Requirements on the Determination of the Beam Energy for E89003

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August 8, 1997

One of the major sources of error in L/T separations is typically the determination of the beam energy. For Expt. 89003, the beam energy sensitivity is greatly reduced thanks to the sieve slit measurement on ^{1}H(e,e$'$p). This measurement allows matching of q, ω, p_r and e_m for each of the kinematics involved in the separation measurement. In addition, an absolute momentum calibration of the hadron spectrometer gives knowledge of the magnitude of $\vec{q}$, since for hydrogen elastic scattering, $\vec{q} = \vec{p}$ (where $\vec{p}$ is the proton momentum). The proton energy also allows direct determination of ω, since $\omega = T_p$, the proton kinetic energy. Thus, hydrogen elastic scattering in (e,e$'$p) in combination with a sieve slit in the electron arm, provides knowledge of $q = |\vec{q}|$ for a fiducial ray in the electron arm. For the ^{16}O(e,e$'$p) measurement, one must then relate the rays for all other events to this fiducial ray in order to determine q for each event. The coincidence cross section varies only slightly with beam energy for fixed q, ω, p_r and e_m. In what follows, the uncertainty in the extracted response functions due to uncertainties in the proton absolute momentum (and hence q and ω) and beam energies is determined.

In order to estimate the sensitivities, a simple case of an L/T separation at two incident energies is examined. We have for the coincidence cross sections:

$$\sigma_i = \sigma_{mi}\big(v_L R_L + v_{Ti} R_T\big) \tag{1}$$

where σ_m is the Mott cross section, $v_L = (Q^2/q^2)^2$, $v_T = Q^2/2q^2 + \tan^2\theta/2$ and $i = 1,2$ corresponds to quantities for the $e = 0.845$ GeV and $e = 2.445$ GeV kinematics respectively. Here, $Q^2 = q^2 - \omega^2$ and σ_i are the measured cross sections and are assumed to be properly normalized (*i.e.* the uncertainties in these cross sections are neglected here).

Uncertainties in p and e affect our determination of R_L and R_T at two levels:

- For errors in p and/or e, there is an extraction error for the response functions (*i.e.* a vertical scale error).

- For an error in p, the response functions are extracted at the wrong q and ω (*i.e.* a horizontal scale error).

From the above expressions for the cross sections we can write:

$$\frac{R_L}{\sigma_2} = \left[\frac{\sigma_1}{\sigma_2}\frac{v_{T2}}{\sigma_{m1}} - \frac{v_{T1}}{\sigma_{m2}}\right] / \left[v_L(v_{T2} - v_{T1})\right] \tag{2}$$

$$\frac{R_T}{\sigma_2} = \left[\frac{1}{\sigma_{m2}} - \frac{\sigma_1}{\sigma_2}\frac{1}{\sigma_{m1}}\right] / \left[v_{T2} - v_{T1}\right] \tag{3}$$

In computing the errors, it is assumed that q is the same for both measurements, since the sieve technique matches them. Also, the ratio σ_1/σ_2 is determined in a simple factorized model. Note that this ratio is fixed in computing the error derivatives, since these are the measured cross sections. The derivatives are computed numerically.

The factorized model gives:

$$\sigma = K\sigma_{ep}^{\text{free}} S(p_r, e_m) \tag{4}$$

where σ is either σ_1 or σ_2, $K = pE$ (where E is the proton total energy), $\sigma_{ep}^{\text{free}}$ is the free electron-proton cross section, and S is the nuclear spectral function (assumed fixed, since p_r and e_m are directly determined via the sieve measurement on hydrogen). For the ep cross section, we have:

$$\sigma_{ep}^{\text{free}} = \sigma_m \left(v_L k_L G_E^2 + v_T k_T G_M^2\right) \tag{5}$$

where $k_L = q^2/Q^2$ and $k_T = Q^2/2m_p^2$ (m_p is the proton mass). For the proton form factors, the dipole model is used:

$$G_E = \left(1 + \frac{Q^2}{0.71\ \text{GeV}^2}\right)^{-2} \tag{6}$$

$$G_M = \mu_p G_E \tag{7}$$

with $\mu_p = 2.79$. The Mott cross section can be written as:

