Radiation Effects in Simulation

Jie Liu
University of Virginia
jie@jlab.org

May 12th, 2015

Abstract

The g_2^p experiment requires a high precision acceptance of the High Resolution Spectrometers (HRS) for determining the cross section. A simulation package was developed to help understand the acceptance and other related issues including yields drifts and packing fraction study. When charged lepton scatter off material, they would have energy loss due to ionization, external bremesstrahlung and internal bremestrahlung. These radiation effects would change the lepton momentum and affect the dp acceptance. In this document, these radiative effects are carefully examined and the simulation results will be reported.

1 Introduction

A charge particle will experience several kinds of interactions when pass through material. It will lose energy and deflected from initial direction. These interactions include: inelastic collisions with atomic electrons in the material, scattering off nuclei, bremesstrahlung, emission of cherenkov radiation, nuclear reactions. The bremesstrahlung can be classified as external and internal one, depends on whether the interactions occurs on the scattering atom, or some other atoms. For g2p experiment, most of these processes are electromagnetic effects and are calculable in QED. However, To study the detector acceptance, the QED calculation involves the analytical integration over the photonic phase space is complicated or not desirable because of specifics of detector geometry and resolutions. The Monte Carlo simulation which provides the radiative effects is very crucial to help study this issue. Ionization, internal and external bremesstrahlung are the most contributing radiative effects in addition to the primary scattering.

2 Ionization

Ionization is due to the charged particle colliding with other atomic electrons. The average energy loss for heavy charged hadron is calculated from well-known Bethe-Block formula:

$$\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}[\ln(\frac{2m_e\gamma^2 v W_{max}}{I^2}) - 2\beta^2 - \delta - 2\frac{C}{Z}]$$

$\frac{dE}{dx}$	mean energy loss per unit path length
N_A	Avogadro‘s number $(6.02 * 10^{23} mol^{-1})$
r_e	classical electron radius (2.818fm)
$m_e c^2$	electron rest energy (0.511MeV)
I	mean excitation potential of material
ρ	density of absorb material
Z	atomic number of absorb material
A	atomic weight of absorb material
z	charge of incident particle in unit of e
β	v/c incident of particle
γ	$1/\sqrt{(1-\beta^2)}$
δ	shell correction
W_{max}	maximum energy transfer in a single collision

The density effect is due to the the fact that the particle will polarize the atom along its path and this polarization will shield the full electric field intensity for the electrons far away from the path. Then less energy loss will happen for collisions with these outer lying electrons. The shell correction is very important in low energies where the assumption that the atomic electron is stationary respect to particle will not be valid.

For electrons, the basic collision energy loss mechanism is the same. But the electron is light, so a large fraction of the incident energy is possible. Moreover, Due to the scattering off identical particles, the indistinguishability should be taken into consideration in the quantum level. The Bethe-Block formula for electron is:

$$\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}[\ln(\frac{\tau^2(\tau+2)}{2(I/(m_e))^2} - F(\tau) - \delta - 2\frac{C}{Z}]$$

where τ and $F(\tau)$ are:

$$\tau = \frac{E - m_e c^2}{m_e c^2}$$

$$F(\tau) = 1 - \beta^2 + \frac{\tau^2/8 - (2\tau+1)\ln 2}{(\tau+1)^2}$$

2.1 Ionization Straggling Model

For any given particle pass through media, the energy loss will not always equal to the mean value given by the Bethe-Block Formula because of the statistical nature in number of collisions and the associated energy loss in each collision. The ionisation fluctuations are characterised by the significance parameter κ, which is defined as

$$\kappa = \frac{\xi}{E_{max}}$$

where E_{max} is the maximum allowed energy transfer in a single collision with an atomic electron:

$$E_{max} = \frac{2m_e\beta^2\gamma^2}{1 + 2\gamma m_e/m_x + (m_e/m_x)^2},$$

and ξ relates to the Rutherford scattering cross section:

$$\xi = \frac{2\pi z^2 e^4 N_{Av} Z\rho\delta x}{m_e\beta^2 c^2 A} = 153.4\frac{z^2}{\beta^2}\frac{Z}{A}\rho\delta x \quad \text{keV},$$

Depends on the value of κ, ionisation fluctuations can be approximately distinguishes to several regimes and treated by different theory models as follows:

1. $\kappa < 0.01$, Landau distribution, particle travels through extreme thin absorbers. Landau theory assumes that the number of these collisions is high. If ξ/I is too small, special models need be used to take account of the atomic structure. material are used

2. $0.01 < \kappa < 10$, Vavilov distribution, particle travels through thin absorbers.

