

OPTICS CALIBRATION FOR THE E06-007 EXPERIMENT

1 – DESCRIPTION OF THE DATABASE WITH THE TENSOR ELEMENTS

The Transport Tensor links the focal plane coordinates to the target coordinates. The relationship between the focal plane and target coordinates can be written (in a first-order approximation) as:

$$\begin{bmatrix} \delta \\ \theta \\ y \\ \phi \end{bmatrix}_{tg} = \begin{bmatrix} \langle \delta | x \rangle & \langle \delta | \theta \rangle & 0 & 0 \\ \langle \theta | x \rangle & \langle \theta | \theta \rangle & 0 & 0 \\ 0 & 0 & \langle y | y \rangle & \langle y | \phi \rangle \\ 0 & 0 & \langle \phi | y \rangle & \langle \phi | \phi \rangle \end{bmatrix} \cdot \begin{bmatrix} x \\ \theta \\ y \\ \phi \end{bmatrix}_{fp} \quad (1.1)$$

In practice, the expansion of the focal plane coordinates is performed up to the fifth order. A set of tensors link the focal plane coordinates to target coordinates according to:

$$\begin{aligned} y_{tg} &= \sum_{jkl} \left[\sum_i C_i^{Y_{jkl}} \cdot x_{fp}^i \right] \cdot \theta_{fp}^j \cdot y_{fp}^k \cdot \phi_{fp}^l \\ \theta_{tg} &= \sum_{jkl} \left[\sum_i C_i^{T_{jkl}} \cdot x_{fp}^i \right] \cdot \theta_{fp}^j \cdot y_{fp}^k \cdot \phi_{fp}^l \\ \phi_{tg} &= \sum_{jkl} \left[\sum_i C_i^{P_{jkl}} \cdot x_{fp}^i \right] \cdot \theta_{fp}^j \cdot y_{fp}^k \cdot \phi_{fp}^l \\ \delta_{tg} &= \sum_{jkl} \left[\sum_i C_i^{D_{jkl}} \cdot x_{fp}^i \right] \cdot \theta_{fp}^j \cdot y_{fp}^k \cdot \phi_{fp}^l \end{aligned} \quad (1.2)$$

Because of mid-plane symmetry of the spectrometers (k+l) should be odd for Y and P and even for D and T. The coefficients are stored in the database. For example, for δ :

CURRENT DATABASE – db_L.vdc- LEFT ARM) → deltaP

$$\begin{aligned}
 & \text{j k l} \\
 D_i 0 0 0 & \rightarrow 0.0 + 0.083831x_{fp} + 0.01032x_{fp}^2 \\
 D_i 1 0 0 & \rightarrow \theta_{fp} \cdot [-0.00501 + 0.2406x_{fp} + 0.015912x_{fp}^2] \\
 D_i 2 0 0 & \rightarrow \theta_{fp}^2 \cdot [-1.0088 + 1.0400x_{fp} + 0.40717x_{fp}^2] \\
 D_i 0 0 2 & \rightarrow \phi_{fp}^2 \cdot [-0.12537 + 0.64795x_{fp}] \\
 D_i 0 2 0 & \rightarrow y_{fp}^2 \cdot [0.06862 - 0.10922x_{fp}] \\
 D_i 0 1 1 & \rightarrow y_{fp} \cdot \theta_{fp} \cdot [0.98321 + 1.6114x_{fp}] \\
 D_i 1 0 2 & \rightarrow \theta_{fp} \cdot \phi_{fp}^2 \cdot [-17.765] \\
 D_i 1 1 1 & \rightarrow \theta_{fp} \cdot y_{fp} \cdot \phi_{fp} \cdot [-39.851] \\
 D_i 2 0 2 & \rightarrow \theta_{fp}^2 \cdot \phi_{fp}^2 \cdot [-1018.8] \\
 D_i 2 1 1 & \rightarrow \theta_{fp}^2 \cdot \phi_{fp} \cdot y_{fp} \cdot [-1653.8] \\
 D_i 0 2 2 & \rightarrow \phi_{fp}^2 \cdot y_{fp}^2 \cdot [535.78]
 \end{aligned}$$

CURRENT DATABASE – db_R.vdc- RIGHT ARM)

