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The observed/measured asymmetry is defined and calculated as

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

where σ_R and σ_L are the scattering cross sections of right and left handed incident electrons, respectively.

This observed asymmetry measurement contains a number of sources of false asymmetries, including helicity-correlated beam position movements. There is beam motion associated with the 30Hz helicity oscillation of the beam, that is, when the beam's helicity reverses, the beam itself moves slightly. Depending on the detectors' response to these beam position shifts, this can contribute to the measured asymmetry value. These false components of the asymmetry measurement must be removed to extract the true parity-violating physics asymmetry.

Beam position monitors (BPMs) record the beam's x and y position in the CODA data file for each run. The Parity Analyzer (PAN) produces two ROOT TTrees from this CODA data file, the "raw tree" (R-Tree) and the "pair tree" (P-Tree). The R-Tree contains the raw beam position and beam current monitor values. The P-Tree contains the asymmetries, calculated as

$$A^X = \frac{X_R - X_L}{X_R + X_L},$$

or differences, calcuated as

$$D^X = X_R - X_L,$$

of these monitors, where R and L indicate the helicity of the incident electrons and X is the monitor.

The false contributions to the observed asymmetry are eliminated using regression. The asymmetries of the detector channels are regressed against the beam position differences in x and y, and the correlations are subtracted to leave only the actual parity-violating physics asymmetry.

When the helicity correlated beam motion is plotted against the detector channel (or, in these test cases, against one of the other BPMs) the correlation is measured by the correlation factor, which is one for a perfect correlation and zero for no correlation. Shifts in detector motion that correlate well with shifts in helicity-correlated beam motion could be an indication that the beam motion is contributing to the measured asymmetry. It is a mantra of Statistics 101, however, that correlation is not causation, a question that is addressed by "dithering", discussed further on. Assuming that there is in fact a causal relationship between the helicity-correlated beam motion and the asymmetry in the detectors, one can calculate the "amount" of correlation using regression and subtract it from the detector asymmetry, leaving a regressed detector signal that is no longer correlated to the beam position motion, and therefore, ideally, contains only the physics asymmetry.

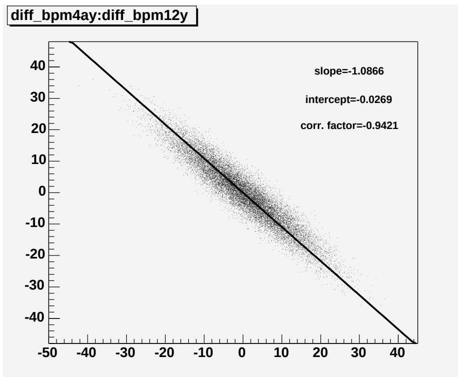

Figure 1: Before Regression

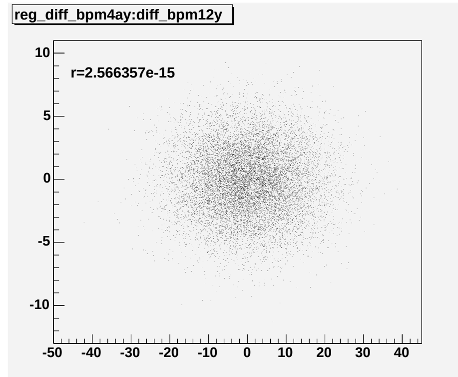

Figure 2: After Regression

Figures 1 and 2 show BPM4ay vs. BPM12y before and after regression for run # 1507, $I_{beam} = 5\mu A$. The correlation factor after regression is almost zero.

To test this procedure, and also to determine the intrinsic resolution of the BPMs at different beam currents, five BPMs are used, 8, 10, 12, 4a, and 4b, 4b being the closest to the target position.

The following four runs from the January and February 2003 run of E01-012 were used, two each at $I_{beam} = 5\mu A$ and $I_{beam} = 12\mu A$.

run #	date	I_{beam}	Energy	A^Q (start/end)
1363	01/18/03	$12\mu A$	5 GeV	2.1/2.3
1507	01/26/03	$5\mu A$	5.02 GeV	9.14/0.35
1519	01/27/03	$12\mu A$	5.02 GeV	1.79/-9.89
1638	02/02/03	$5\mu A$	5.02 GeV	-28.47/5.94

The pre-regression beam position differences are plotted against each other to look at the correlations. (See Fig. 1)

The files are then analyzed using the Regression and Dithering Analysis (REDANA), which, when the correct dependent and independent variables are set in the "control.conf" configuration file, produces a regressed dataset of the dependent variable(s), regressing against all the independent variables listed in the configuration file.

