veto amplitude was used in addition to time when selecting veto hits to associate to quasi-elastic events.

While it would be nice to have a detector with uniform neutron detection efficiency, this was not the case for Big Hand. The GLA detectors did not have the resolution to identify quasi-elastic neutrons as the rest of the neutron arm could, and the horizontal edges were not adequately covered by the veto detectors. Two additional cuts, on the neutron arm's  $x$  and  $y$  fiducial regions were used to select the region of the detector that had a high neutron detection efficiency. This selection is



name	$Q^2 = 2.5 \text{ GeV}^2$	$Q^2 = 3.5 \text{ GeV}^2$	$Q^2 = 1.7 \text{ GeV}^2$
Amplitude	$A_V < 200$	$A_V < 200$	$A_V < 200$
Spatial (m)	$ X_V - X_C  < 0.3$	$ X_V - X_C  < 0.3$	$ X_V - X_C  < 0.3$
Time (ns)	$ T_V - T_C  < 10$	$ T_V - T_C  < 10$	$ T_V - T_C  < 10$
Y1 Cut (m)	$Y_C < 0.076 \& Y_V < 0$	$Y_C < 0.076 \& Y_V < 0$	$Y_C < 0.076 \& Y_V < 0$
Y2 Cut (m)	$Y_C > -0.774 \& Y_V > 0$	$Y_C > -0.774 \& Y_V > 0$	$Y_C > -0.774 \& Y_V > 0$

Table 5.2: A table of selection criteria used to determine which veto hits to consider when doing charge identification. The spatial cuts of Y1, Y2, and X determine which bars are to be included, events were only differentiated by time and amplitude within a veto detector which was within range of the hadron hit. The subscript  $V$  signifies veto, while  $C$  signifies the cluster in the neutron arm.

shown in Figures 5.19 and 5.20.

The area of the veto used to determine the charge of the hadron is determined by looking at the coincidence between the hadron hit and the veto hit, see Appendix A.5 for more details.

## 5.5 Dilution of the neutron sample by protons

Between the target and the veto detectors, there were several centimeters of lead and other material [60]. Hadrons that leave the target could be observed as if they were in the other isospin state, through hadronic interactions. This would create a dilution in the neutral candidate sample due to quasi-elastic events still being detected as hadrons, but with the wrong charge. This is especially important in the case of protons observed as neutrons, which creates a significant dilution (see Table 5.5) in the neutron sample. This conversion can be understood, and accounted for, using the ratios of observed protons and neutrons for different targets. The three targets used were hydrogen, which is a single proton,  $^3\text{He}$ , which is the target of

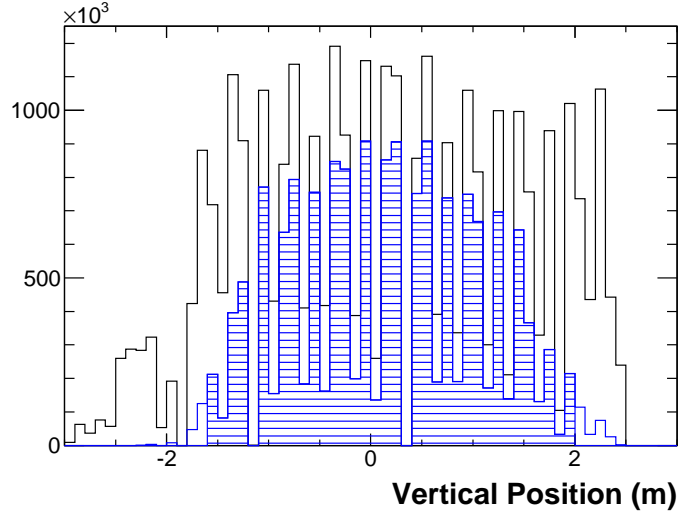


Figure 5.19: Vertical position (m) within the Big Hand detector. This is for before cuts (black), after event selection cuts (blue), with the selected region of  $x$  shaded. In the vertical direction the neutron arm was segmented, giving the segmented structure observed. GLA bars are removed, as seen in the most negative region. See Table 5.6 for cuts. This distribution is for  $Q^2 = 3.5 \text{ GeV}^2$ .

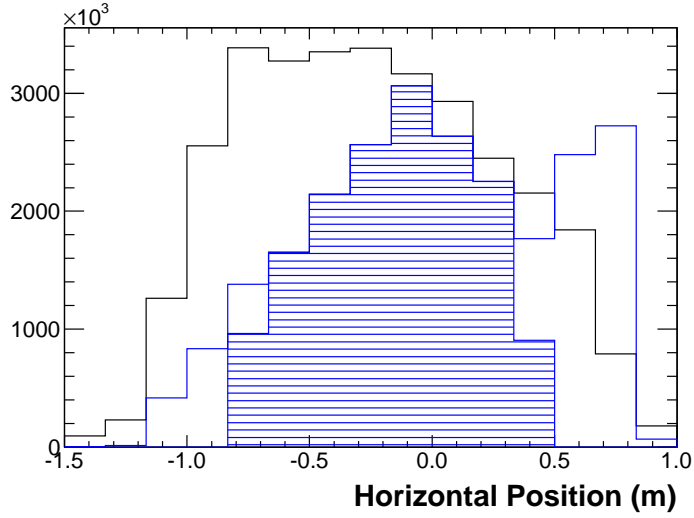


Figure 5.20: Horizontal position (m) within the Big Hand detector. This is for before cuts (black), after event selection cuts (blue), with the selected region of  $y$  shaded. In the horizontal direction the neutron arm was not entirely covered by the veto planes, giving the peaks on the two edges. See Table 5.6 for cuts. This distribution is for  $Q^2 = 3.5 \text{ GeV}^2$ .

interest, and nitrogen, which has equal numbers of protons and neutrons. These ratios are referred to as  $R_{tgt}$ ,

$$R_{tgt} = \frac{f_n}{f_p} . \quad (5.15)$$

Here the subscript  $tgt$  stands for target, and is replaced by  $N$  for nitrogen,  $H$  for hydrogen, and  $He$  for  $^3\text{He}$ . The fraction  $f_{p(n)}$  is that of protons(neutrons) to the total number of hadrons.

The purity factor,  $D_n$ , is the ratio of observed neutrons to the neutrons knocked out of the target at the interaction point with the electron beam. Similarly,  $D_p$  is the ratio of observed protons to those starting from the target. For the observed  $R_{tgt}$ , the  $f_{p(n)}$  are observed fractions. However, as mentioned, it is possible that interactions between the target and detector can cause a conversion in the observed iso-spin state. To account for this, the ratio can be expressed in terms of mixing coefficients. These provide the numbers of observed  $N^{p(n)}$  resulting from an initial target population  $N_{p(n)}$ . Expressed in this manner

$$R_{tgt} = \frac{N_n^n + N_p^n}{N_p^p + N_n^p} = \frac{\sigma_n \nu_n^n + \frac{Z_{tgt}}{N_{tgt}} \sigma_p \nu_p^n}{\sigma_n \nu_n^p + \frac{Z_{tgt}}{N_{tgt}} \sigma_p \nu_p^p} . \quad (5.16)$$

As an example,  $N_p^n$  is the number of observed neutrons from initial protons. The above can be presented more compactly as:

$$R_{tgt} = \frac{\frac{\sigma_n}{\sigma_p} \frac{\nu_n^n}{\nu_p^n} + \frac{Z_{tgt}}{N_{tgt}} \frac{\nu_p^n}{\nu_p^p}}{\frac{\sigma_n}{\sigma_p} \frac{\nu_n^p}{\nu_p^p} + \frac{Z_{tgt}}{N_{tgt}}} . \quad (5.17)$$

Here  $\sigma_{p(n)}$  are the single nucleon cross sections,  $\nu$  are the mixing coefficients, and  $\frac{Z_{tgt}}{N_{tgt}}$  is the ratio of protons to neutrons (unique for each target, and may be dependent on perpendicular and parallel missing momentum) within the target. Obviously

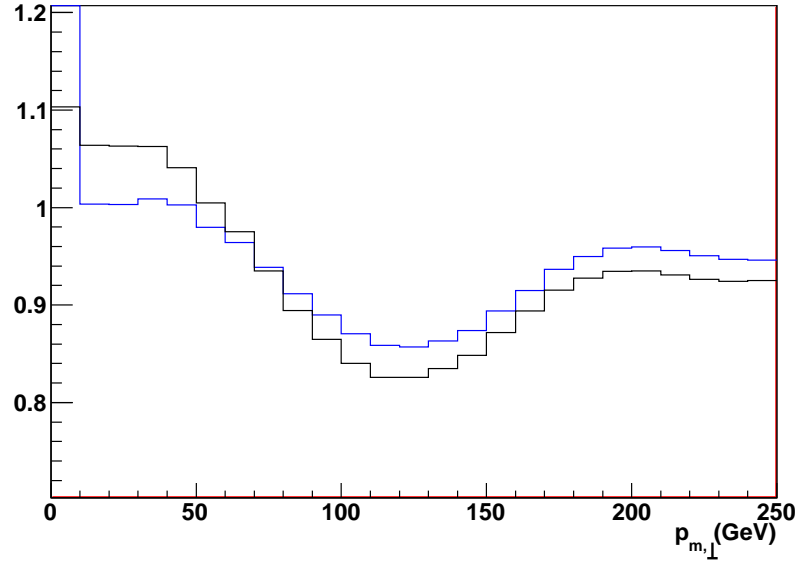


Figure 5.21: Presented is the ratio of protons to neutrons within the target for nitrogen. Shown are two models, one a simple extension to the plane wave impulse approximation (in blue), and the other is a model by Udias (in black). Shown is the ratio for various cuts of  $p_{\perp}$ . A cut of 150 MeV was used in this experiment. This is for  $Q^2 = 3.5 \text{ GeV}^2$ .

$N_{tgt} = 0$  for hydrogen,  $\frac{Z_N}{N_N} \approx 1$  for nitrogen, and  $\frac{Z_{He3}}{N_{He3}} \approx 2$  for the  $^3\text{He}$ . In particular models [11], the ratio was investigated for the applied cuts and kinematics (see Figure 5.21 and 5.22). This provided values for these two coefficients (see Table 5.5). In terms of these mixing coefficients, the purity factor is then

$$D_n = \frac{N_n^n}{N_n^n + N_p^n} = \frac{\frac{\sigma_n}{\sigma_p} \frac{\nu_n^n}{\nu_p^n}}{\frac{Z_{tgt}}{N_{tgt}} \frac{\nu_p^n}{\nu_p^n} + \frac{\sigma_n}{\sigma_p} \frac{\nu_n^n}{\nu_p^n}} . \quad (5.18)$$

Using this information, it is possible to solve for the mixing ratios from the ratios of hydrogen ( $\text{H}_2$ ),  $^3\text{He}$ , and nitrogen ( $\text{N}_2$ )

$$\frac{\nu_p^n}{\nu_p^n} = R_H , \quad (5.19)$$

$$\frac{\nu_n^p}{\nu_p^p} = \frac{\sigma_p}{\sigma_n} \left[ \frac{R_{He} \frac{Z_{He3}}{N_{He3}} - \frac{Z_N}{N_N} R_N + \left( \frac{Z_N}{N_N} - \frac{Z_{He3}}{N_{He3}} \right) R_H}{R_N - R_{He}} \right] , \quad (5.20)$$

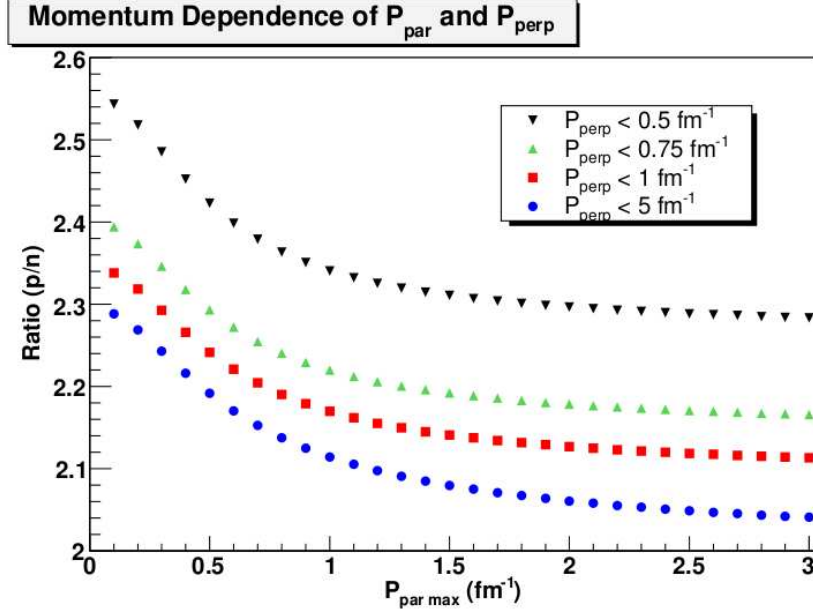


Figure 5.22: Presented is the ratio of protons to neutrons within the target for  $^3\text{He}$ . Shown are from cuts on  $p_\perp$ . The applied cut on  $p_\perp$  was 0.15 GeV with  $P_\parallel$  cuts of 400, 250, and 200 MeV for kinematics of  $Q^2 = 3.5, 2.5$ , and  $1.7 \text{ GeV}^2$ . Shown is the ratio for various cuts of inverse distance. This study was done by Aidan Kelleher.

$$\frac{\nu_n^n}{\nu_p^p} = \frac{\sigma_p}{\sigma_n} \left[ R_N \left( \frac{Z_{He3}}{N_{He3}} - \frac{Z_N}{N_N} \right) \frac{R_{He} - R_H}{R_N - R_{He}} - \frac{Z_N}{N_N} R_H \right]. \quad (5.21)$$

Therefore the dilution due to the proton to neutron conversion in the sample,

$$D_n = \frac{R_{He} \left[ \left( \frac{Z_{He3}}{N_{He3}} - \frac{Z_N}{N_N} \right) R_N + \frac{Z_N}{N_N} R_H \right] - \frac{Z_{He3}}{N_{He3}} R_N R_H}{R_{He} \left( \frac{Z_{He3}}{N_{He3}} - \frac{Z_N}{N_N} \right) (R_N - R_H)}. \quad (5.22)$$

This accounts for mixing due to conversions in the air and other material between the target and the veto detectors. However, there are additional considerations due to the veto detectors before the final dilution factor, due to protons observed as neutrons, can be determined.

As mentioned in section 5.4, the veto detector efficiency was related to large dead-time in the veto detector TDCs and high rate within the veto detectors. The dead-time could be as high as 110 ns for some paddles, while the rate in the indi-

vidual veto detectors was on the order of 1 MHz. Additionally, the hydrogen and nitrogen data were acquired at different rates than the  $^3\text{He}$  data. Because of this, it is desirable to account for the rate dependence in the charge identification. This is complicated for a couple of different reasons. First, the veto was segmented into left and right and into two planes, which made the veto difficult to model. Also, because of the high activity in the veto, every event had multiple random hits within the veto planes.

The observed protons and neutrons detected in the veto can be represented in the following equations, where the  $f_{p(n)}$  stands for the fraction of protons(neutrons) incident on the veto detectors over the total number of hadrons incident on the veto detectors; which is equivalent to the fraction when there is no deadtime. In these equations  $r$  represents the rate in the detectors.

$$N_n^p(r) + N_p^p(r) = f_p T(r) + f_n T(r) P_{busy}^{veto}(r) \quad (5.23)$$

$$N_n^n(r) + N_p^n(r) = f_n T(r) (1 - P_{busy}^{veto}(r)) \quad (5.24)$$

A charged hit is determined by a hit in the proper range of either veto plane (see Table 5.4). So the rate dependent number of observed neutrons is the rate dependent total number of hadrons,  $T(r)$ , incident on the neutron detector multiplied by the fraction,  $f_n$ , of the number of neutrons to hadrons multiplied by the probability that there was no accidental background hit in the veto detector to make it appear charged  $(1 - P_{busy}^{veto})$ . The rate dependent number of observed protons is the total number of hadrons multiplied by the fraction of protons over hadrons,  $f_p$ , plus

the fraction of neutrons misidentified by the accidental background.

These probabilities can be determined by modeling them using the Poisson distribution, and then adding up all the probabilities for each hadron event. This gives real numbers for everything but the fractions incident on the veto detectors. The observed ratios, at a given frequency, are

$$R_{tgt}(r) = \frac{N_n^n(r) + N_p^n(r)}{N_n^p(r) + N_p^p(r)} . \quad (5.25)$$

Using this formula twice, once for the observed rate  $r$  and once for the desired rate  $d$ , gives a formula to relate every observed ratio  $R_{tgt}(r)$  to a desired ratio  $R_{tgt}(d)$ :

$$R_{tgt}(r) = \frac{R_{tgt}(d)T(d)[F^r(\beta)]}{R_{tgt}(d)T(d)[T(r) - F^r(\beta)] + [F^d(\beta) - R_{tgt}(d)(T(d) - F^d(\beta))]T(r)} . \quad (5.26)$$

The fitting parameter  $\beta$  accounts for correlations in hits in different veto bars in a given event. The function  $F$  depends on  $\beta$  and  $\xi$ , where  $\xi$  is the total rate dependent probability for veto detectors to be busy in an event:

$$F^r(\beta) = \sum_0^{T(r)} P_{busy}(\beta, r) . \quad (5.27)$$

here  $\xi$  is a useful quantity to define for the calculation and fit of

$$F^r(1) = \xi(r) = \sum_0^{T(r)} P_{busy}(\beta = 1, r) , \quad (5.28)$$

which is just  $F^r(\beta)$  assuming no correlation between veto hits. The probability of an accidental hit,  $P_{busy}(\beta)$  with  $r$  being the rate and  $\tau$  being the dead-time (or busy time) of the bar, in a given event is

$$P_{busy}(\beta, r) = (e^{-\sum r\tau})^\beta . \quad (5.29)$$

The functional form of  $F^r(\beta)$  can be approximated by assuming that the rate  $r$  multiplied by the deadtime  $\tau_d$  is approximately the same for all bars, so the quantity  $A = e^{-\sum r\tau_d}$  is the same for each bar. This is a good assumption, since the rate multiplied by dead-time is the same order of magnitude for all veto bars. From this,

$$F^r(1) = \xi(r) \approx T(r)A(r) . \quad (5.30)$$

Therefore a good approximation for  $F(\xi, \beta)$  is

$$F^r(\beta) \approx T(r) \left( \frac{\xi(r)}{T(r)} \right)^\beta . \quad (5.31)$$

This allows the ratio at a specific rate of the different targets to be calculated using the fractions of uncharged and charged particles incident on the veto. Using these ratios in equation 5.22 gives the rate independent dilution correction for proton to neutron conversion:

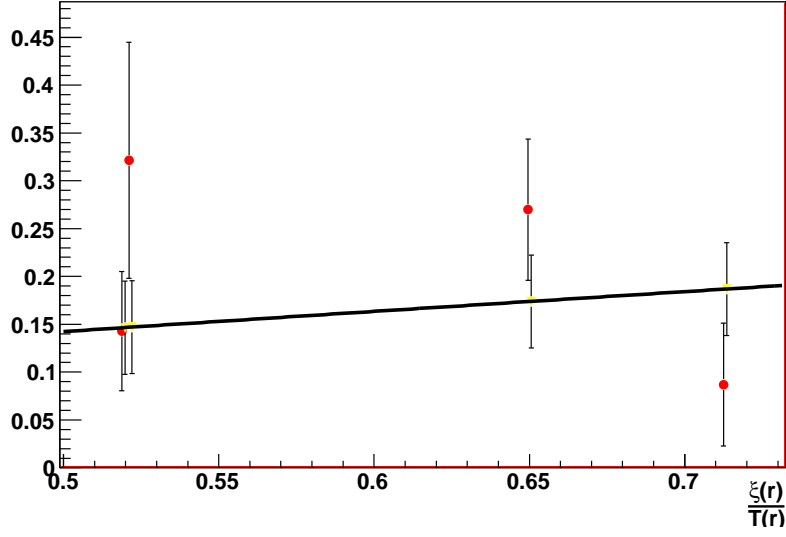
$Q^2 =$	Ratio for $^3\text{He}$	Ratio for Hydrogen	Ratio for Nitrogen	Purity Factor
2.5 GeV <sup>2</sup>	$0.102 \pm 0.001$	$0.019 \pm 0.002$	$0.193 \pm 0.010$	$0.8 \pm 0.03$
3.5 GeV <sup>2</sup>	$0.073 \pm 0.001$	$0.014 \pm 0.002$	$0.107 \pm 0.053$	$0.865 \pm 0.053$
1.7 GeV <sup>2</sup>	-	-	-	-

Table 5.3: A table of the ratios for different criteria. Over the course of a kinematic setting, runs varied in rate. This had a small effect on the purity factor, the results presented in this table is for a normal run.

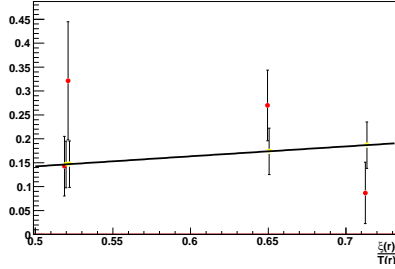
name	Helium Ratio	Nitrogen Ratio
Kin 2	2.15	1.07
Kin 3	2.31	1.07
Kin 4	2.15	1.07

Table 5.4: The effective ratio of  $\frac{Z}{N}$  for the target and cuts.