3. $\kappa > 10$ A large number of collisions involving the loss of all or most of the incident particle energy during the traversal of an absorber, the fluctuation follows gauss distribution. This normally happen in thick material.

In the simulation package, the ionization energy loss is divided to discrete and continuous energy loss. The discrete energy loss represents the energy loss of a charged particle due to the explicit production of a δ-ray. The cross section for generating a δ-ray is determined by the δ-ray production threshold. The randomly sampling of the δ-ray energy is based on [1][2]. The discrete energy loss is above the δ-ray production threshold.

The cumulative effect of ionization below the δ-ray production threshold is accounted as continuous energy loss. For a given step, firstly obtain the mean energy loss below the production threshold according to restricted Bethe-Block Formula and then apply energy loss fluctuations on top base on different theories. A parameterised model by L. Urbán is applied for fluctation.It is assumed that the atoms have only two energy levels with binding energy E_1 and E_2. The particle–atom interaction will then be an excitation with energy loss E_1 or E_2, or an ionisation with an energy loss distributed according to a function $g(E) \sim 1/E^2$.

$$g(E) = \frac{(E_{max} + I)I}{E_{max}}\frac{1}{E^2} \tag{1}$$

This model is widely used in geant4 and approaches smoothly the Landau as the condition approaches the limit of the validity of Landau's theory, as shown in Fig.1-4. These figures are generated from a simple simulation that electron pass through fixed thickness carbon with fixed initial beam energy for each configuration. Fig.1 and Fig.2 have different production threshold, while Fig.3 and Fig.4 use different thickness of carbon. We can see they have a good agreement for carbon thickness 1mm or above. It is fast and it can be used for any thickness of a medium [1][2].

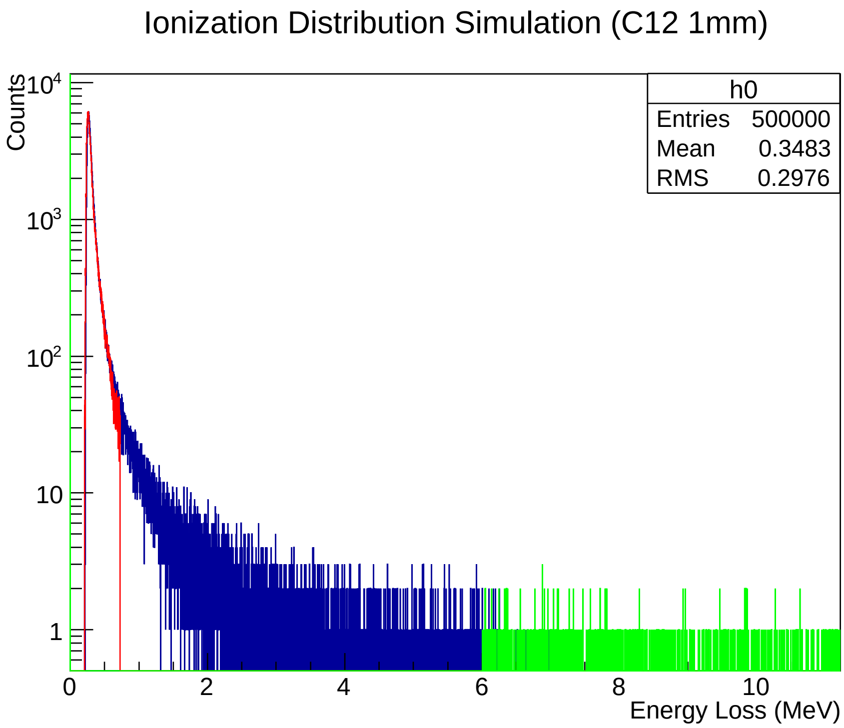

Figure 1: Ionization distribution simulation. Red: Landau distribution; Blue: Urbán model; Green: δ-ray. Beam 2.253GeV, Carbon thickness 1mm, Production threshold 6MeV.