For these tests, BPMs 4a and 4b are regressed separately against the other four BPMs. Two different regression algorithms are implemented in REDANA: iterative linear regression and matrix inversion.

Both regression methods define a dependent variable, y, and n independent variables, x_i. The variable y is regressed against each x_i and the correlation is subtracted.

Iterative linear regression is a simple χ^2-minimization, iterated over each independent variable. First, the correlation coefficient is calculated:

$$b_i = \frac{\Sigma(y - \bar{y})(x_i - \bar{x}_i)}{\Sigma(x_i - \bar{x}_i)},$$

where the summations are over the events for a single independent variable, x_i. Then x_i is regressed from y, leaving a new y, y_{reg}:

$$y_{reg} = y - \bar{y} - b_i(x_i - \bar{x}_i).$$

This is then iterated over all the independent variables, using the y_{reg} as the new y, and x_{i+1}.

The matrix inversion algorithm declares the same independent and dependent variables, but also defines an independent variable matrix of variances and covariances:

$$X_{ij} = \Sigma(x_i - \bar{x}_i)(x_j - \bar{x}_j)$$

and a dependent variable vector of covariances:

$$Y_i = \Sigma(x_i - \bar{x}_i)(y - \bar{y})$$

A vector of correlation factors $\boldsymbol{B}$ is calculated by inverting the independent $\boldsymbol{X}$ matrix:

$$\boldsymbol{B} = \boldsymbol{Y}\boldsymbol{X}^{-1}.$$

The regression is then calculated as:

$$y_{reg} = y - \bar{y} - \sum_i B_i(x_i - \bar{x}_i).$$

The regressed beam position differences are again plotted against each other to confirm that the regression algorithm did in fact eliminate any correlations.(see Fig. 2)

The pre- and post-regression beam position differences are then plotted on the same graph, and fitted with a Gaussian, to determine the change in mean and RMS/σ after the regression. (see Fig. 3, and 5-7)

Finally, the standard deviation, σ, from the Gaussian fits is plotted vs. beam current. (Fig. 4 and 8)

The before and after widths for BPM4b are tabulated here:

		Before Regression		After Regression	
run #	I_{beam}	σ_x	σ_y	σ_x	σ_y
1507	$5\mu A$	9.1003	7.2708	3.7865	2.8820
1638	$5\mu A$	115599	7.5288	3.8029	3.0403
1363	$12\mu A$	7.6024	8.1165	2.1559	2.1702
1519	$12\mu A$	9.0303	6.9403	2.3719	1.8047

The "dithering" part of REDANA is a process where the beam parameters are varied on purpose, that is, the beam is moved on purpose in a controlled manner and the correlations between the beam motion and the detector response is measured. This is to test the reliability of the regression algorithm, since it should not matter whether the beam motion is artificially introduced or randomly occuring.