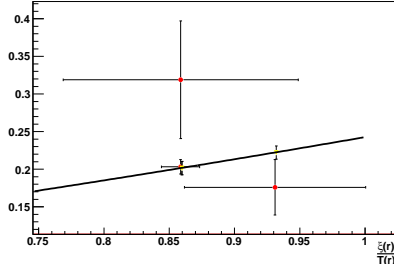




(a) uncharged to charged ratio for  $Q^2 = 2.5 \text{ GeV}^2$

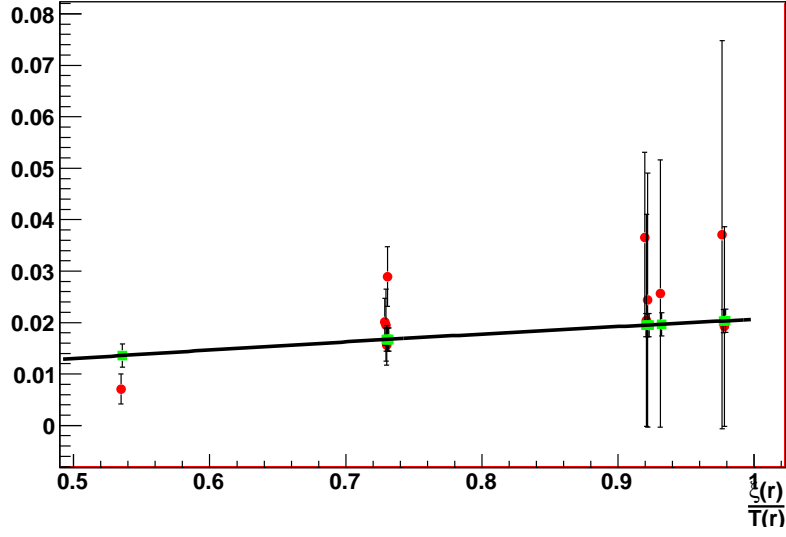


(b) uncharged to charged ratio for  $Q^2 = 3.5 \text{ GeV}^2$

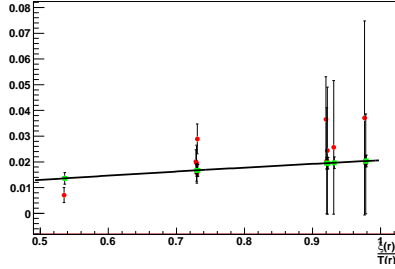


(c) uncharged to charged ratio for  $Q^2 = 1.7 \text{ GeV}^2$

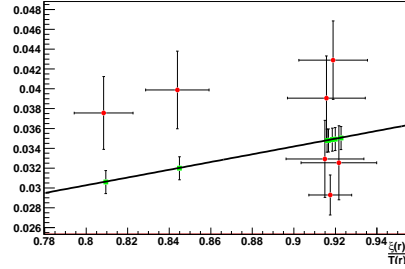
Figure 5.23: Shown in these figures is the measured uncharged to charged ratio, the uncharged to charged ratio within the model, and the fit line of the model in nitrogen. These are plotted against the natural dependence  $\frac{\xi}{T}$ .



(a) uncharged to charged ratio for  $Q^2 = 2.5 \text{ GeV}^2$

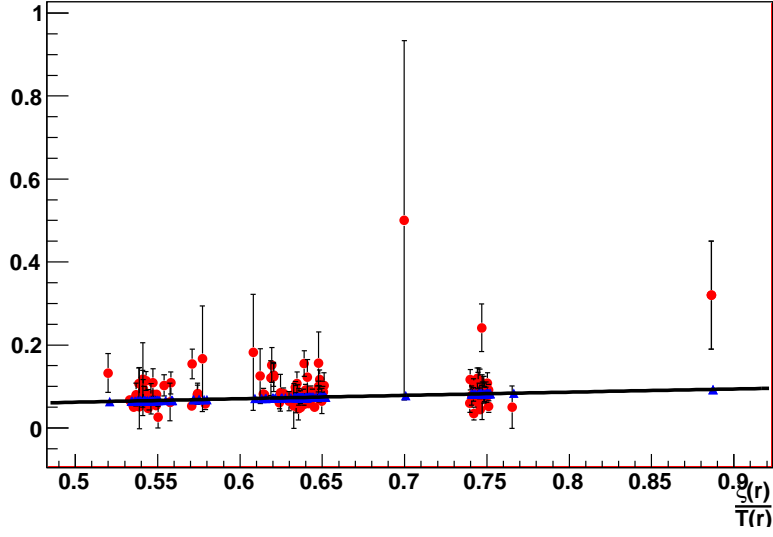


(b) uncharged to charged ratio for  $Q^2 = 3.5 \text{ GeV}^2$

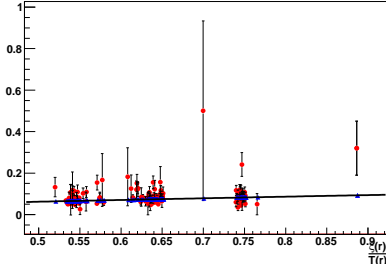


(c) uncharged to charged ratio for  $Q^2 = 1.7 \text{ GeV}^2$

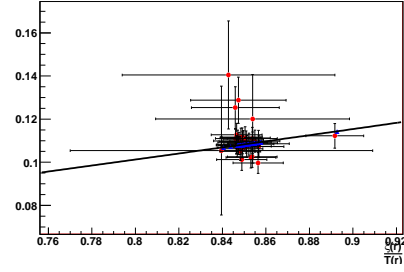
Figure 5.24: Shown in these figures is the measured uncharged to charged ratio, the uncharged to charged ratio within the model, and the fit line of the model in hydrogen. These are plotted against the natural dependence  $\frac{\xi}{T}$ .



(a) uncharged to charged ratio for  $Q^2 = 2.5 \text{ GeV}^2$



(b) uncharged to charged ratio for  $Q^2 = 3.5 \text{ GeV}^2$



(c) uncharged to charged ratio for  $Q^2 = 1.7 \text{ GeV}^2$

Figure 5.25: Shown in these figures is the measured uncharged to charged ratio, the uncharged to charged ratio within the model, and the fit line of the model in  $^3\text{He}$ . These are plotted against the natural dependence  $\frac{\xi}{T}$ .

$$D_n = \frac{R_{He}(d)[(\frac{Z_{He3}}{N_{He3}} - \frac{Z_N}{N_N})R_N(d) + \frac{Z_N}{N_N}R_H(d)] - \frac{Z_{He3}}{N_{He3}}R_N(d)R_H(d)}{R_{He}(d)\left(\frac{Z_{He3}}{N_{He3}} - \frac{Z_N}{N_N}\right)(R_N(d) - R_H(d))}. \quad (5.32)$$

### 5.5.1 Single Track Analysis and Background Correction

For approximately 10% of the detected events, the hadron interacted in the lead resulting in multiple events in the neutron apparatus or an accidental background hit that could not be differentiated from the hadron. If there were two or more hits in the region of parameter space used to select quasi-elastic events (hereafter known as the quasi-elastic region), the charge becomes impossible to determine since just one "hit" in the veto detectors associated with the quasi-elastic region will often set all hits in the quasi-elastic region as charged.

By restricting the analysis to only those events which have a single hit identified as quasi-elastic, the problem of charge identity and multiple hits in the quasi-elastic region is solved. Using this single track analysis (SQE), however, causes the determination of the background to become more complicated.

When multi-hit events are eliminated, the spectrum far from the quasi-elastic hit is clean (see Figure 5.29). Using the perpendicular transferred momentum,  $q_\perp$ , the distance far from the quasi-elastic hit can be parametrized in the same variable which is used to select quasi-elastic events. In this band of  $q_\perp$ , the ratio of charged to uncharged background is relatively constant (see Figure 5.28). This gives the ratio of charged to uncharged for the background which is unrelated to the quasi-elastic events. The only determination that is made in this region is the charge ratio of the

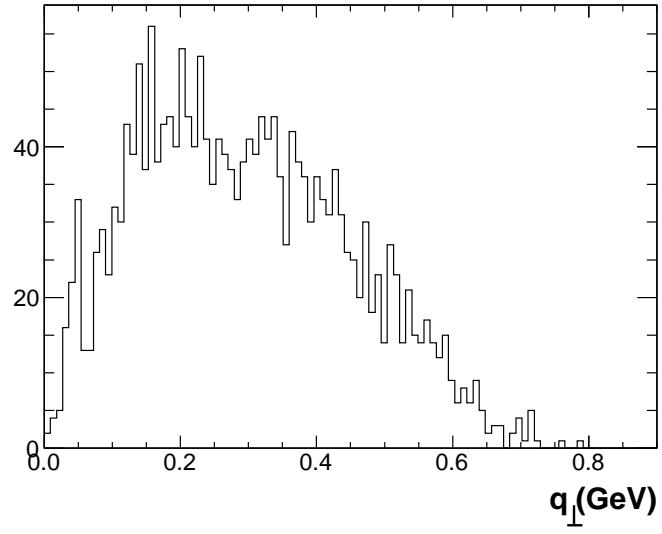


Figure 5.26: Plot  $q_{\perp}$  for a region in time. This is 5 ns removed from the peak region.

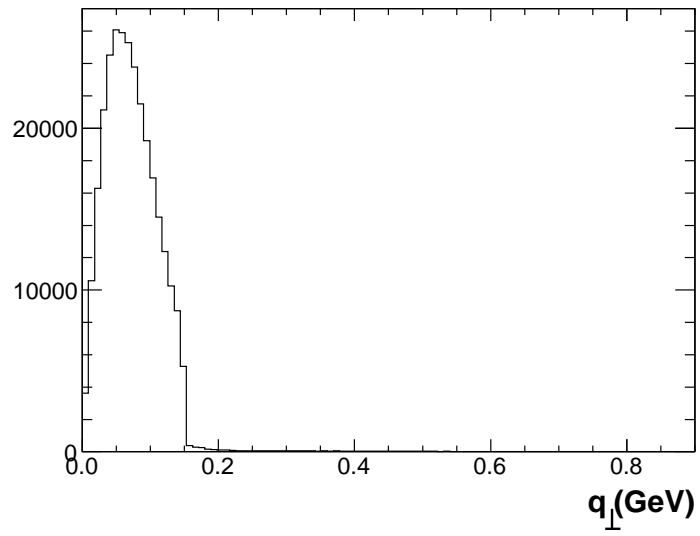


Figure 5.27: Plot  $q_{\perp}$  for a region in time. This is at the quasi-elastic region.

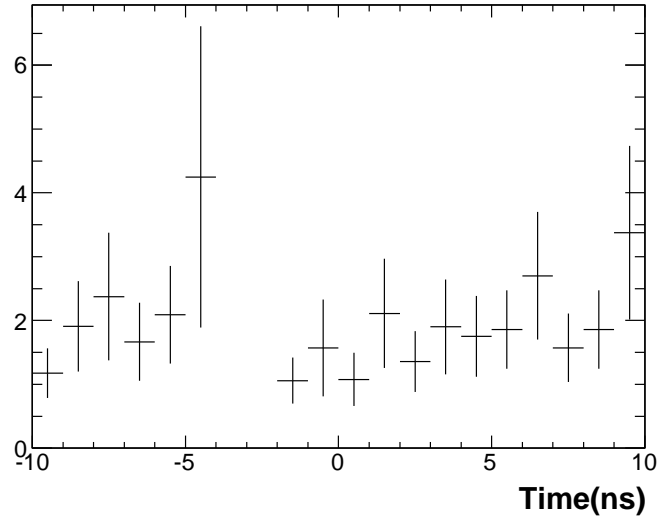


Figure 5.28: Plot of the ratio of uncharged to charged for the region of  $q_{\perp}$  between 0.55 and 0.60  $GeV/c$ . The region near 3  $ns$  is removed due to the photon peak being there. Add axis information time (ns), update description (changed plot)

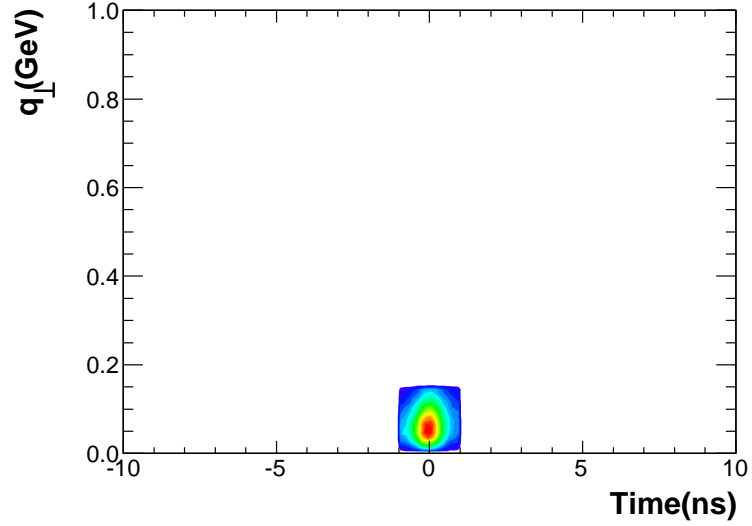


Figure 5.29: Plot of  $q_{\perp}$  versus time-of-flight for events with only a single hit in the  $|t| < 7$  and  $q_{\perp} < 0.15$  region. The area in the spectrum of events coming from  $k$  photons via  $\pi_0$  production was removed (between 2 and 4 ns). This is for runs 4090-4099.

background. The total background count is determined via a different mechanism.

Since it is required that every hit in the quasi-elastic region is either a single quasi-elastic hit, or a single background hit, the total number of counts (and events) is:

$$N_{SQE}^{A+B} = N_{back}^A + N_{QE}^A + N_{back}^B + N_{QE}^B , \quad (5.33)$$

where  $A$  and  $B$  are two regions within the SQE region. The SQE region is chosen so as to include all of the quasi-elastic events within it. It is useful to define region  $B$  as being a region where  $N_{QE}^B = 0$ .

This is a lot simpler than that of the full events, which can be expressed as the following:

$$N_{full}^{A+B} = N_{back}^A + N_{QE}^A + N_{QE|back}^A + N_{back|back}^A + N_{back|back}^B + N_{QE|back}^B + \\ N_{QE|QE}^A + N_{back|QE}^A + N_{back}^B + N_{back|QE}^B ,$$

here  $|_{back}$  is the number of hits which also have at least one background hit,  $|_{QE}$  is the number of hits with at least one  $QE$  hit. Another class of hits are  $N_{QE|QE}^A$  which are the number of  $QE$  hits with at least one other  $QE$  hit; these would be from fragmentation caused by interaction between the target and the final hit location in the neutron arm. Here  $A$  is the SQE region with the quasi-elastic hits within it and  $B$  is a region in the SQE region without any quasi-elastic hits.

If the region is restricted to being outside the QE region, the expression is left as:

$$N_{full}^B = N_{back}^B + N_{back|back}^B + N_{back|QE}^B . \quad (5.34)$$

Obviously it is just desired to remove the  $N_{back}^B$  from the QE sample in region A.

Since equation 5.33 is true for SQE analysis, outside of the QE region the expression for the counts is

$$N_{SQE}^B = N_{back}^B , \quad (5.35)$$

due to  $N_{back}^B|_{back}$  and  $N_{back}^B|_{QE}$  events not occuring because only one hit can occur in the SQE region. Once the regions  $A$  and  $B$  are scaled to be the same size, the number of quasi-elastic events in SQE analysis can be determined

$$N_{QE} = N_{A,SQE} - N_{B,SQE} , \quad (5.36)$$

here  $B$  is a sub region of the SQE region where the QE events don't reside, and  $A$  is the sub region of the SQE region where QE events do reside.

For times earlier than the quasi-elastic window, as described above the hits are all background, coming from an accidental coincidence between the electron and hadron (see Figure 5.29). This is in the -4 ns to -7 (or -13) ns window. Using the counts (scaled properly) from this background region provides the background in single quasi-elastic hit analysis. The same area in  $q_{\perp}$  is used to make scaling easier. To determine the neutral accidental background, the earlier determined background ratio term is used:

$$N_{neut,back} = N_{back} f_{back} , \quad (5.37)$$

here  $f$  is the fraction of neutrals to hadrons in the background.

These background counts, neutral or charged, are used to determine the accidental background dilution and asymmetry used in the determination of the physical asymmetry. The neutral background dilution of a sample ( $N$ ) can be expressed as

$$D_{back} = 1 - \frac{N_{neut,back}}{N} . \quad (5.38)$$



Similarly the background asymmetry can be defined in the standard way

$$A_{back} = \frac{N_{back}^+ - N_{back}^-}{P_{^3He}N} . \quad (5.39)$$

### 5.5.2 Nitrogen Dilution

The  $^3\text{He}$  inside the target cell is diluted by unpolarized nitrogen, used as a buffer gas to decrease interactions between the polarized  $^3\text{He}$  and glass cell walls and to aid in the polarization process (Section 4.4.2). This contamination creates a dilution of the  $^3\text{He}$  asymmetry, and is measured by looking at events which appear to be quasi-elastic in a  $N_2$  target and scaling by the relative densities and total charge on target. This dilution factor is

$$D_{N_2} = 1 - \frac{N_{N_2} - N_{back,N_2}}{N - N_{back}} \frac{Q_{^3He}\rho_{^3He}}{Q_{N_2}\rho_{N_2}} . \quad (5.40)$$

The values used to calculate this dilution factor are provided in Table 5.5. Quasi-elastic event selection, discussed earlier in the chapter, was used to select the events used for this analysis.

$\rho_{QE}$	$Q^2$	$\rho_{N_2}$	$Q_{^3He}$	$Q_{N_2}$	$D_{N_2}$	$N$	$N_{back}$	$N_{N_2}$	$N_{back,N_2}$
0.162	2.5	4.4	18399	14020	0.949	2089.1	46.2	220.5	14.9
0.162	3.5	3.6	1441010	37920	0.982	2686.2	52.6	28.5	5.5
99	1.7	99	99	99	99	99	99	99	99

Table 5.5: A table for the Nitrogen Dilution factor determination. Uncertainties are discussed in Appendix B. Densities are determined from measurements of the target [51][46][68].

### 5.5.3 RF correction

While the timing of the coincidence event relative to the beam crossing cannot be determined absolutely, it can be determined up to an arbitrary constant. Event by event variations about this constant can be determined from the 499 MHz RF signal, which can be used to improve the hadron time-of-flight measurement between the target and detectors.

The signal from the RF is put into the F1 TDC, which wraps (starts counting again at 0) at  $W = 65526$  channels, and has  $t_{res} = 0.1183$  ns resolution. The RF signal repeats in a  $T_{win} = 2$  ns window. In the following formula  $t_{RF}$  = RF time,  $t_{ref}$  = reference time,  $t_{L1A}$  = level 1 accept time, and the aforementioned time-of-flight (equation 5.9).

The RF correction is calculated using the following formula

$$RF_{corr} = \frac{V_{tgt}}{c} + t_{res} \left( t_{RF} - t_{ref} - \left( \left[ \frac{t_{RF} - t_{ref}}{0.5W} \right] \mathbf{mod} 1 \right) W \right), \quad (5.41)$$

$$t_{rf} = \left[ (t_e - t_{L1A} - RF_{corr} - t_{rfshift}) \mathbf{mod} \left( \frac{1}{0.499} \right) + \frac{2}{0.499} \right] \mathbf{mod} \left( \frac{1}{0.499} \right). \quad (5.42)$$

This  $t_{rf}$  is the shift used for correcting the time-of-flight in equation 5.9. The  $t_{rfshift}$  is the peak location shown in Figure 5.30. This shift is plotted versus run number in Figure ???. This correction is due time of the scattering within the target.

## 5.6 Asymmetry calculation

After the selection of quasi-elastic neutron events (Table 5.6), the measured asymmetry is corrected to give the physical asymmetry, which gives  $G_E^n$  as presented

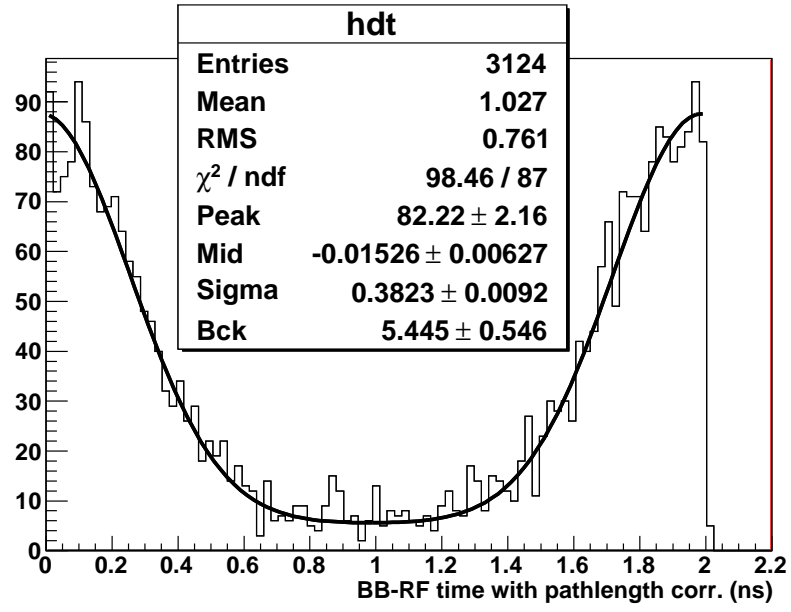


Figure 5.30: Spectrum used to determine the RF correction. This is for run 3888. The peak location is determined from the Mid parameter, which is  $t_{rfshift}$ .

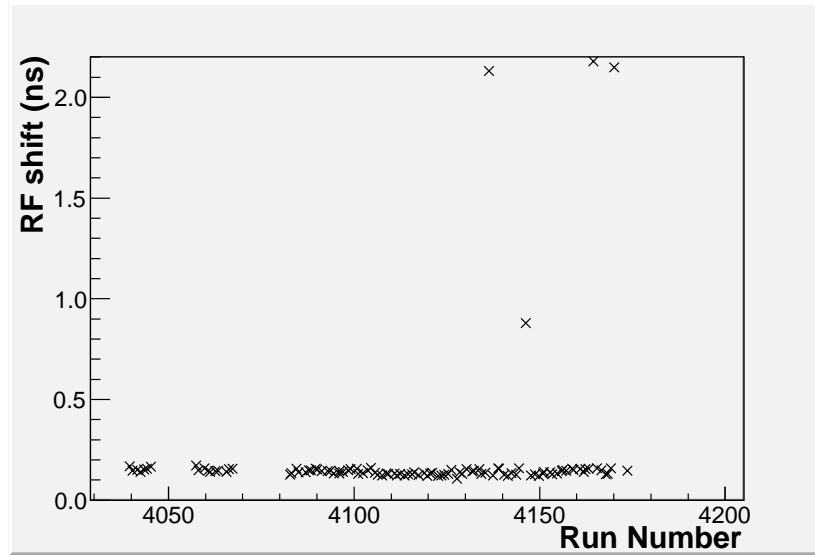


Figure 5.31: RF correction versus run number for  $Q^2 = 2.5 \text{ GeV}^2$ . Note that since the plotted function is  $\text{mod} \left( \frac{1}{0.499} \right)$  that the values of 2 ns and 0 ns are almost equivalent.

