Neutron Spin Structure Functions and Moments at Low Q^2

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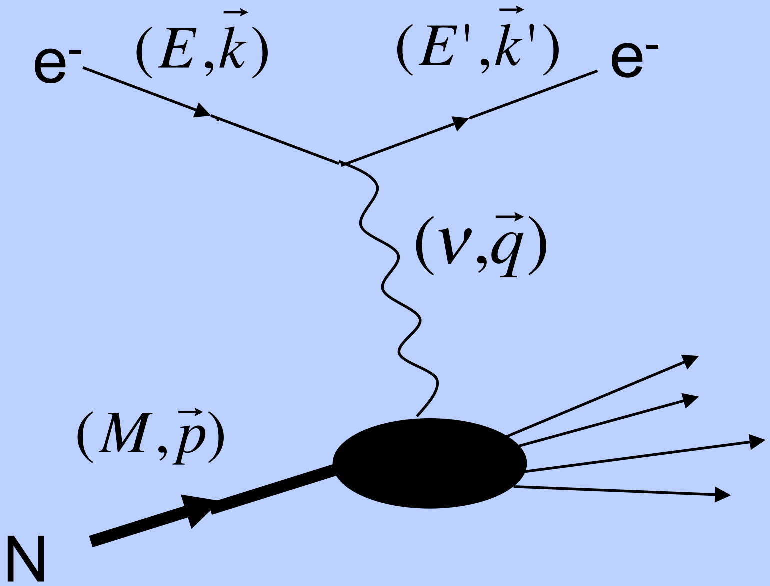
on behalf of the Jefferson Lab
Hall A and Polarized ^{3}He Collaborations

Polarized Inclusive Electron Scattering

- Scattering longitudinally polarized electrons from polarized nuclei.
- E = 1-5.7 GeV
- Virtual photon probe of quark/nuclear structure.
- At large Q^2, interaction dominated by scattering by single, asymptotically-free quark.
- At lower Q^2, sensitive to nucleon dynamics/QCD.

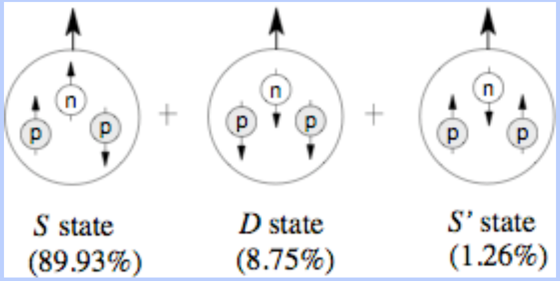


$$Q^2 = \vec{q}\cdot\vec{q} - \nu^2 \qquad -\text{(four - momentum transfer squared)}$$

$$x = \frac{Q^2}{2M\nu} \qquad \text{(fractional momentum of struck quark)}$$

Experimental Method

- Inclusive polarized electron scattering from a ^3He target polarized longitudinal or transverse to electron helicity.
- Polarized ^3He --> polarized neutron target.
- ^3He ground state:



- Measure e-N asymmetries and un-polarized cross-sections--extract polarized cross-section differences:

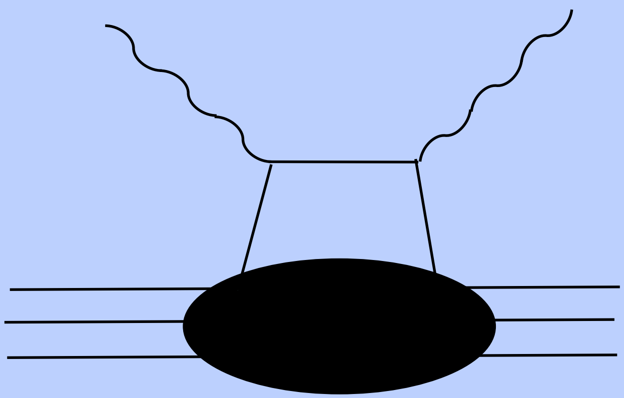
$$\Delta\sigma_{\parallel}(\nu,Q^2) = A_{\parallel}\sigma_0 \qquad \Delta\sigma_{\perp}(\nu,Q^2) = A_{\perp}\sigma_0$$

Spin structure functions

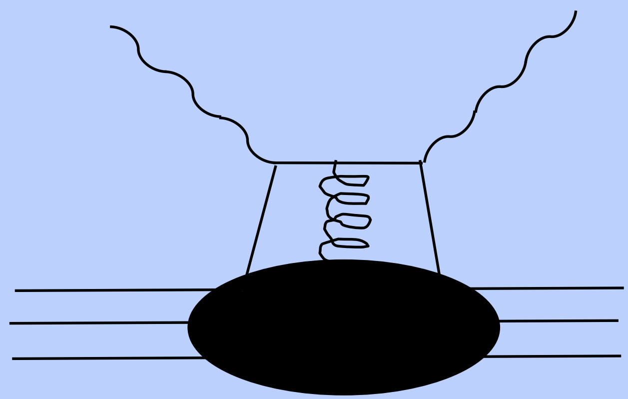
$$g_1(\nu,Q^2),\ g_2(\nu,Q^2) \quad \propto \quad \Delta\sigma_{\parallel},\ \Delta\sigma_{\perp}$$

$$g_1(x,Q^2)=\frac{1}{2}\sum_i e_i^2 \Delta q_i(x,Q^2)$$ ⇒ At large Q^2, related to polarized quark PDF's. Non-pQCD higher-twist contributions suppressed by powers of 1/Q.

$$g_2(x,Q^2)$$ ⇒ Asymptotically-free AND higher-twist contributions enter at same order for any Q^2.



leading twist



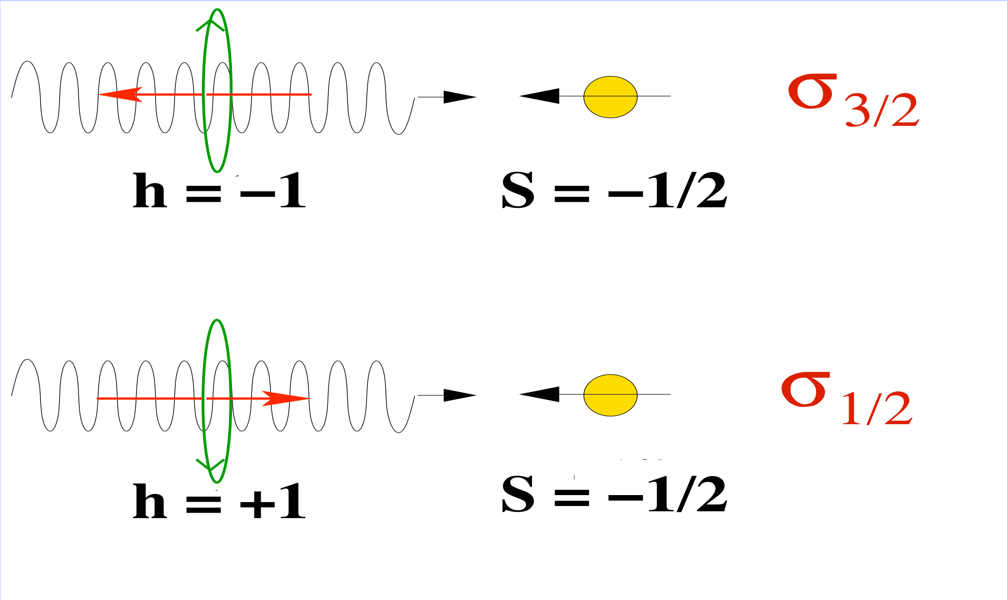
twist-3

Virtual Photon-Nucleon Polarized Cross Sections

$$\sigma_{TT}(\nu,Q^2) \equiv \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{2}$$
$$= \frac{4\pi^2\alpha}{MK}(g_1 - \gamma^2 g_2)$$

absorption of transversely polarized virtual photons by polarized nuclei

K is the virtual photon flux—convention dependent



- Longitudinal-transverse interference cross-section

$$\sigma_{LT}(\nu,Q^2) = \frac{4\pi^2\alpha}{MK}(g_1 + g_2)$$

- Note $\Delta\sigma^{eN}_{ll,\perp} \propto \sigma_{TT}\,,\ \sigma_{LT}$

- If we assume Δ resonance is purely transverse,

$$\sigma_{LT} \approx 0 \quad \Rightarrow \quad g_2 \approx \text{-}g_1$$

- At large Q^2, this is also expected in DIS region from the Wandzura-Wilczek relation, derived using the OPE:

$$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 dy \frac{g_1(y,Q^2)}{y} + \bar{g}_2(x,Q^2)$$

Gerasimov-Drell-Hearn Sum Rule for Real Photons

- Begin with the spin dependent part of the forward Compton amplitude, S_1
- Use the following dispersion relation and three assumptions

$$\mathrm{Re}\ S_1(\nu) = \frac{2\nu}{\pi}\int_{\nu_0}^{\infty} d\nu' \frac{\mathrm{Im}\ S_1(\nu')}{\nu'^2 - \nu^2}$$

- Optical Theorem $\qquad \mathrm{Im}\ S_1(\nu) = \frac{\nu}{8\pi}\sigma_{TT}(\nu)$
- Low Energy Theorem $\qquad \mathrm{Re}\, S_1(\nu) = -\frac{e^2\kappa^2}{8\pi M^2}\nu \qquad$ where κ is the target anomalous magnetic moment

$$\boxed{\int_{\nu_0}^{\infty} \frac{\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)}{\nu} d\nu = -2\pi^2\alpha\left(\frac{\kappa}{M}\right)^2}$$

- Unsubtracted Dispersion Relation: One assumption is convergence of the dispersion integral.

- Sum Rule valid for any spin ½ nucleon/nucleus.