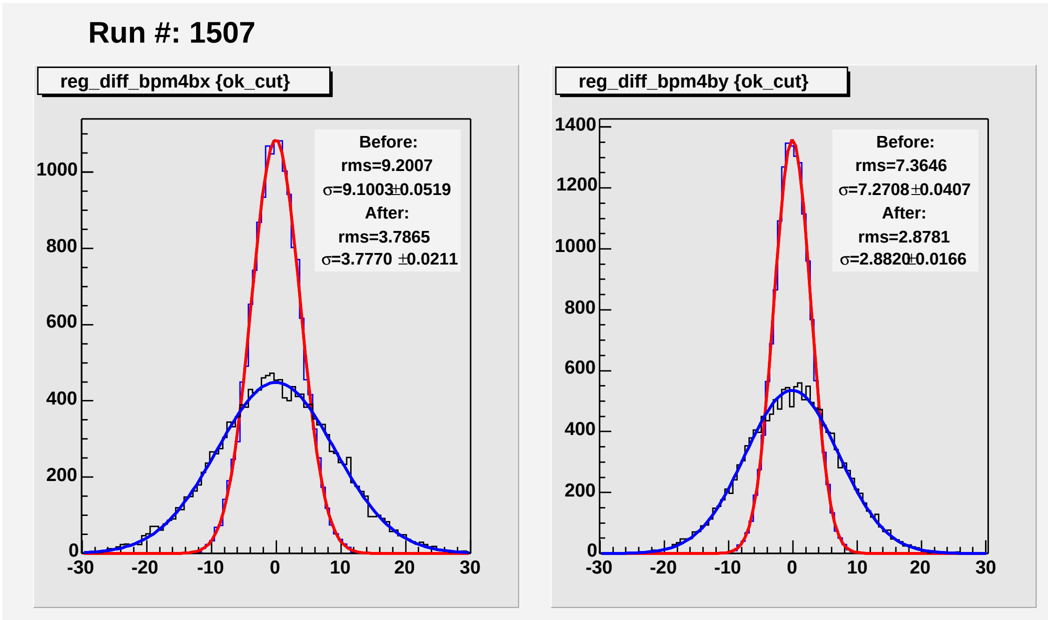

Figure 3: BPM4b, run #1507, $I_{beam} = 5\mu A$

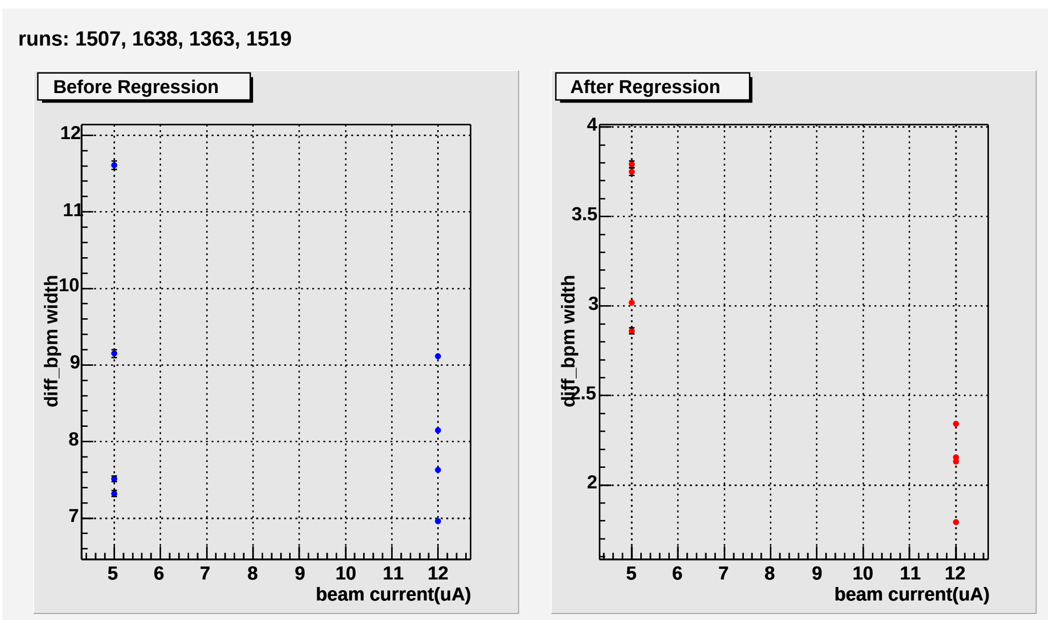

Figure 4: BPM4b