Name	$Q^2 = 2.5 \text{ GeV}^2$	$Q^2 = 3.5 \text{ GeV}^2$	$Q^2 = 1.7 \text{ GeV}^2$
Time (ns)	$ t  < 2$	$ t  < 1$	$ t  < 1$
Invariant Mass (GeV)	$ w - 0.9  < 0.2$	$ w - 0.775  < 0.175$	$ w - 0.9  < 0.2$
Q Perp (GeV)	$q_{\perp} < 0.15$	$q_{\perp} < 0.15$	$q_{\perp} < 0.15$
SQE region B	$ t - 8.5  < 4.5$	$ t - 8.5  < 4.5$	$ t - 8.5  < 4.5$
Missing Mass (GeV)	$M_{miss} < 2$	$M_{miss} < 2$	$M_{miss} < 2$
Vertical Cut (m)	$ X + 0.2  < 1.7$	$ X + 0.2  < 1.7$	$ X + 0.2  < 1.7$
Horizontal Cut (m)	$ Y + 0.183  < 0.7$	$ Y + 0.183  < 0.7$	$ Y + 0.183  < 0.7$

Table 5.6: Table of quasi-elastic event selection cuts. The Single QE cut is that there is one and only one hit in the associated region of time and  $q_{\perp} < 0.15$ . This is in addition to the region of time and  $q_{\perp}$  which are selected as quasi-elastic event candidates. The vertical cut and horizontal cut are neutron arm fiducial cuts.

in section 3.3.3. This is the general expression, as mentioned in the previous sections there are corrections which need to be done for nitrogen dilution in the target cell (section 5.5.2), proton to neutron conversion between the target and scintillator bars (section 5.5), accidental background asymmetry and dilution (section 5.5.1), neutron polarization in the nucleus [32], beam polarization (section 4.3), target polarization (section 4.4), and final state interactions (section 5.7.1). The corrected expression is

$$A_{Phys} = \frac{A_{sum} - A_{proton} - A_{back}}{P_3 P_{beam} P_n D_{N_2} D_n D_{FSI} D_{back}} . \quad (5.43)$$

Since the proton contamination of the neutron sample ( $D_n$ ), proton asymmetry ( $A_{proton}$ ), nitrogen dilution ( $D_{N_2}$ ), background asymmetry ( $A_{back}$ ), and background dilution ( $D_{back}$ ) all depend on similar quantities, corrections need to be applied as presented in sections 5.7 and 5.5. The nitrogen dilution is corrected for the background dilution and the proton contamination dilution is corrected for both the nitrogen dilution and the background dilution.

The physical asymmetry, corrected for the discussed dilutions and asymme-

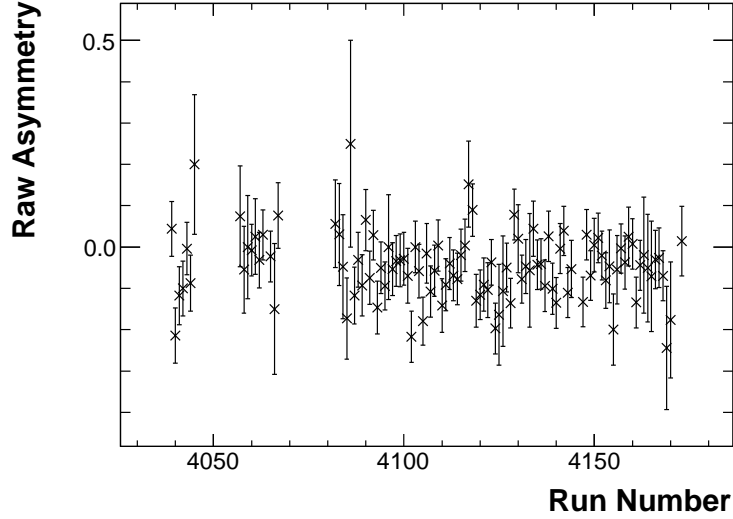


Figure 5.32: Shown is the run number versus raw asymmetry for  $Q^2 = 2.5 \text{ GeV}^2$ .

tries, is presented in Table 5.7. The dilutions can be expressed in terms of those due to polarization ( $P$ ), those due to theoretical calculations like FSI ( $D_{fsi}$ ) and neutron polarization ( $P_n$ ), and experimental corrections not due to polarization ( $V$ ). Expressed this way

$$P = P_{3He}P_{beam} \quad (5.44)$$

and

$$V = D_n D_{N_2} D_{back} . \quad (5.45)$$

At times during the measurements at  $Q^2 = 2.5$  and  $3.5 \text{ GeV}^2$ , the target cell and beam were changed. This meant that the  $P_{beam}$ ,  $P_{3He}$ , and  $D_n$  could change during the run. To account for this, the target polarization corrected the asymmetry on a run by run basis. Using standard error analysis (and Appendix B), the summation was

$$A_{sum} = \frac{\sum A_i \frac{1}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}} , \quad (5.46)$$

$Q^2 =$	1.7 GeV <sup>2</sup>	2.5 GeV <sup>2</sup>	3.5 GeV <sup>2</sup>
$A_{meas}$	$-0.0437 \pm 0.0096$	$-0.0449 \pm 0.0055$	$-0.0356 \pm 0.0081$
$N$	10882	3301	15381
$P_{nucl}$	$0.4928 \pm 0.02$	$0.436 \pm (0.02)$	$0.477 \pm (0.02)$
$P_{beam}$	$0.835 \pm (0.03)$	$0.835 \pm (0.03)$	$0.835 \pm (0.03)$
$P_n$	$0.86 \pm (0.02)$	$0.86 \pm (0.02)$	$0.86 \pm (0.02)$
$D_{back}$	0.994	$0.9792 \pm 0.0003$	$0.9732 \pm 0.0005$
$A_{back}$	$0.0002 \pm 0.03$	$0.0013 \pm 0.0001$	$0.0013 \pm 0.00003$
$D_n$	$0.823 \pm 0.033$	$0.8 \pm 0.023 \pm (0.04)$	$0.865 \pm 0.046 \pm (0.043)$
$A_{proton}$	$-0.0003 \pm 0.0001$	$-0.00089 \pm 0.0002$	$-0.0006 \pm 0.0001$
$D_{N_2}$	$0.946 \pm 0.001$	$0.949 \pm 0.004 \pm (0.005)$	$0.982 \pm 0.004 \pm (0.002)$
$P$	-	0.364	0.398
$V$	-	0.744	0.826
$A_{phys}$	$-0.190 \pm 0.040$	$-0.198 \pm 0.024 \pm (0.022)$	$-0.130 \pm 0.031 \pm (0.014)$

Table 5.7: Determination of the physical asymmetry from the measured experimental asymmetry. A full discussion and analysis of errors is presented in Appendix B.

$$\sigma_{sum} = \frac{1}{\sqrt{\sum \frac{1}{\sigma_i^2}}} , \quad (5.47)$$

where the above equation was used for statistical uncertainties and the  $A_i$  are the measured asymmetries for each run. After this summation, the physical asymmetry is calculated using

$$A_{Phys} = \frac{A_{sum} - A_{proton} - A_{back}}{P_{beam} P_n D_{N_2} D_n D_{FSI} D_{back}} . \quad (5.48)$$

Many runs were taken over the course of a measurement at a given  $Q^2$ . The stability of the asymmetry over the course of the run, for a given running condition, was stable as shown in Figure 5.32.

While the other asymmetries and dilutions can be corrected for without using acceptance information, the FSI dilution and proton asymmetry require knowledge of the kinematics to be properly applied.

The framework using knowledge of the kinematics to determine the ratio of

the electric and magnetic form factors, and so determine  $G_E^n$ , is based on equation 3.15. This provides the relationship between kinematic variables, the asymmetry, and the form factor ratio. The relation between the physical asymmetry and the ratio,  $\lambda$ , can be expressed as:

$$A = \frac{B\lambda + C}{\lambda^2 + D} , \quad (5.49)$$

where  $\lambda = \frac{G_E}{G_M}$  is the form factor ratio, and  $B$ ,  $C$ , and  $D$  are the following kinematic quantities

$$B = -2\sqrt{\tau(1+\tau)} \tan\left(\frac{\theta_e}{2}\right) \sin\theta \cos\phi , \quad (5.50)$$

$$C = -2\tau \sqrt{1+\tau + (1+\tau)^2 \tan^2\left(\frac{\theta_e}{2}\right)} \tan(\theta_e) \cos\theta , \quad (5.51)$$

$$D = -\tau + 2\tau(1+\tau) \tan^2\left(\frac{\theta_e}{2}\right) . \quad (5.52)$$

and the quantities  $\tau$ ,  $\theta_e$ ,  $\theta$ , and  $\phi$  were defined in section 4.4.2. These quantities can be averaged from all events, and are used in the determination of the proton asymmetry correction, the determination of  $Q^2$ , and the determination of  $\lambda$  from the physical asymmetry.

The dilution corrections are presented explicitly in section 5.5. The asymmetry corrections will be presented explicitly here. To make the best use of the good proton form factor measurements [63], values for the proton form factor ratio were used to determine the  $A_{proton}$ . The equations 5.50, 5.51, and 5.52 were used to calculate the proton physical asymmetry

$$A_{phys,proton} = \frac{(B\lambda_{proton} + C)}{\lambda_{proton}^2 + D} . \quad (5.53)$$

This can then be corrected for the nitrogen dilution of the charged events ( $D_{ch,N_2}$ ), the proton polarization ( $P_p$ ), background dilution ( $D_{back}$ ), and the beam polarization

( $P_{beam}$ ) to give the proton asymmetry correction

$$A_{proton} = \frac{1 - D_N}{D_{back} D_{N_2}} P_{beam} P_p A_{phys,proton} . \quad (5.54)$$

By using the plot of this ratio at different values of the physical asymmetry, the error and value for the ratio can be determined. However, if instead of being plotted,  $\lambda$  is solved for, an inversion must be used. This can be done using the following method [31]. Using equation 5.50, 5.51 and 5.52, we can express 5.49 as:

$$\frac{\Delta(\theta, \phi)}{\Sigma(\theta, \phi)} = A_{phys} = \frac{B\lambda + C}{D + \lambda^2} = \frac{C}{D} + \frac{B}{D}\lambda - \frac{C}{D^2}\lambda^2 - \frac{B}{D^2}\lambda^3 + \frac{C}{D^3}\lambda^4 + \frac{B}{D^3}\lambda^5 . \quad (5.55)$$

$Q^2 =$	1.7 GeV <sup>2</sup>	2.5 GeV <sup>2</sup>	3.5 GeV <sup>2</sup>
$A_{phys}$	$-0.190 \pm 0.040$	$-0.198 \pm 0.024 \pm 0.022$	$-0.130 \pm 0.031 \pm 0.014$
T0	-0.0156	-0.0128	-0.0752
T1	0.8362	0.8353	0.9844
T2	0.0134	0.0114	0.0927
T3	-0.6832	-0.6811	-1.1977
T4	-0.0116	-0.0102	-0.1159
T5	0.5660	0.5640	1.4805
$\lambda$	$-0.218 \pm 0.054$	$-0.232 \pm 0.033 \pm 0.031$	$-0.250 \pm 0.047 \pm 0.021$

Table 5.8: Expansion Coefficients in the determination of  $\lambda$  from the physical asymmetry. Uncertainty in  $\lambda$  calculated as discussed in Appendix B.

These expansion coefficients can be expressed as  $T_0 = \frac{C}{D}$ ,  $T_1 = \frac{B}{D}$ ,  $T_2 = \frac{C}{D^2}$ ,  $T_3 = \frac{B}{D^2}$ ,  $T_4 = \frac{C}{D^3}$ , and  $T_5 = \frac{B}{D^3}$ . In this expansion, these provide all the acceptance and kinematic information for a given event. This allows the expression of  $A_{phys}$  as a polynomial in  $\lambda$  with coefficients being equal to the averaged  $T_n$  values. For specific averaged values,  $\lambda$  can be solved for giving the ratio of the electric and magnetic form factors for that specific kinematic. To first order, this lambda is

$$\lambda_0 = \frac{A_{phys} - \langle T_0 \rangle}{\langle T_1 \rangle} . \quad (5.56)$$



Using these terms, the value for  $Q^2$  was also corrected for finite acceptance:

$$Q^2 = \frac{\sum T_{1,i} Q_i^2}{T_1} . \quad (5.57)$$

The value for  $\lambda$ ,

$$\lambda = \frac{G_E^n}{G_M^n} , \quad (5.58)$$

provides the electric form factor of the neutron as a ratio of the magnetic form factor.

Using this corrected value for  $Q^2$  and using the value of the magnetic form factor as determined from a common parameterization [48], the value of the electric form factor was determined. This calculation is presented in Table 5.9. The calculation of the uncertainty is presented in appendix B.

The results and discussion of them in relation to previous measurements and selected models are presented in Chapter 6.

$Q^2 =$	1.7 GeV <sup>2</sup>	2.5 GeV <sup>2</sup>	3.5 GeV <sup>2</sup>
$\lambda$	$-0.218 \pm 0.054(0.030)$	$-0.232 \pm 0.033 \pm 0.031$	$-0.250 \pm 0.047 \pm 0.021$
$Q^2$	1.75 (GeV <sup>2</sup> )	2.49 (GeV <sup>2</sup> )	3.48 (GeV <sup>2</sup> )
$G_M^n$	$-0.096 \pm 0.005$	$-0.0955 \pm 0.0018$	$-0.0557 \pm 0.0011$
$G_E^n$	$0.021 \pm 0.005(0.003)$	$0.0221 \pm 0.0032 \pm 0.0030$	$0.0139 \pm 0.0026 \pm 0.0012$

Table 5.9: Displayed is the calculated values for  $\lambda$  and  $Q^2$ , and the value of  $G_M^n$  as arrived at from a parameterization [48]. From this the value of  $G_E^n$  can be determined. Determination of uncertainty is discussed in Appendix B.

## 5.7 Corrections

### 5.7.1 FSI Corrections

As mentioned previously, in section 3.5, to first order there are four types of interactions at lower momentum transfer. Shown in Figure 5.33, these are the

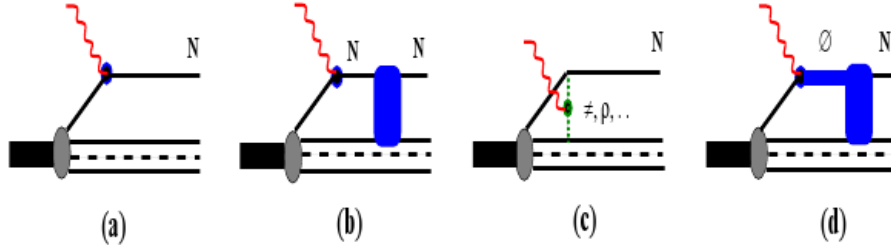


Figure 5.33: The four dominant lowest order diagrams in single nucleon scattering. Shown is (a) impulse approximation, (b) meson exchange currents, (c) isobar contributions, and (d) FSI. [75]

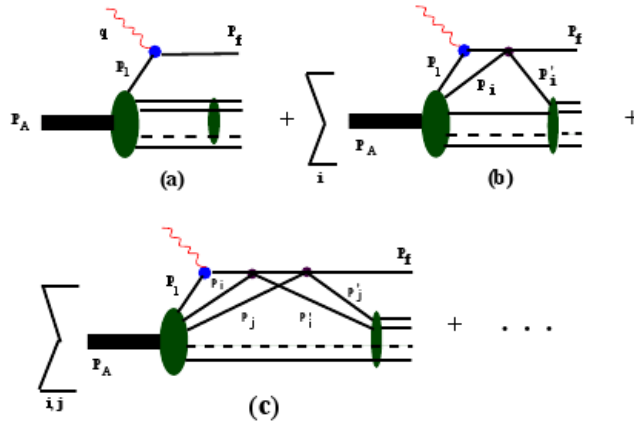


Figure 5.34: In nuclei with more than a single nucleon, the plane wave impulse approximation can be used to give the description of the interaction. Shown are the leading order diagrams for  $^3\text{He}$  which is the PWIA diagram, single re-scattering diagram, and double re-scattering diagram. [75]

impulse approximation (IA), isobar current (IC), meson exchange currents (MEC), and final state interactions (FSI). As has already been argued, meson exchange currents and isobar currents are small for  $Q^2 > 1 \text{ GeV}^2$ . This is due to them scaling as  $\frac{1}{Q^4}$ . This leaves just final state interactions as a necessary correction to the physical asymmetry, which was arrived at from the impulse approximation. The final state interactions are based on nucleon-nucleon cross sections, which are flat above 2 GeV [75]. To lowest order, these FSI will be either single or double re-scattering between the spectator hadrons and the detected hadron (see Figure 5.34).

There are two similar approximations made in computing the effects of these FSI. These are the Glauber approximation and the generalized eikonal approximation (GEA). In the Glauber approximation the incident particles energy is assumed to be much larger than that of the potential, and is large enough so that the wavelength of the incident particle is smaller than the potential width. This means that the Glauber approximation truncates the terms after double scattering due to the spectator hadrons being considered at rest. In the generalized eikonal approximation, the high momentum particle can not interact with a slow hadron a second time after interacting with another bound hadron.

The eikonal approximation reduces to the Glauber approximation in the limit of zero longitudinal momentum transfer,  $\Delta = \frac{q_0}{|q|}(E_s - m) \rightarrow 0$ . Here  $q$  is transferred momentum, the subscript  $s$  refers to spectator, and  $m$  is the recoiling nucleon mass. This is due to the Glauber approximation assuming that all nucleons are stationary while the GEA allows for nonzero initial momentum of the recoiling nucleon [75].

### 5.7.2 Inelastic Contribution

The inelastic contribution to the quasi-elastic asymmetry is the final correction to be developed. This contribution can be limited by removing some of the quasi-elastic region which is closest to the delta resonance (at  $W = 1.4$  GeV). The observed width of the quasi-elastic peak in the invariant mass spectrum is increased due to the motion of the nucleon within  $^3\text{He}$  and due to the resolution of the detectors. The inelastic resonance width is similarly increased. This inelastic contribution increases the tail in the quasi-elastic sample towards greater invariant mass as demonstrated in Figures 5.10(a), 5.9(a), 5.8(a), and ?? for  $Q^2 = 2.5$  GeV<sup>2</sup>.

The inelastic contribution will be deconvoluted using a Monte Carlo (MC). This Monte Carlo is described in section A.2 and simulates the quasi-elastic and inelastic spectrum near the quasi-elastic spectrum. Using the inelastic counts under the quasi-elastic peak and the inelastic asymmetry from the simulation, the measured quasi-elastic asymmetry can have the contribution from inelastic events removed [69]. This Monte Carlo is being analyzed by another collaborator. When this is completed, the invariant mass cuts in Table 5.6 can be widened to 1.2 GeV.

The contribution of the inelastic sample can be empirically estimated by considering the asymmetry as a function of invariant mass. This shows the asymmetry both in the inelastic region, the quasi-elastic region, and the region with a significant inelastic contribution to the quasi-elastic events. This is demonstrated in Figure 5.35.

Preliminary values for the inelastic asymmetry contribution and dilution are presented Table 5.10. The presented values are calculated in the framework of a

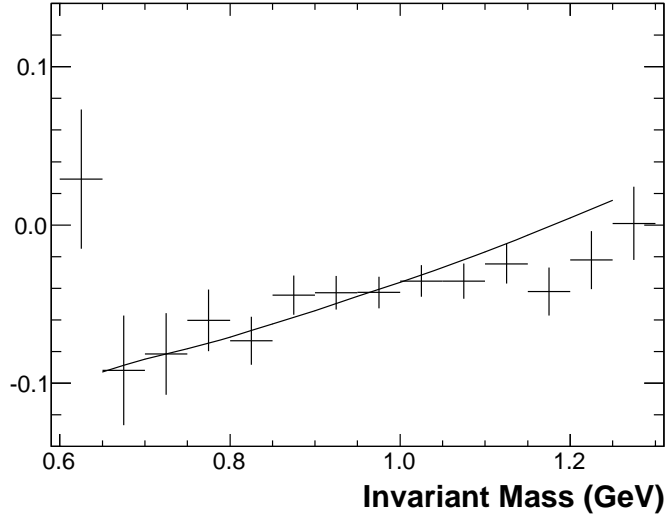


Figure 5.35: The asymmetry as a function of invariant mass. This shows that the invariant mass doesn't over the region selected (see Table 5.6).

separate analysis with a different set of cuts and charge selection [68]. These are not included in the presented analysis here, but give an estimation to the contribution in the quasi-elastic event sample. The correction to the physical asymmetry due to the inelastic asymmetry and dilution is

$$A_{corr} = \frac{A_{phys} - A_{inelas}}{D_{inelas}} . \quad (5.59)$$

$Q^2$	$D_{inelas}$	$A_{inelas}$	$A_{phys}$
1.7 GeV <sup>2</sup>	0.9839	-0.0038	-0.271
2.5 GeV <sup>2</sup>	0.9834	-0.0030	-0.190
3.5 GeV <sup>2</sup>	0.9726	-0.0050	-0.159
3.5 GeV <sup>2</sup> *	0.905	-0.0143	-

Table 5.10: Shown are preliminary inelastic asymmetries and dilutions for the various kinematic [69]. The \* row relates to an expanded selection of quasi-elastic events, with a maximum invariant mass of 1.2 GeV rather than the usual maximum of 1.05 (see Table 5.6).

As seen, with the cuts presented in Table 5.6 the contribution of the inelastic

events is small. The likliest source of a change in the results presented in Table 5.9 is due to increased theoretical understanding of  $^3\text{He}$  nucleus. The results presented and discussed in Chapter 6 will be based off the results presented in Table 5.9.

## Chapter 6

### Results and Discussion

It is common in the literature to use  $t = -Q^2 = -\Delta^2$  rather than  $Q^2$  when talking about fits and models. This practice will be followed in this chapter. Additionally, unless otherwise mentioned, the scaling factor used is  $\mu^2 = 2\text{GeV}$ .

#### 6.1 Framework Formalism

As noted earlier, in section 1.3, QCD can not be solved analytically in the regime where the quarks can be described as confined. The theory is also difficult to discuss analytically since Lorentz boosts of such a wavefunction is as complicated as solving the theory. By putting the problem into the Light Front gauge, a more simple description can be discussed. In Light Front Quantization, QCD is quantized at a fixed light front time  $\tau = t + \frac{z}{c}$  giving the following quantities

$$P^\pm = P^0 \pm P^Z , \quad (6.1)$$

and  $\vec{P}^\perp$  as the new four-momentum. Here  $P^+$  is the longitudinal momentum and  $\vec{P}^\perp$  is the transverse momentum. The wavefunctions of this formulation describe the nucleon in an arbitrary moving frame. To describe the quarks and gluons within the nucleon

$$p_i^+ = x_i P^+ , \quad (6.2)$$

and

$$\vec{p}_i^\perp = x_i P^\perp + \vec{k}_i^\perp , \quad (6.3)$$

are used where  $x_i = \frac{p_i^+}{P^+}$  and  $\vec{k}_i^\perp$  represent the relative momentum coordinates. The parton model is an approximation in a general reference frame, but is exact in the light front quantization.