The GDH Sum Rule at Q^2=0

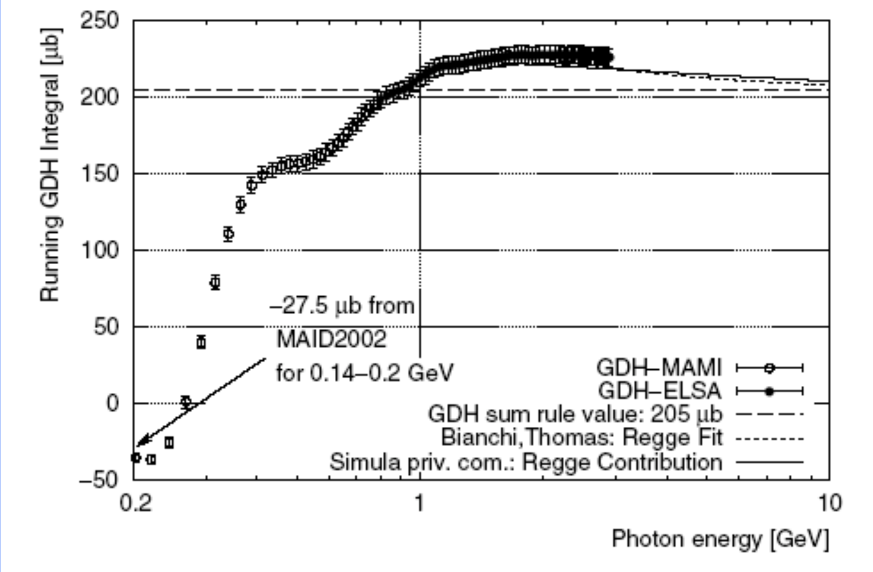
$$I_{GDH} \equiv \int_{\nu_{thr}}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} d\nu = -2\pi^2\alpha \frac{\kappa^2}{M^2}$$

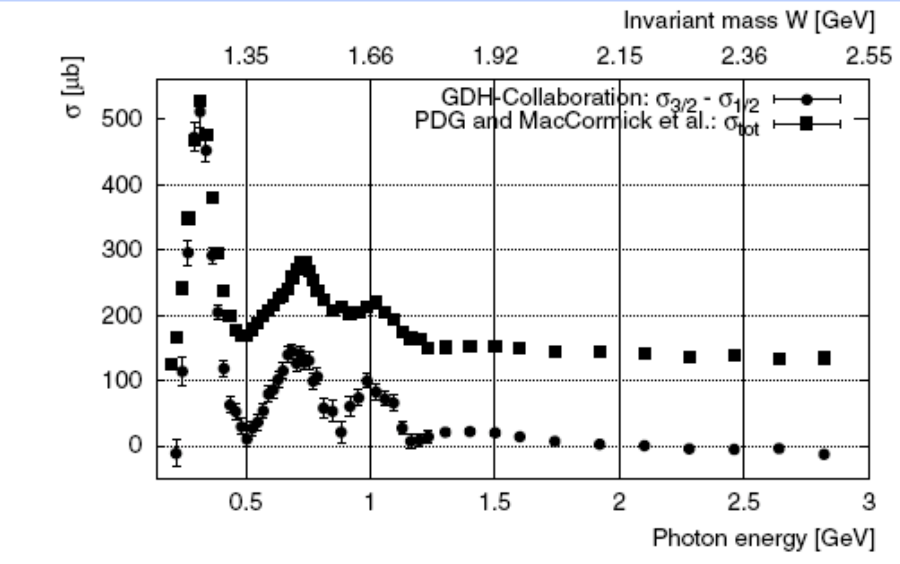
S. Gerasimov, Yad. Fiz. 2 (1965) 598, Sov. J. Nucl. Phys. 2 (1966) 930.
S.D. Drell and A.C. Hearn, PRL 16 (1966) 908.

$$I^p = -205\ \mu b \qquad I^n = -233.5\ \mu b \qquad I^{^3He} = -496\ \mu b$$

(^{3}He must receive large contribution below pion production threshold)

Proton data from MAMI/ELSA, PRL 93 (2004) 032003.





Extended GDH Sum for $Q^2>0$

LHS: Replace real photon cross sections with transversely polarized virtual photon cross sections.

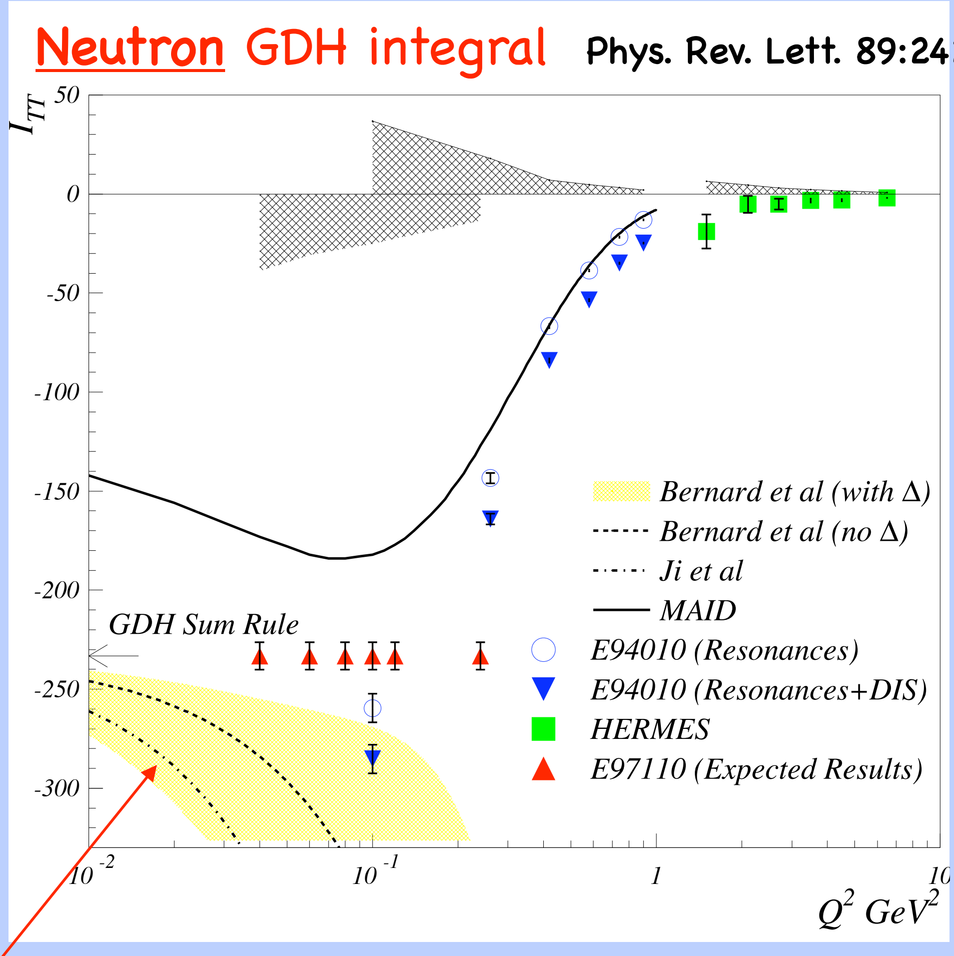
RHS: Calculate amplitudes using models.

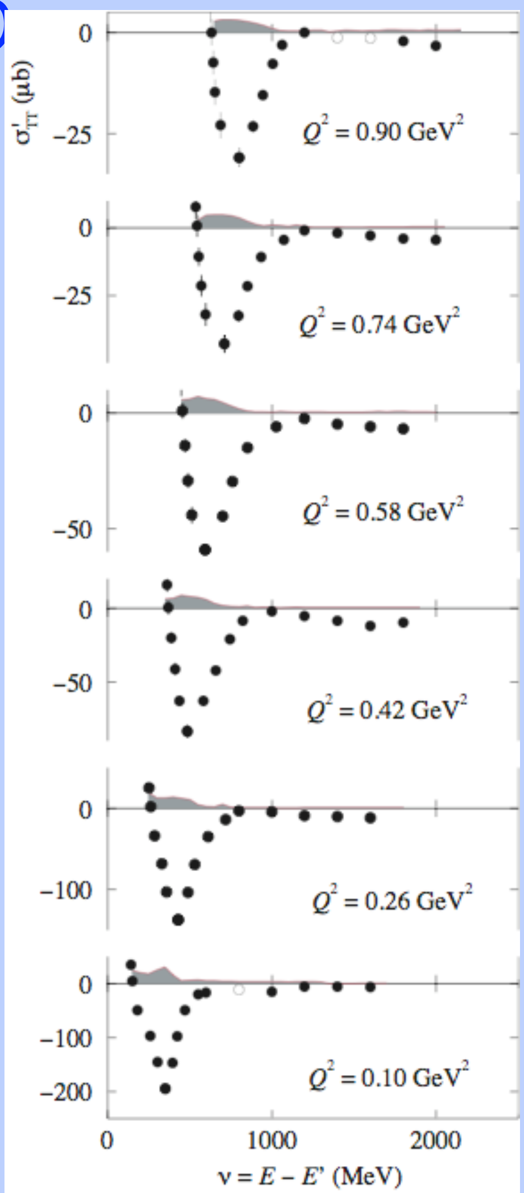
$$\int_{\nu_{thr}}^{\infty} \frac{\sigma_{1/2}^{T}(\nu,Q^2)-\sigma_{3/2}^{T}(\nu,Q^2)}{\nu} d\nu = S_1(Q^2)$$

$\sigma_i^T(\nu,Q^2) =$ transversely-polarized virtual photon absorption cross-sections.

- Q^2-evolution provides information about nucleon transition from hadronic to partonic degrees of freedom.
- At small Q^2, calculate $S(Q^2)$ using Chiral-PT.
- At large Q^2, calculate $S(Q^2)$ using pQCD, twist-expansion.
- Phenomenological models (e.g. MAID), lattice calculations for transition region, Q^2=0.1 - 1.0 GeV^2.

First Jefferson Lab Experiment E94-010 Neutron Results (0.1 < Q^2 < 1.0 GeV^2)

