Measurement of the Proton Elastic Form Factor Ratio At Low $Q^2$
Proposal PR-08-007 (PR-07-004 Update)

J. Arrington, D. Day, D. Higinbotham, R. Gilman, G. Ron,
A. Sarty spokespersons
a Hall A Collaboration experiment

PAC33, Jan 14-17 2008

- 2 part, high-precision ($<1\%$) measurement of the proton EM form factor ratio $\mu_P G_E / G_M$.
- 2 different methods used.
- Access very low $Q^2$.
- Direct measurement of proton structure, many implications for analysis of other experiments.
The PR-08-007 Collaboration

- Argonne National Lab
- Jefferson Lab
- Rutgers University
- St. Mary's University
- Tel Aviv University
- UVa
- CEN Saclay
- Christopher Newport University
- College of William & Mary
- Duke University
- Florida International University
- Institut de Physique Nuclaire d'Orsay
- Kent State University
- MIT
- Norfolk State University
- Nuclear Research Center Negev
- Old Dominion University
- Pacific Northwest National Lab
- Randolph-Macon College
- Seoul National University
- Temple University
- Université Blaise Pascal
- University of Glasgow
- Jožef Stefan Institute and University of Ljubljana
- University of Maryland
- University of New Hampshire
- University of Regina
- University of South Carolina

And the Hall A Collaboration
Review of Proton Form Factors

Cross section for scattering from a spinless, point-like particle

\[ \frac{d\sigma_{\text{Mott}}}{d\Omega} = \frac{\alpha^2}{Q^2} \left( \frac{E'}{E} \right)^2 \cot^2 \theta_e \]

For a spin-\(\frac{1}{2}\) particle with internal structure

\[ \frac{d\sigma}{d\Omega} = \frac{d\sigma_{\text{Mott}}}{d\Omega} \frac{1}{1 + \tau} \left[ G_E^2 + \frac{\tau}{\varepsilon} G_M^2 \right] \]

\[ \tau = \frac{Q^2}{4M^2}, \quad \varepsilon = \left[ 1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right]^{-1} \]

Lowest order perturbation theory in QED, elastic ep scattering is given by single photon exchange (Born Approximation).
**Review of Proton Form Factors**

- FFs describe the proton internal structure. Related (NR) to the charge and magnetization densities (Fourier).
- FFs Approximately follow Dipole Form
  \[ G_D = \left(1 + \frac{Q^2}{0.71}\right)^{-2} \]
- Define \( R \equiv \mu_P \frac{G_E}{G_M} \). From normalization \( R(Q^2 = 0) = 1 \). If both FFs follow dipole \( R = 1 \).

\[ j_\mu = \bar{u}(k')\gamma_\mu u(k) \]

\[ e(k) \quad \text{Time} \quad e(k') \]

\[ G_E \equiv F_1 - \tau F_2 \ ; \ G_M \equiv F_1 + F_2 \]

\[ d\sigma \quad d\sigma_{\text{Mott}} \quad \frac{1}{1 + \tau} \left[ G_E^2 + \frac{\tau}{\varepsilon} G_M^2 \right] \]

Sachs FF:
Rosenbluth and Polarization methods do not agree at high $Q^2$. Mostly explained by $2\gamma$ exchange.

Deviation from $R = 1$ indicated at low $Q^2$. Virtual meson cloud? (Friedrich & Walcher).


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Deviation from $R = 1$ indicated at low $Q^2$. Virtual meson cloud? (Friedrich & Walcher).
Our focus is on the low $Q^2$ region.

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Friedrich & Walcher Analysis

- **2003** - Bump/Dip structure in all 4 FFs. Plot shows FF data vs. fit: 2-dipoles + bump (deviations are model-dependent and hard to interpret).

- **2007** - LEDEX & Bates BLAST data show deviations from unity. Inconsistent with the F & W analysis.

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Extracting the Individual FFs

Can combine high precision $R$ and cross section data:


Extract individual FFs $(Q^2 = 0.389 \text{ GeV}^2)$:

Complementary to the Mainz XS Measurement

- Mainz experiment has taken data.
- Measured cross sections down to $Q^2 \approx 0.01 \text{GeV}^2$.
- Having cross sections + polarizations:
  - Reduces correlations between $G_E$ and $G_M$.
  - Reduces correlations between $Q^2$ points.
  - Checks experimental consistency (eg. $2-\gamma$).
- Ratio + Mainz data $\rightarrow$ Dataset of individual FFs with unprecedented precision!
Individual FFs (our R + Mainz XS) vs. Mainz alone

Projected uncer. on $G_M^P / G_D$ vs. Mainz (assuming 1% XS)

Projected uncer. on $G_E^P / G_D$ vs. Mainz (assuming 1% XS)
Improved proton+neutron gives improved isoscalar and isovector form factors.