The transferred momentum in this frame is  $q^+ = 0$ ,  $q^- = 0 \frac{2q\dot{P}}{P^+}$  and  $\vec{q}^\perp$ . Obviously  $Q^2 = -t = \vec{q}_\perp^2$  and is often referred to as  $\Delta_\perp^2$ .

In non-relativistic physics the natural framework for discussion of quantities is the center of mass, in the relativistic framework of light front QCD the natural framework is the center of transverse momentum,  $\vec{R}^\perp$ . The obvious, parton, definition is

$$\vec{R}^\perp = \sum_i x_i \vec{r}_i^\perp . \quad (6.4)$$

The conjugate position to the momentum  $\vec{k}_i^\perp$  is  $\vec{b}_i^\perp$ . It is in terms of this position that the distribution of the quarks can be described. Here

$$\sum_i \vec{b}_i^\perp = 0 . \quad (6.5)$$

In this mixed reference frame with longitudinal momentum and transverse position, the GPDs have a density interpretation. The probability to find a quark with  $x$  and  $\vec{b}^\perp$  minus the probability to find an antiquark is given by

$$q_v(x, \vec{b}^\perp) = \int \frac{d^2 \Delta}{4\pi^2} e^{-\vec{b}^\perp \cdot \Delta} H_v^q(x, t = -\Delta^2) \quad (6.6)$$

Using equations ????, the parton charge density in this mixed reference frame



is

$$\rho_0(b) = \sum_q e_q \int dx q(x, \vec{b}) = \int \frac{d^2 q}{4\pi^2} F_1(Q^2 = \Delta^2) e^{i\Delta \cdot \vec{b}} \quad (6.7)$$

This integral can also be expressed in terms of the Sachs form factors

$$\rho(\vec{b}) = \int_0^{\text{inf}} \frac{dQ Q}{2\pi} J_0(Qb) \frac{G_E(Q^2) + \tau G_M(Q^2)}{1 + \tau} \quad (6.8)$$

For a nucleon polarized in the  $xy$  direction, the transverse density can also be defined

$$\rho_T(\vec{b}) = \rho_0(b) - \sin(\theta_b - \theta_S) \int_0^{\text{inf}} \frac{dQ Q^2}{2\pi 2M_N} J_1(bQ) F_2(Q^2) \quad (6.9)$$

here  $\vec{b} = b(\cos \theta_b \hat{e}_x + \sin \theta_b \hat{e}_y)$  and  $\theta_S$  similarly defines the transverse polarization direction.

## 6.2 Results

### 6.3 Pauli and Dirac Form Factors

As presented in equation ?? the Pauli and Dirac form factors can be related to the Sachs electromagnetic form factors. Since the Sachs magnetic form factor,  $G_M^n$ , is already well known, it is just the electric form factor which needs to be known to present  $F_1$  and  $F_2$ . Because of the dependence in equation ??,  $F_1$  has much more dependence on the electric form factor than  $F_2$  does. Using a dipole approximation for  $G_M^n$

$$G_M^n = \mu_n G_D, \quad (6.10)$$

it is possible to determine the Pauli and Dirac form factors using only electric form factor data.

-

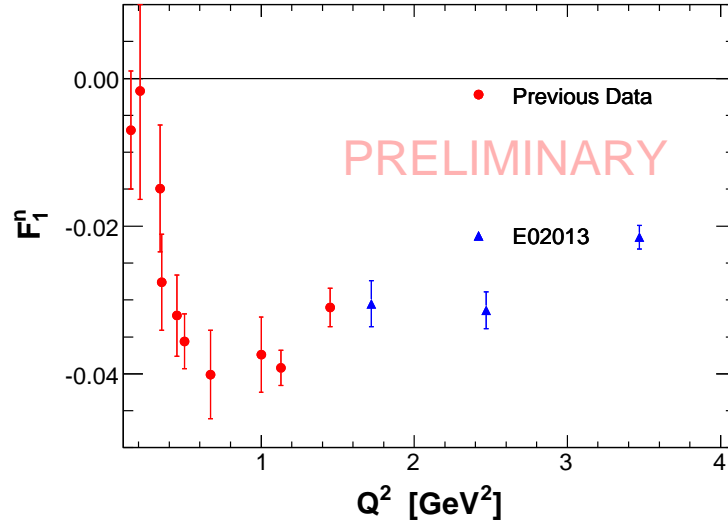


Figure 6.1: Pauli form factor ( $F_2$ ) dependent on  $G_E$  and the dipole approximation for  $G_M$ .

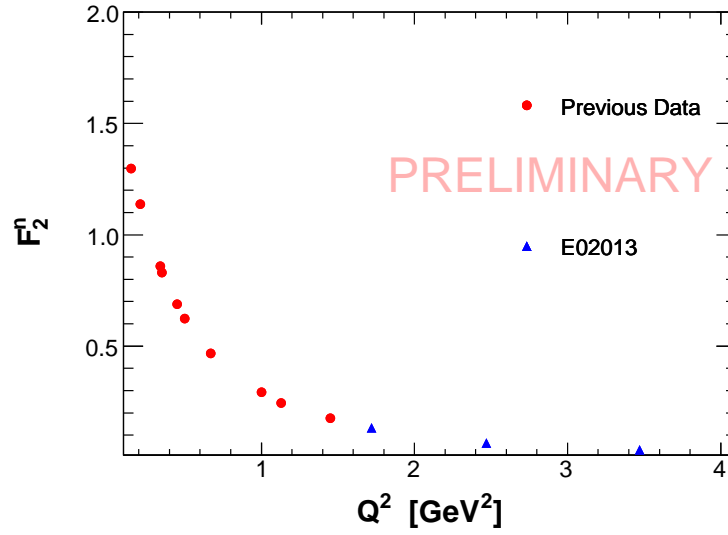


Figure 6.2: Dirac form factor ( $F_1$ ) dependent on  $G_E$  and the dipole approximation for  $G_M$ .

### 6.3.1 Flavor Form Factors

If certain assumptions are made, the flavor form factors can be deconvoluted from the nucleon form factors. Iso-spin symmetry being conserved at these energies, in the neutron the down quarks switch places with the up quarks in the proton. For the proton, since the charge of the proton is defined to be one (see equation ??), the equation for the proton form factors in terms of the flavor form factors is

$$F_{1,2}^p = \frac{2}{3}F_{1,2}^u + \frac{-1}{3}F_{1,2}^d . \quad (6.11)$$

The formula for the neutron is similar, but due to the up and down quarks being switched (position wise) within the neutron, the formula is

$$F_{1,2}^n = \frac{-1}{3}F_{1,2}^u + \frac{2}{3}F_{1,2}^d . \quad (6.12)$$

For both the magnetic form factors of the neutron and proton form the dipole approximation is used. For the electric form factor of the proton, a modified dipole is used [?]

$$G_E^p = (1.06 + 0.14t)G_D . \quad (6.13)$$

This allows the Pauli and Dirac form factors for both the proton and the neutron to be calculated, and the flavor form factors deconvoluted, with presented uncertainty only dependent on the neutron electric form factor data. Since theorists predict the strange form factor is not differentiable from 0 for these nucleons, it is ignored in this deconvolution[].

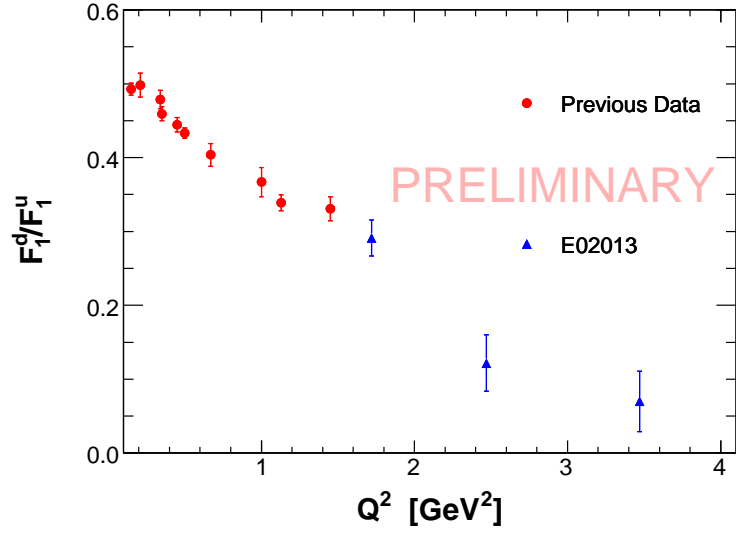


Figure 6.3: Ratio of the down and up quark flavor Pauli form factors. The form factors not dependent on  $G_E^n$  are just dipole approximations.

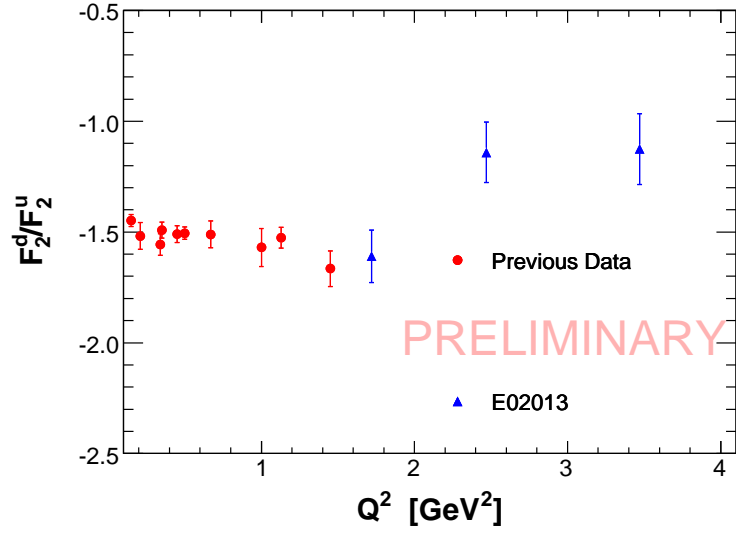


Figure 6.4: Ratio of the down and up quark flavor Dirac form factors. The form factors not dependent on  $G_E^m$  are just dipole approximations.

## 6.4 GPD Models

A popular framework to consider nuclear interactions and structure is that of generalized parton distributions. These were discussed earlier in section ???. They parametrize the non-forward bits of the light front operators. They have the advantage in being able to describe, in part, elastic processes, deep inelastic processes, deep virtual compton scattering, as well as physical properties like quark orbital angular momentum. In terms of form factors, the GPDs give the quark contribution at a given  $x$  to the the flavor form factors.

$$F_1^q(t) = \int_0^1 dx H^q(x, \xi, t) \quad (6.14)$$

$$F_2^q(t) = \int_0^1 dx E^q(x, \xi, t) \quad (6.15)$$

There are two other GPDs, which are related to the pseudoscalar and axial form factors. These aren't really effected by our measurement, and are more difficult to constrain by current measurements.

### 6.4.1 Impact Parameter Space

Because GPDs divide the momentum into longitudinal and transverse components,  $t$  can be transformed (similar to a Fourier transform) into space to provide a description of the nucleon in terms of impact parameters. This is the distance from the center of momentum of the nucleon. From this, the distribution of the flavor can be determined for different longitudinal momentum fractions ( $x$ ).

$$H^q(x, \vec{b}) = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} e^{-i\vec{b} \cdot \vec{\Delta}} H^q(x, -\vec{\Delta}^2) \quad (6.16)$$

$$E^q(x, \vec{b}) = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} e^{-i\vec{b} \cdot \vec{\Delta}} E^q(x, -\vec{\Delta}^2) \quad (6.17)$$

This  $H^q(x, \vec{b})$  is actually  $q(x, \vec{b})$  the distribution of partons as a function of the transverse position  $\vec{b}$  from the center of momentum and light-cone momentum fraction  $x$ .

Because the impact parameter space corresponds to locations within the hadron, the density interpretation of GPDs is used to create conditions that must be satisfied by the GPDs, for example giving a relation between the impact parameter and the longitudinal momentum fraction. Obviously any nucleon is going to have the allowed region of impact parameter space go to 0 as the longitudinal momentum fraction goes to 1 [].

$$\frac{1}{2} |\nabla_{\vec{b}} E^q(x, \vec{b})| \leq H^q(x, \vec{b}) \quad (6.18)$$

In impact parameter space, the averaged square of the relative distance between the struck hadron and the spectator system can be described by

$$d_q^2(x) = \frac{\langle \vec{b}^2 \rangle_x^q}{(1-x)^2} \quad (6.19)$$

The probability interpretation of  $E^q(x, \vec{b})$  is in the transverse basis rather than the longitudinal basis. The distribution is then

$$q_v^X(x, \vec{b}) = q_v(x, \vec{b}) - \frac{b^y}{m} \frac{\partial}{\partial \vec{b}^2} E_v^q(x, \vec{b}) \quad (6.20)$$

where  $q_v^X(x, \vec{b})$  is the probability to find an unpolarized quark with momentum fraction  $x$  and impact parameter  $\vec{b}$  in a proton polarized in the  $X$ -direction minus the probability to find an antiquark.

#### 6.4.2 Fit to our data

As mentioned previously, many of the models[?][?] have parameters which are dependent on the nucleon form factors. Since 2004, new data on the nucleon form factors, in particular the neutron form factor, has arisen. This allows for new fits of the models.

The models for the helicity conserving GPD take the form of

$$H = q_v(x)e^{f_q(x)t} \quad (6.21)$$

where  $q_v(x)$  is the quark density function. This is gotten from global fits to DIS data and such, the fits used by these models were the MRST2002 (Guidal *et al.*[]) and the CRET (Deihl *et al.*[]). New data is also available for these fits.

The form of  $f_q(x)$  is another parameter. Deihl *et al.* fit the nucleon data and use the Regge phenomenology to get a class of fits. Guidal *et al.* on the otherhand just use Regge phenomenology as it has behavior which is close to correct both as  $x \rightarrow 1$  and  $x \rightarrow 0$ . Guidal *et al.* use

$$f_q(x) = \alpha' \ln \left( \frac{1}{x} \right) \quad (6.22)$$

while Deihl *et al.* use

$$f_q(x) = \alpha'(1-x)^2 \ln \left( \frac{1}{x} \right) + A_q(1-x)^2 + B_q(1-x)^3 \quad (6.23)$$

Deihl *et al.* argue that expecting  $f_q(x)$  to be described at high  $t$  by  $\alpha$  does not work as Regge phenomenology does not apply at that regime.

The helicity flip GPD has an additional component meant to change its behavior as a function of  $t$  compared to  $H$ . It should drop as  $\frac{1}{t}$  or faster, and so Guidal *et al.* solves this by putting in a term  $(1-x)^{\eta_q}$ . This gives the Guidal *et al.*  $E$  GPD as

$$E = N_{\eta_q} q_v(x) (1-x)^{\eta_q} e^{f_q(x)t} \quad (6.24)$$

where  $N_{\eta_q}$  is a normalization constant to account for the added term.

Deihl *et al.* use a different profile function

$$g_q(x) = \alpha' (1-x)^2 \ln\left(\frac{1}{x}\right) + C_q (1-x)^2 + D_q (1-x)^3 \quad (6.25)$$

which gives the  $E$  GPD as

$$E = \frac{\Gamma(2-\alpha+\beta_q)}{\Gamma(1-\alpha)\Gamma(1+\beta_q)} \kappa_q x^{-\alpha} (1-x)^{\beta_q} e^{tg_q(x)} \quad (6.26)$$



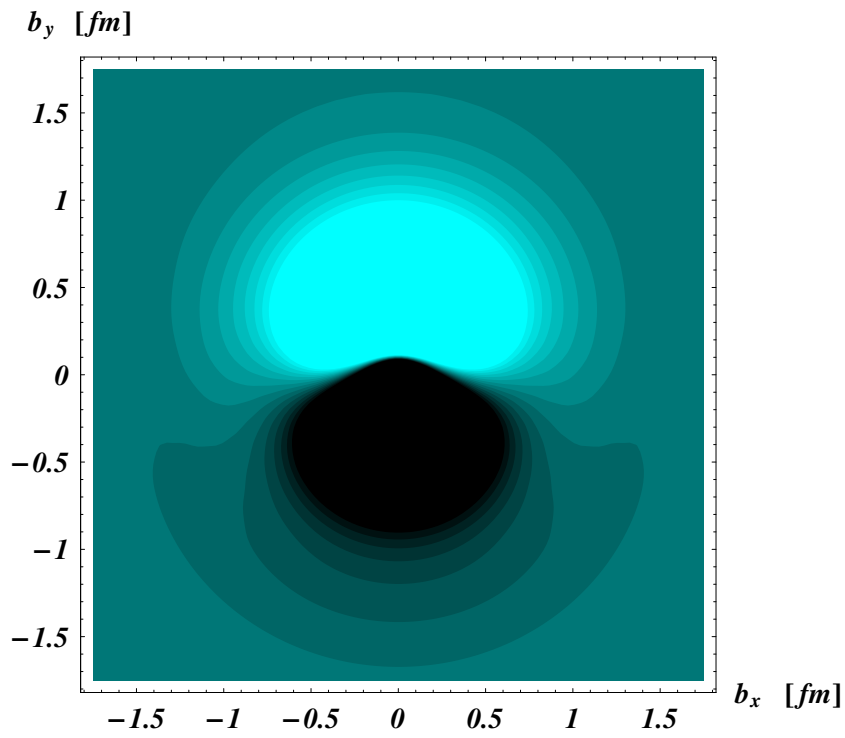


Figure 6.5: Diagram showing the transverse charge density of a simple model where the nucleon is polarized along the x-axis. Lighter is greater density.

## Appendix A

### Neutron Arm Analysis

?? include information on interactions in thesis

The experiment that the neutron arm was developed for was E02-013, a measurement of the electric form factor of the neutron via  ${}^3\overline{\text{He}}(e, e'n)$ . The position, active area, and shielding were developed to match the electron spectrometer at the highest measured value of  $Q^2$  and to provide the highest possible neutron detection efficiency at this value [21].

#### A.1 A Neutron Arm Analysis

?? define the NA

The neutron detector, known as Big Hand, is a large detector made up of 244 neutron bars totaling 976 ADC and TDC channels and an additional 192 single sided veto bars. In addition, there are four 'marker' counters which were placed between the neutron detectors and the veto detectors and were used for calibration. Important to neutron identification is the vertical position, the horizontal position, the time of flight, and charge of the event within the neutron detector. The neutron detector was highly segmented to aid in vertical resolution (known as  $x$  within the neutron arm). The horizontal resolution and the time of flight resolution of the neutron arm was determined by a time calibration of all the bars, with the 'marker'

counters playing a crucial role in the horizontal calibration.

## A.2 Simulations

?? Introduce simulations, why are they needed?

Detailed simulations were undertaken of the neutron arm using Geant 4. This simulation used Maid data and an interaction simulation for Big Bite to create the matching events and included the target, neutron arm, and all surrounding material. The goals of this simulation was to understand the inelastic contribution in the invariant mass and time spectrums, to come up with a theoretical value for the proton to neutron conversion between the target and neutron bars, and to determine the result of the threshold change for the on this conversion for some of the data taken at  $Q^2 = 2.5 \text{ GeV}^2$ . Additionally, it can provide neutron detection efficiency, hadron detection efficiency, and veto efficiency numbers which are needed to understand the neutron arm.

?? what was included in the simulation

The simulation needed to include all the material in between the target and the neutron bars in order to properly simulate the events within the neutron detector. This included the iron box, the ends of the target cell, the material for the target ladder, all holes in the target inclosure. A document prepared by Aidan shows all these components of the target system, which were included []. On the neutron arm side, all bars, PMT light guides, shielding, cassettes, as well as detector response were included. Details of the geometry were documented by Tim Ngo []. The Big

Bite acceptance was modelled using Big Bite geometry and the preshower/shower threshold and resolution, and is in good agreement with observed data.

?? what models/assumptions were included in the simulation

In this simulation, elastic and quasi-elastic scattering are simulated in the impulse approximation, where  $^3\text{He}$  is modeled using momentum distributions [?]. Parameterizations of the nucleon form factors [?] are used to calculate elastic cross sections and asymmetries. Pion production is a dominant inelastic contribution for these kinematics, and cross sections and asymmetries are provided by the MAID parameterization [1]. External radiative effects caused by objects in the vicinity like the beamline are included and internal radiative effects are modeled <sup>1</sup> [?].

The neutron arm response was simulated using the events above on an event-by-event basis with a GEANT4 simulation. The best matching hadronic model package, XXXX, was selected. Additional software was developed to reproduce PMT signals and electronics responses, and mimic the NA trigger. The simulated data then had the virtual thresholds changed to match the real data, and the virtual data was processed similarly to the real data [?].

### A.3 Calibrations

?? Goals for the NA

Because of the segmentation within the Neutron Arm, the bars all must be calibrated relative to each other in order to achieve the desired resolution needed to

---

<sup>1</sup>Mo and Tsai using the peaking approximation.

identify the good quasi-elastic hits within the neutron detector. For the measurement at  $Q^2 = 3.5\text{GeV}^2$  the required parallel momentum resolution was ??? which corresponds to a time of flight resolution of ??? at those settings [21].

## ?? Introduction to calibration

Before the experiment began, during the commissioning period, the discriminators were set, the cables were emplaced, and the high voltage (HV) for the detector bars was adjusted. During the experiment, the threshold for the NA was adjusted, and the HV was adjusted many times. This created a number of different 'periods' with different NA properties. The remainder of this section will be on the calibrations needed to get the desired NA properties.

## ?? Goal of software calibration

The desired situation is that the neutron arm provides as good of time of flight resolution as possible. As explained, this is necessary for quasi-elastic event selection and neutron identification. Important in the time calibration of the neutron arm is not only achieving a good time resolution, but also achieving an accurate and precise horizontal position. The HV tune gives ADC signals that are the same within 10% for cosmic type events. Differences in electronics and cables cause for each PMT signal read out of the TDC to have some particular delay. While events from cosmic sources are all similar, the differences in events during the experiment was greater. Because of this, events taken during running conditions created dissimilar signals within the ADC and caused the discriminator to trigger off a different point in the amplitude spectrum.

## ?? Types of calibrations

These considerations imply that two important calibrations need to be done. A time offset calibration of the time for each PMT, and an amplitude correction to the mean time and horizontal position of the event within the bar.

### A.3.1 HV Tune

?? Goals for the HV tune

The HV for the counters was set with the goal of having each neutron counter demonstrate similar behavior during operation. This behavior should be well behaved for hadronic events, and for other events which are expected within the detector during normal operation. This allows the analysis to treat all bars similarly when calculating time of flight, identifying charged particles, and calculating perpendicular missing momentum (due to the calculated hit position within the neutron arm), and the other quantities needed for quasi-elastic neutron identification.