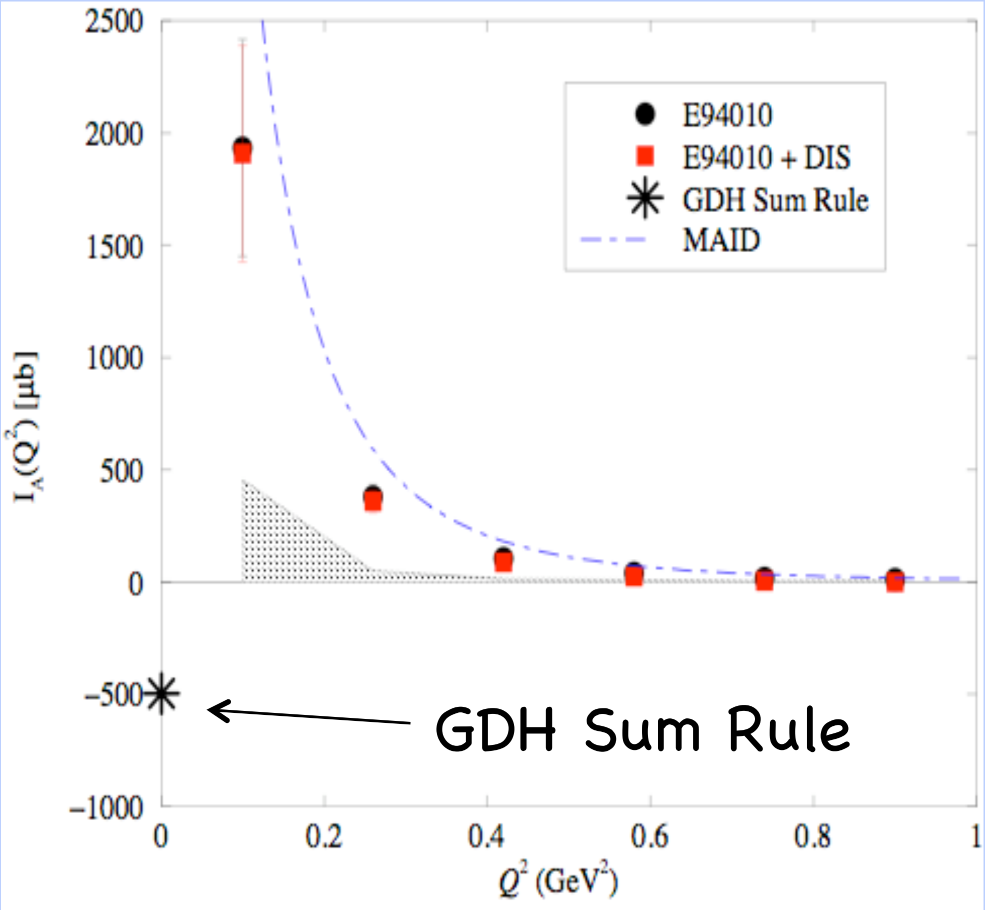


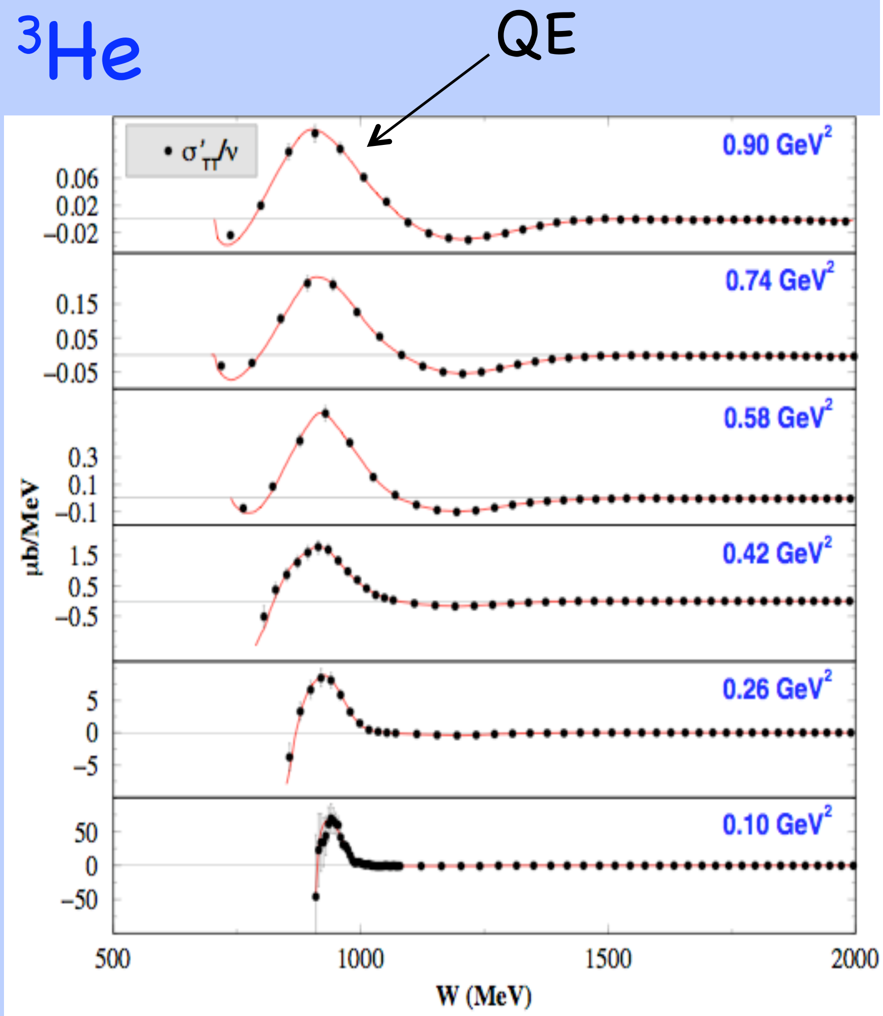


- Chi-PT calculations predict negative slope
- Implies dramatic behavior for $Q^2 < 0.1$ GeV^2

σ_{TT} for ^{3}He

GDH on ^{3}He





- Large, positive contribution from QE asymmetry.
- Implies dramatic behavior below Q^2=0.1 GeV^2
- Karl Slifer, Temple Univ., Ph.D. Thesis, arXiv:0803.2267, accepted for pub.

$g_1(x,Q^2)$ and $g_2(x,Q^2)$ integrals

- Could also use: $\boxed{\sigma_{TT} = \frac{4\pi^2\alpha}{MK}\left(g_1 - \gamma^2 g_2\right)}$

$$\bar{\Gamma}_1(Q^2) \equiv \int_0^{x_{th}} g_1(x,Q^2)\,dx = \frac{Q^2}{8}\bar{S}_1(Q^2) \quad \text{(no elastic contribution)}$$

- Note: $\bar{\Gamma}_1(0) = 0$

- GDH integral: $I_A \equiv \frac{16\pi^2\alpha}{Q^2}\int_0^{x_{th}}\left[g_1(x,Q^2) - \frac{16M^2x^2}{Q^2}g_2(x,Q^2)\right]dx$

- $x_{th} =$ two-body break-up for ^{3}He

- Calculate $\bar{S}_1(Q^2)$ near Q^2=0 using Chiral Perturbation Theory

Higher-twist Contributions to Nucleon Structure

- OPE expansion of first moment of g_1 at <u>large Q^2</u>,

$$\Gamma_1(Q^2) \equiv \int_0^1 g_1(x,Q^2)dx \;=\; \mu_2(Q^2) + \frac{\mu_4(Q^2)}{Q^2} + \frac{\mu_6(Q^2)}{Q^4} + \cdots$$

- μ_2 = leading-twist contribution; related to polarized PDF's, $\Delta q_i/q_i$; gives quark contribution to nucleon spin, $\Delta\Sigma$

$$\mu_4 = \frac{M^2}{9}\left(a_2 + 4d_2 + 4f_2\right)$$

$$\left.\begin{array}{l}\bullet\ a_2\text{=twist-2 target mass correction.}\\ \bullet\ d_2\text{=twist-3 reduced matrix element.}\\ \bullet\ f_2\text{=twist-4 reduced matrix element.}\end{array}\right\}\ \text{non-trivial quark-gluon correlations}$$

- d_2 and f_2 related to nucleon color susceptibilities; nucleon response to color electric and magnetic fields:

$$\chi_E = \frac{2}{3}(2d_2 + f_2), \quad \chi_M = \frac{1}{3}(4d_2 - f_2)$$

Quantifying HT contributions

- Goal: Use new precision data to calculate moments and quantify higher-twist contributions.

- a_2 given by x^2 moment of $g_1^{twist\text{-}2}$; use fit to world data; a_2^n=-0.0031(20) at Q^2=5 GeV^2.

- d_2 given by x^2 moment of $g_2^{twist\text{-}3}$; sensitive to quark-gluon interactions; calculable measured data using:

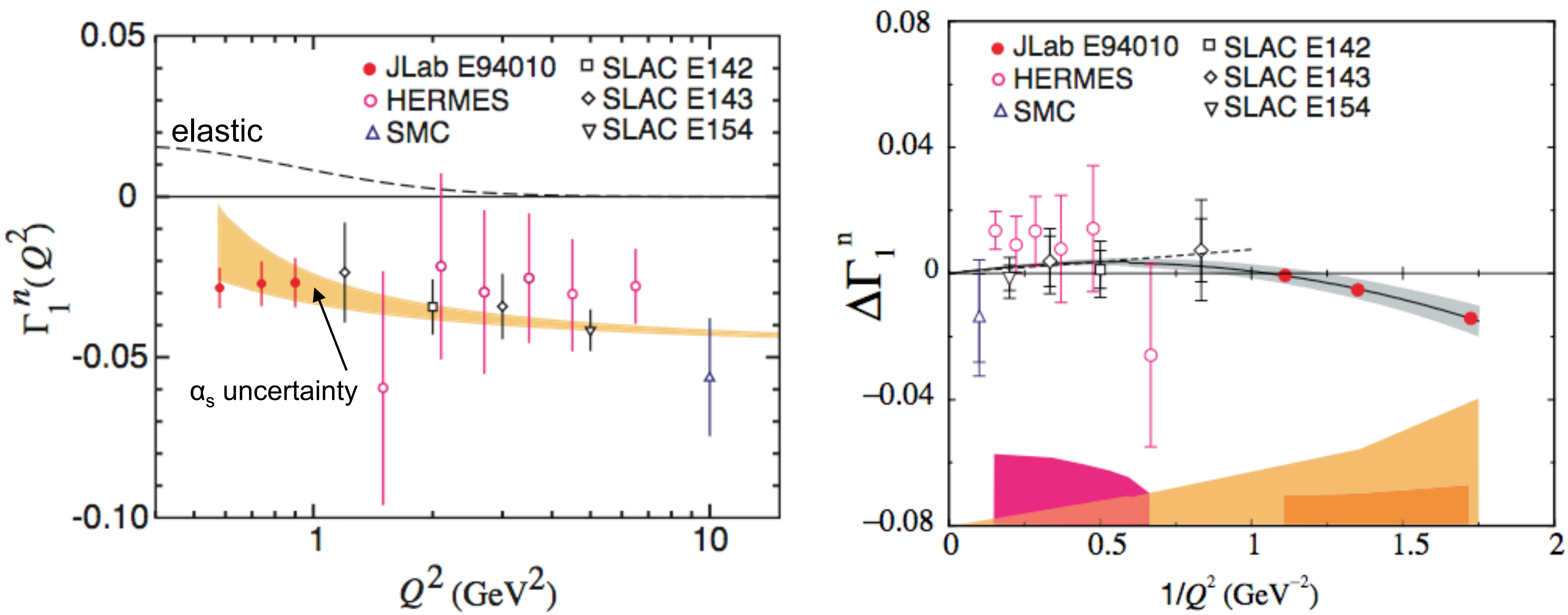
$$d_2(Q^2) = \int_0^1 x^2\left[2g_1(x,Q^2) + 3g_2(x,Q^2)\right]dx$$

- SLAC E155x: d_2^n=0.0079(48) at Q^2=5 GeV^2

- Unknowns: f_2, μ_6.

Quantifying HT contributions

- Analysis of world data using Bianchi-Thomas for low-x extrap; new JLab data to constrain high-x contrib.
- Fit Γ_1^n for Q^2>5 GeV^2 assuming no HT effects
- Evolve using DGLAP to Q^2< 5 GeV^2; subtract from measured Γ_1^n to get $\Delta\Gamma_1^n$ $(1/Q^2)$; new precision data at low-Q^2 from JLab.



