False Asymmetries due to Iron Collimators

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1 INTRODUCTION

This document presents an estimate of the false asymmetry caused by polarized electrons being collimated in the spectrometer by a slit made of magnetized iron. For HAPPEX, iron at the front of the spectrometer dipole acts as a collimator (it limits the acceptance near the ends of the 15 cm long target.) Some of the electrons lose some energy by Moller scattering from the polarized electrons in the iron, and the Moller cross section is spin dependent. Electrons that lose too much energy no longer reach the detector, and this loss of events is therefore helicity dependent.

2 Penetration through iron

The geometry we are assuming is given in Fig. 1. The arrow indicates the type of track that is of concern. It just clips the edge of the collimator and passes through a small amount of iron. If it passed through more iron, it would lose too much energy due to bremsstrahlung to be accepted by the detector. It may also lose some energy by Moller scattering. For one helicity, the Moller cross section is larger, and the spin-dependent Moller scattering may cause the otherwise good track to be lost. For the calculation, we will integrate over all such tracks. We approximate the tracks to be parallel. Also, we assume that the electrons have a uniform distribution that extends beyond the gap into the iron.

We define the following quantitles:

I=Uniform flux of electrons per unit distance.

w=width of gap between slits.

$\theta \ll 1 \sim 1/20$ =angle electrons make with side of collimator. See Fig. 1.

ΔE = maximum energy loss that still is within acceptance. We will assume $/DeltaE$=100MeV.

x =distance inside edge of collimator. (See Fig. 1.) $x < 0 \Rightarrow$ electrons pass through slit. $x > 0 \Rightarrow$ electrons pass through some iron.

$t \sim \theta$ = approximate thickness of iron penetrated by a given event for the geometry of Fig. 1.

$P(t)$ =probability that electron passes through t cm of iron without losing more than ΔE.

L_R=radiation length of iron =1.75 cm.

ϕ =angle B field makes with electron helicity. Here we assume a conservative $\cos\phi = 0.2$.

R_0 = 2.818 fm= classical electron radius.

$\delta\sigma$= difference in cross section for longitunaly polarized electorns.

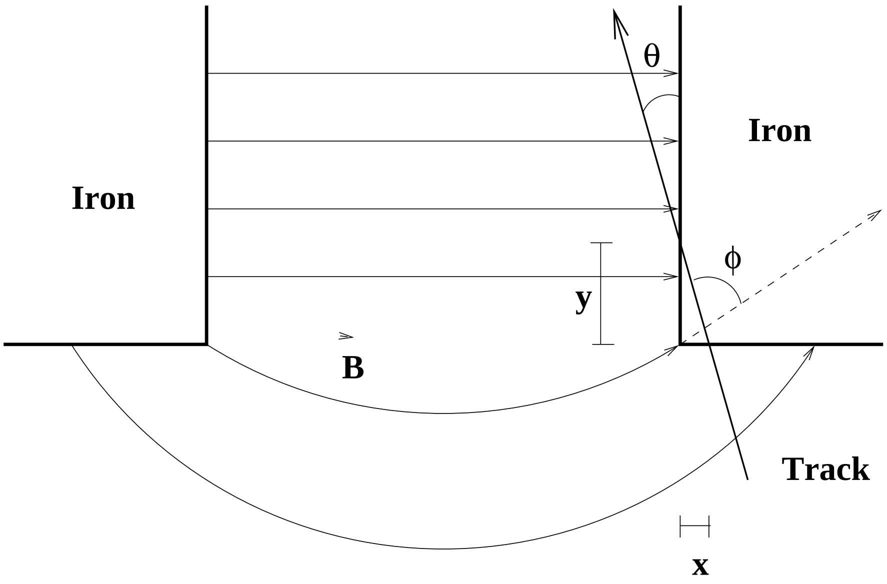

Figure 1: Geometry for iron collimators

For the general case, the asymmetry $\mathcal{A}$ due to a cross section difference $\delta\sigma$ is

$$\mathcal{A} \equiv \frac{\delta N}{N_i} = t(\delta\sigma)\frac{\rho N_0}{A}$$

where N_i is the incident flux, ρ is the density if iron, N_0 is Avagadro's number, and $A \sim 55$ is the atomic number.

In our case, $\mathcal{A}$ is the ratio of δN, the difference in the (small) number of events clippping the collimator, divided by the number of good events $=Iw$ that pass through the slits. In addition, t varies with x for parallel tracks:

$$\mathcal{A} = \frac{\delta N}{Iw} = \frac{2}{Iw} \int_0^\infty dx It P(y) (\delta\sigma) \frac{\rho N_0}{A}$$

thus we have a small difference in attenuation for the small number of electrons clipping the collimator.