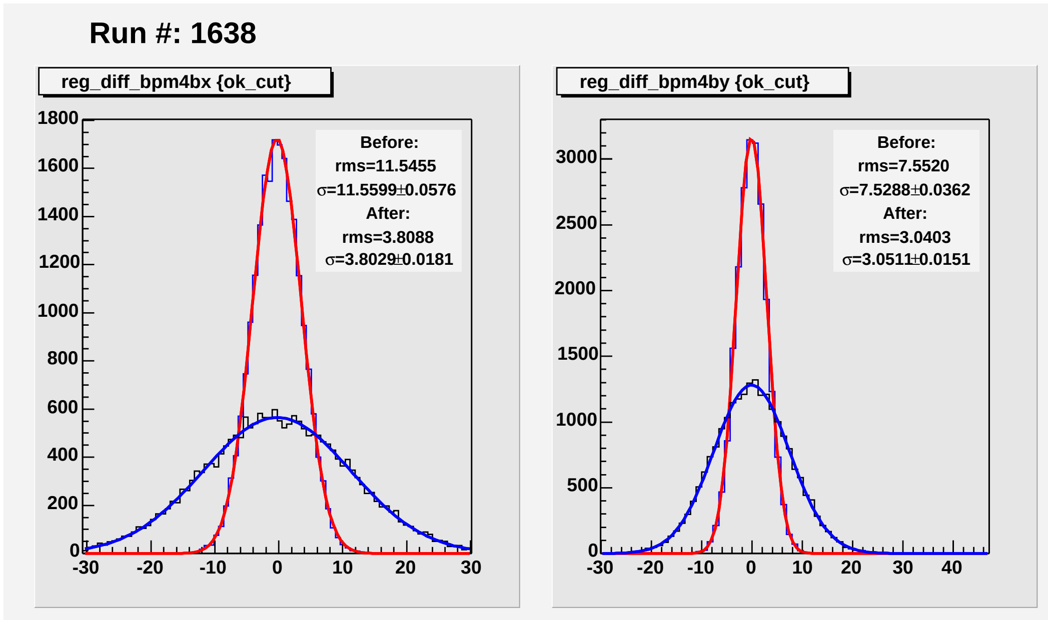

Figure 5: BPM4b, run #1638, $I_{beam} = 5\mu A$

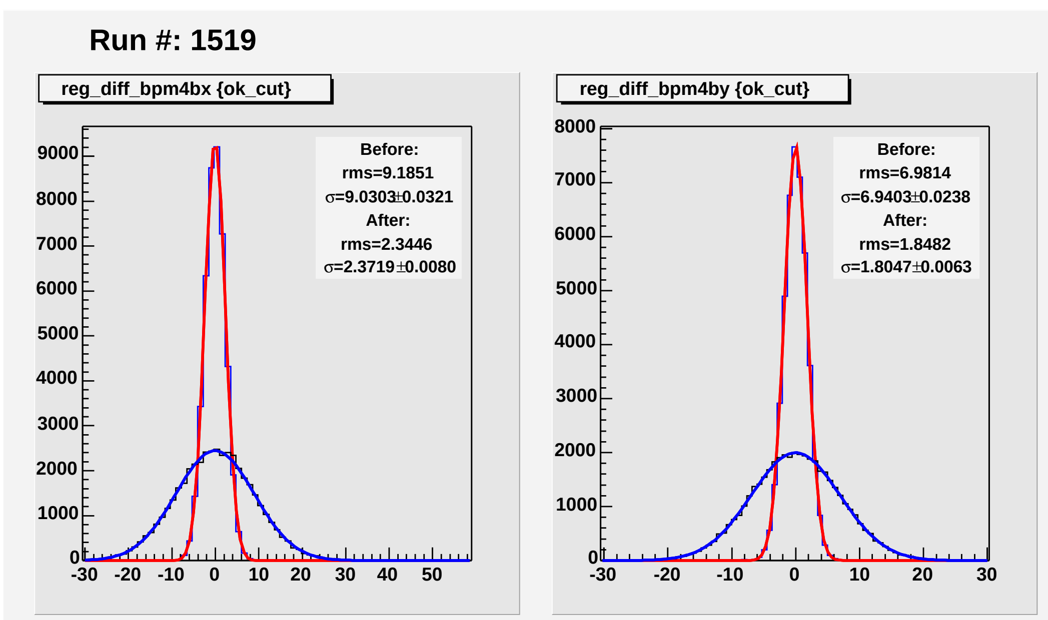

Figure 6: BPM4b, run #1516, $I_{beam} = 12\mu A$