HT contributions and color susceptabilities

- Results of fits show small HT contributions to g_1^n at Q^2=1 GeV^2:

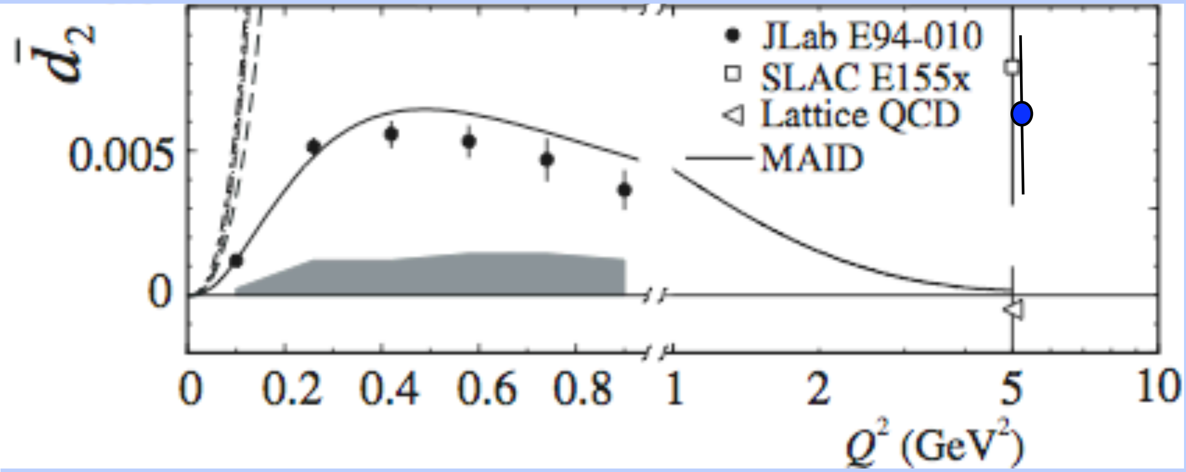
$$f_2^n = 0.034 \pm 0.043, \qquad \mu_6^n = (-0.019 \pm 0.017) M^4$$

- Extract color susceptabilities at Q^2=1 GeV^2:

$$\chi_E^n = 0.033 \pm 0.029, \quad \chi_M^n = -0.001 \pm 0.016$$

- Z.-E. Meziani *et al.,* PLB, 613 (2005) 148.

d_2^n in the resonance region



JLab high-Q^2 result, E99-117

d_2^n = 0.0062 +/- 0.0028 at Q^2=5 GeV^2

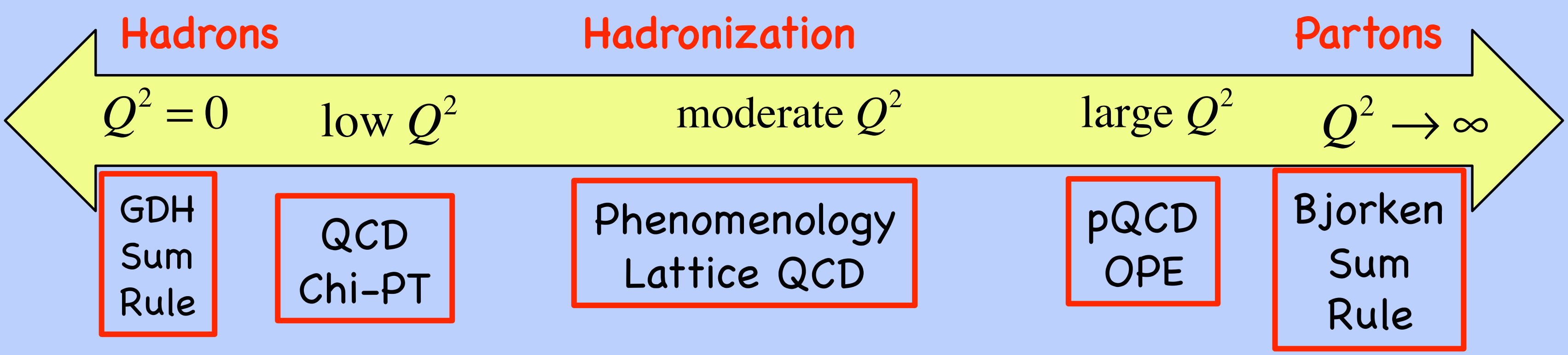
- Also calculate d_2^n below Q^2=1 GeV^2.
- Agrees with MAID.
- Trend towards lattice result.
- However, note disagreement of exp't and lattice at Q^2=5 GeV^2; upcoming experiment to measure d_2^n at larger Q^2.
- **Phys. Rev. Lett. 92 (2004) 022301.**

Bjorken Sum Rule

- Can be derived using extended GDH formalism.
- Originally derived before QCD--rigorous test of quark-parton model as $Q^2 \rightarrow \infty$.
- Rigorous test of pQCD at moderate Q^2.

$$\Gamma_1^{p-n}(Q^2) \equiv \int_0^1 dx \left(g_1^p(x,Q) - g_1^n(x,Q^2) \right) =$$

$$\frac{g_A}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.58\frac{\alpha_s^2}{\pi^2} - 20.21\frac{\alpha_s^3}{\pi^3} + \cdots \right] + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}(Q^2)}{Q^{2i-2}}$$



HT from Bjorken Sum Rule at low-Q^2

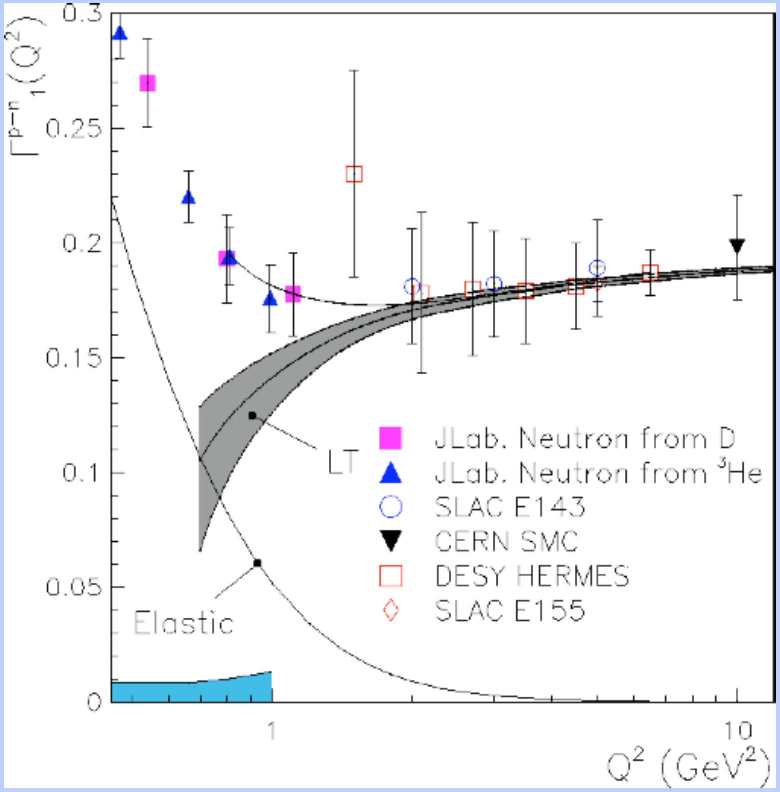
- Combine ^{3}He data with p,d data from CLAS to study Q^2-evolution of BJ sum rule, Γ_1^{p-n}. Less sensitive to low-x behavior, resonance contribs.

- At Q^2=1.0 GeV^2, μ_4^{p-n} = -0.06+/-0.02, μ_6^{p-n}= 0.09+/-0.02

- Both HT contributions are non-zero, but the sum is zero.

- OPE not converging at order μ_6^{p-n}/Q^4.

- PRL 93 (2004) 212001

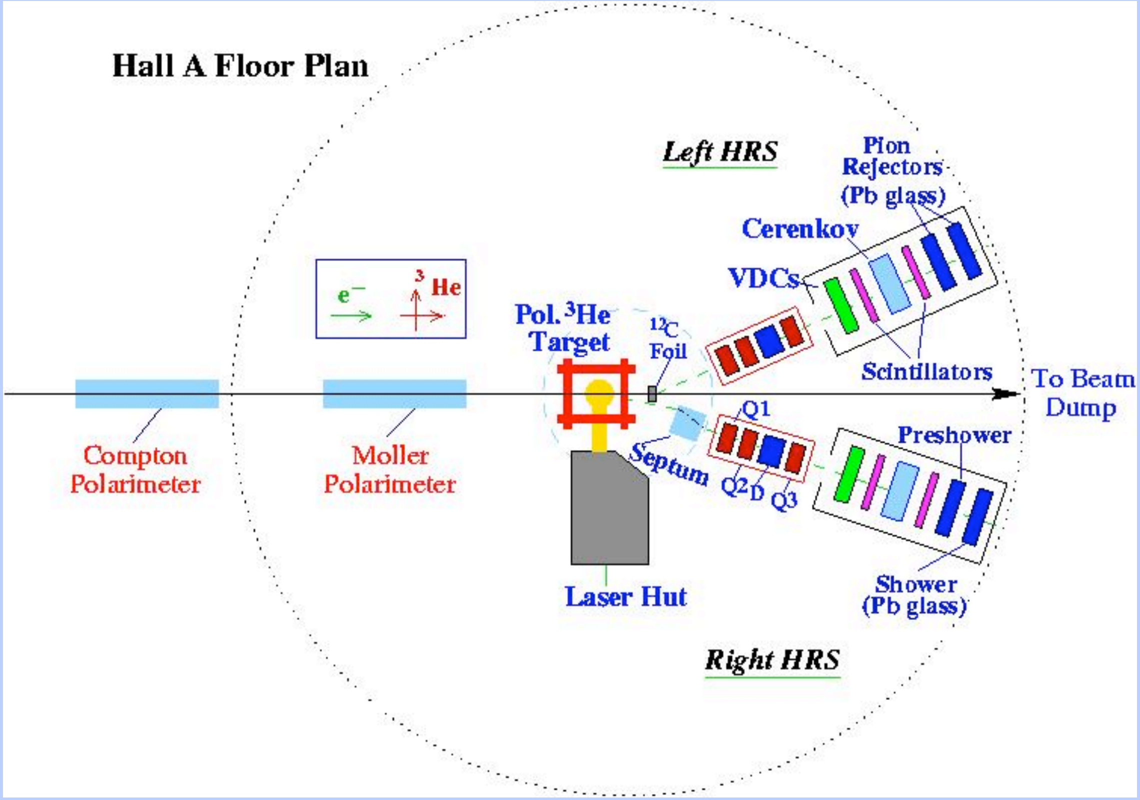


See new data in talk by S. Kuhn

elastic + inelastic data

Jefferson Lab Hall A

- Data needed at lower Q^2 to test Xhi-PT calculations and allow extrapolation to Q^2=0.
- New measurement of g_1^n, g_2^n at very low Q^2