$$\sigma_m = \frac{\alpha^2 \cot^2\theta/2}{Q^2}\left(1 - \frac{\omega}{e}\right) \tag{8}$$

which shows the explicit dependence on the beam energy, e, for fixed Q^2. In this model we have for the response functions:

$$R_L = K k_L G_E^2 S \tag{9}$$

$$R_T = K k_T G_M^2 S \tag{10}$$

which gives $R_L/R_T = (2m_p^2/\mu_p^2)(q^2/Q^4) = 0.343$ for hydrogen kinematics ($\omega = Q^2/2m_p$ with $q = 1$ GeV/c). Then the cross section ratio is:

$$\frac{\sigma_1}{\sigma_2} = \frac{\sigma_{m1}}{\sigma_{m2}} \frac{v_L \frac{R_L}{R_T} + v_{T1}}{v_L \frac{R_L}{R_T} + v_{T2}} = 0.0538 \tag{11}$$

where we have used scattering angles of 100.76° and 23.36° for kinematics 1 and 2 respectively (with $\omega = 0.433$ GeV).

The same factorized model can be used to estimate the variation of the response functions with respect to q and ω. Note that the errors in q and ω are assumed to be correlated since they both arise from our uncertainty in the absolute proton momentum. That is, the sieve slit measurement with H(e,e′p) gives $q = p$ and $\omega = T_p$. Then for a proton momentum uncertainty of 10^{-3}, we get:

$$\frac{\delta R_L}{R_L} = 2 \times 10^{-3} \tag{12}$$

$$\frac{\delta R_T}{R_T} = 4 \times 10^{-4} \tag{13}$$

where the spectral function S is assumed fixed since p_r and e_m are matched via the sieve slit measurement. These are the errors we would make by placing the data points at the wrong q and ω. In addition, we have the R_L and R_T extraction errors mentioned previously.

We now determine the extraction errors by varying q and each of the two beam energies in turn, using the previous expressions for R_L/σ_2 and R_T/σ_2. The following table gives the response functions (divided by σ_2) for the nominal kinematics and for 10^{-3} variations in p (giving rise to variations in $q = p$ and $\omega = T_p$) and in each of the two incident energies. Only one quantity is varied each time, keeping the others at their nominal values. The numbers in parentheses give the resulting relative errors in R_L or R_T (*i.e.* $\delta R/R$).

Kinematics	R_L/σ_2 ($\times 10^5$ fm^{-2})	R_T/σ_2 ($\times 10^5$ fm^{-2})
Nominal	0.103100	0.301686
q up by 10^{-3}	0.103007 (-9×10^{-4})	0.302933 ($+4\times10^{-3}$)
e_{01} up by 10^{-3}	0.103418 ($+3\times10^{-3}$)	0.301219 (-2×10^{-3})
e_{02} up by 10^{-3}	0.103013 (-8×10^{-4})	0.301717 ($+1\times10^{-4}$)

Table 1: The effects of small variations in proton momentum and beam energy on the structure function separations.

In conclusion, a 1% error in the proton absolute momentum gives 1% and 4% errors in R_L and R_T respectively. A 1% error in either of the two beam energies, gives no more than 3% errors in the response functions. **Therefore, if we want to limit the errors in R_L and R_T to $\leq 2\%$ from these sources, 5×10^{-3} determination of the proton momentum and beam energies is required.** This is not expected to pose a problem. Based on analysis of J. LeRose, the proton absolute momentum should be determined to better than $2{\times}10^{-3}$ by dead reckoning from field map data, NMR field values and surveys of the positions of the magnetic elements. The beam energy can be determined either from the opening angle in H(e,e′p) elastic scattering or from the absolute spectrometer momentum for elastic scattering at a known angle. It is important that any variations in beam energy during the course of the experiment be accounted for in order to apply the sieve slit calibrations to the production data. Provided this is done, an absolute determination of the proton momentum in combination with the sieve slit data greatly reduces our sensitivity to uncertainty in the beam energy.