From QED, the number of electrons losing less that ΔE when passing througn matter is:

$$P(t) = \left(\frac{\Delta E}{E}\right)^{(t/L_R)} = \exp\left[\ln\left(\frac{\Delta E}{E}\right)\left(\frac{x}{\theta L_R}\right)\right] = e^{-x/b}$$

where

$$b \equiv -\theta L_R / \ln(\Delta E/E) = .05 \times 1.75 / \ln(100MeV/3.2GeV) = 0.025cm$$

Thus:

$$\mathcal{A} = \frac{\delta N}{Iw} = \frac{2}{Iw} \int_0^\infty dx \frac{x}{\theta} e^{-x/b} \delta\sigma \frac{\rho N_0}{A} = \frac{2b^2}{w\theta} \delta\sigma \frac{\rho N_0}{A}$$

3 Moller Sacttering

First, some kinematics must be done. We compute the minimun CM scattering angle θ_{min} corresponding to an energy loss of ΔE. Then θ_{min} provides the needed cutoff for the divergent integrals that commonly arise in computing the total cross section for Moller scattering.

$$E_B = 2\gamma E_{CM}$$

$$E' = E_{CM}\gamma(1 + \cos\theta_{CM} \sim 2E_{CM}\gamma(1 - \theta^2/2)$$

$$\nu = E - E' = E_{CM}\gamma\theta^2/2 = E_B\theta_{CM}^2/4$$

$$\theta_{CM} \sim \sqrt{4\nu/E_B} \sim 21^\circ \sim \theta_{min}$$

The Moller asymmetry is:

$$A_M = \frac{\sin^2\theta(8-\sin^2\theta)}{(4-\sin^\theta)^2} < \frac{8}{9}\sin^2\theta$$

and the cross section is:

$$\frac{d\sigma}{d\Omega} = \left[R_0\sqrt{\frac{m}{2E}}\frac{4-\sin^2\theta}{\sin^2\theta}\right]^2 < R_0^2\frac{m}{2E}\frac{16}{\sin^4\theta}$$

We have introduced simple approximations that make the integrals easier to do.

Thus the integrated asymmetry $\delta\sigma$ is:

$$\delta\sigma = \int A_M\frac{d\sigma}{d\Omega}d\Omega < 2\pi\frac{8}{9}\frac{8R_0^2m}{E}\int\frac{d\theta}{\sin\theta} = \frac{128\pi}{9}\ln(\tan[\theta_{min}/2])\frac{R_0^2m}{E} = 10^{-3}barn$$

4 Results

Finally, two more factors must be included: $\cos\phi$ and $P_e \sim 5\%$ for iron:

$$\mathcal{A} = \frac{2b^2}{w\theta}(\delta\sigma)\frac{\rho N_0}{A}\cos\phi P_e$$

$$= \frac{2\times(0.025cm)^2}{15cm\times 0.05}10^{-3}barns\frac{8g/cm^3 6\times 10^{23}}{55}\times 0.2\times 0.05 \sim 10^{-9}$$

In conclusion, the contribution is negligible. There is no need to add a tungsten collimator to alleviate the problem.

5 More realistic geometry

The pole tips are actually rounded as shown in Fig. 2 with a radius of curvature R. We redefine x to be the nearest distance of the track to the pole tip, then:

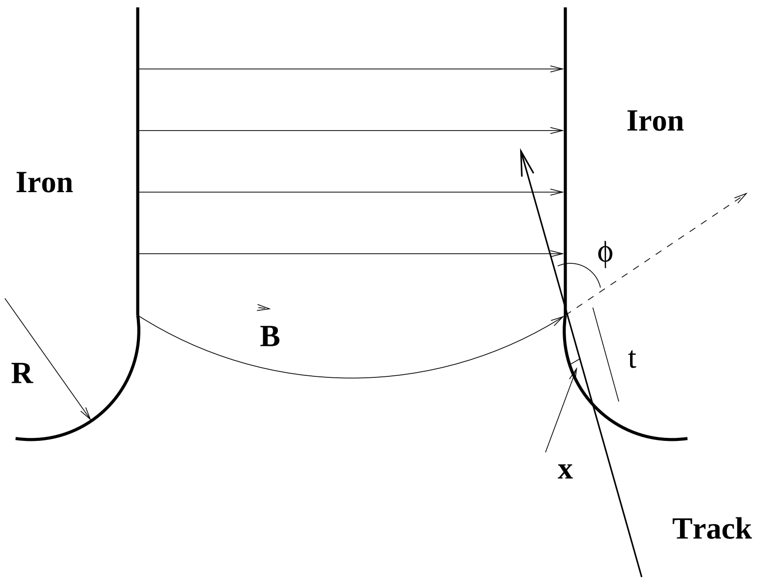

Figure 2: Geometry for rounded iron collimators

$$t = \sqrt{2Rx}: \quad dx = \frac{t}{R}dt$$

$$P(t) = \left(\frac{\Delta E}{E}\right)^{(t/L_R)} = \exp\left[\ln\left(\frac{\Delta E}{E}\right)\left(\frac{t}{L_R}\right)\right] = e^{-x/b'}$$

where

$$b' \equiv -L_R/\ln(\Delta E/E) = 1.75/\ln(100MeV/3.2GeV) = 0.5cm; \quad b = b'\theta$$

Thus:

$$\mathcal{A} = \frac{\delta N}{Iw} = \frac{2}{Iw}\int_0^\infty dt\frac{t^2}{R}e^{-x/b'}\delta\sigma\frac{\rho N_0}{A} = \frac{4b'^3}{wR}\delta\sigma\frac{\rho N_0}{A}$$

The change form the previous result is that θ is replaced by $2b'/R$, which increases the estimate by a factor of 2. On the other hand, $\cos\phi$ should be smaller. Thus the conclusion is the same.