Simulation update

Packing fraction

Last time

Correction factor

- Radiation effect δ

- Electron straggling in the target δ_t

$$\frac{d\sigma}{d\Omega}|_{meas} = \frac{d\sigma}{d\Omega}|_{Rosenbluth} e^{(\delta+\delta_t)}$$

- For our system, $Q^2 \approx 0.5\ GeV^2$, the internal bremsstrahlung equals two external radiator with t ≈ 0.01

- Target thickness t ≈ 0.03 straggling effect is very important

Last time

Radiation effect

$$
\begin{aligned}
\delta = \frac{-\alpha}{\pi}\Bigg(& \tfrac{28}{9} - \tfrac{13}{6}\ln\left(\frac{-q^2}{m^2}\right) + \left(\ln\frac{-q^2}{m^2} - 1 + 2Z\ln\eta\right)\left(2\ln\frac{E_1}{\Delta E} - 3\ln\eta\right) - \Phi\left(\frac{E_3-E_1}{E_3}\right) - Z^2\ln\frac{E_4}{M} \\
& + Z^2\ln\frac{M}{\eta\Delta E}\left(\frac{1}{\beta_4}\ln\frac{1+\beta_4}{1-\beta_4} - 2\right) + \frac{Z^2}{\beta_4}\left\{\tfrac{1}{2}\ln\frac{1+\beta_4}{1-\beta_4}\ln\frac{E_4+M}{2M} - \Phi\left[-\left(\frac{E_4-M}{E_4+M}\right)^{1/2}\left(\frac{1+\beta_4}{1-\beta_4}\right)^{1/2}\right]\right\} \\
& + Z\left[\Phi\left(-\frac{M-E_3}{E_1}\right) - \Phi\left(\frac{M(M-E_3)}{2E_3E_4-ME_1}\right) + \Phi\left(\frac{2E_3(M-E_3)}{2E_3E_4-ME_1}\right) + \ln\left|\frac{2E_3E_4-ME_1}{E_1(M-2E_3)}\right|\ln\left(\frac{M}{2E_3}\right)\right] \\
& - Z\left[\Phi\left(-\frac{E_4-E_3}{E_3}\right) - \Phi\left(\frac{M(E_4-E_3)}{2E_1E_4-ME_3}\right) + \Phi\left(\frac{2E_1(E_4-E_3)}{2E_1E_4-ME_3}\right) + \ln\left|\frac{2E_1E_4-ME_3}{E_3(M-2E_1)}\right|\ln\left(\frac{M}{2E_1}\right)\right] \\
& - Z\left[\Phi\left(-\frac{M-E_1}{E_1}\right) - \Phi\left(\frac{M-E_1}{E_1}\right) + \Phi\left(\frac{2(M-E_1)}{M}\right) + \ln\left|\frac{M}{2E_1-M}\right|\ln\left(\frac{M}{2E_1}\right)\right] \\
& + Z\left[\Phi\left(-\frac{M-E_3}{E_3}\right) - \Phi\left(\frac{M-E_3}{E_3}\right) + \Phi\left(\frac{2(M-E_3)}{M}\right) + \ln\left|\frac{M}{2E_3-M}\right|\ln\left(\frac{M}{2E_3}\right)\right]\Bigg) \\
- \frac{\alpha}{\pi}\Bigg(& -\Phi\left(\frac{E_1-E_3}{E_1}\right) + \frac{Z^2}{\beta_4}\left\{\Phi\left[\left(\frac{E_4-M}{E_4+M}\right)^{1/2}\left(\frac{1-\beta_4}{1+\beta_4}\right)^{1/2}\right] - \Phi\left[\left(\frac{E_4-M}{E_4+M}\right)^{1/2}\right] + \Phi\left[-\left(\frac{E_4-M}{E_4+M}\right)^{1/2}\right]\right\}\Bigg).
\end{aligned}
$$