- E97-110 measured at 6° and 9°; 0.02 < Q^2 < 0.3 GeV^2

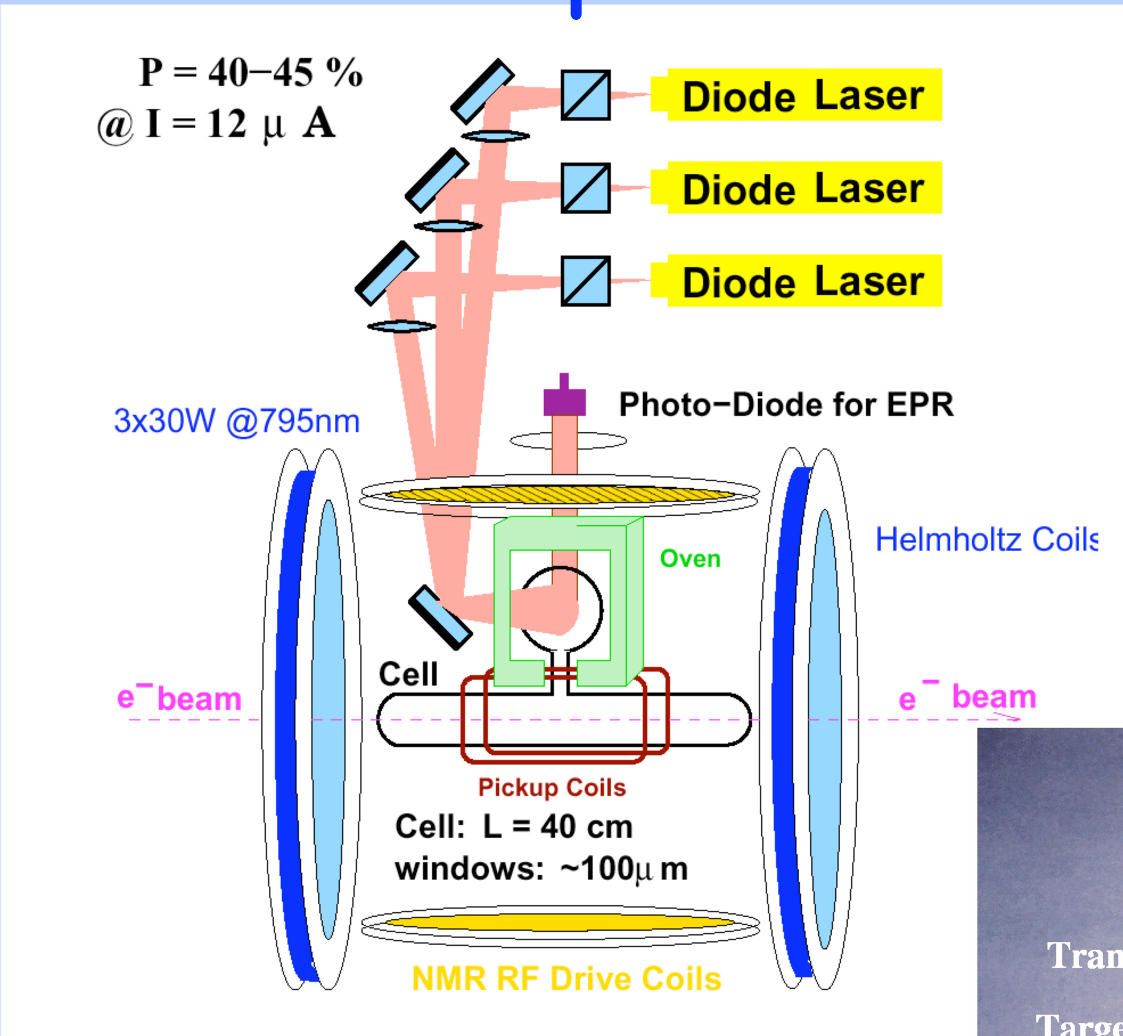
Septum Magnets

Two new superconducting septum magnets used to reach scattering angles down to 6^{o}

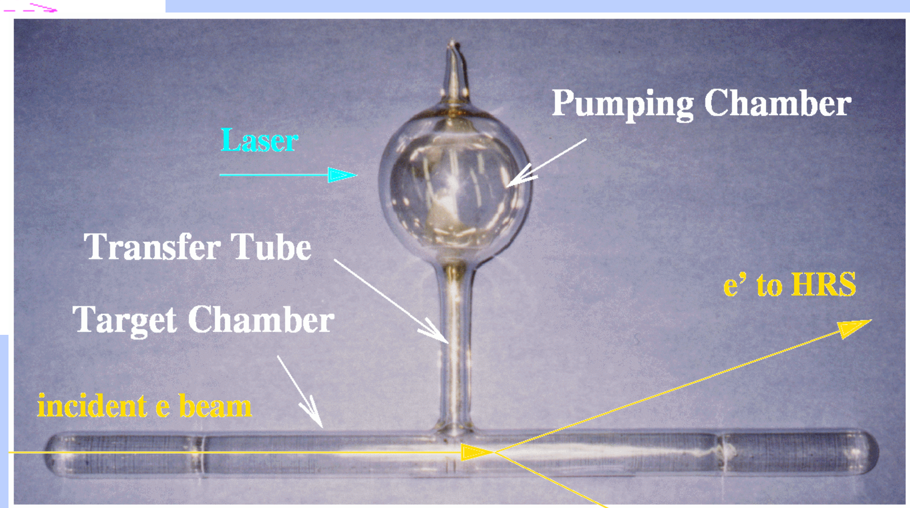
Minimum Q^2=0.02 GeV^2

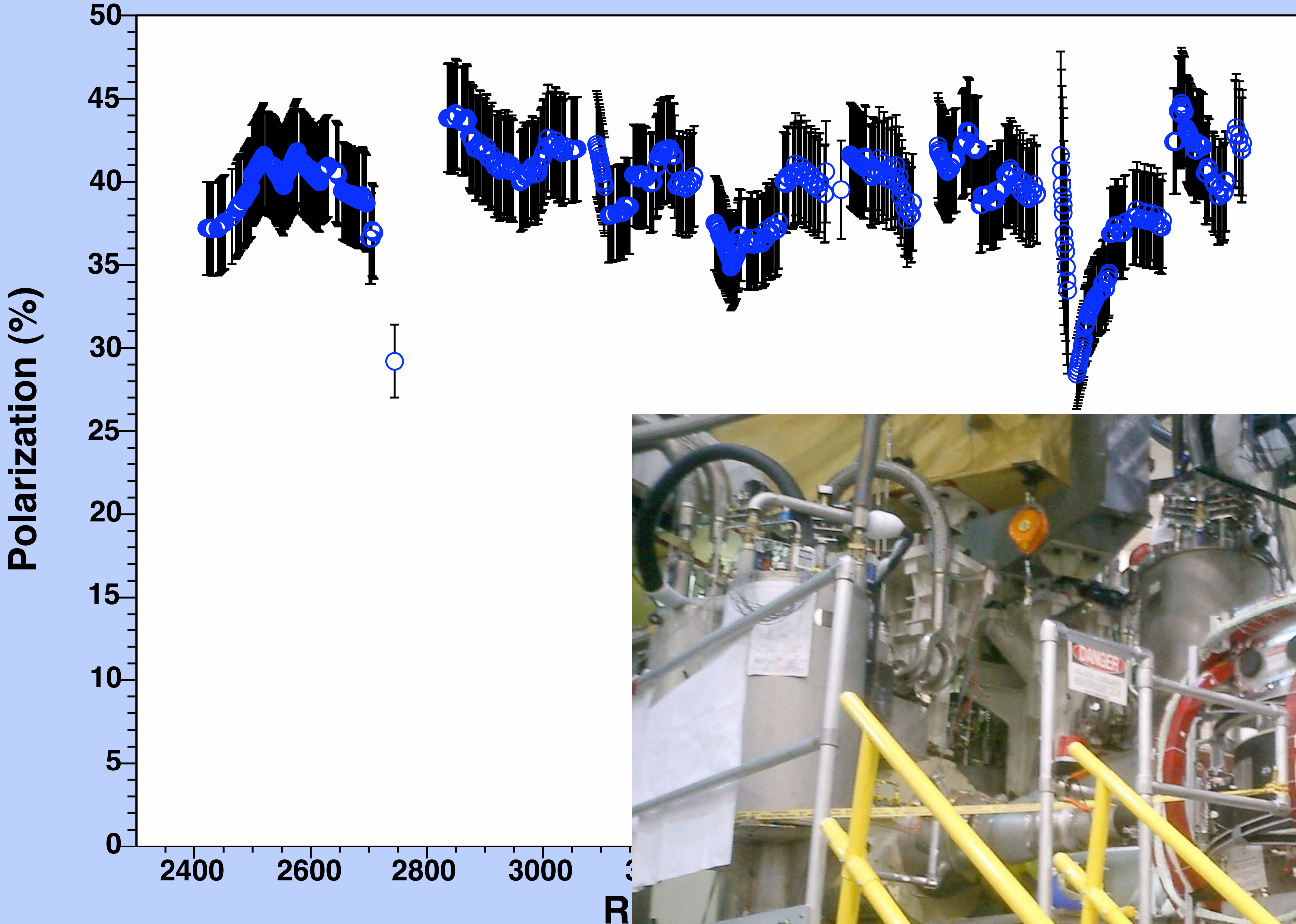
"nearly-real photons"

Hall A polarized ^{3}He target



- Both longitudinal and transverse polarization (and soon vertical)
- NMR and EPR for polarimetry