?? Idea

The HV tune was done prior to the start of the experiment using cosmic particles. All cosmic rays have similar signatures within the detector, but their signals are not as uniform as the protons and neutrons from the target were. Because of high voltage (HV) drift and the difficulties in calibrating all photo-multiplier tubes (PMTs), a threshold of 10% different from the average was used to determine whether the PMT's HV needed adjustment. During the experiment cosmics continued to be used to adjust the HV when beam was not available, if a PMT drifted outside of this 10% threshold.

?? HV script

A high voltage script was written in Java for online monitoring of the high voltage and for changing the high voltage. This allowed different files to be saved with high voltage settings, and for a high voltage to be turned on and off as needed, either for individual bars or for sets of bars. Additionally, it allowed the resetting of HV during a trip or other event. Each colored set of bars in figure 4.19 used the same high voltage, in addition to using the same amplifier and discriminator. This high voltage script was written by Eugene Chudakov.

???? (Put in table of HV changes?) ????

?? Process

A script was created to realize the HV calibration<sup>2</sup>. The resulting files could be fed into the previously described HV script to change the HV. Diagnostic files showing the fitted spectrums for the PMTs were produced, spectrums were divided based on detector type, plane, and side.

?? Tune script

In this HV calibration script, long cosmic events were selected. These were events which were detected in a bar and its vertical neighbors. The amplitudes were tuned towards a common goal (G), which was determined based on the height of the bars relative to each other. This was 1000 for the UVA and GLA bars, and 1500 for the CMU bars. Two different techniques were used to set the gain, the first <sup>3</sup> using simply the amplitude of the bars. This peak was directly compared to the goal to

---

<sup>2</sup>Written by Igor Rachek, and modified by Jon Miller, Rob Feuerbach, and Pavel.

<sup>3</sup>Developed by Igor Rachek.

determine the gain to apply.

A second technique <sup>4</sup> used logarithms. This used two items of information determined from amplitude, a mean  $A_M = \frac{\ln A_L + \ln A_R}{2}$  and a position  $A_D = \ln A_R - \ln A_L$ . Here the subscript  $L$  stands for left,  $R$  stands for right,  $A$  is the amplitude while  $T$  is the time. In this case the gain was determined by fitting  $A_D$  and  $A_M$ , and was given by  $e^{-\ln G + A_M \pm 0.5 A_D}$  where  $+$ ( $-$ ) was for the right(left) gain.

The desired resolution is 300  $ps$  (?) this is due to needing to determine the hits in the neutron arm down to a momentum of 140 MeV for the identification of quasi-elastics in kinematic 3.

Another difficulty with using particles from cosmic events is that the path they travel through the neutron arm is less straight than events caused by interactions in the target. This causes a differently shaped peak which is more broad than the hadron events.

### A.3.2 Time Offsets

?? Steps to final Time calibration

To call two times aligned, the spectrums of two (often neighboring) bars would be shifted to coincide after the relevant corrections were applied. To do the offset calibration of the PMTs, three steps were used. First, the left times and right times were roughly aligned together. This was done so that the left and right times could be associated properly together to form a mean time. Not only the mean time of the bar could be calculated, but also the time difference of the bar (left time minus

---

<sup>4</sup>Developd by Pavel



right time) could be calculated, and was used to give the horizontal position. In the second phase of the calibration, the peaks for the mean time and the time difference were aligned for neighboring bars. This allowed for the creation of clusters, which are hits in neighboring bars with similar mean times and are identified as coming from the same event. In the final calibration, these clusters were used to improve the calibration using charged events (in  $^3\text{He}$ ). This only was used to do the final calibration of the first plane, later planes had their mean and horizontal position calibration improved using events which travel deep into the neutron detector. The first plane also had its horizontal position calibrated using the known information of marker bars.

???? End Time offset General summary ????

?? Goals as part of this process

The required time resolution of the neutron arm must be less than 500 ps for the most restrictive of kinematic points (kinematic 3). This resolution depends on the Big Bite scintillator also, the dependence on Big Bite was due to using it as the stop time for the TDCs and thus as the reference time of the neutron arm event. The combined resolution of the entire neutron detector needs to be less than 500 ps, so the peaks of events within bars need to be given the correct mean time for their vertical position and depth.

?? limitations of resolution

The resolution of the time of an event in the neutron arm (NA) is a convolution of the alignment of the bars and the internal resolution of the bars, which provides a minimum resolution for the experiment (using individual PMT amplitudes can

be used to improve the internal resolution of the bars). Alignment means that the spectrum in a bar, after corrections for position within the NA and after selecting hadronic events, will be similar to others bars after such corrections. For example, if a particle passes through two neighboring bars, they are aligned if the time that the particle passed through the bar is the same when corrected for the distance between the two bars that the particle traveled. The time of an event in a neutron bar or counter is just the mean time as determined from the PMTs.

The PMTs in each neutron bar determine two main quantities. These are the mean time of the hit  $t_{mean}$  and the horizontal position of the hit  $y_{pos}$ . The horizontal position of the hit is determined by the difference in time,  $t_{diff}$ , between the measured signals in the two PMTs for a hit. These two quantities, only corrected for offsets, are

$$t_{mean} = A((t_{l,raw} - t_{l,offset}) + (t_{r,raw} - t_{r,offset})) , \quad (A.1)$$

$$t_{diff} = B((t_{l,raw} - t_{l,offset}) - (t_{r,raw} - t_{r,offset})) . \quad (A.2)$$

In these relationships  $t_{l,raw}$  is the left raw time (from the TDC) and  $t_{l,offset}$  is the software offset applied to the left TDC value. Here  $A$  and  $B$  are constants that are about  $\frac{1}{2}$  and defined to be so in all calculations.

The best understood particles that are detected from the target are protons (from  $H_2$ ), although muons, once detected, will not deviate noticeably while traveling through the detector. Initially the calibration of the neutron detector was kept entirely internal to the neutron detector. In the final calibration the front plane was calibrated relative to Big Bite using quasi-elastic protons (from  $^3He$ ), with the later

planes calibrated relative to the first plane (N1).

### A.3.3 Initial Calibration

During the course of the experiment the high voltage was changed many times. Although this was the case, high voltage changes did not change values of the amplitude by much more than 10% and so rough values for the mean time and time difference for a bar will be independent of the high voltage setting. Because of this, the initial calibration was done once, with later calibrations fine tuning the initial calibration of neighboring PMTs on the same side for each HV setting calibrated.

The first improvement in calibration was achieved by comparing two bars in neighboring planes and subtracting their mean times[30]. This created a peak for events when there is a hit in both planes at that vertical ( $x$ ) position. This peak was then adjusted to give the proper time of flight for near  $c$  particles and so all later planes were calibrated relative to the first plane (N1). The horizontal position calibration was realized by using the time difference in neighboring bars. This was done similarly to the mean time calibration.

A final difficulty of this technique is that these events might not travel through the entire neutron bar, but the shower from the cosmic might start or end within the bar. This is different then during running conditions, because the converter layer in front of each neutron scintillator bar results in most detected hadrons beginning interaction directly in front of the scintillator paddle where initial detection occurs.

While this improvement provided the necessary accuracy and resolution to

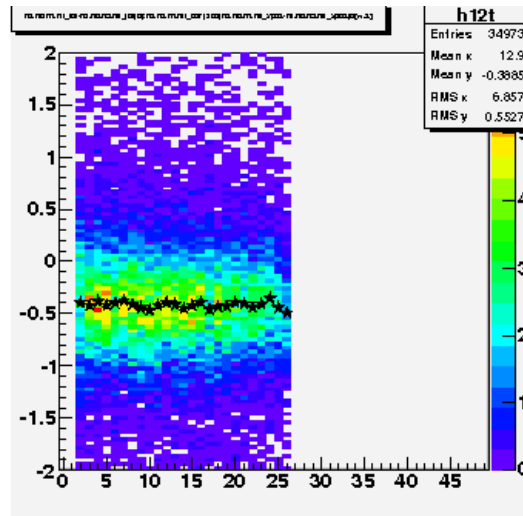


Figure A.1: Calibration of second plane to first plane using Rob's script.

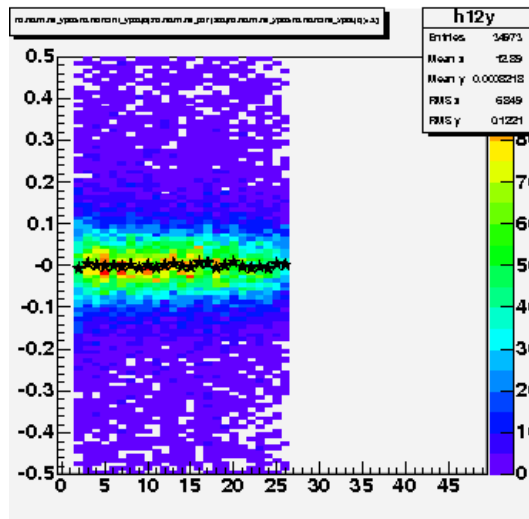


Figure A.2: Horizontal calibration of second plane to first plane using Rob's script.

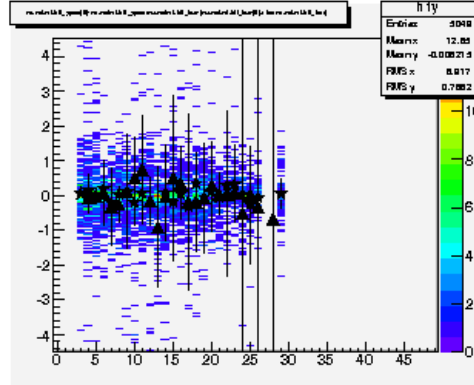


Figure A.3: Horizontal calibration of the first plane using Rob's script.

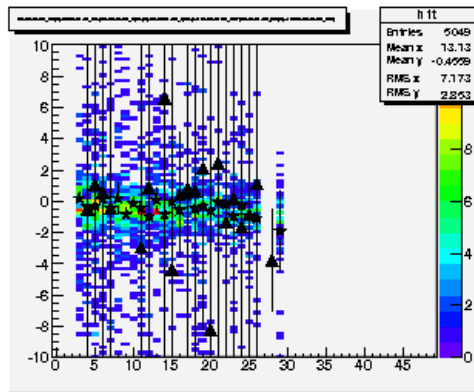


Figure A.4: Vertical calibration of the first plane using Rob's script.

arrive at a preliminary result for the  $Q^2 = 1.5 \text{ GeV}^2$  kinematic, it was not robust enough to provide final results. The next improvement to the calibration was done by using a better selection of events. These better events were proton and muon events and selected by selecting long events that pass through the neutron arm, where a particle lost little of its momentum prior to being observed in the NA.

Because of the better identification of protons and muons, the peak shape will be more similar for individual bars, and the calibration will be better. This improved internal calibration calibrated the later planes relative to the first, by shifting  $t_{central}$  to 0:

$$t_{central} = t_{1st} - t_{Nth} - t_{1st \rightarrow Nth} . \quad (\text{A.3})$$

Here  $t_{1st}$  refers to the mean time of the first plane the particle passed through, and  $t_{Nth}$  refers to the mean time of the  $n$ th plane the particle passed through (at the same  $x$  as the first hit). Subtracted from this difference is the time for the particle to travel between the first and  $n$ th planes ( $t_{1st \rightarrow Nth}$ ). The time between planes was calculated using the distance between the planes, and  $c_n$  to approximate the speed of a relativistic particle (since the particle is unknown). These well behaved events were also used to calibrate the horizontal position of the hit within the Neutron Arm, aligning all later planes to the first plane. This completes the internal calibration.

The final improvements to the neutron arm calibration were to achieve an accurate horizontal ( $y$ ) position and to determine the overall offsets for the vertical position ( $x$ ) using particles. The best understood particles within the neutron arm

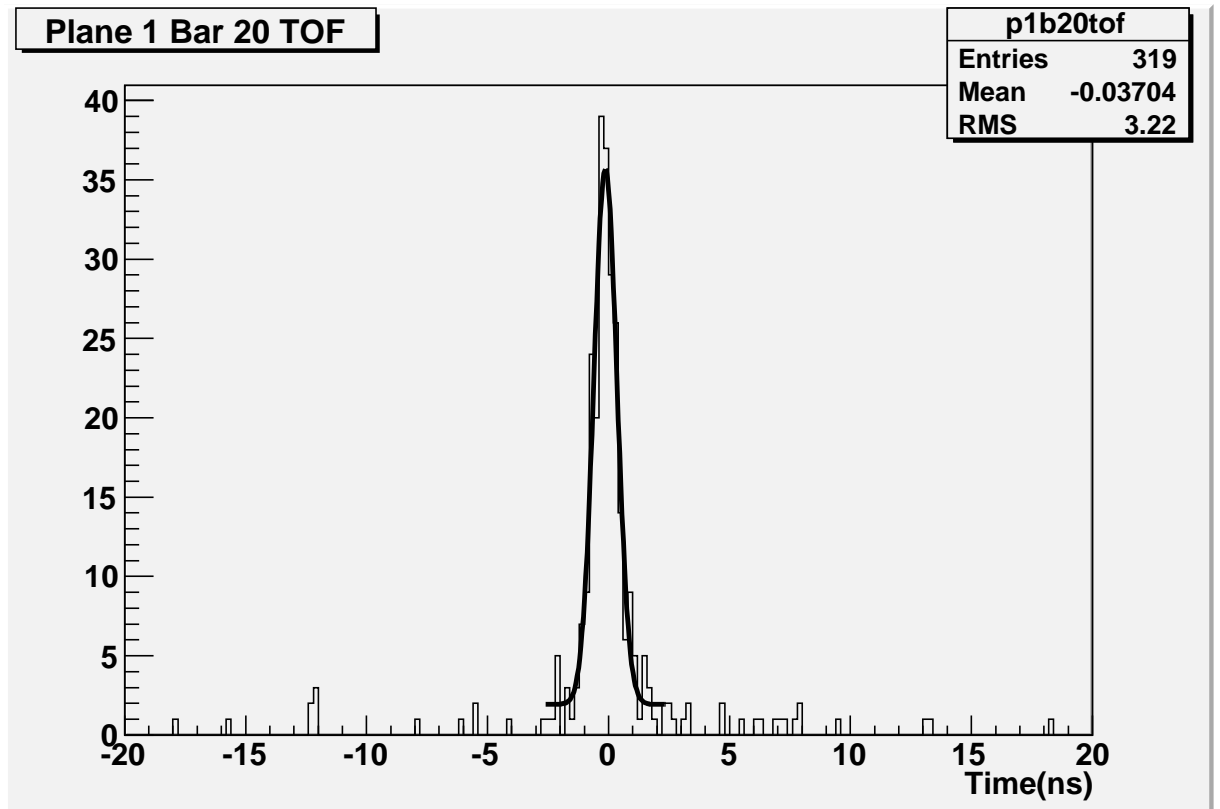


Figure A.5: Calibration tof plane 1 bar 20. This shows the time of flight peak for quasi-elastic protons between the BigBite scintillators and the front plane of the neutron arm.

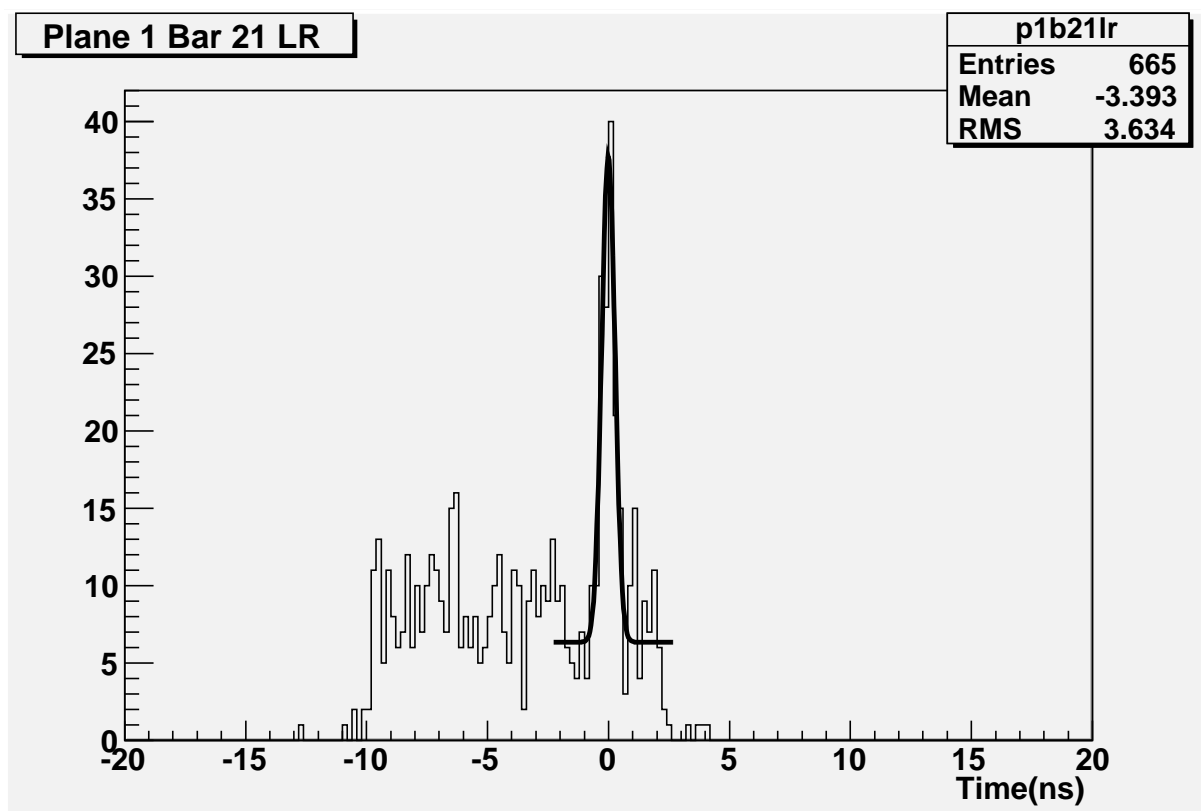


Figure A.6: Calibration lr plane 1 bar 20. This histogram shows the coincidence peak in the horizontal bars time, with coincidence with the marker counters.



(NA) are elastic protons from hydrogen. These elastic protons form well defined peaks in the mean time spectrum of bars in the first plane (N1). After corrections (detailed in equation 5.9) based on the position of the hit within the neutron arm and the time of the event within Big Bite, these spectra were aligned:

$$T_{c,TOF} = t_{1st} + t_{L1A} - t_e - t_h - t_{peak} + t_{rf} . \quad (\text{A.4})$$

Here  $t_{c,TOF}$  refers to the centered time of flight,  $t_{1st}$  refers to the mean time as measured in plane 1 (N1),  $t_{L1A}$  refers to the level one accept time,  $t_e$  refers to the time for the electron,  $t_h$  is the expected time for the hadron to travel to the neutron bar,  $t_{peak}$  is the time for the hadron to travel to the neutron bar, and  $t_{rf}$  is a correction to the known time of interaction due to RF information about the beam.

The neutron arm was placed to measure the maximum number of quasi-elastic events as possible in coincidence with Big Bite, not to measure as many elastics as possible. Because of this elastic events did not cover the entire face of the neutron arm, but only a portion of it. To calibrate the entire detector, quasi-elastic events were used rather than the cleaner elastic sample. These still give a tight time of flight peak and have events which pass through the detector.

Four marker counters were installed vertically within the neutron detector. These counters had poor resolution and were not meant to be used to determine the vertical position of a hit within the neutron bar. These counters were used to perform an absolute calibration of the horizontal position for all bars in the first plane. This was done by using the coincidence between a hit in the marker

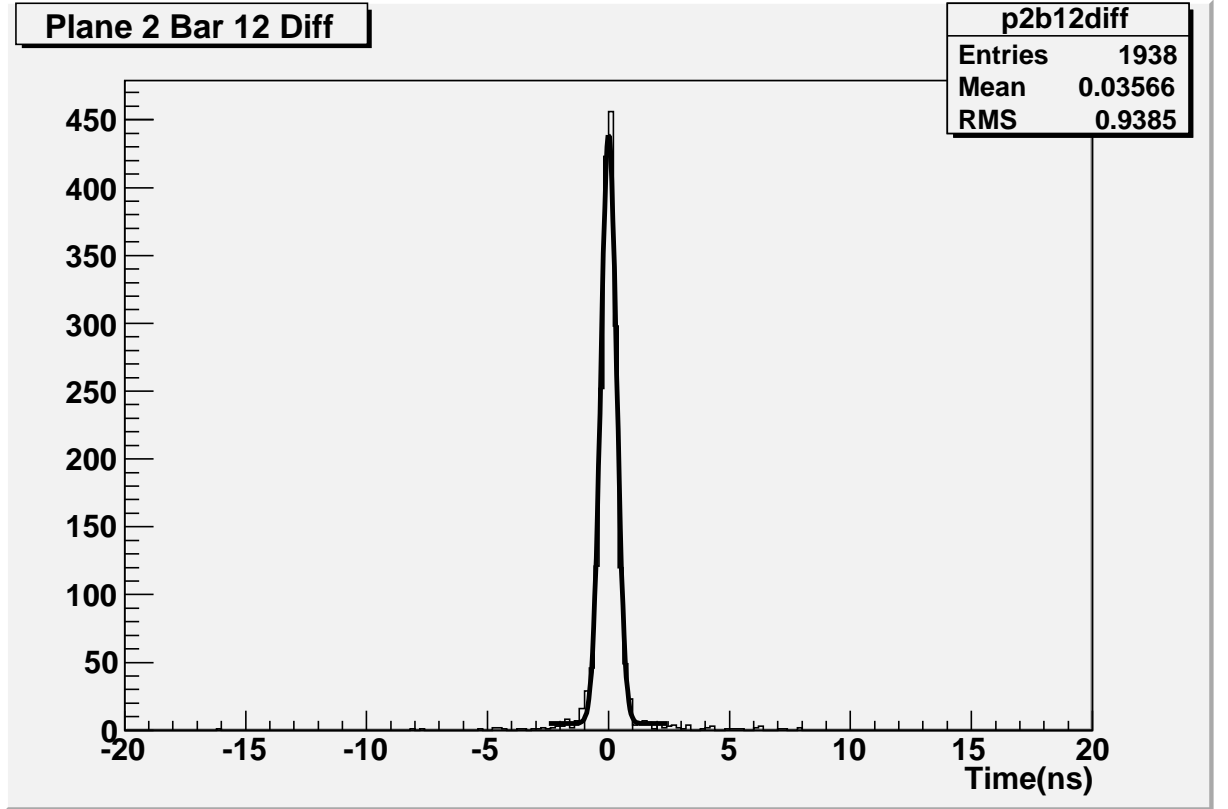


Figure A.7: Calibration diff plane 2 bar 12. This is the difference between neighboring bars in different planes. In particular, this is the difference between plane 1 bar 12 and plane 2 bar 12.

counters and a hit in the neutron arm for a given event. For a single marker PMT the number of counts is small. By using the geometry information for the marker counters[59], the coincident peaks for all four marker counters can be combined ( $y_{center} = y_{diff} - y_{marker}$ ), providing the necessary statistics. This  $y_{center}$  is the position within the neutron arm for hits where there is a nearby well-timed marker bar hit. Here  $y_{center}$  is the peak location,  $y_{diff}$  is the original position along the bar, and  $y_{marker}$  is the geometric position of the marker counter.