Vacuum , vertex, recoil effect, dynamical effect

Rev. Mod. Phys, 1969

Last time

Straggling effect

$$\delta_t = -\left\{\left[b_w t_{iw} + \tfrac{1}{2} b T\right] \ln\left(E_1/\eta^2 \Delta E\right) + \left[b_w t_{fw} + \tfrac{1}{2} b T\right] \ln\left(E_3/\Delta E\right)\right\}$$

- t_{fw}: final windows thickness
- t_{iw}: initial windows thickness
- T : target thickness

In units of radiation length

- b_w b : depends on material, close to 4/3

- $\delta_t \approx -0.33$
- $\delta \approx -0.26$

Straggling Effect

$$\sigma_b \equiv \left(\frac{d^2\sigma}{d\Omega\, dE_p}\right)_b$$

$$= \frac{M_T + 2(E_s - \omega_s)\sin^2(\frac{1}{2}\theta)}{M_T - 2E_p \sin^2(\frac{1}{2}\theta)}$$

$$\times \tilde{\sigma}_{\mathrm{el}}(E_s - \omega_s)\left[\frac{bt_b}{\omega_s}\,\phi(v_s) + \frac{\xi}{2{\omega_s}^2}\right]$$

$$+ \tilde{\sigma}_{\mathrm{el}}(E_s)\left[\frac{bt_a}{\omega_p}\,\phi(v_p) + \frac{\xi}{2{\omega_p}^2}\right],$$

- Straggling effect due to real bremesstrahlung and ionization
- Energy peak
- Elastic Tail
- Phys.Rev. D VO.12 NO.7
- Thanks Ryan

Internal Bremesstrahlung

- one-photon exchange mechanism
- interference terms between the electron bremsstrahlung and the hadron bremsstrahlung are ignored
- only the scattered electrons are detected.

$$\frac{d^2\sigma_{jr}}{d\Omega dp} = \frac{\alpha^3}{(2\pi)^2}\left(\frac{E_p}{E_s}\right) M^{-1} \int_{-1}^{1} \frac{\omega d(\cos\theta_k)}{2q^4(u_0 - |\mathbf{u}|\cos\theta_k)}$$

$$\times \Bigg(M^2 F_j(q^2) \left\{ \frac{-2\pi a m^2}{(a^2-b^2)^{3/2}} \left[2E_s(E_p+\omega) + \frac{q^2}{2} \right] - \frac{2\pi a' m^2}{(a'^2-b'^2)^{3/2}} \left[2E_p(E_s-\omega) + \frac{q^2}{2} \right] - 4\pi \right.$$

$$+ 4\pi \left(\frac{\nu}{(a^2-b^2)^{1/2}} - \frac{\nu'}{(a'^2-b'^2)^{1/2}} \right) \{ m^2(sp-\omega^2) + (sp)[2E_sE_p - (sp) + \omega(E_s-E_p)] \}$$

$$+ \frac{2\pi}{(a^2-b^2)^{1/2}} \left[2(E_sE_p + E_s\omega + E_p^2) + \frac{q^2}{2} - (sp) - m^2 \right]$$