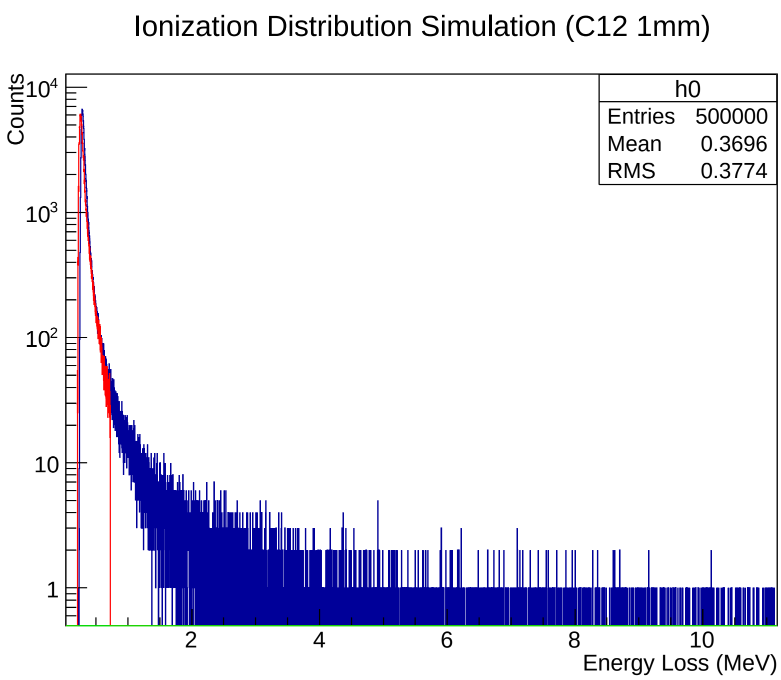

Figure 2: Ionization distribution simulation. Red: Landau distribution; Blue: Urbán model; Green: δ-ray. Beam 2.253GeV, Carbon thickness 1mm, Production threshold 2.253/2.0GeV.

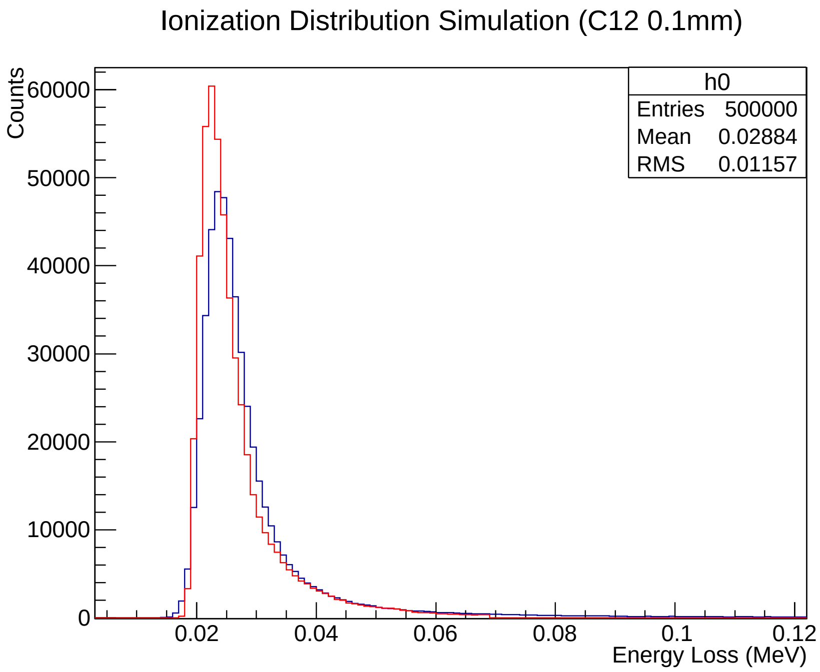

Figure 3: Ionization distribution simulation. Red: Landau distribution; Blue: Urbán model. Beam 2.253GeV, Carbon thickness 0.1mm

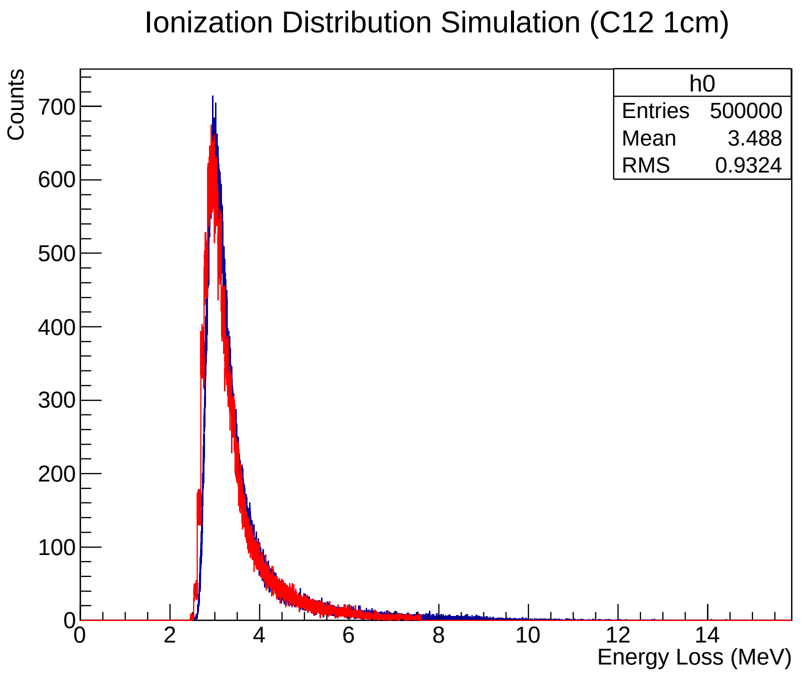

Figure 4: Ionization distribution simulation. Red: Landau distribution; Blue: Urbán model. Beam 2.253GeV, Carbon thickness 1cm