The results of the calibration is summarized in Figure A.9 using a single plot

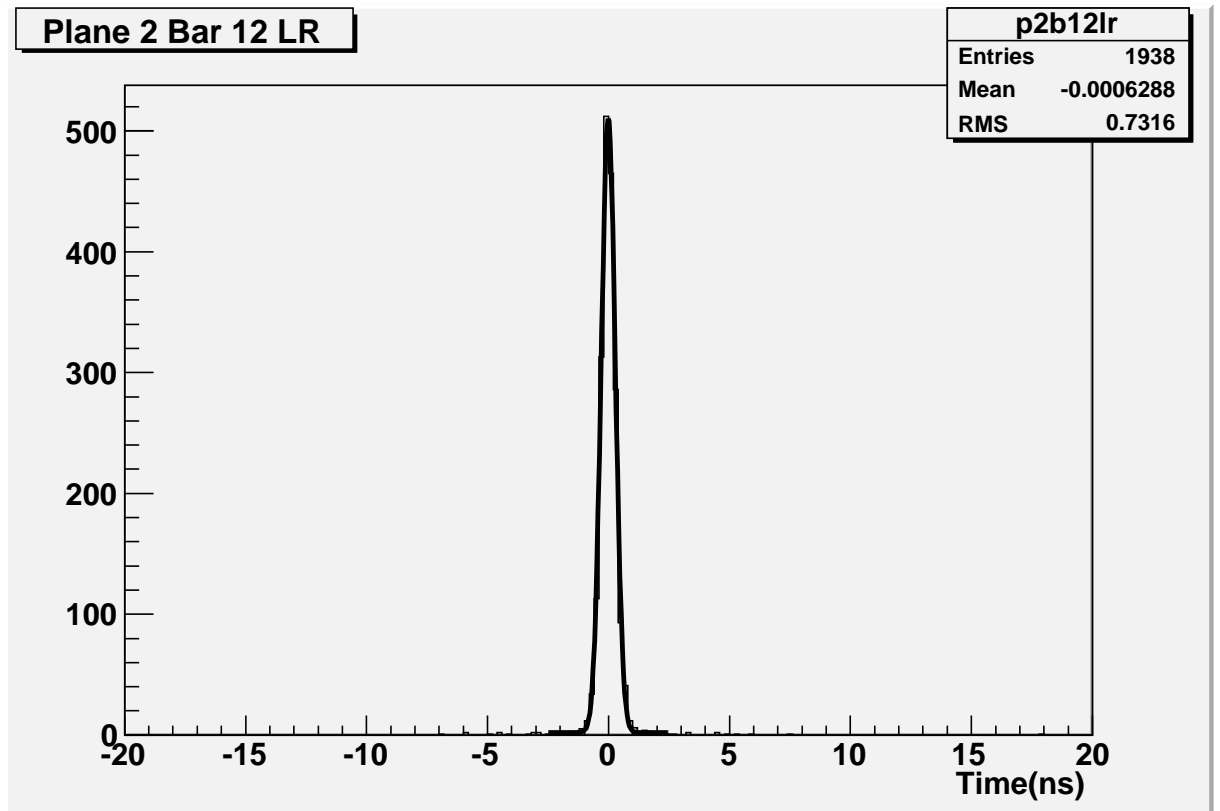


Figure A.8: Calibration lr plane 2 bar 12. This is the horizontal position (transformed to time) of the hits in plane 1 bar 12 and plane 2 bar 12 for events with well timed hits within the two bars.

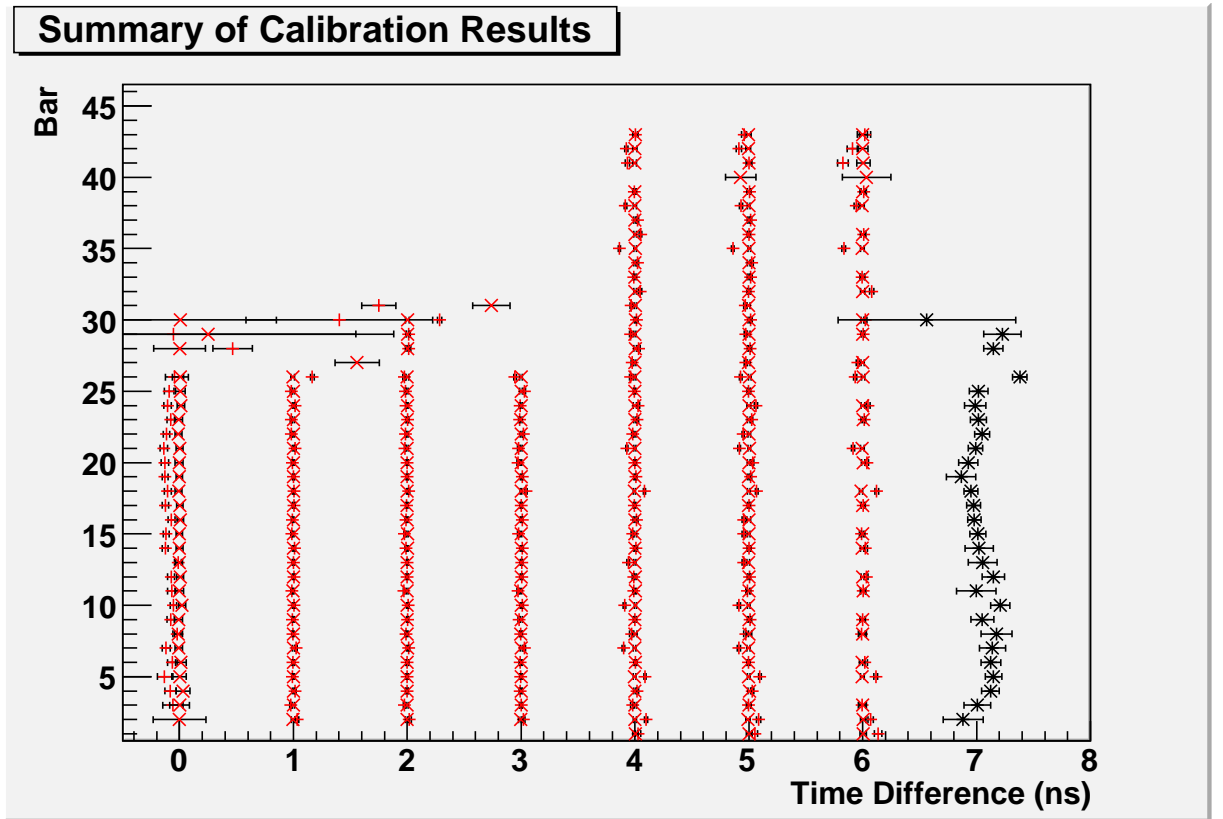


Figure A.9: Calibration summary plot for run 3975 after pass 3 calibration. In this plot, X is the result, + is the before calibration. On the x axis, 0 is for plane 1 and 1-6 show the differences between planes 1 and planes 2-7). At 7 on the x axis is the horizontal absolute position calibration for plane 1.

showing the main points of calibration: the mean times of planes N2 through N7, the time of flight for N1, and the  $y_{center}$  position for the absolute horizontal calibration. In this plot, some of the bars did not have high enough statistics to form a proper peak and are shown as poorly calibrated. These bars have greater uncertainty, and so for these the respective histograms are checked by eye.

### A.3.4 Amplitude Correction

The amplitude spectrum was discriminated to provide a clear time for the TDC. However, since there was a threshold for this discrimination, the time measured by the TDC is depended on the shape and size of the amplitude. The mean time and horizontal position for an event were corrected for this amplitude dependence. This relationship for the initial amplitude calibration was

$$t_{corr} = t_i - C_{par}(A_{adc})^{exp} + C_{par}(A_{ref})^{exp} . \quad (\text{A.5})$$

The *exp* is constant and set to  $-1$  for all detectors. In the initial calibration the reference paramters,  $A_{ref}$ , were defined to be 0. The values for  $C_{par}$  were determined individually for each PMT.

This initial calibration was improved using a model where the amplitude correction was a function of the hit location ( $\Delta Y$ ) within the bar. In this model, the amplitude dependent behavior should be the same for both sides of the bar. Additionally, because the threshold was set to be the same for all bars, this behavior should also be independent of which particular bar was hit. The correction can be approximated as (with a  $A$  and  $\delta Y$  dependence as information from the PMT):

$$\frac{(C_1 + D_1 \Delta Y)^2}{A} \approx \frac{C_2 + D_2 \Delta Y}{A} . \quad (\text{A.6})$$

The desire is to determine the constants which make a narrow peak in the time of flight between the Neutron Arm and Big-Bite. To do this minuit<sup>1</sup> was used to minimize the following:

$$T_{diff,rf} + \frac{C_2}{2} \left( \frac{1}{A_L} + \frac{1}{A_R} \right) + \frac{D_2}{2} \left( \frac{Y + Y_L + \frac{Y_L + Y_R}{2}}{A_L} + \frac{Y + Y_R + \frac{Y_L + Y_R}{2}}{A_R} \right) \quad (\text{A.7})$$

here  $Y$  is the horizontal hit location in the neutron bar,  $Y_R$  is the position of right PMT, and  $Y_L$  is the position of the left PMT.  $Y$  is obviously dependent on both  $C$  and  $D$  but this is ignored as higher order.

While there were many quasi-elastic events where the first plane of interaction was in the first plane, and some where it was in the second plane, the later planes were less often the plane of first interaction. By using quasi-elastic events the first two planes were corrected for amplitude dependence. These bars were compared to bars in later planes to fit their amplitude dependence. To do this, the following formula was used, where  $T_{N+2}$  is the time of flight for a hadron in the desired plane,

$$T_{N+2} = 2T_{N+1} - T_N . \quad (\text{A.8})$$

### A.3.5 Energy Calibration

### A.3.6 Veto Calibration

The veto was calibrated relative to the neutron arm first plane (N1). This calibration was achieved by using events with coincident hits in two veto bars with similar  $x$  position in both veto planes. For example, a left(right) veto bar was calibrated relative to a left(right) in the other plane. Doing this step by step the entire veto was internally calibrated, using the fact that the two veto planes were shifted relative to each other. See figure 4.19 showing that each veto bar covers vertical space with two bars on the other veto plane. These planes were also offset horizontally relative to each other, the coincident region allowed left times and right times to be aligned. This provided an overall, intra-veto calibration and alignment.

A central veto detector provided an overall offset compared to the neutron plane one (N1) using coincident events between it and a neutron bar, completing the initial veto offset calibration.

This initial calibration was improved by looking at long events and aligning the resultant peaks in the veto to those in the neutron detectors at corresponding x position. The horizontal position within the bar can be corrected for, but this would provide resolution beyond that needed due to the interactions before the veto in the shielding.

These long tracks have hits in many planes within the neutron arm and almost always have hits in the veto as well. These charged tracks were used to align the veto peak with the observed peak in plane one (N1). The relationship between veto amplitude and time was used to correct the time in the veto for this alignment. The relationship in equation A.3 was used, except the two veto planes were substituted in for the Nth neutron plane.

## A.4 Neutron Software

### A.4.1 Cluster formation

The data is first pulled from raw coda files in the process of decoding and analysis. The reference hit, which was read into a specific channel of the F1 TDC, must exist for the event to be decoded. The decoding process loops through all TDC and ADC channels, and creates independent arrays of hits for raw analysis. These include the corrections to change the PMT values from channels into time,

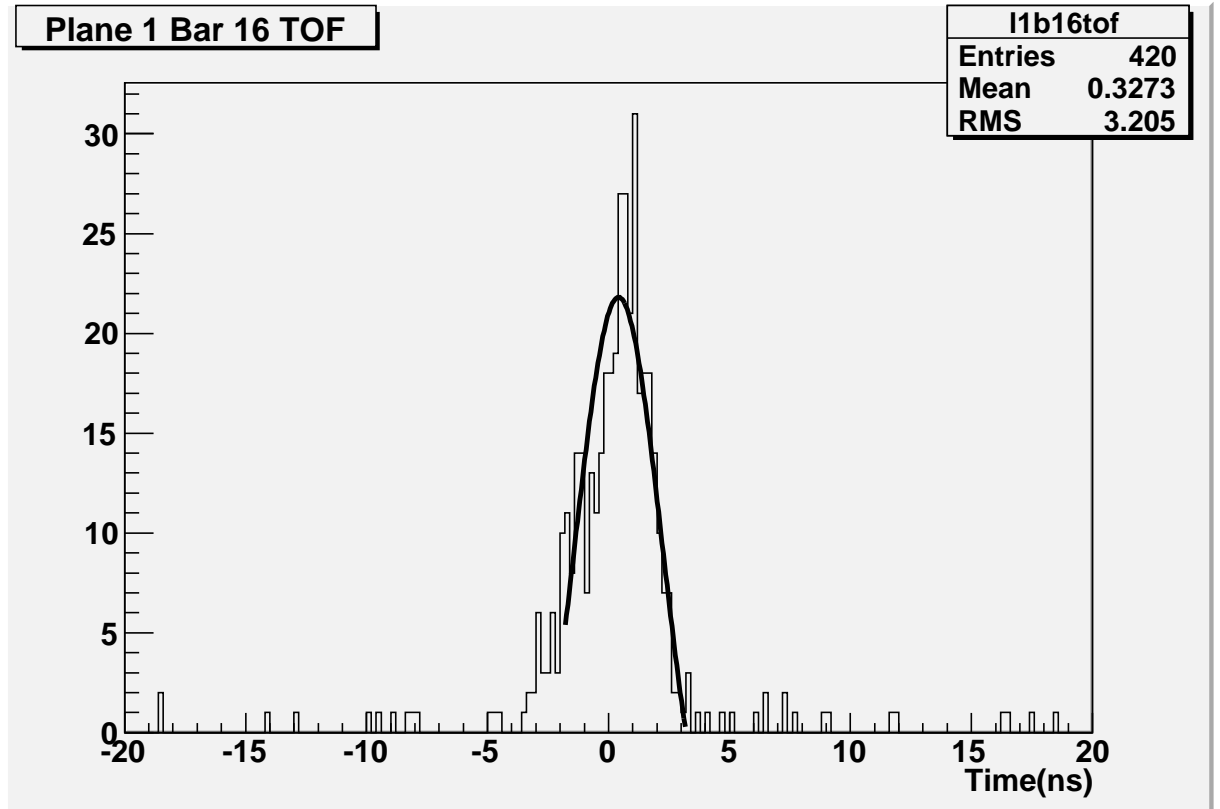


Figure A.10: Calibration veto tof left 1 bar 16



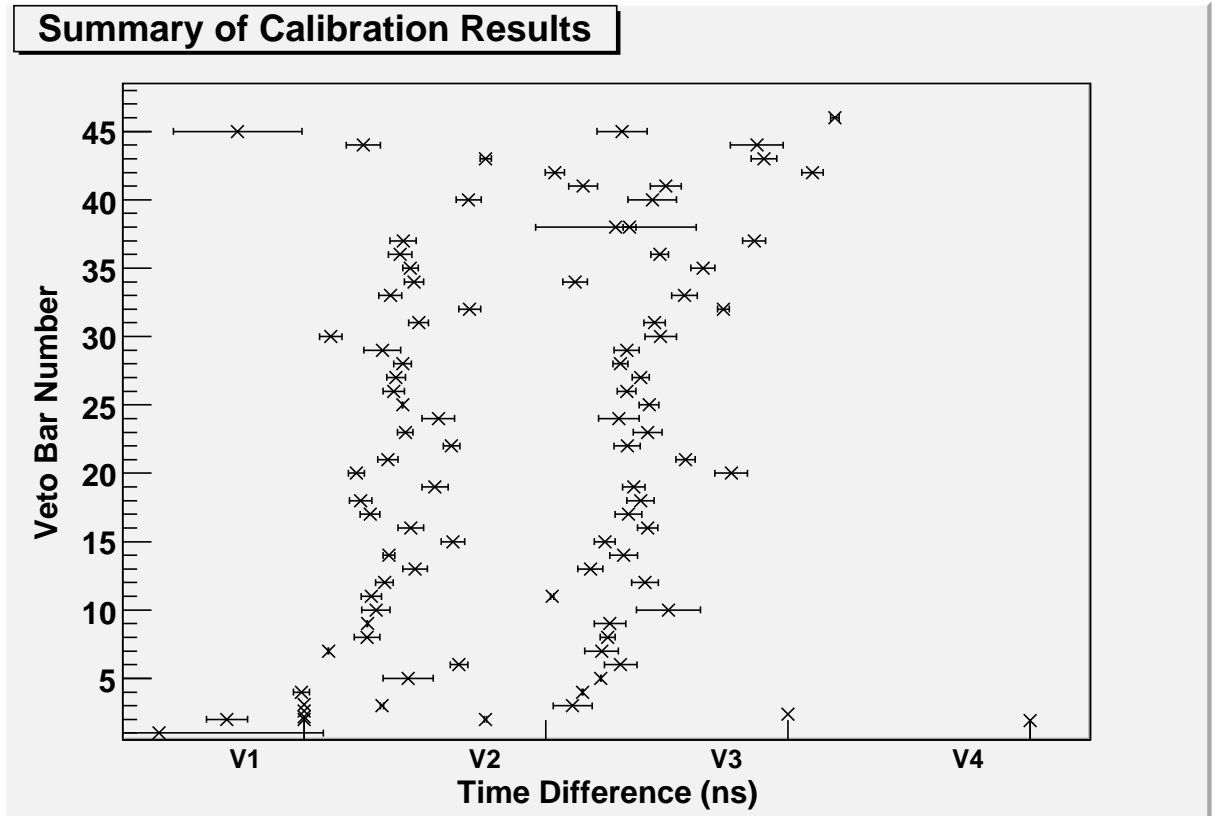


Figure A.11: Calibration of one side of the veto detector.

but does not allow the determination of mean time or horizontal position. An offset is applied to the PMT time ( $t_{off}$ ) to correct the time relative to the reference time

$$t_{off} = t_{ref} \pm wrap * res . \quad (A.9)$$

In this formula  $+$ ( $-$ ) is selected when the difference between time and reference time is greater(less) than  $+$ ( $-$ ) half the *wrap*. Here *wrap* is the number of channels in the F1 TDC, when this number is reached the TDC starts counting over again. The trigger provides a delayed signal to serve as this reference time ( $t_{ref}$ ). Because of this all neutron arm times are given compared to the trigger time (see L1A discussion in section 4.7). *res* is the resolution which was 0.118 ns for the F1 TDC. The F1 TDC was constantly acquiring hits, keeping track of the information in each channel relative to the reference time which was determined by the trigger time. The trigger also determined when the TDC was read, but the reference time was required to reconstruct the channels read. The total number of channels in the F1 TDC (the *wrap*) is 135558. The PMT time also included the time offsets, read in from the database. All indices are separate at this stage, even for ADC and TDC information for the same PMT.

In the coarse processing, the hits are sorted by bar number and value. The hits are looped through and checked that the PMT time is within a limited range and that TDC hits exist on both sides as well as an amplitude. Hits that pass these checks are considered 'complete'. At this stage the mean time is corrected for the amplitude correction. This is also where the mean time and the horizontal position

of a hit is calculated.

$$\begin{aligned}
t_{diff} &= \frac{(t_r - t_l)}{2} \\
t_{mean} &= \frac{(t_r + t_l)}{2} \\
y_t &= t_{diff} c_n
\end{aligned} \tag{A.10}$$

$$GetAmpl = \frac{A_l}{A_r}$$

$$A_{bar} = \sqrt{A_l * A_r}$$

Here  $t_{mean}$  is the mean time of the hit (and  $t_l$  and  $t_r$  are corrected left and right times), while  $t_{diff}$  is the difference between times of hits in the left and right PMTs. This difference allows the calculation of the horizontal position ( $y = y_t + y_{offset}$ ) of the hit.  $c_n$  is just the propagation speed of the signal within the bar. Also,  $A_l$  and  $A_r$  are the left and right amplitudes, respectively, while  $A_{bar}$  gives the 'energy' for the full bar.

The last part of the neutron arm decoding and processing, the fine processing is done after all coarse processing. In this process the horizontal position of the hit is checked to see if it exists within the neutron arm. After this check, the clusters are created. This is done by sorting the hits and then looping through hits with each hit placed in only a single cluster. In this loop, the first hit is put into a cluster, the AddNeighbors method is called (recursively) which collects (and removes from further consideration) neighboring (in time and position) hits. A hit is classified as a neighbor if it is one bar away and within 10 ns. These clusters are then sorted by energy. A track is created from the cluster (using the initial hit information) back to the target, and this is used further in analysis. The path length and momentum

direction variables (but momentum is not constructed at this stage) are also created.

## A.5 Veto Analysis

As previously described (section 4.6.2), the veto is made up of two planes of detectors with 48 detectors per plane. These detectors are segmented into left and right veto counters, providing a total of 192 separate PMTs which form the veto detector. The two veto planes were also shifted vertically and horizontally relative to each other [59]. This vertical and horizontal segmentation improves the localization of veto detector hits for association with neutron detector hits. Because of the shift between the two planes, the edges of the neutron detector are only covered by a single plane of veto, which reduced the detection efficiency of the veto and hence provide different charge identification efficiency in these edges compared to the central region of the neutron arm.

The horizontal segmentation of the veto detectors allows for the identification of charged hits in the neutron detectors based upon horizontal position of the cluster. If the hadron hit is far to one side of the detector, than a charged identification in the veto on the other side of the detector would not identify a charged hadron. Additionally, only veto bars within a vertical region are considered as hits to identify a charged hadron (see figure A.12).

The veto dead-time is different for the individual veto counters. This can be determined by measuring the least amount of time between sequential hits.