$$\begin{aligned}
 & \text{j k l} \\
 D_i 0 0 0 & \rightarrow 0.0 + 0.083113x_{fp} + 0.012361x_{fp}^2 + 0.0015953x_{fp}^3 - 0.0039872x_{fp}^4 \\
 D_i 1 0 0 & \rightarrow \theta_{fp} \cdot [-0.019043 + 0.25641x_{fp} + 0.021779x_{fp}^2 + 0.13072x_{fp}^3] \\
 D_i 2 0 0 & \rightarrow \theta_{fp}^2 \cdot [-1.0864 + 0.88496x_{fp} - 1.9431x_{fp}^2] \\
 D_i 0 0 2 & \rightarrow \phi_{fp}^2 \cdot [0.019538 + 0.30371x_{fp}] \\
 D_i 0 2 0 & \rightarrow y_{fp}^2 \cdot [0.28829 - 0.29092x_{fp} - 0.31134x_{fp}^2] \\
 D_i 0 1 1 & \rightarrow y_{fp} \cdot \theta_{fp} \cdot [0.45943 - 0.074564x_{fp} + 0.89922x_{fp}^2] \\
 D_i 3 0 0 & \rightarrow \theta_{fp}^3 \cdot [-12.167 - 13.129x_{fp}] \\
 D_i 1 2 0 & \rightarrow \theta_{fp} \cdot y_{fp}^2 \cdot [-8.2555 + 20.051x_{fp}] \\
 D_i 1 0 2 & \rightarrow \theta_{fp} \cdot \phi_{fp}^2 \cdot [-12.167 - 13.129x_{fp}] \\
 D_i 1 1 1 & \rightarrow \theta_{fp} \cdot y_{fp} \cdot \phi_{fp} \cdot [-39.851]
 \end{aligned}$$

CURRENT DATABASE – db_L.vdc- LEFT ARM) → Theta

$$\begin{aligned}
 & \quad j \quad k \quad l \\
 T_i 0 0 0 & \rightarrow -1.0018 - 0.33183x_{fp} - 0.043355x_{fp}^2 \\
 T_i 1 0 0 & \rightarrow \theta_{fp} \cdot [-2.3427 + 0.60230x_{fp} + 0.12252x_{fp}^2 - 0.085444x_{fp}^2] \\
 T_i 2 0 0 & \rightarrow \theta_{fp} \cdot [-3.9319 - 0.87557x_{fp} + 12.270x_{fp}^2] \\
 T_i 1 0 2 & \rightarrow \theta_{fp} \cdot \phi^2 \cdot [83.781 - 100.86x_{fp}] \\
 T_i 3 0 0 & \rightarrow \theta_{fp}^3 \cdot [41.392 - 370.70x_{fp}]
 \end{aligned}$$

CURRENT DATABASE – db_R.vdc- RIGHT ARM)

$$\begin{aligned}
 & \quad j \quad k \quad l \\
 T_i 0 0 0 & \rightarrow -1.0046 - 0.33492x_{fp} - 0.040787x_{fp}^2 \\
 T_i 1 0 0 & \rightarrow \theta_{fp} \cdot [-2.3551 + 0.56775x_{fp} - 0.044475x_{fp}^2 - 0.13842x_{fp}^2] \\
 T_i 2 0 0 & \rightarrow \theta_{fp}^2 \cdot [-4.2266 + 1.0975x_{fp} + 3.2612x_{fp}^2] \\
 T_i 0 0 2 & \rightarrow \phi_{fp}^2 \cdot [0.63757 + 0.93859x_{fp} - 0.60328x_{fp}^2] \\
 T_i 0 2 0 & \rightarrow y_{fp}^2 \cdot [0.68616 - 2.2501x_{fp} - 3.3522x_{fp}^2] \\
 T_i 1 2 0 & \rightarrow \theta_{fp} \cdot y_{fp}^2 \cdot [-17.294 - 131.87x_{fp}] \\
 T_i 3 0 0 & \rightarrow \theta_{fp}^3 \cdot [205.35 - 125.10x_{fp}] \\
 T_i 2 0 2 & \rightarrow \theta_{fp}^2 \cdot \phi^2 \cdot [-1279.9] \\
 T_i 4 0 0 & \rightarrow \theta_{fp}^4 \cdot [-1084.4]
 \end{aligned}$$

CURRENT DATABASE – db_L.vdc- LEFT ARM) → Phi

$$\begin{aligned}
 & \text{j k l} \\
 P_i 0 0 0 & \rightarrow -0.0014434 - 0.0018217x_{fp} - 0.0012462x_{fp}^2 \\
 P_i 0 0 1 & \rightarrow \phi_{fp} \cdot [-0.6538 - 0.097331x_{fp} + 0.14350x_{fp}^2 - 0.019313x_{fp}^3] \\
 P_i 0 0 3 & \rightarrow \phi_{fp}^3 \cdot [6.0409 - 77.950x_{fp}] \\
 P_i 0 1 0 & \rightarrow y_{fp} \cdot [-0.35366 + 0.29185x_{fp} - 0.047389x_{fp}^2 + 0.15419x_{fp}^3] \\
 P_i 0 1 2 & \rightarrow y_{fp} \cdot \delta_{fp}^2 \cdot [-6.1321 + 14.877x_{fp}] \\
 P_i 0 2 1 & \rightarrow y_{fp}^2 \cdot \phi_{fp} \cdot [-55.248 - 9.8969x_{fp}] \\
 P_i 0 3 0 & \rightarrow y_{fp}^3 \cdot [26.034 + 16.507x_{fp}] \\
 P_i 1 0 1 & \rightarrow \theta_{fp} \cdot \phi \cdot [4.4915 + 1.8355x_{fp}] \\
 P_i 1 0 3 & \rightarrow \theta_{fp} \cdot \phi^3 \cdot [47.409 + 65.451x_{fp}] \\
 P_i 1 1 0 & \rightarrow \theta_{fp} \cdot y_{fp} \cdot [4.2693 + 1.4865x_{fp}] \\
 P_i 1 1 2 & \rightarrow \theta_{fp} \cdot y_{fp} \cdot \phi_{fp}^2 \cdot [-1331.1] \\
 P_i 1 2 1 & \rightarrow \theta_{fp} \cdot y_{fp}^2 \cdot \phi_{fp} \cdot [-943.41] \\
 P_i 1 3 0 & \rightarrow \theta_{fp} \cdot y_{fp}^3 \cdot [1031.0] \\
 P_i 2 0 1 & \rightarrow \theta_{fp}^2 \cdot \phi_{fp} \cdot [3.6884 - 158.65x_{fp}] \\
 P_i 2 1 0 & \rightarrow \theta_{fp}^2 \cdot y_{fp} \cdot [-0.81345 + 68.379x_{fp}] \\
 P_i 3 0 1 & \rightarrow \theta_{fp}^3 \cdot \phi_{fp} \cdot [-890.98] \\
 P_i 3 1 0 & \rightarrow \theta_{fp}^3 \cdot [-2574.5]
 \end{aligned}$$