3 Internal Bremesstrahlung

The internal radiative correction to the elastic peak is well know from Mo and Tsai's work [3]. The radiative correction factor is:

$$\delta = \frac{-\alpha}{\pi}(\frac{28}{9} - \frac{13}{6}\ln\frac{-q^2}{m^2} + (\ln\frac{-q^2}{m^2} - 1 + 2Z\ln\eta)(2\ln\frac{E_1}{\Delta E} - 3\ln\eta) - \phi(\frac{E_1 - E_3}{E_3}) - Z^2\ln\frac{E_4}{M}$$

$$+Z^2\ln\frac{M}{\eta\Delta E}(\frac{1}{\beta_4}\ln\frac{1+\beta_4}{1-\beta_4} + 2) + \frac{Z^2}{\beta_4}\frac{1}{2}\ln\frac{1+\beta_4}{1+\beta_4}\ln\frac{E_4+M}{2M} - \phi(-(\frac{E_4-M}{E_4+M})^{1/2}\frac{1+\beta_4}{1-\beta_4}^{1/2})$$

$$+Z[\phi(-\frac{M-E_3}{E_1}) - \phi(\frac{M(M-E_3)}{2E_3E_4 - ME_1}) + \phi(2\frac{E_3(M-E_3)}{2E_3E_4 - ME_1}) + \ln|\frac{2E_3E_4 - ME_1}{E_1(M-2E_3)}|\ln\frac{M}{2E_3}]$$

$$-Z[\phi(-\frac{E_4-E_3}{E_3}) - \phi(\frac{M(E_4-E_3)}{2E_1E_4 - ME_3}) + \phi(2\frac{E_1(E_4-E_3)}{2E_1E_4 - ME_3}) + \ln|\frac{2E_1E_4 - ME_3}{E_3(M-2E_1)}|\ln\frac{M}{2E_1}]$$

$$-Z[\phi(-\frac{M-E_1}{E_1}) - \phi(\frac{M-E_1}{E_1}) + \phi(\frac{2(M-E_1)}{M}) + \ln|\frac{M}{2E_1 - M}|\ln\frac{M}{2E_1}]$$

$$+Z[\phi(-\frac{M-E_3}{E_3}) - \phi(\frac{M-E_3}{E_3}) + \phi(\frac{2(M-E_3)}{M}) + \ln|\frac{M}{2E_3 - M}|\ln\frac{M}{2E_3}])$$

$$+\frac{-\alpha}{\pi}(\phi(-\frac{E_1-E_3}{E_3}) + \frac{Z^2}{\beta_4}\phi[(\frac{E_4-M}{E_4+M})^{1/2}\frac{1+\beta_4}{1-\beta_4}^{1/2}] - \phi[(\frac{E_4-M}{E_4+M})^{1/2}] + \phi[-(\frac{E_4-M}{E_4+M})^{1/2}])$$

Here $E_1, E_3, and E_4$ are energies of incident electron, scattered electron, and the recoil nucleus, respectively while m and M are the masses of the electron and the target particle. The step function p is defined by MY as $\beta(x) = \ln(x)^2\theta(l|x)$; β_4 is the velocity of the recoil particle in units of the velocity of light,$\eta = \frac{E_1}{E_3}$, and $\phi(x)$ is the Spence function defined as: $\phi(x) = \int_0^x \frac{\ln|1-y|}{y}dy$. The δ can be decomposed to vacuum correction, vertex correction and internal bremesstrahlung correction:

$$\delta_{vac} \approx \frac{2\alpha}{\pi}(-\frac{5}{9} + \frac{1}{3}\ln\frac{-q^2}{m^2})$$

$$\delta_{vertex} \approx \frac{2\alpha}{\pi}(-1 + \frac{3}{4}\ln\frac{-q^2}{m^2})$$

$$\delta_{internbre} = \delta - \delta_{vac} - \delta_{vertex}$$

The ΔE dependence term will relate to the elastic tail. For the g2p experiment, the hadron internal bremesstrahlung can be ingorned. So the significant term is as follows:

$$\delta_{rad} = \frac{-\alpha}{\pi}(\ln\frac{-q^2}{m^2} - 1)(2\ln\frac{E_1}{\Delta E})$$

$$\delta_{rad} \approx \frac{-\alpha}{\pi}(\ln\frac{-q^2}{m^2} - 1)(\ln\frac{E_1E_3}{\Delta E^2})$$