Veto detection efficiency depends on the horizontal, time, and amplitude cuts

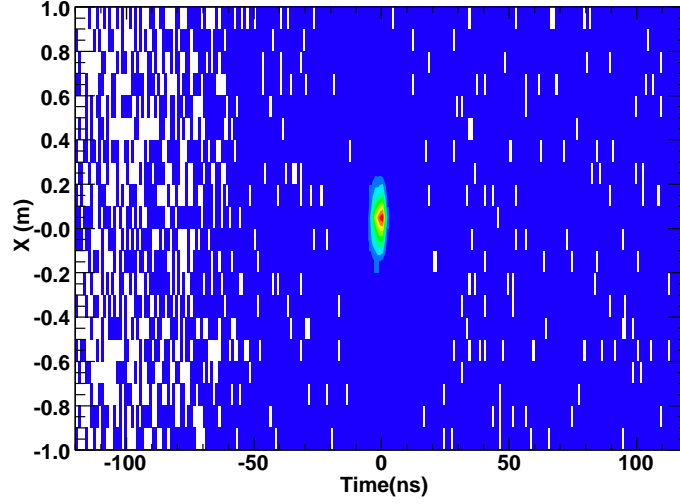


Figure A.12: Plot of X position in the veto subtracted by the X position in the neutron arm versus  $\Delta t$ . These are for quasi-elastic hits in the neutron arm. Plotted on the x axis is the time difference between the neutron arm and the veto arm. This identifies a region of 0.3-0.5 m to consider in the veto detector to identify charge hits.

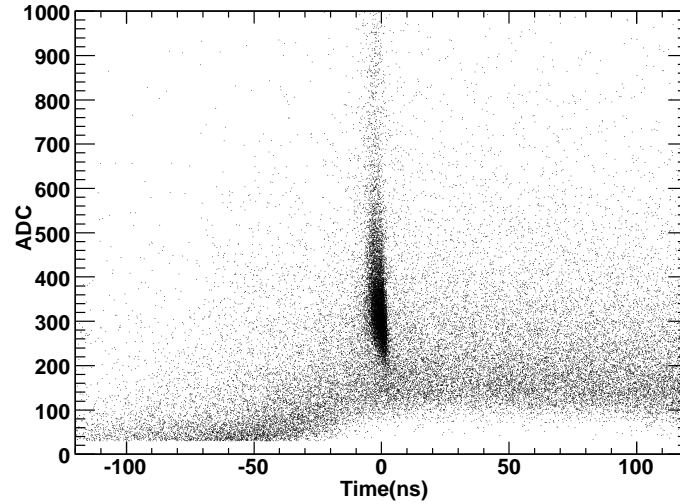


Figure A.13: Plotted is the time difference between the veto hit and the neutron arm hit for a single veto plane versus the amplitude in the veto bar. This figure shows all bars in the plane. The gate is seen to start around 40 ns before the coincidence hit in the neutron arm. The deadtime of the various veto detectors is between 40 and 110 ns. During this dead time, the hits are identified as charged if the amplitude is above 200. For a region of  $\pm 10$  ns around 0, all events are identified as charged.

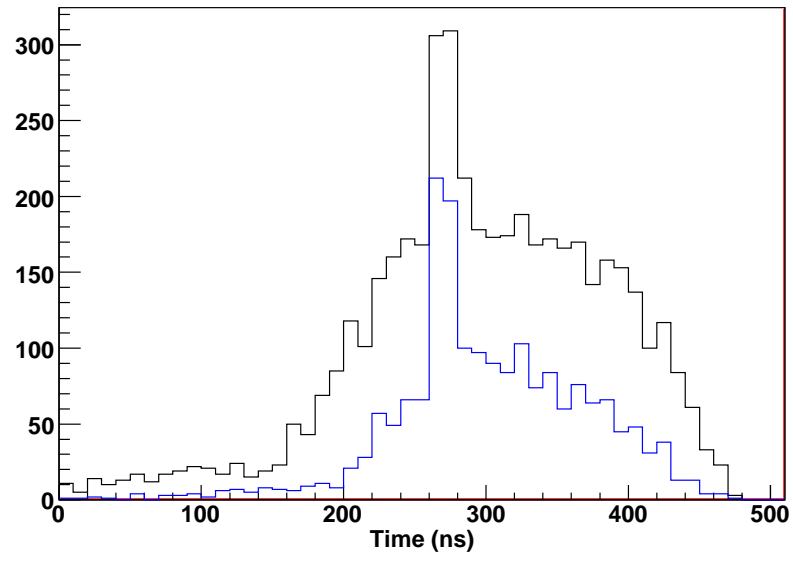


Figure A.14: Plot showing the rate both with and without a 200 channel ADC cut for the veto detector L1-14. The ADC cut rate is in blue. The window between 300 and 350 ns was used to calculate the rate.

used to identify charged particles.

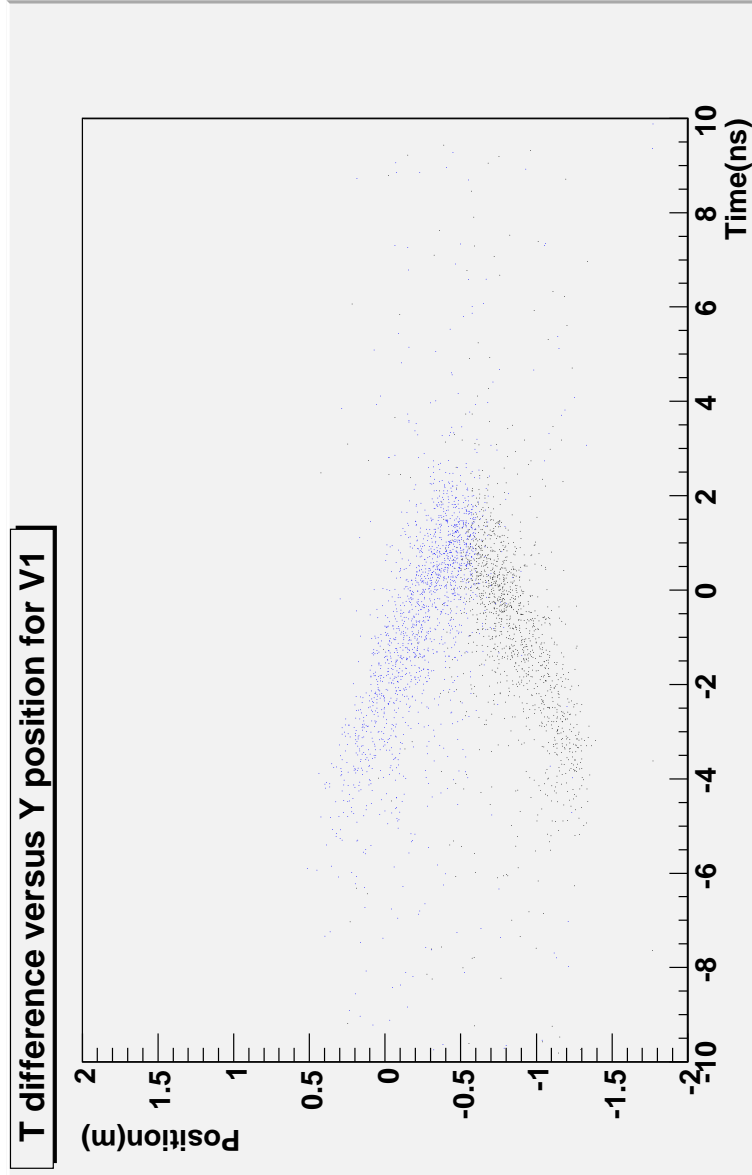


Figure A.15: Plot of hits within the Veto detector. Colors are given to show coincidence between the two separate halves of the V1 veto detector.

Identifier	Date	Comment	
new_crate19	May 6		Large
new_crate23	May 3		hat
new_crate23	May 3		
new_crate22	May 3		
new_crate21	May 3		
new_crate20	May 3		hat
new_crate19	Apr 29		PS1:2004 S5 + S9 and PS1:20
new_crate18	Apr 29	4335-4338	PS1:2004 S5 + S9 + S11 +
new_crate17	Apr 29		
new_crate16	Apr 23	At 4151 (see HALOG)	PS1:2002 S9 and
new_crate15	Apr 23		
new_crate14	Apr 19	At 4068	
new_crate13	Apr 19		
new_crate12	Apr 18	At 4050? (See Halog)	
new_crate11	Apr 17	At 4016 (See Halog)	
new_crate10	Apr 17		
new_crate9	Apr 17		Only PS1:2004 S11 + S1
045.set	Apr 12		Only PS1:2002 S14 and PS1:200
new_crate8	Apr 6	At 3714? (See Halog)	Only PS1:200
new_crate7	Apr 4		Only I
new_crate6	Apr 4		
new_crate6.set	Apr 4	At 3665 roughly	
new_crate5.set	Apr 4		
new_crate5	Apr 4		
new_crate4	Apr 4		
new_crate4.set	Apr 4		
new_crate3.set	Apr 3		
044_new.set	Apr 3	At 2621 roughly	
006_new.set	Apr 3		
006_crate3.set	Apr 3		
006.set	Apr 3		
044	Apr 1	At 3570 (see Halog)	
043	Mar 24	Somewhere in 3320s/3330s	
042	Mar 23		
041	Mar 23	I think something like 3290/3270	
040	Mar 22	I think 3268	
039	Mar 22		Only I
03-04-2006	Mar 15	3016, I think	
038	Mar 1	2477 (See Halog)	
037	Feb 27		Co
036	Feb 27		Co
035	Feb 24		Co

Table A.1: Table of the list of high voltage changes (HV) for the neutron arm in the E02-013 experiment.



Number of Events	A only	A with $> 1$ A	B with $> 1$ A
0	35.5%	92.7%	08.5%
At least 1	59.7%	06.8%	84.0%
At least 2	04.4%	00.4%	06.8%

Table A.2: A table of the correlation between hits in Veto plane 1 and Veto plane 2. The is for all events within a single run in kinematic 2A (3190).

## Appendix B

### Error Analysis

In the measurement of the electric form factor of the neutron at  $Q^2 = 3.5 \text{ GeV}^2$  statistical uncertainty is the main component of the error. The double polarization method insures small systematic uncertainties (as described in section ??). Due to the small asymmetry, large statistics are required. The primary cause of the small asymmetry is the small size of  $G_E^n$ .

The error analysis for E02-013 entails propagation of correlated and uncorrelated systematic and statistical uncertainties for the large number of equations used to calculate  $G_E^n$  from the measured quantities. Equation 5.43 gives the factors that contribute to  $G_E^n$ . The uncertainty in a few of these factors give the most important contributions to the final precision of the  $G_E^n$  measurement.

For the propagation of the uncertainties, standard error techniques are used [79]. Assuming the errors are uncorrelated the expression to determine the uncertainty in a function  $q$  which depends on the components  $x, \dots, z$  is

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x\right)^2 + \dots + \left(\frac{\partial q}{\partial z} \delta z\right)^2}. \quad (\text{B.1})$$

When  $q$  is proportional to the product or quotient of the components,

$$\frac{\delta q}{q} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \dots + \left(\frac{\delta z}{z}\right)^2}. \quad (\text{B.2})$$

The correlated systematic errors also contribute to the final uncertainty. As an example, there were not independent measurements of polarization for every run, so

the systematic error was defined to be the same fractional error as the polarization after the summation, rather than be included in the summation error calculation (equation 5.47). Explicit discussion of the error analysis using formula B.1 follows.

For statistical error, assuming a poisson distribution, the formula

$$\frac{\delta N}{N} = \frac{1}{\sqrt{N}} \quad (\text{B.3})$$

is used, where  $N$  is the pertinent statistics. For small asymmetries and large statistics method of determination of the uncertainty is valid [68]

$$\delta A \approx \frac{1}{\sqrt{N}} - \frac{(N^+ - N^-)^2}{N^{\frac{5}{2}}} \approx \frac{1}{\sqrt{N}} . \quad (\text{B.4})$$

The formula B.4 is used to determine the statistical error in empirically measured asymmetries like the raw accidental background asymmetry and raw neutron asymmetry. The statistical uncertainty for quasi-elastic event counts, background counts, and other counts is determined using equation B.3.

The following dilution factors, polarizations, and asymmetry corrections are used in the calculation of the physical asymmetry (equation ??):  $D_{back}$ ,  $D_{N_2}$ ,  $A_{back}$ ,  $D_{N_2}^p$ ,  $A_{sum}$ ,  $D_p$ ,  $D_n^p$ ,  $P_n$ ,  $P_{beam}$ ,  $P_{3He}$  (for description see below). Some of these depend on the same sources of uncertainty. Common sources of uncertainty in these factors and corrections are a systematic uncertainty due to the charge determination in the accidental background technique (see section 5.5.1) and statistical uncertainties due to the raw neutral counts and the raw background counts. While the systematic uncertainty due to the charged to uncharged background ratio is a component of almost all dilutions and corrections, the small number of raw background makes it of minimal importance. This is observed in Figure B where the change

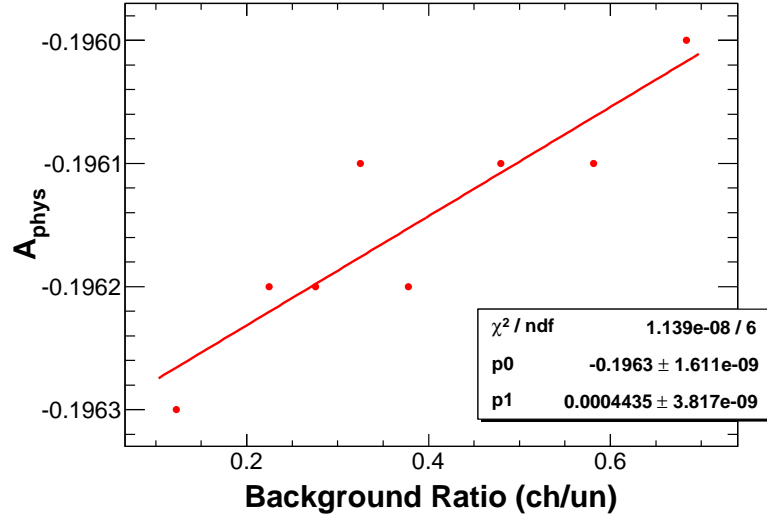


Figure B.1: The physical asymmetry as the accidental background ratio of charged to uncharged used in the calculation is changed. This figure shows that the dependence on this ratio is very small, with an estimated fractional uncertainty on the accidental background charge ratio of 20%. This estimated fractional uncertainty corresponds to a fractional uncertainty of less than .2%. Plotted is a linear fit, showing the small dependence. This is for an analysis of Kinematic 2(b).

of neutron physical asymmetry as a function of the charge ratio of the background is plotted for kinematic 2. This ratio will be treated as a constant for the rest of this error analysis. The value for this ratio and accidental background counts is presented in Table B.3.

As described in section 4.4.2 the polarization changes run to run, and a modified asymmetry is calculated on a run per run basis, and then used in a weighted sum for the entire kinematic (as derived in Appendix C). The only components which change significantly on a per run basis are the target polarization and neutron purity factor. The neutron purity factor and the rest of the dilutions are corrected for after

the summation (see equation 5.48)

$$A_{sum} = \frac{\sum_i \frac{A_{raw,i}}{P_{3He,i}} N_i P_{3He,i}^2}{\sum_i N_i P_{3He,i}^2} . \quad (\text{B.5})$$

The factor  $A_{back}$  can be found by using equation

$$A_{back} = \frac{N_{back}^+ - N_{back}^-}{N \langle P_{3He} \rangle} = A_{back,raw} \frac{N_{neut,back}}{N \langle P_{3He} \rangle} . \quad (\text{B.6})$$

The accidental background asymmetry correction contributes to the statistical uncertainty through the counts in the raw accidental background and quasi-elastic samples and through the accidental background asymmetry. As described in section 5.5.1, the neutral accidental background is related to the raw accidental background by  $N_{neut,back} = f_{back} N_{back}$ . Additionally, it contributes to the systematic uncertainty through the target polarization.

The polarization of the target and nucleus and beam are presented in sections 4.3 and 4.4.2 and in [32]. The target polarization ( $P_{3He}$ ) uncertainty is correlated between kinematics 3, 4, and 2(b). As mentioned in 5.7.1, the values for the polarizations of the protons and neutron within the helium nucleus depend on the theoretical model used to calculate the nuclear corrections. Changes in these values will be correlated. The value used for the beam polarization ( $P_{beam}$ ) and systematic uncertainty is arrived at from inspecting Table 4.2.

The dilution caused by the accidental background (equation 5.38) is written as

$$D_{back} = 1 - \frac{N_{neut,back}}{N} . \quad (\text{B.7})$$

The sources statistical uncertainty are the counts of the raw accidental background

and quasi-elastic samples, similar to the accidental background asymmetry correction.

The dilution from the nitrogen in the neutron sample (equation 5.40 gives the method to properly calculate this) can be written as

$$D_{N_2} = 1 - \frac{N_{N_2}}{N - N_{neut,back}} . \quad (\text{B.8})$$

with a dependence on the uncertainties in the constants shown in Table B.1. The dominant sources of systematic uncertainty in this factor are from the  $N_2$  and  $N_2$  in the  $^3\text{He}$  target densities. The dominant source of statistical uncertainty is due to the nitrogen quasi-elastic events and background. The uncertainties for this dilution were propagated using equation B.1. The results for this error analysis are also presented in Table ?? . This uncertainty is treated as independent from the other uncertainties, although there is a small contribution from the  $^3\text{He}$  quasi-elastic sample.

The dilution of the protons in the neutron sample (equation 5.32) can be expressed as

$$D_n = 1 - \frac{N_{proton}}{N - N_{neut,back} - N_{N_2}} . \quad (\text{B.9})$$

Here the sources of uncertainty are statistical from the background, charged quasi-elastic, neutral quasi-elastic in  $\text{H}_2$ ,  $\text{N}_2$  and  $^3\text{He}$ , and systematic from the technique and model. It is assumed that the statistical uncertainty from Minuit and the systematic uncertainty from the technique dominates all other sources of error. By considering other analyses, a fractional systematic error of 5% was arrived at. The cited statistical uncertainty is that given by Minuit fitting [?].

Finally, the dilution caused by other final state interactions within the neutron sample can be expressed as

$$D_{FSI} = 1 - \frac{N_{FSI}}{N - N_{neut,back} - N_{N_2} - N_{proton}} . \quad (B.10)$$

Because the dilution due to final state interactions is undetermined but guided by theory [?][75], an estimation is determined for the systematic uncertainty that accounts for the likely values of this dilution.

The proton asymmetry has similar dilutions as the neutral asymmetry, with a couple of important differences. This formula 5.54 is

$$A_{proton} = \frac{1 - D_n}{D_{back} D_{N_2}} P_{beam} P_{prot} A_{p,phys} . \quad (B.11)$$

The polarization of protons within the nucleus,  $P_{prot}$  is provided by theory []. The proton physical asymmetry is calculated from the proton form factor ratio as described in equation 5.49:

$$A_{p,phys} = \frac{B\lambda_{proton} + C}{\lambda_{proton}^2 + D} . \quad (B.12)$$

Here the uncertainty in  $\lambda_{proton}$  is determined from the parameterization [48]. The calculation and uncertainties for the quantities used in the determination of  $A_{proton}$  are in Table B.2.

A reminder that the expression for the physical asymmetry is (equation 5.43)

$$A_{phys} = \frac{A_{sum} - A_{back} - A_{proton}}{P_n P_{beam} D_{back} D_{N_2} D_p D_{fsi}} . \quad (B.13)$$

By applying equation B.1 to equation 5.43, the statistical and systematic uncertainty of the physical asymmetry is calculated. These results, in addition to the

	$D_{N_2}$			$D_{N_2}^c$		
$Q^2 =$	2.5 GeV <sup>2</sup>	3.5 GeV <sup>2</sup>	1.5 GeV <sup>2</sup>	2.5 GeV <sup>2</sup>	3.5 GeV <sup>2</sup>	1.5 GeV <sup>2</sup>
$\rho_{^3He}$	0.164	0.164	0.164	0.164	0.164	0.164
$\rho_{N_2}$	4.4	3.6	9.1	4.4	3.6	9.1
$Q_{^3He}$	184399	1441010	—	184399	1441010	—
$Q_{N_2}$	14020	37920	—	14020	37920	—
$N_{^3He}$	2089.1	2686.2	—	19076.7	30331	—
$N_{N_2}$	220.5	28.5	—	1123.2	263.5	—
$D_{N_2}$	0.9493	0.9818	—	0.9714	0.9851	—
Statistical	0.0036	0.0035	—	0.0009	0.0009	—
Systematic	0.0047	0.0015	—	0.0027	0.0013	—

Table B.1: A table showing the measurements and error calculations for the nitrogen dilution of the uncharged sample, for all three kinematics.  $N_{^3He}$  is the number of background corrected counts for the selected  $^3He$  runs,  $\rho_{N_2}$  is the nitrogen density,  $Q_{^3He}$  is the total charge of the selected  $^3He$  runs. This calculation is as described in section 5.5.2.

components contribution to the uncertainties, are presented in Table ?? for kinematic 2a, Table B.5 for kinematic 2b, Table B.6 for kinematic 3, and Table B.7 for kinematic 4.

The uncertainty in the ratio of neutron form factors, or  $\lambda$ , is determined using equation B.1 and 5.55. This is a straight forward calculation [31], but the value for  $\lambda$  is only determined for the central value of  $Q^2$ . To determine the value for the electric form factor of the neutron, equations B.2 and 5.58 were used. Results are



<i>parameter</i>	$Q^2 = 2.5 \text{ GeV}^2$	$Q^2 = 3.5 \text{ GeV}^2$	$Q^2 = 1.5 \text{ GeV}^2$
$G_E^p$	$0.0330 \pm 0.0051$	$0.018 \pm 0.004$	$0.077 \pm 0.007$
$G_M^p$	$0.1378 \pm 0.0049$	$0.089 \pm 0.004$	$0.262 \pm 0.006$
$A_{p,phys}$	$0.177 \pm 0.028$	$0.182 \pm 0.030$	$0.0031$
$P_{prot}$	$-0.028 \pm 0.004$	$-0.028 \pm 0.004$	$-0.028 \pm 0.004$
$P_{beam}$	$0.835 \pm (0.03)$	$0.835 \pm (0.03)$	$0.835 \pm 0.03$
$D_{N2}$	$0.949 \pm 0.004 \pm (0.004)$	$0.982 \pm 0.004 \pm (0.002)$	$0.946 \pm 0.002$
$D_{back}$	$0.9792 \pm 0.0003$	$0.9732 \pm 0.0005$	$0.993$
$D_n$	$0.8 \pm 0.023 \pm (0.04)$	$0.865 \pm 0.046 \pm (0.043)$	$0.677 \pm 0.025$
$A_{proton}$	$-0.0009 \pm 0.0002$	$-0.0006 \pm 0.0001$	$-0.0005 \pm 0.0002$

Table B.2: A table of the errors for the calculation of  $A_{proton}$ . The calculation is calculated according to equation 5.54. The uncertainty in  $G_M^p$  and  $G_E^p$  is determined using a parameterization [48] and the propagation of the error is according to equation B.1.

displayed in Table B.4.