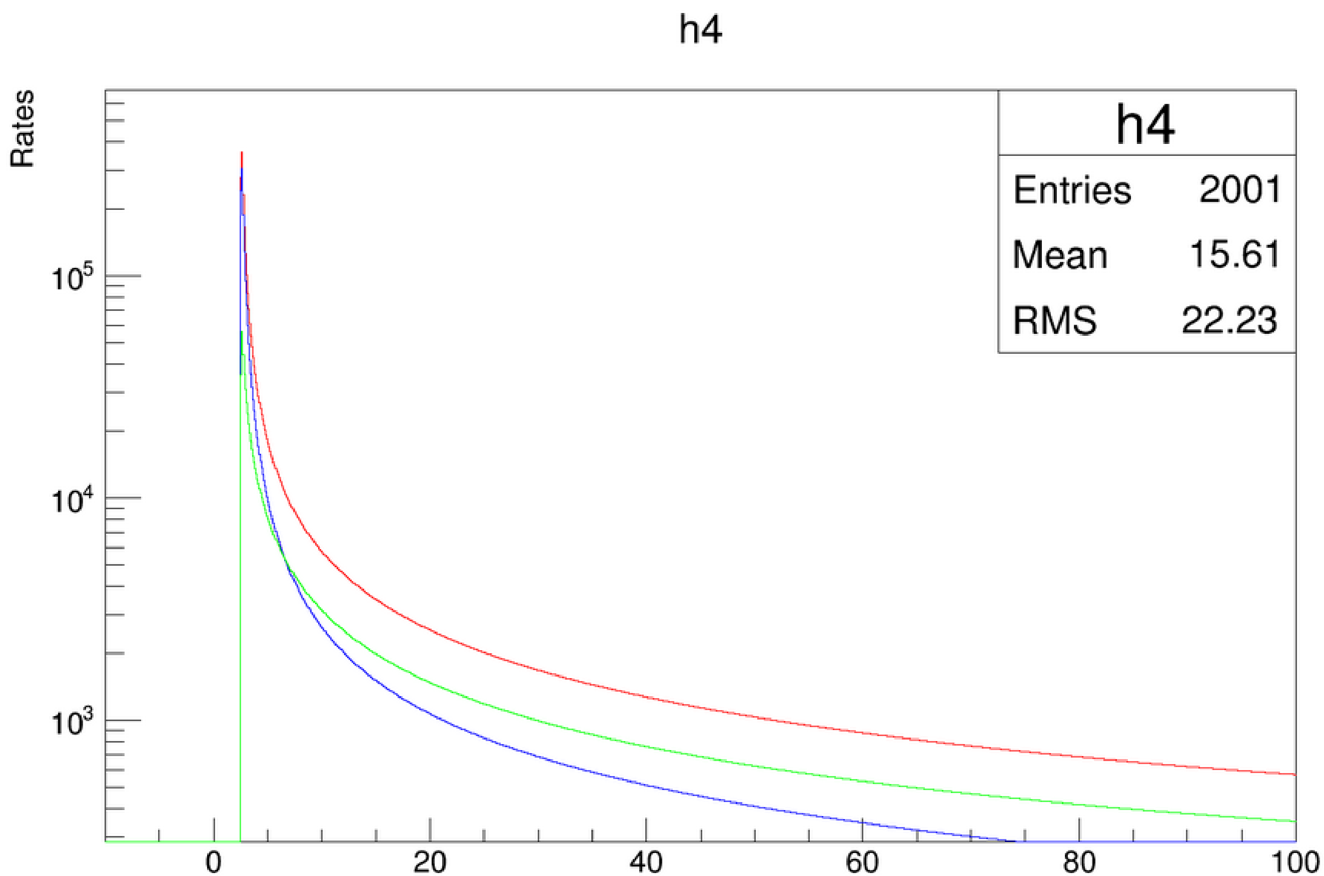
$$\left. - \frac{2\pi}{(a'^2-b'^2)^{1/2}} \left[2(E_sE_p - E_p\omega + E_s^2) + \frac{q^2}{2} - (sp) - m^2 \right] \right\}$$

$$+ G_j(q^2) \left[\left(\frac{2\pi a}{(a^2-b^2)^{3/2}} + \frac{2\pi a'}{(a'^2-b'^2)^{3/2}} \right) m^2(2m^2+q^2) + 8\pi \right.$$

$$\left. + 8\pi \left(\frac{\nu}{(a^2-b^2)^{1/2}} - \frac{\nu'}{(a'^2-b'^2)^{1/2}} \right) (sp)(sp-2m^2) + 2\pi[(a^2-b^2)^{-1/2} - (a'^2-b'^2)^{-1/2}](2sp+2m^2-q^2) \right] \Bigg)$$