- Absolute uncertainties on proton/neutron FFs are what really matters. Proton has better relative measurements than neutron, but comparable absolute uncertainties.

- Improved proton+neutron gives improved 2 quark flavor decomposition (assumed strange FFs equal 0).
Possible Impacts on other experiments - PV

- Determination of strange quark form factors by HAPPEX and GO parity violation experiments depends on knowledge of the EMFF.
- G. Ron et al. PRL 99, 202002 (2007), adjusts the expected HAPPEX-I non-strange asymmetry by about -0.5ppm, corresponding to a smaller effect from the strange quarks, by about $1/2 \sigma$.
- New results could shift the expected HAPPEX-III result by one standard deviation.
- Knowledge of the effect on GO requires precise form factors over a wide $Q^2$ range.
Zemach Radius

- $E_{\text{hfs}}(e^- p) = (1 + \Delta_{QED} + \Delta_R^p + \Delta_{\mu\nu p}^p + \Delta_{\mu\nu p}^p + \Delta_{\text{weak}}^p + \Delta_S) E_F^p$

- Structure dependent term
  \[ \Delta_S = \Delta_Z + \Delta_{\text{pol}}, \quad \Delta_Z = -2\alpha m_e r_Z (1 + \delta_Z^{\text{rad}}) \]

- Zemach radius: \[ r_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[ G_E(Q^2) \frac{G_M(Q^2)}{1 + \kappa_P} - 1 \right] \]

- A leading theoretical uncertainty from $\Delta_Z$: $r_Z = 1.05 \pm 0.02$ fm leads to $\Delta_Z = 40 \pm 1$ ppm.

- Low $Q^2$ evaluation relies on assumptions of extrapolation to $Q^2 = 0$, parametrizations all basically enforce similar $G_E$ and $G_M$ low $Q^2$ dependence by having \( \approx \) linear extrapolation.

- New measurements would reduce the Zemach radius uncertainty by \( \approx 2 \).
The Proposed Measurement
Part I - Recoil Polarimetry
Polarization Transfer - Scatter
polarized electrons off unpolarized protons \( \rightarrow \) measure recoil proton polarization.

\[
I_0 P_x = -2\sqrt{\tau(1 + \tau)} \tan \frac{\theta E}{2} G_E G_M
\]

\[
I_0 P_z = \frac{E + E'}{M} \sqrt{\tau(1 + \tau)} \tan^2 \frac{\theta_e}{2} G_M^2
\]

\[
R \equiv \mu_P \frac{G_E}{G_M} = -\mu_P \frac{E + E'}{2M} \tan \frac{\theta_e}{2} \frac{P_x}{P_z}
\]
Part I - Overview

Part I conditionally approved in PAC31 (PR-07-004).

- **Hall A FPP**, $E_e \sim 0.85\text{GeV}$, 80% polarization
- **PRL 99, 202002 (2007) data** took 12-18 hours / Data point with $P_e = 40\%$, we request 1 day / Point (2 days at 0.25 GeV$^2$)
- **Systematics** $\sim 0.4\%$ at 0.5 GeV$^2$, better for lower $Q^2$
- **Standard Hall A setup**

<table>
<thead>
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<th>$Q^2$ (GeV$^2$)</th>
<th>$(\Delta \text{Ratio/Ratio})_{\text{stat.}}$ (%)</th>
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Opportunity for Part I With a 1.2 GeV Beam
(May 2008 - No existing approved experiment can run)

- Experiment entirely consistent with $HRS_L$ (protons) + BigBite (electrons) setup.
- Statistics slightly worse.
- Systematics similar.
- Total about the same.
- Senior PhD. Student (involved in BigBite detector package construction) interested.
- Available beam time $\approx$ our Part I request.

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<td>0.43</td>
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<tr>
<td>0.7</td>
<td>0.38</td>
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From the PAC31 report on PR-07-004: “Since Mainz is presently running an experiment which using Rosenbluth separation can determine the same ratio in the same region of Q2, consideration should be given to these results and especially their level of uncertainties before approval to proceed with this proposal is given.”