<i>parameter</i>	$Q^2 = 2.5 \text{ GeV}^2$	$Q^2 = 3.5 \text{ GeV}^2$	$Q^2 = 1.5 \text{ GeV}^2$
Hadron Ratio	$0.325 \pm 0.065$	$0.26 \pm 0.04$	$0.33 \pm 0.05$
Hadons	2495	4201	2762
Neutral	1883.2	1092.3	911.5
Charged	611.8	3108.7	1850.5

Table B.3: A table for all three kinematics showing the measurements for the charged and uncharged background and the uncertainty in the background ratio.

<i>parameter</i>	$Q^2 = 2.5 \text{ GeV}^2$	$Q^2 = 3.5 \text{ GeV}^2$	$Q^2 = 1.5 \text{ GeV}^2$
GEn/GMn	-0.232	-0.250	0.0031
Error	$\pm 0.0333 \pm (0.031)$	$\pm 0.047 \pm (0.021)$	$\pm 0.0004 \pm 0.0004$
GMn	$-0.0955 \pm 0.0018$	$-0.0557 \pm 0.0011$	0.00356
GEn	0.0221	0.0139	0.01552
Error	$\pm 0.0032 \pm (0.0030)$	$\pm 0.0026 \pm (0.0012)$	$\pm 0.0058 \pm 0.0030$

Table B.4: A table of the errors for the calculation of  $G_E^n$ . Uncertainty in the ratio of form factors was calculated from the physical asymmetry as expressed in section 5.6 and Table 5.9 and Table 5.8. The uncertainty in  $G_M^n$  is determined from a parameterization [48] and propagation of the error is according to equation B.1.

quantity	value	$\sigma_{stat}$	contrib. $A_{phys}$	$\sigma_{syst}$	contrib. $A_{phys}$
$A_{meas}$	-0.0449	$\pm 0.0055$	0.024	—	-
$P_{3He}$	0.4362	—	-	$\pm 0.02$	0.018
$A_{back,raw}$	0.0279	$\pm 0.0004$	0.00004	—	-
$D_{FSI}$	0.95	—	-	$\pm 0.01$	0.002
$D_n$	0.80	$\pm 0.023$	0.006	$\pm 0.04$	0.010
$A_{p,phys}$	0.177	—	-	$\pm 0.028$	0.0003
$P_{beam}$	0.835	—	-	$\pm 0.03$	0.007
$N_{total}$	33001	$\pm 182$	0.0001	—	-
$N_{back}$	4201	$\pm 65$	0.0003	—	-
$P_n$	0.86	—	-	$\pm 0.02$	0.005
$P_{prot}$	-0.028	—	-	$\pm 0.004$	0.0003
$D_{N_2}$	0.949	$\pm 0.0036$	0.00074	$\pm 0.0036$	0.00074
$A_{phys}$	-0.198	-	$\pm 0.024$	-	$\pm 0.022$

Table B.5: A table of the uncertainties that contribute to the physical asymmetry ( $A_{phys}$ ). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation ???. This table shows the errors for  $Q^2 = 2.5$  GeV<sup>2</sup>.

quantity	value	$\sigma_{stat}$	contrib.	$\sigma_{syst}$	contrib.
$A_{meas}$	-0.0356	$\pm 0.0081$	0.030	—	-
$P_{3He}$	0.477	—	-	$\pm 0.02$	0.011
$A_{back,raw}$	0.0225	$\pm 0.0004$	0.00004	—	-
$D_{FSI}$	0.95	—	-	$\pm 0.01$	0.001
$D_n$	0.865	$\pm 0.046$	0.006	$\pm 0.043$	0.006
$A_{p,phys}$	0.182	—	-	$\pm 0.030$	0.0002
$P_{beam}$	0.835	—	-	$\pm 0.03$	0.005
$N_{total}$	15381	$\pm 124$	0.0001	—	-
$N_{back}$	2762	$\pm 53$	0.0003	—	-
$P_n$	0.86	—	-	$\pm 0.02$	0.003
$P_{prot}$	-0.028	—	-	$\pm 0.004$	0.0002
$D_{N_2}$	0.982	$\pm 0.0035$	0.0004	$\pm 0.0015$	0.0002
$A_{phys}$	-0.130	-	$\pm 0.031$	-	$\pm 0.014$

Table B.6: A table of the uncertainties that contribute to the physical asymmetry ( $A_{phys}$ ). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation ???. This table shows the errors for  $Q^2 = 3.5$  GeV<sup>2</sup>.

quantity	value	$\sigma_{stat}$	contrib.	$\sigma_{syst}$	contrib.
$A_{meas}$	-0.05118	$\pm 0.00677$	0.02596	—	-
$P_{3He}$	0.49368	—	-	$\pm 0.02$	0.016078
$A_{back,raw}$	0.05411	$\pm 0.00008$	0.00108	—	-
$D_{FSI}$	0.95	—	-	$\pm 0.05$	0.01047
$D_n$	0.80	$\pm 0.02332$	0.00554	$\pm 0.07287$	0.01730
$A_{p,phys}$	0.17667	—	-	$\pm 0.02755$	0.00024
$P_{beam}$	0.835	—	-	$\pm 0.03$	0.00720
$N_{total}$	21804	$\pm 147.662$	0.00014	—	-
$N_{back}$	2495	$\pm 49.95$	0.00042	—	-
$P_n$	0.86	—	-	$\pm 0.02$	0.00463
$P_{prot}$	-0.028	—	-	$\pm 0.004$	0.00022
$D_{N_2}$	0.975	$\pm 0.0015$	0.00030	$\pm 0.0023$	0.00047
$A_{phys}$	-0.19888	$\pm 0.02655$	-	$\pm 0.02722$	-

Table B.7: A table of the uncertainties that contribute to the physical asymmetry ( $A_{phys}$ ). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation ???. This table shows the errors for kinematic 4.

## Appendix C

### Calculations

#### C.1 Asymmetry Best Value

The formula to determine the best value of a set of gaussian measurements is the one presented in undergraduate textbooks, and the method to determine the best value of a distribution is also presented, but the formula for an asymmetry is not. This method used is known as the maximum likelihood method. For this asymmetry, there are two poisson distributions of  $N_+$  and  $N_-$ .

A poisson distribution is decribed by[79]

$$P_\mu(\nu) = e^{-\mu} \frac{\mu^\nu}{\nu!} , \quad (\text{C.1})$$

where  $\nu$  is the number of occurances and  $\mu$  is the expected number of occurances. The asymmetry is completely described by the number of positive counts with positive helicity and negative helicity,  $N_+^i$  and  $N_-^i$  ( $i$  is the  $i$ th experiment). This gives the likelyhood

$$L = P_{\mu_+}(N_+^i) \times P_{\mu_-}(N_-^i) \times \cdots . \quad (\text{C.2})$$

The  $\mu$ , or expected number of occurances, for some run is identified as

$$\mu_\pm = (N_+^e + N_-^e) \frac{1 \pm A^e}{2} = \frac{N_+^e + N_-^e \pm (N_+^e - N_-^e)}{2} = N_\pm^e . \quad (\text{C.3})$$

Here the index  $e$  gives the expected value for that given run period, while  $i$  is the

ith measured value. The expected value of counts for  $+$  and  $-$  give the best value for the asymmetry.

There are many measurements (given by index  $i$ ), all having their own best value for the counts to give the best asymmetry. This allows us to have

$$\mu_{\pm,i} = \kappa_{\pm,i} = \kappa_i \frac{1 \pm A_i^e}{2} , \quad (\text{C.4})$$

where  $\kappa$  is the correct  $N$  to get the best asymmetry. Following the previous work for the poisson distribution

$$\sum \frac{\partial \ln L}{\partial A} = (-1 + \frac{N_{\pm i}}{\mu_{\pm i}})(\pm \frac{\kappa_i P_i}{2}) = \sum \frac{N_{i+} P_i}{1 - A P_i} - \frac{N_{i-} P_i}{1 + A P_i} = 0 . \quad (\text{C.5})$$

Using the standard expansion

$$(1 \pm A P_i)^{-1} = 1 - \pm P_i A , \quad (\text{C.6})$$

the relationship

$$\sum P_i (N_{i+} - N_{i-}) = - \sum P_i^2 N_i A \quad (\text{C.7})$$

is arrived at, which gives

$$A = \frac{- \sum P_i (N_{i+} - N_{i-})}{\sum P_i^2 N_i} . \quad (\text{C.8})$$

Putting it into a more recognizable form

$$A = \frac{- \sum P_i^2 N_i \frac{A_i}{P_i}}{\sum P_i^2 N_i} . \quad (\text{C.9})$$

Which leaves the gaussian definitions of sigma and measurements

$$W_i = \frac{1}{\sigma_i^2} = P_i^2 N_i . \quad (\text{C.10})$$

## Bibliography

- [1] *Techniques for Nuclear and Particle Physics Experiments: A How-to Approach*. Oxford University Press, 2009.
- [2] Sergey Abramhof. personal communication, 2006-2009.
- [3] J. Alcorn et al. Basic Instrumentation for Hall A at Jefferson Lab. *Nucl. Instrum. Meth.*, A522:294–346, 2004.
- [4] C. Amsler et al. Review of particle physics. *Phys. Lett.*, B667:1, 2008.
- [5] Hall A analyzer. <http://hallaweb.jlab.org/root/index.html>.
- [6] H. Arenhovel. ON DEUTERON BREAKUP BY ELECTRONS AND THE MOMENTUM DISTRIBUTION OF NUCLEONS IN THE DEUTERON. *Nucl. Phys.*, A384:287–301, 1982.
- [7] H. Arenhovel. ON THE DETERMINATION OF THE ELECTRIC NEUTRON FORM-FACTOR IN D (E POLARIZED), E-PRIME (N POLARIZED) P. *Phys. Lett.*, B199:13–16, 1987.
- [8] H. Arenhovel, W. Leidemann, and E. L. Tomusiak. THE ROLE OF THE NEUTRON ELECTRIC FORM-FACTOR IN D (E, E- PRIME N) N INCLUDING POLARIZATION OBSERVABLES. *Z. Phys.*, A331:123–138, 1988.
- [9] H. Arenhovel, W. Leidemann, and E. L. Tomusiak. General formulae for polarization observables in deuteron electrodisintegration and linear relations. *Few Body Syst.*, 15:109–127, 1993.
- [10] R. G. Arnold, Carl E. Carlson, and Franz Gross. Polarization transfer in elastic electron scattering from nucleons and deuterons. *Phys. Rev.*, C23:363, 1981.
- [11] J. Becker et al. Determination of the neutron electric form factor from the reaction  $\text{he-3}(e, e' \text{ n})$  at medium momentum transfer. *Eur. Phys. J.*, A6:329–344, 1999.
- [12] Andrei V. Belitsky, Xiang-dong Ji, and Feng Yuan. A perturbative qcd analysis of the nucleon’s pauli form factor  $f_2(q^2)$ . *Phys. Rev. Lett.*, 91:092003, 2003.
- [13] J. Bermuth et al. The neutron charge form factor and target analyzing powers from  $\text{he-3}(\text{pol.})(e(\text{pol.}), e' \text{ n})$  scattering. *Phys. Lett.*, B564:199–204, 2003.
- [14] M. S. Bhagwat, I. C. Cloet, and C. D. Roberts. Covariance, Dynamics and Symmetries, and Hadron Form Factors. 2007.
- [15] R. Bijker and F. Iachello. Re-analysis of the nucleon space- and time-like electromagnetic form factors in a two-component model. *Phys. Rev.*, C69:068201, 2004.



- [16] B. Blankleider and R. M. Woloshyn. Quasielastic scattering of polarized electrons on polarized he-3. *Phys. Rev.*, C29:538, 1984.
- [17] Stanley J. Brodsky. Light-front QCD. 2004.
- [18] Stanley J. Brodsky and Guy F. de Teramond. Light-front hadron dynamics and AdS/CFT correspondence. *Phys. Lett.*, B582:211–221, 2004.
- [19] Stanley J. Brodsky, Susan Gardner, and Dae Sung Hwang. Discrete symmetries on the light front and a general relation connecting nucleon electric dipole and anomalous magnetic moments. *Phys. Rev.*, D73:036007, 2006.
- [20] Matthias Burkardt. Impact parameter space interpretation for generalized parton distributions. *Int. J. Mod. Phys.*, A18:173–208, 2003.
- [21] G. Cates, K. McCormick, B. Reitz, and B. Wojtsekhowski. Jlab proposal 02-013. 2002.
- [22] I. C. Cloet and C. D. Roberts. Form Factors and Dyson-Schwinger Equations. *PoS*, LC2008:047, 2008.
- [23] T. De Forest. Off-shell electron nucleon cross-sections. the impulse approximation. *Nucl. Phys.*, A392:232–248, 1983.
- [24] M. Diehl. Generalized parton distributions from form factors. *Nucl. Phys. Proc. Suppl.*, 161:49–58, 2006.
- [25] M. Diehl. Generalized parton distributions: Recent results. *AIP Conf. Proc.*, 842:294–302, 2006.
- [26] T. W. Donnelly and A. S. Raskin. Considerations of polarization in inclusive electron scattering from nuclei. *Annals Phys.*, 169:247–351, 1986.
- [27] S. Kadlecik T. G. Walker E. Babcock, I. A. Nelson. *Phys. Rev. A*, 71:013414, 2005.
- [28] T. Eden et al. Electric form-factor of the neutron from the h-2 (e (polarized), e-prime n (polarized) ) h-1 reaction at  $q^{*2} = 0.255\text{-(gev/c)}^{*2}$ . *Phys. Rev.*, C50:1749–1753, 1994.
- [29] W. Fabian and H. Arenhovel. ELECTRODISINTEGRATION OF DEUTERIUM INCLUDING NUCLEON DETECTION IN COINCIDENCE. *Nucl. Phys.*, A314:253–286, 1979.
- [30] Robert Feuerbach. personal communication, 2007.
- [31] G.B. Franklin. Gen asymmetry corrections for finite angular acceptance., 2006.
- [32] J.L. Friar et al. Neutron polarization in polarized  $^3\text{he}$  targets. *Phys. Rev. C*, 42:2310–2314, 1990.

- [33] S. Galster et al. Elastic electron - deuteron scattering and the electric neutron form-factor at four momentum transfers  $5\text{-fm}^{*-2}$  ;  $q^{*2}$  ;  $14\text{-fm}^{*-2}$ . *Nucl. Phys.*, B32:221–237, 1971.
- [34] Alan Gavalya. personal communication, 2006-2009.
- [35] D. I. Glazier et al. Measurement of the electric form factor of the neutron at  $q^{*2} = 0.3\text{-}0.8 \text{ (gev/c)}^{*2}$ . *Eur. Phys. J.*, A24:101–109, 2005.
- [36] J. Golak, G. Ziemer, H. Kamada, H. Witala, and Walter Gloeckle. Extraction of electromagnetic neutron form factors through inclusive and exclusive polarized electron scattering on polarized he-3 target. *Phys. Rev.*, C63:034006, 2001.
- [37] M. Guidal, M. V. Polyakov, A. V. Radyushkin, and M. Vanderhaeghen. Nucleon form factors from generalized parton distributions. *Phys. Rev.*, D72:054013, 2005.
- [38] F. Halzen and A. Martin. *Quarks and Leptons*. John Wiley and Sons, 1984.
- [39] C. Herberg et al. Determination of the neutron electric form factor in the  $d(e,e' n)p$  reaction and the influence of nuclear binding. *Eur. Phys. J.*, A5:131–135, 1999.
- [40] Coda homepage. <http://coda.jlab.org>.
- [41] E. S. Hrycyshyn and L. Krause. *Canadian Journal of Physics*, 48:2761, 1970.
- [42] R. FRISCH O. STERN I. ESTERMANN. Magnetic Moment of the Proton. *Nature*, 132:169–170, 1933.
- [43] Hall A Information. <http://hallaweb.jlab.org>.
- [44] C. E. Jones-Woodward et al. Determination of the neutron electric form-factor in quasielastic scattering of polarized electrons from polarized he-3. *Phys. Rev.*, C44:571–574, 1991.
- [45] A. Kelleher. *A Measurement of the Neutron Electric Form Factor at Very large Momentum Transfer Using Polarized Electrons Scattering from A Polarized Helium-3 Target*. PhD thesis, College of William and Mary, 2009.
- [46] Aidan Kelleher. personal communication, 2006-2009.
- [47] J. J. Kelly. Nucleon knockout by intermediate-energy electrons. *Adv. Nucl. Phys.*, 23:75–294, 1996.
- [48] J. J. Kelly. Simple parametrization of nucleon form factors. *Phys. Rev.*, C70:068202, 2004.
- [49] F. Klein and H. Schmieden. Electromagnetic nucleon form-factors in the space - like region. *Nucl. Phys.*, A623:323c–332c, 1997.

- [50] A. Kolarkar. *Precision Measurements of the Neutron Electric Form Factor at High Momentum Transfers*. PhD thesis, University of Kentucky, 2007.
- [51] Ameya Kolarkar. personal communication, 2006-2007.
- [52] C. W. Leemann, D. R. Douglas, and G. A. Krafft. The Continuous Electron Beam Accelerator Facility: CEBAF at the Jefferson Laboratory. *Ann. Rev. Nucl. Part. Sci.*, 51:413–450, 2001.
- [53] G. Peter Lepage and Stanley J. Brodsky. Exclusive Processes in Perturbative Quantum Chromodynamics. *Phys. Rev.*, D22:2157, 1980.
- [54] Earle L. Lomon. Effect of recent R(p) and R(n) measurements on extended Gari-Kruempelmann model fits to nucleon electromagnetic form factors. *Phys. Rev.*, C66:045501, 2002.
- [55] R. Madey et al. Measurements of  $g(e)(n)/g(m)(n)$  from the  $h-2(e(pol.),e' n(pol.))h-1$  reaction to  $q^{*2} = 1.45-(\text{gev}/c)^{*2}$ . *Phys. Rev. Lett.*, 91:122002, 2003.
- [56] M. Meyerhoff et al. First measurement of the electric form-factor of the neutron in the exclusive quasielastic scattering of polarized electrons from polarized  $he-3$ . *Phys. Lett.*, B327:201–207, 1994.
- [57] Gerald A. Miller. Light front cloudy bag model: Nucleon electromagnetic form factors. *Phys. Rev.*, C66:032201, 2002.
- [58] Gerald A. Miller. Charge Density of the Neutron. *Phys. Rev. Lett.*, 99:112001, 2007.
- [59] T. Ngo. *Neutron arm study and calibration for the gen experiment at thomas jefferson national laboratory*. PhD thesis, California State University, 2007.
- [60] T. Ngo. Geometry document. 2008.
- [61] M. Ostrick et al. Measurement of the neutron electric form factor  $g(e,n)$  in the quasifree  $h-2(e(pol.),e' n(pol.))p$  reaction. *Phys. Rev. Lett.*, 83:276–279, 1999.
- [62] I. Passchier et al. The charge form factor of the neutron from the reaction  $h-2(pol.)(e(pol.),e' n)p$ . *Phys. Rev. Lett.*, 82:4988–4991, 1999.
- [63] C. F. Perdrisat, V. Punjabi, and M. Vanderhaeghen. Nucleon electromagnetic form factors. 2006.
- [64] B. Plaster. *The Neutron Electric Form Factor to  $Q^{*2} = 1.45-(\text{GeV}/c)^{*2}$* . PhD thesis, MIT, 2004.
- [65] B. Plaster et al. Measurements of the neutron electric to magnetic form factor ratio  $g(e(n))/g(m(n))$  via the  $h-2(e(pol.),e' n(pol.))h-1$  reaction to  $q^{*2} = 1.45-(\text{gev}/c)^{*2}$ . *Phys. Rev.*, C73:025205, 2006.

- [66] Igor Rachek. personal communication, 2005.
- [67] A. S. Raskin and T. W. Donnelly. Polarization in coincidence electron scattering from nuclei. *Ann. Phys.*, 191:78, 1989.
- [68] S. Riordan. *Measurements of the Electric Form Factor of the Neutron at  $Q^2 = 1.7$  and  $3.5$  (GeV/c) $^2$* . PhD thesis, Carnegie Mellon University, 2008.
- [69] Seamus Riordan. personal communication, 2006-2009.
- [70] C. D. Roberts. Hadron Properties and Dyson-Schwinger Equations. *Prog. Part. Nucl. Phys.*, 61:50–65, 2008.
- [71] D. Rohe et al. Measurement of the neutron electric form factor  $g_{\text{en}}$  at 0.67-(gev/c) $^2$  via  $\text{he-3}(\text{pol.})(\text{e}(\text{pol.}), \text{e}' \text{ n})$ . *Phys. Rev. Lett.*, 83:4257–4260, 1999.
- [72] M. V. Romalis and G. D. Cates. *Phys. Rev. A*, 58:3004, 1998.
- [73] Cern root homepage. <http://root.cern.ch>.
- [74] R. G. Sachs. High-energy behavior of nucleon electromagnetic form factors. *Phys. Rev*, 126(6):2256–2260, jun 1962.
- [75] Misak M. Sargsian. Selected topics in high energy semi-exclusive electro- nuclear reactions. *Int. J. Mod. Phys.*, E10:405–458, 2001.
- [76] R. Schiavilla and I. Sick. Neutron charge form factor at large  $q^2$ . *Phys. Rev.*, C64:041002, 2001.
- [77] H. Schmieden. Double polarization experiments at intermediate energy. *Nucl. Phys.*, A663:21–28, 2000.
- [78] Jaidrep Singh. personal communication, 2006-2009.
- [79] John R. Taylor. *Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*. University Science Books, 1996.
- [80] A. K. Thompson et al. Quasielastic scattering of polarized electrons from polarized he-3 and measurement of the neutron’s form- factors. *Phys. Rev. Lett.*, 68:2901–2904, 1992.
- [81] Bogdan W. personal communication, 2007-2009.
- [82] G. Warren et al. Measurement of the electric form factor of the neutron at  $q^2 = 0.5\text{-gev/c}^2$  and  $1.0\text{-gev/c}^2$ . *Phys. Rev. Lett.*, 92:042301, 2004.