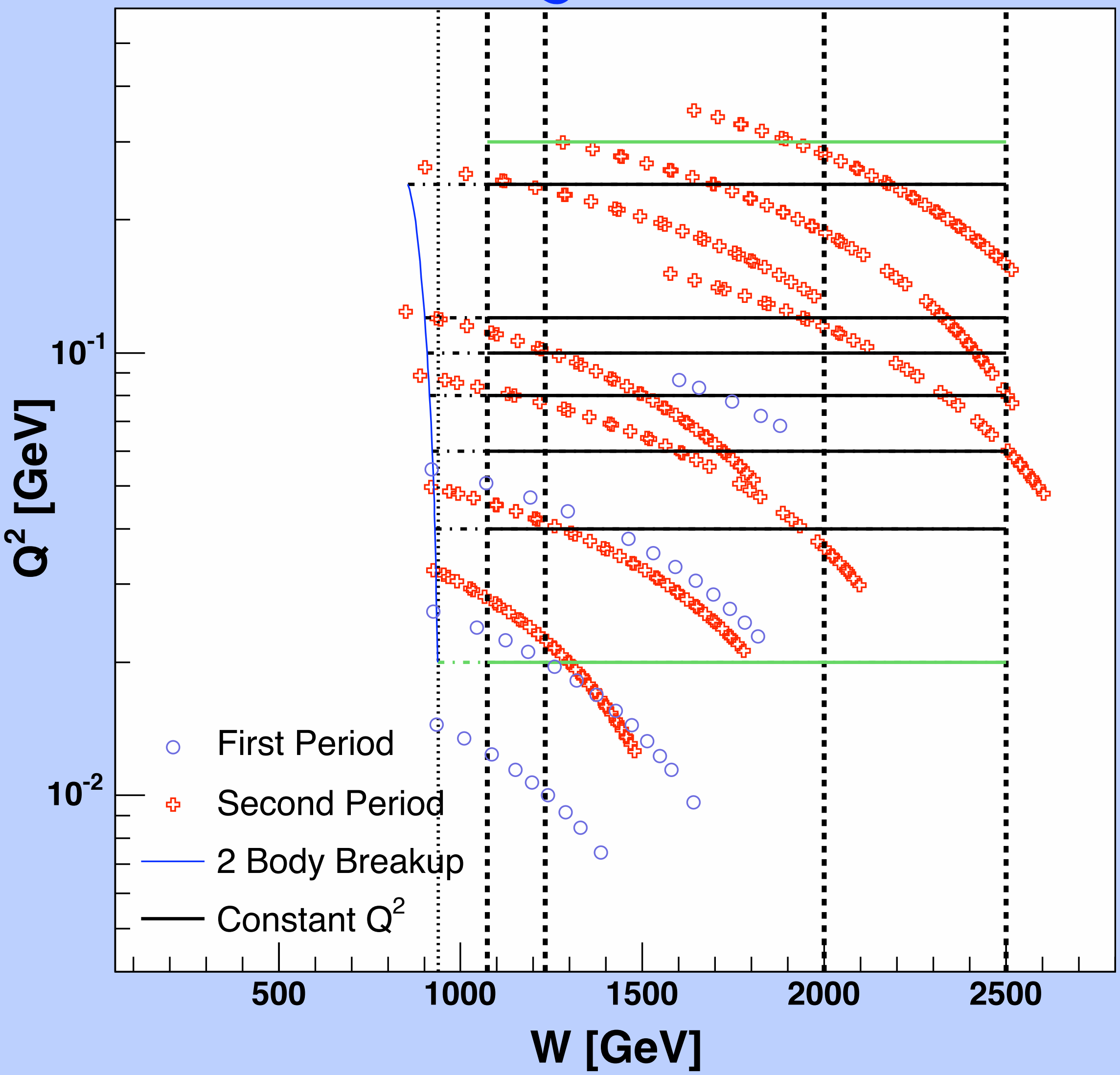




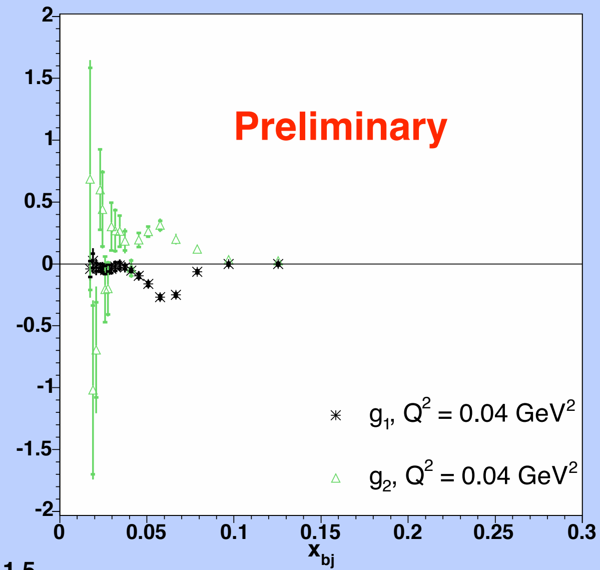
Polarization 35-45% in-beam

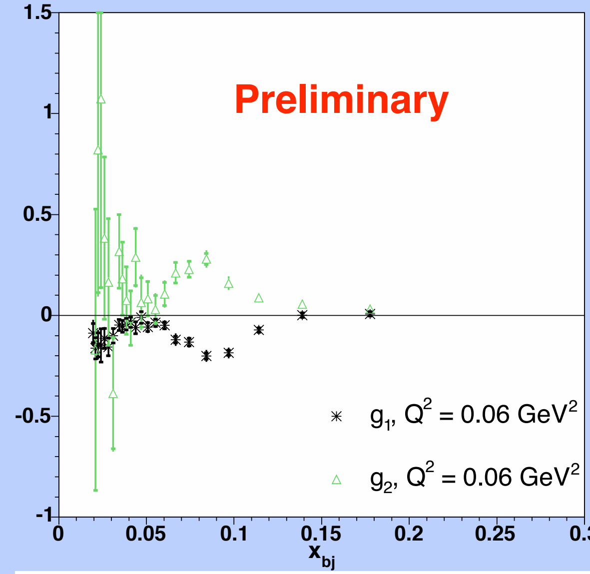
Recent development: new hybrid K/Rb cells give faster spin-up, polarization > 50% !!!

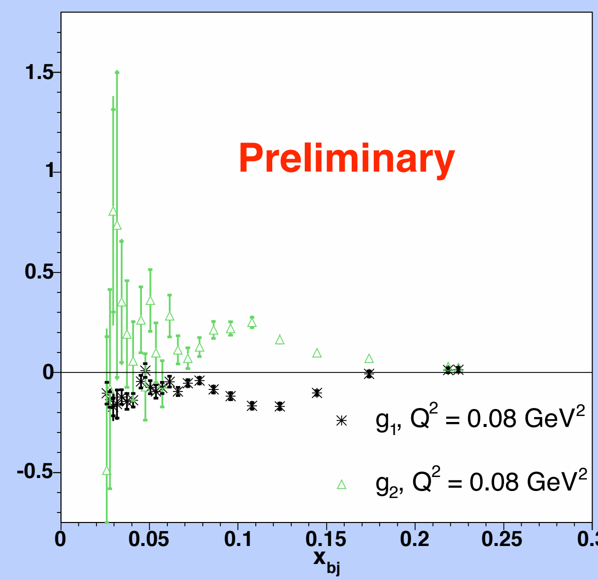
Second Experiment, E97-110
Kinematic Coverage 0.02 < Q < 0.3 GeV^2

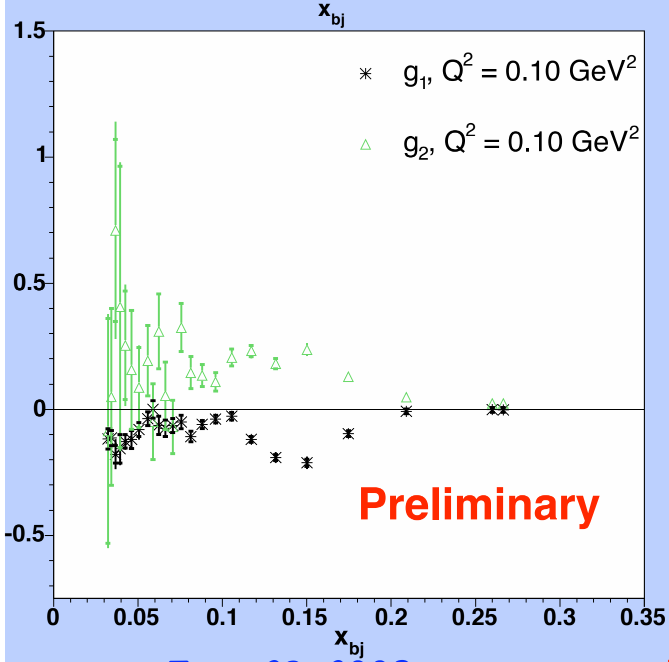


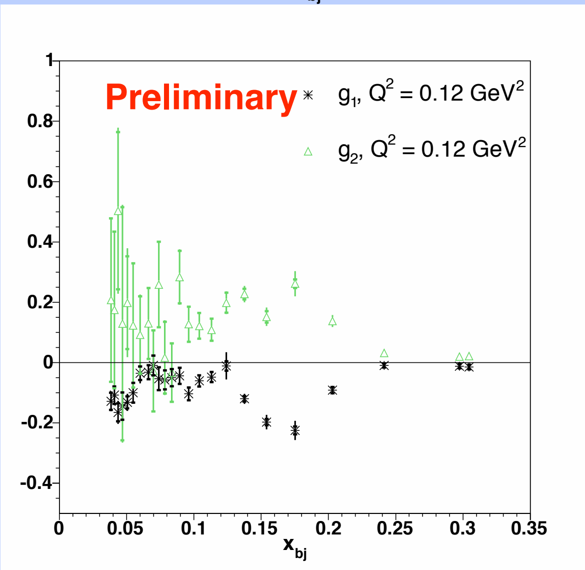
g_1, g_2 versus x at constant Q^2 — ^{3}He

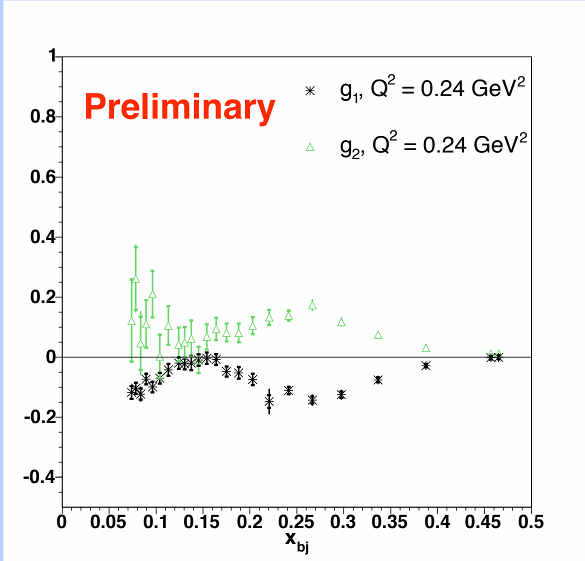




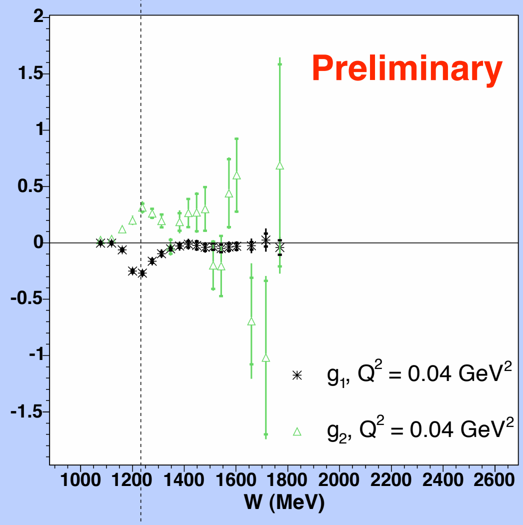


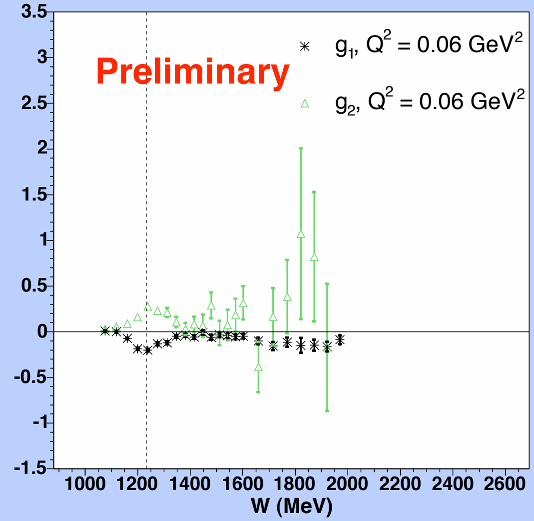


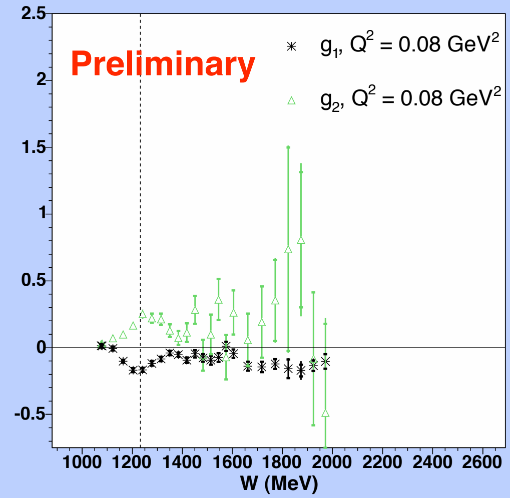




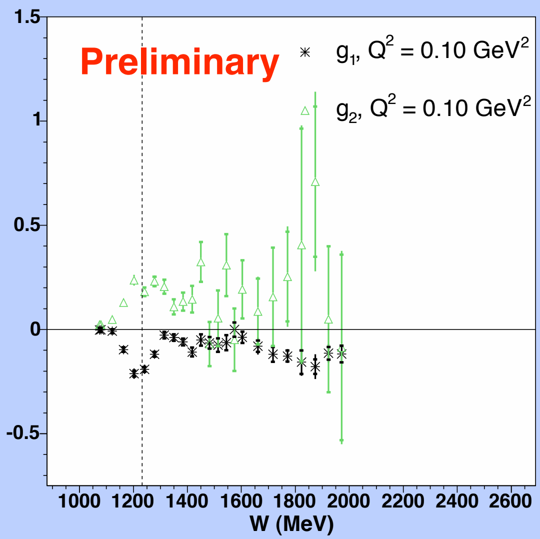
g_1, g_2 versus W at constant Q^2 — ^{3}He