- Mainz experiment has taken data.
- Planned $\sim 1\%$ stat. uncertainties lead to gray error band.
- Plot compares our TOTAL expected uncertainties to Rosenbluth extraction of the form factor ratio from the Mainz data (up to 5 times better in the “bump” region).
The Proposed Measurement

Part II - Double Spin Asymmetry
Part II - Overview

- **Measure asymmetry in** $\vec{p}(\vec{e}, e')$ **simultaneously in both HRSs** (equal acceptance).

- **Take the ratio of asymmetries** → **Systematics cancel out.**

\[
\frac{G_E^P}{G_M^P} = -\mu_P \frac{a(\tau, \theta) \cos \theta_1 - \frac{f_2}{f_1} \Gamma a(\tau, \theta) \cos \theta_2}{\cos \phi_1 \sin \theta_1 - \frac{f_2}{f_1} \Gamma \cos \phi_2 \sin \theta_2}
\]

\[
a(\tau, \theta) = \sqrt{\tau(1 + (1 + \tau) \tan^2(\theta_e/2))}
\]

$\theta_1^\ast$($\phi_1^\ast$) - polar (azimuthal) angle of the target spin with respect to the $\vec{q}$ in the $i^{th}$ spectrometer. $\Gamma = \frac{A_1}{A_2}$. $f_1 \approx f_2$.

- **With septa** → **reach VERY low $Q^2$** while keeping scattered electron at high momentum (less effect from target field).
Part II - Systematics

- Mostly cancel out when taking the ratio of asymmetries.
- Beam and Target polarization identical for both HRSs (and constant when considering small time slices).
- Only second order effect from dilution factor.
- Main systematic uncertainty is scattering angle reconstruction → use accurate target field map and perform optics study of septum magnets with target field (expect little degradation in resolution, $E'_e > 1$ GeV/c).
- High rate (low $Q^2$) → uncertainties dominated by systematics.
Part II - Requirements

- 11 days of 80% polarized beam in Hall A.
- 3 Angles at 1 pass beam, 4 at 2 pass, 5 at 3 pass.
- Installation of UVa polarized target.
- Installation of septa on HRSs.
- Upstream chicane for beam deflection.
- Installation of local beam dump.
- All installations also required for PR-08-027 ($g_2^P$).
From the PAC31 report on PR-07-004: “Since Mainz is presently running an experiment which using Rosenbluth separation can determine the same ratio in the same region of $Q^2$, consideration should be given to these results and especially their level of uncertainties before approval to proceed with this proposal is given.”

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Summary

Part I - Recoil Polarization

Part II - Double Spin Asymmetry

14 Days of 80% polarized beam

11 Days of 80% polarized beam

Hall A is uniquely suited for this experiment!
Summary

Part I - Recoil Polarization

14 Days of 80% polarized beam

Part II - Double Spin Asymmetry

11 Days of 80% polarized beam

- Ratio up to 5 times better than Mainz.
- Significant model discrimination.
- High precision mapping of low $Q^2$ region (where $G_M$ poorly known).
- Improved magnetic and Zemach radii, IS/IV/u/d form factors.
- $Q$ Range: $0.23 < \langle R^P \rangle_{ch} < 1.6$ fm.

$11 + 14 = 23$ Days, from reducing overlap.
Backup Slides
Compatibility with $Q_{weak}$

- Beam polarization fine – with longitudinal 1-pass beam in Hall C, Hall A polarization 95, 90, 85% of Hall C for 1, 2, 3 pass beam.
- Measurements use currents $\sim 85$ nA.
- Uses 1, 2, 3 pass beam.
- Would need 1-pass split to get to lowest $Q^2$. 
Quantifying the Zemach Radius

Different calculations/fits disagree:

- Friedrich & Walcher - 1.0431fm
- Arrington (LT) - 1.0708fm
- Arrington (Pol) - 1.0403fm
- Arrington (new PRC) - 1.0707fm
- Kelly - 1.059fm
- Dipole - 1.0149fm

A 5% variation between fits $\rightarrow \approx 2\text{ppm in HFS}$.
Uncertainty from $\Delta^{\text{pol}} \approx 0.6\text{ppm}$.

$G_M$ largely unmeasured as $Q^2$ to 0. Results are consistent due to (the assumed) nearly identical extrapolations for $G_E$ and $G_M$ for $Q^2 \rightarrow 0$. 
Could this be done elsewhere? - Recoil Polarization

- Our proposed uncertainties on $R$ are 0.5-1.1% (stat.)
- Mainz FPP systematics $\approx$ 4%
- Spin transport favors Hall A. Systematics for Hall C unclear
As Mainz has a low energy electron beam and has spectrometers, we investigated doing this experiment there.