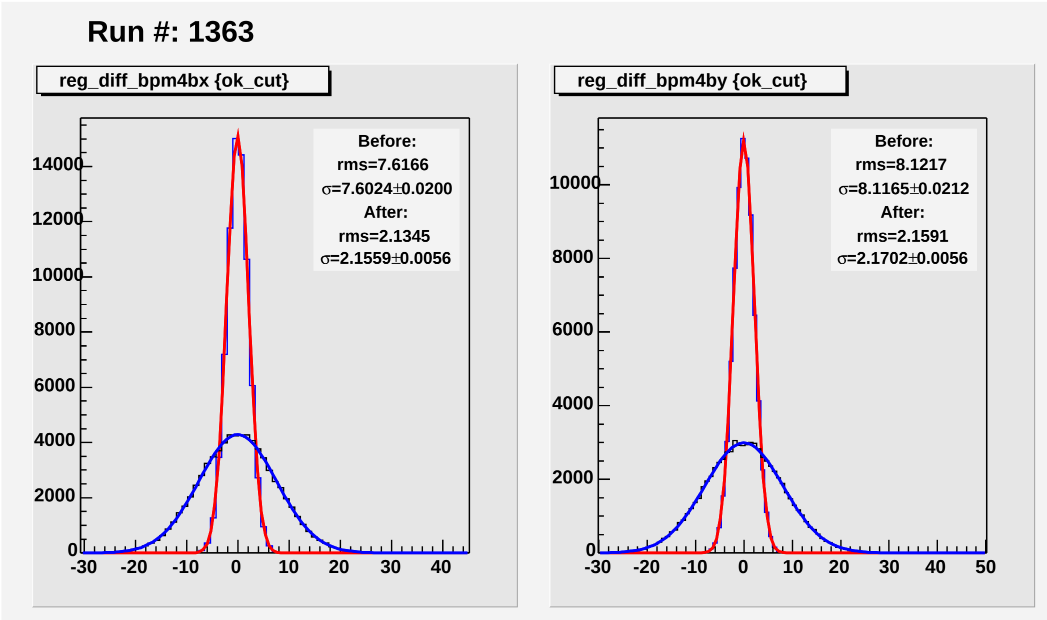

Figure 7: BPM4b, run #1363, $I_{beam} = 12\mu A$

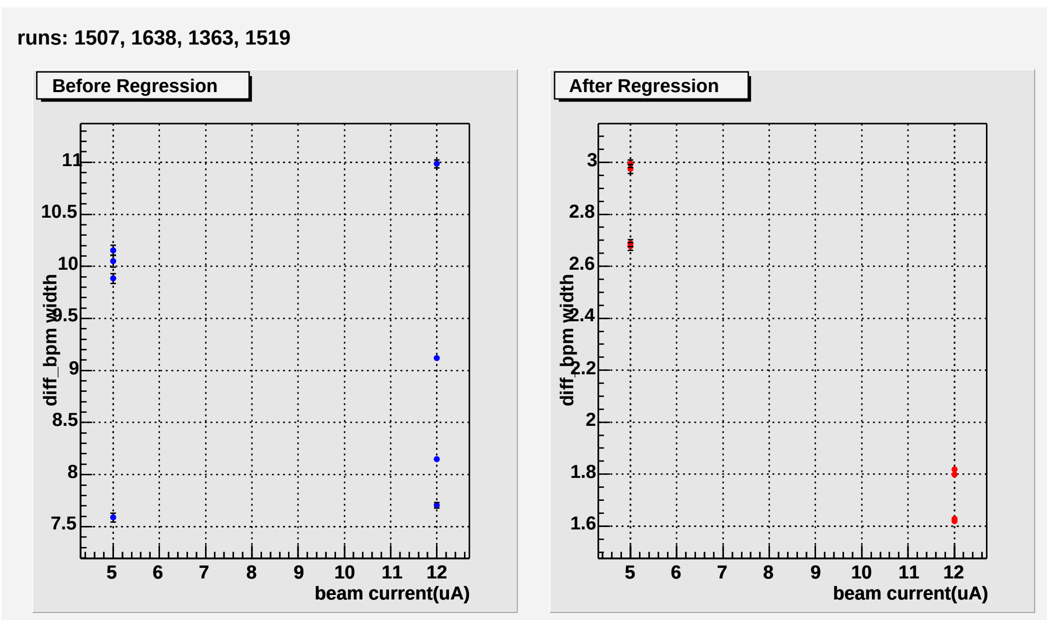

Figure 8: BPM4a