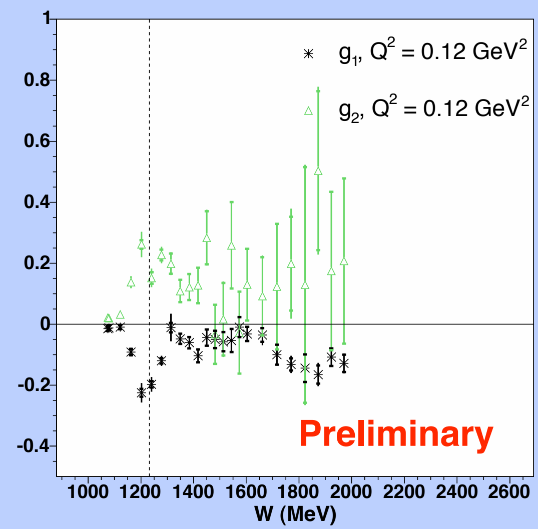


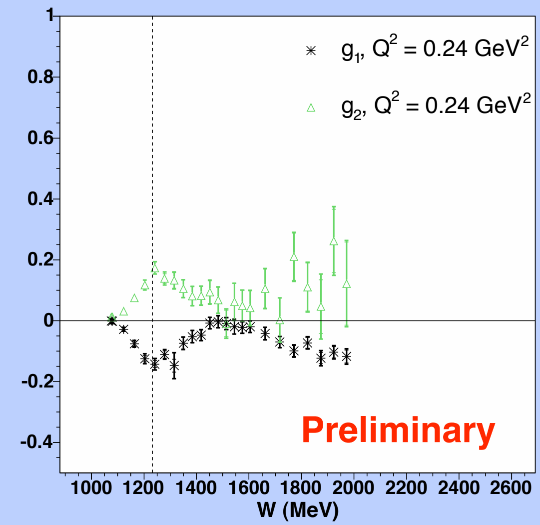




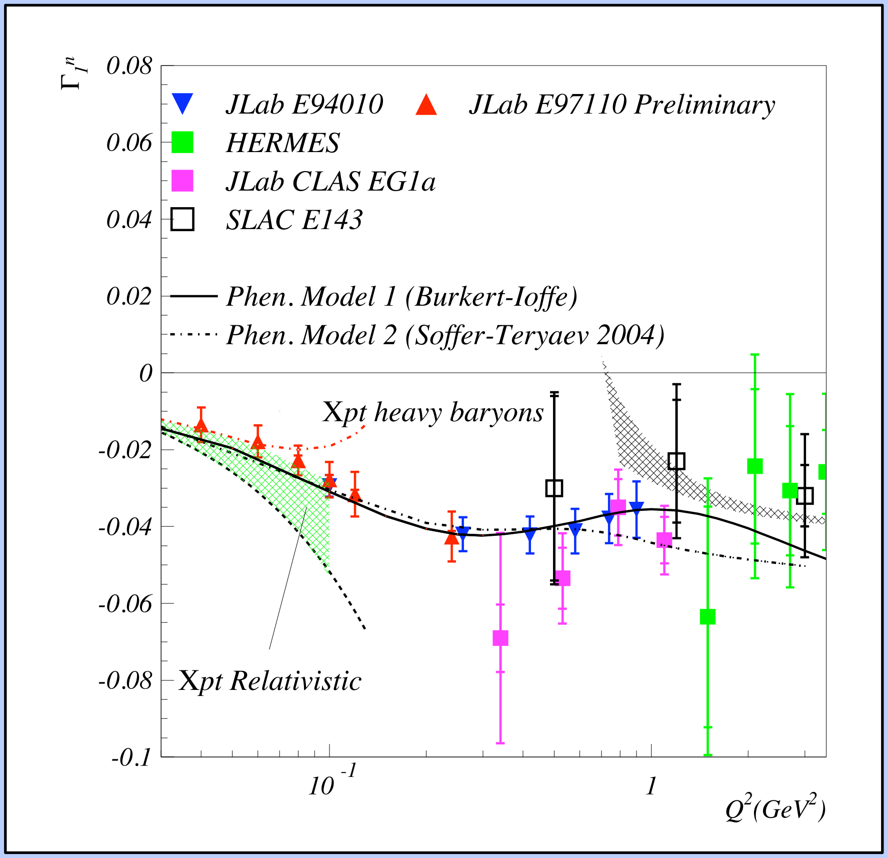
Note $g_2 = -g_1$ at the Δ resonance!

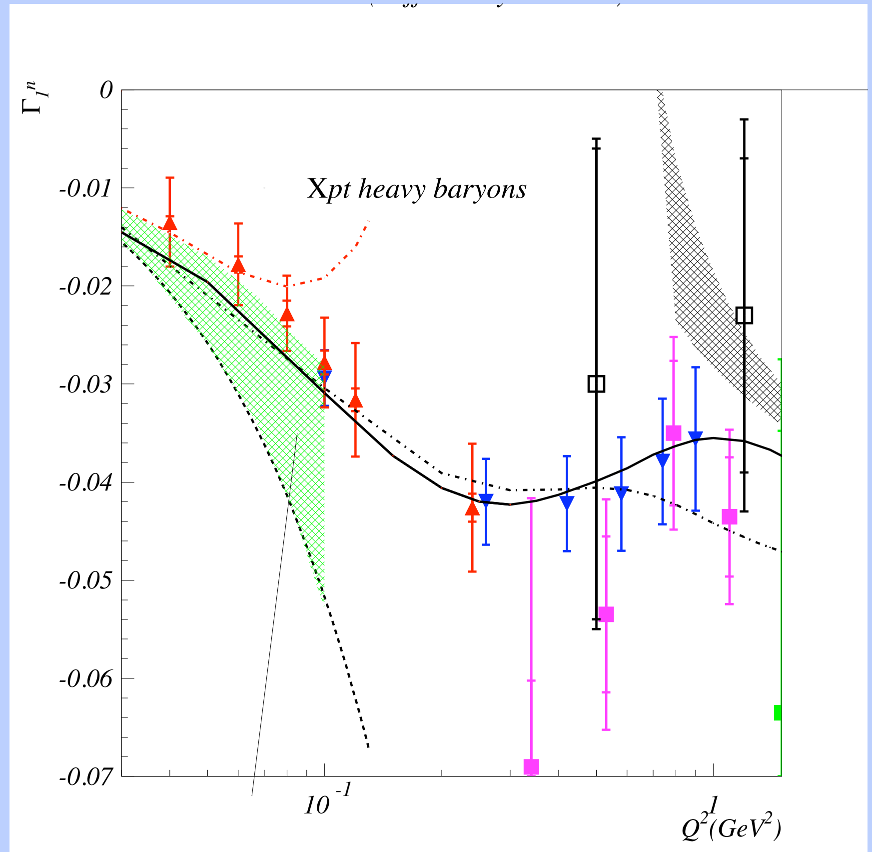






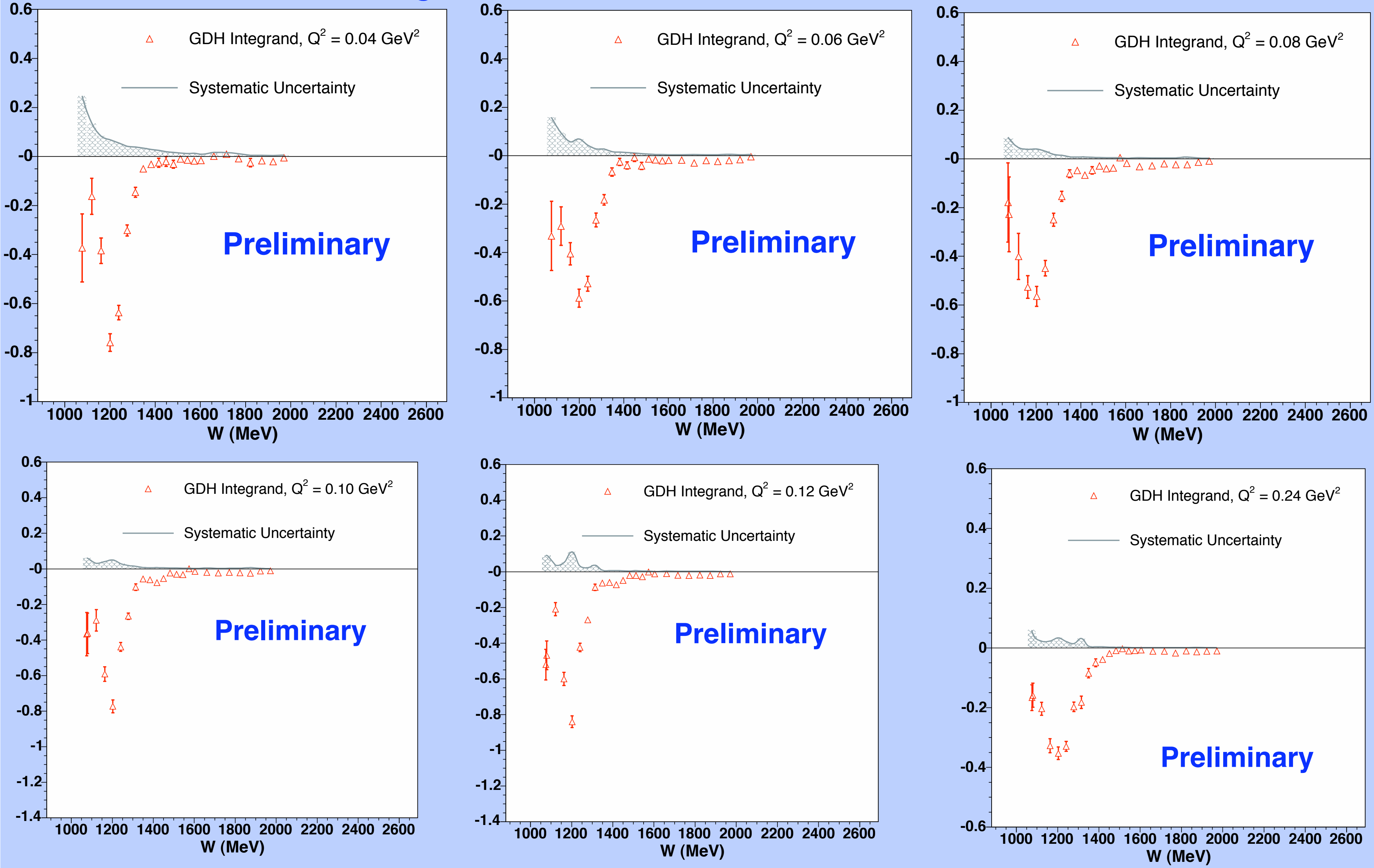
First moment of $g_1^n(Q^2)$--PRELIMINARY





Analysis by V. Sulkosly, Jefferson Lab

GDH integrand σ_{TT}/ν versus W at constant Q^2



Burkhardt-Cottingham Sum Rule

- Use two dispersion relations for Compton amplitude S_2 and same assumptions as for GDH Sum Rule.