Radiation Tail from Carbon target

One Fixed energy and angle



- Red: total
- Green: Internal
- Blue: External

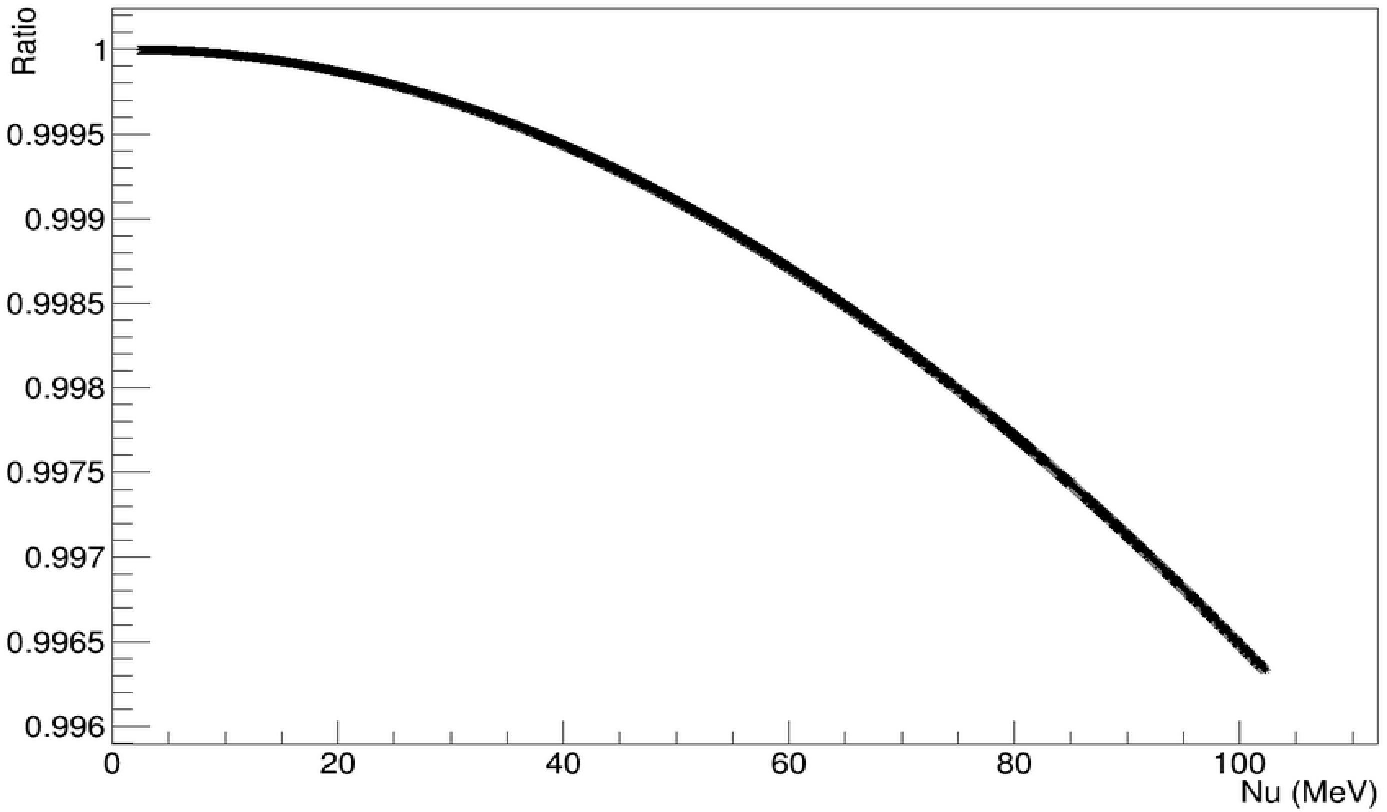
Peak-approximation v.s. exact cacluation

Internal

One Fixed energy and angle

Ratio= peak-approximation method/exact calculation

Agree within 1% in deltaE within 5%



Multi-photon Radiation

One Fixed energy and angle

Ratio= without multi-photon correction/within

Assume soft-photon radiation