CURRENT DATABASE – db_R.vdc- RIGHT ARM)

$$\begin{aligned}
 & \text{j k l} \\
 P_i 0 0 0 & \rightarrow -0.00074355 - 0.0021302x_{fp} - 0.001195x_{fp}^2 \\
 P_i 0 0 1 & \rightarrow \phi_{fp} \cdot [-0.61757 - 0.19659x_{fp} + 0.20069x_{fp}^2 + 0.075009x_{fp}^3] \\
 P_i 1 0 0 & \rightarrow \theta_{fp} \cdot [-0.017992 - 0.0070981x_{fp} + 0.041751x_{fp}^2 + 0.0661x_{fp}^3] \\
 P_i 0 1 0 & \rightarrow y_{fp} \cdot [-0.37403 + 0.34111x_{fp} - 0.069323x_{fp}^2 - 0.10191x_{fp}^3] \\
 P_i 0 1 2 & \rightarrow y_{fp} \cdot \phi_{fp}^2 \cdot [27.226 - 59.994x_{fp}] \\
 P_i 1 1 0 & \rightarrow \theta_{fp} \cdot y_{fp} \cdot [3.2109 - 0.37806x_{fp} + 2.9162x_{fp}^2] \\
 P_i 1 0 1 & \rightarrow \theta_{fp} \cdot \phi_{fp} \cdot [5.0843 + 0.56184x_{fp} - 2.3362x_{fp}^2] \\
 P_i 0 0 3 & \rightarrow \phi_{fp}^3 \cdot [5.5608 + 15.148x_{fp}] \\
 P_i 2 0 1 & \rightarrow \theta_{fp}^2 \cdot \phi_{fp} \cdot [-16.784 - 69.989x_{fp}] \\
 P_i 2 1 0 & \rightarrow \theta_{fp}^2 \cdot y_{fp} \cdot [-82.871 - 9.548x_{fp}] \\
 P_i 0 2 1 & \rightarrow y_{fp}^2 \cdot \phi_{fp} \cdot [-6.635 + 99.351x_{fp}] \\
 P_i 0 3 0 & \rightarrow y_{fp}^3 \cdot [-2.9771 - 41.981x_{fp}] \\
 P_i 1 0 3 & \rightarrow \theta_{fp} \cdot \phi^3 \cdot [-198.33] \\
 P_i 1 3 0 & \rightarrow \theta_{fp} \cdot y_{fp}^3 \cdot [153.73] \\
 P_i 3 0 1 & \rightarrow \theta_{fp}^3 \cdot \phi_{fp} \cdot [-352.6] \\
 P_i 3 1 0 & \rightarrow \theta_{fp}^3 \cdot y_{fp}^1 \cdot [-2089.0] \\
 P_i 1 1 2 & \rightarrow \theta_{fp} \cdot y_{fp} \cdot \phi_{fp}^2 \cdot [-1473.8] \\
 P_i 1 2 1 & \rightarrow \theta_{fp} \cdot y_{fp}^2 \cdot \phi_{fp} \cdot [-2260.0]
 \end{aligned}$$

$$\begin{aligned}
P_i 2 0 0 &\rightarrow \theta_{fp} \cdot [-4.2266 + 1.0975x_{fp} + 3.2612x_{fp}^2] \\
P_i 0 0 2 &\rightarrow \phi_{fp} \cdot [0.63757 + 0.93859x_{fp} - 0.60328x_{fp}^2] \\
P_i 1 2 0 &\rightarrow \theta_{fp} \cdot y_{fp}^2 \cdot [-17.294 - 131.87x_{fp}] \\
P_i 3 0 0 &\rightarrow \theta_{fp}^3 \cdot [205.35 - 125.10x_{fp}] \\
P_i 2 0 2 &\rightarrow \theta_{fp}^2 \cdot \phi^2 \cdot [-1279.9] \\
P_i 4 0 0 &\rightarrow \theta_{fp}^4 \cdot [-1084.4]
\end{aligned}$$