- Also require convergence of S_2 faster than $1/\nu$ (a.k.a. SuperConvergence)

$$\Gamma_2(Q^2) = \int_0^1 g_2(x,Q^2)dx = 0$$

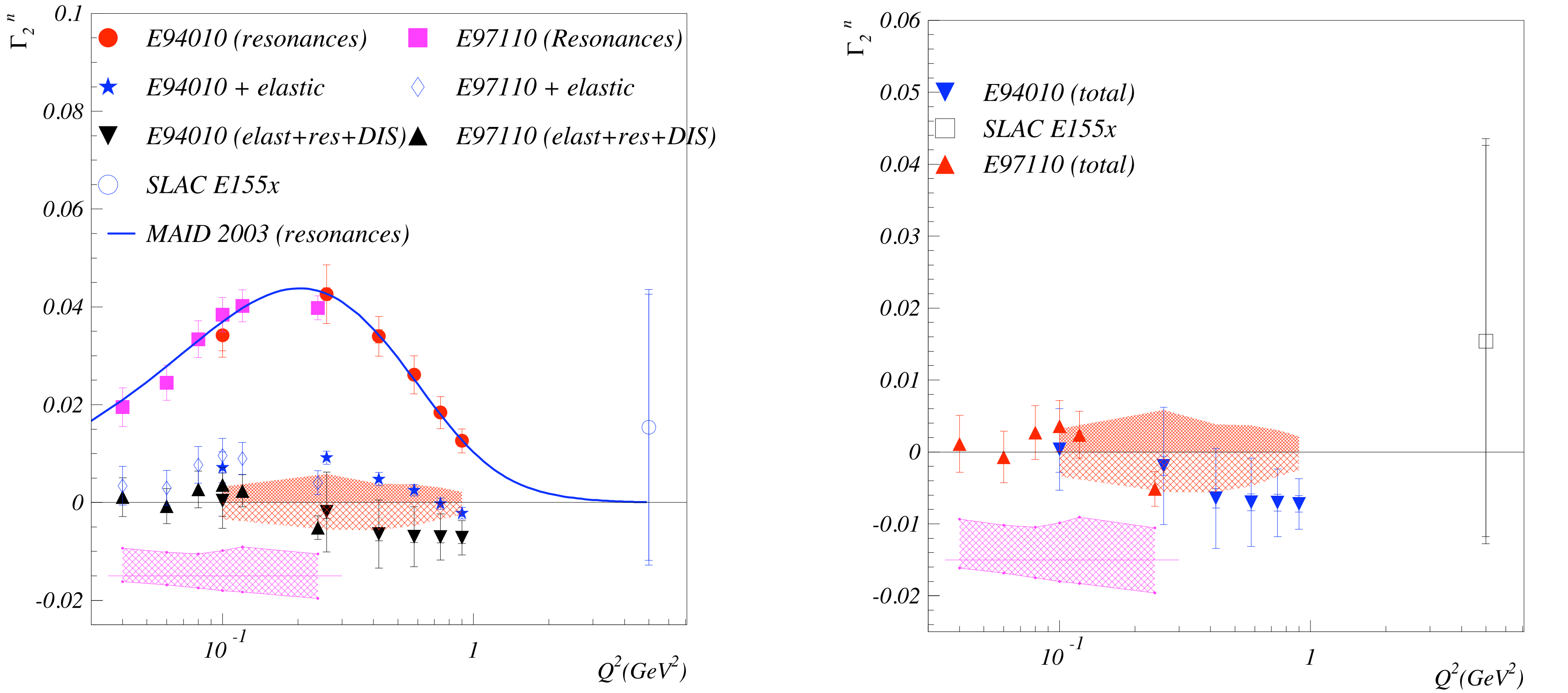
- Valid for all Q^2.

- Can't measure at x=0 where g_2 might diverge...

- Wandzura-Wilczek Relation $g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \overline{g}_2(x,Q^2)$

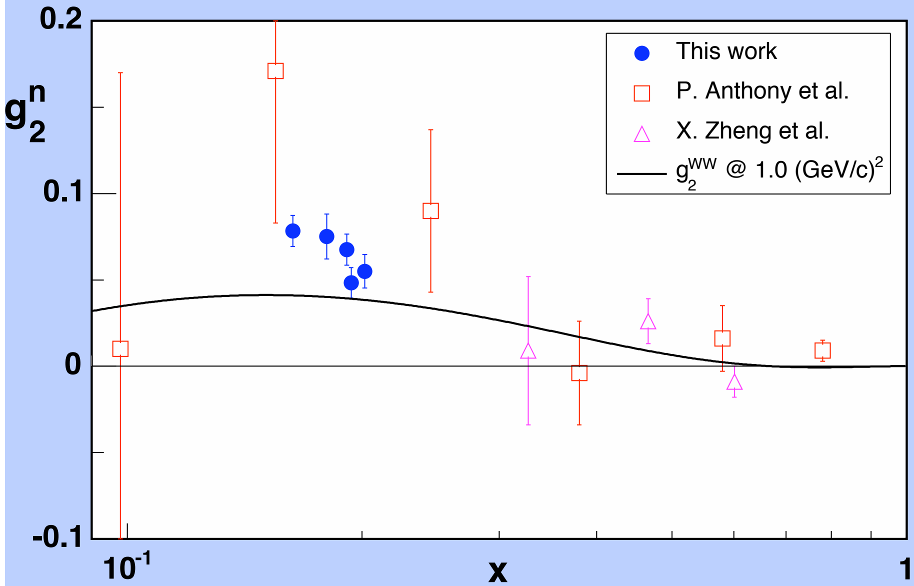
twist-2 higher twist

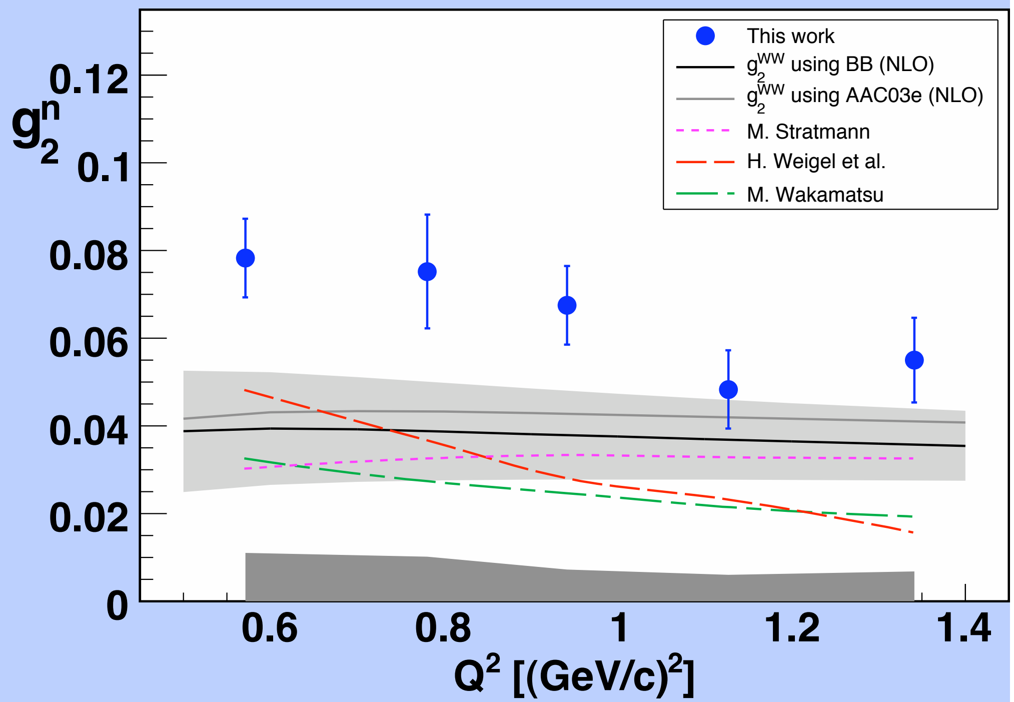
First moment of $g_2^n(Q^2)$--PRELIMINARY
BC Sum Rule



Large inelastic/resonance contribution cancelled by elastic

E97-103: Q^2 dependence of g_2^n





K. Kramer et al.,
arXiv:nucl-ex/0506005, PRL

Twist-3 contribution: $g_2 - g_2^{WW}$

$$C_{tw-3} = 0.0262 \pm 0.0043\ (stat.)$$
$$\pm 0.0080\ (sys.) \pm 0.0099\ (g_2^{WW})$$

Summary and Outlook

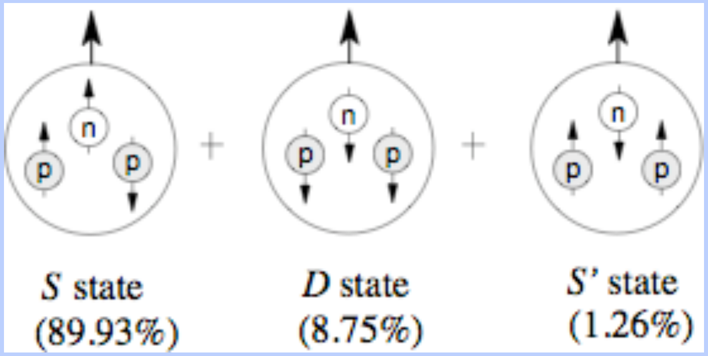
- Precision measurements of polarized and un-polarized cross sections and structure functions g_1 and g_2 for ^3He and neutron from QE to DIS at low Q^2.
- Behavior of GDH g_1 and g_2 integrals measured for ^3He and neutron as $Q^2 \longrightarrow 0$.
- Burkhardt-Cottingham Sum Rule satisfied. Behavior at low x still not known. Inelastic contribution is large at low Q^2.
- New precision measurement of generalized GDH down to Q^2=0.02 GeV^2; results coming soon.
- Reliable extraction of neutron structure functions from ^3He at very low Q^2 will require help from theorists, new data from Jlab in 2009, polarized $^3He(e,e'd)$.

Systematic Uncertainties

Source	Systematic Uncertainty		
Angle	6°	9°	3.775 GeV, 9°
Target density	2.0%		
Acceptance/Effects	5.0%	5.0%	15.0%
VDC efficiency	3.0%	2.5%	2.5%
Charge	1.0%		
PID Detector and Cut effs.	$<$ 1.0%		
$\delta\sigma_{\rm raw}$	6.4%	6.2%	15.5%
Nitrogen dilution	0.2–0.5%		
$\delta\sigma_{\rm exp}$	6.5%	6.3%	15.5%
Beam Polarization	3.5%		
Target Polarization	7.5%		
Radiative Corrections	5–10% in Δ region		
Total on $\Delta\sigma$	11.6–14.5%	11.5–14.4%	18.3–20.2%

Neutron Results from ^{3}He

- Use Bissey et al. formalism, can include contribution from Δ in nucleus



- Biggest nuclear effect due to Fermi motion.
- Integrals less sensitive to nuclear corrections than e.g. structure functions.
- New experiment to study polarized ^{3}He(e,e′d)--^{3}He wave function.