- None of the infrastructure for this experiment currently exists at Mainz (polarized target, septa, chicanes, etc.)
- A1 Hall does not have fully symmetric spectrometers → increases systematic uncertainties.
- Due to larger minimum spectrometer angles, low $Q^2$ requires low electron energies → large $\theta'$ deflection in the target field.

Mainz is clearly not the best facility for this measurement.
Part I - Systematics

- Measurements with quadrupoles turned off.
- Measurement of $R$ at $Q^2 \approx 2.2 \text{ GeV}^2$, in the “spin hole”; variation of spin direction in focal plane very sensitive to spin transport there.
- Done previously with HRS-R for $G_E^D - 1$; never done for HRS-L. Since we need high precision, we plan to redo these tests.
Some Impacts on Proton FFs


- $R(Q^2 = 0.356) = 0.9441 \pm 0.011 - 5\sigma$ from unity!

- In combination with world data:
  - $Q^2 = 0.3 - 0.45\text{GeV}^2 - R = 0.96 \pm 0.007$.
  - $Q^2 = 0.45 - 0.55\text{GeV}^2 - R = 0.987 \pm 0.008$.
  - $3\sigma$ difference between $Q^2$ ranges $\rightarrow$ Hints of narrow structure?
  - Standard fits overpredict $G_E^P(Q^2 = 0.4)$ by $\approx 1-2\%$. 
Charge Densities

From Miller et al.:

- For low $Q^2$, $R \approx 1 - \frac{Q^2}{6} \left( R^*_M - R^*_E \right)$.
- $G_E, G_M$ do not represent true densities due to relativistic effects (Lorentz contraction).
- Move to light-cone variables to get transverse densities:

$$F_1(Q^2) \approx 1 - \frac{Q^2}{4} \langle b^2 \rangle_{Ch}$$

$$F_2(Q^2) \approx \kappa \left( 1 - \frac{Q^2}{4} \langle b^2 \rangle_{M} \right)$$

- Giving: $\langle b^2 \rangle_{M} - \langle b^2 \rangle_{Ch} = \frac{\mu}{\kappa} \frac{2}{3} (R^*_M - R^*_E) + \frac{\mu}{M^2}$
Proton magnetization extends further than proton charge (pion cloud? quark OAM?).
All calculations and fits agree that $\langle b^2 \rangle_M > \langle b^2 \rangle_{ch}$, but different value.
New measurements will challenge some of the fits/calculations.

G. Miller, E. Piasetzky, and G. Ron, Submitted to PRL. [arXiv:0711.0972]
DVCS measurements focus on the high $Q^2$, small $t$ (equivalent to small $Q^2$ in ep elastic) region.

Need elastic scattering results to disentangle $→$ requires knowledge of elastic form factors (at $Q^2_{ep} = -t$).

Knowledge of the FFs is a limiting uncertainty, especially in regions where BH $\gg$ DVS.
Part II with no Septum
(Yes, we can do it)

- $Q^2$ range
  0.015 - 0.4 → 0.06 - 0.4 GeV$^2$.
- Uncertainties roughly similar.
- Use 1, 2 pass beam.
- Still need chicane, local beam dump.

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<td>0.350</td>
<td>0.75</td>
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<tr>
<td>0.400</td>
<td>0.81</td>
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</table>
Mainz Phase Space

\[ Q \text{ [GeV/c]} \times \varepsilon \]

- \( E = 855 \text{ MeV} \)
- \( E > 1.53 \text{ GeV} \)
- \( \theta < 18^\circ \)
- \( \theta > 160^\circ \)
- \( E' > 630 \text{ MeV} \)
Target Schematic
Beamline Schematic
$Q^2$ Acceptances
$Q^2$ Acceptances (Shifted)
Is this a 12 GeV Experiment?
Recoil Polarization

- Requires 1-pass beam.
- Minimum beam energy 2.2 GeV.
- Polarization observables drop sharply with increasing beam energy (forward electron angles).
- To get the same uncertainties with 2.2 GeV beam → 150 hours/point.
Is this a 12 GeV Experiment?

DSA

- Requires 1-pass beam.
- Limits $Q^2$ range to $(0.05 \text{ GeV}^2 \rightarrow)$, assuming speta are installed.
- Maybe possible to run with 1-Linac configuration.
Density Plots