This term will represent the electron internal radiation effect for both incoming and outgoing as follows:

$$e^{\delta_{rad}} = (\frac{E_1}{\Delta E})^{-t_r}(\frac{E_3}{\Delta E})^{-t_r}$$

where

$$t_r = \frac{-\alpha}{\pi}(\ln\frac{-q^2}{m^2} - 1)$$

The sampling distribution for internal bremesstrahlung can be normalized as

$$I_{int}(E, \Delta E) = \frac{t_r}{\Delta E}(\frac{\Delta E}{E})^{t_r}$$

The internal radiative tail simulation was show as black line in Fig.5. The red one comes from the differentiation of Mo and Tsai formula. They are in very good agreement.

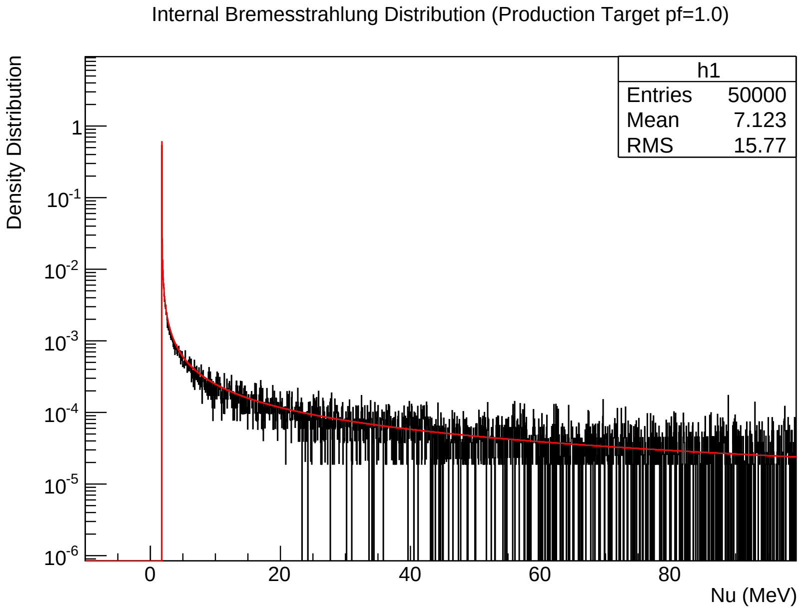

Figure 5: Internal radiation effect simulation, cross section weighted. Beam 2.253GeV, scattering 0.1008rad, production target pf=1. Red: Mo and Tsai model; Black: simulation.

4 External Bremesstrahlung

The External straggling to the elastic peak is well know from Mo and Tsai's work [3]. The radiative correction factor is:

$$\delta_t = -[b_w t_{iw} + 1/2bT]\ln(E_1/(\eta^2\Delta E)) + [b_w t_{fw} + 1/2bT]\ln(E_3/\Delta E)$$

where T, t_{iw}, and t_{fw} are the target, the initial window, and the final window thicknesses, respectively, in units of radiation length. The coefficients b_w and b are weakly Z dependence, very close to 4/3. The distribution of ΔE is given in very good approximation by [4]

$$\delta_t = \frac{bt}{1-0.57772bt}(\frac{\Delta E}{E})^{bt}[\frac{1}{E}(1-\frac{\Delta E}{E}+\frac{3}{4}(\frac{\Delta E}{E})^2)]$$

where E is the energy before bremesstrahlung. The external radiative tail simulation was show as black line in Fig.6. The red one comes from the differentiation of Mo and Tsai formula. They agree each other well.

5 Radiation Effect Simulation vs Model

5.1 Elastic Tail Simualtion vs Model

Take into account the straggling and radiative correction effect, the measured cross section is [3]:

$$\frac{d\sigma}{d\Omega}|_{meas} = \frac{d\sigma}{d\Omega}|_{Rosenbluth}e^{\delta+\delta_t}$$

which is the integral of cross section from 0 to ΔE. The comparison of simulated distribution and model differentiation prediction is conducted point to point level in ΔE. In the simulation package, energy loss is simulated step by step as shown in Fig.7. Fig.8-9 shows the comparison results. The ionization is not included in simulation in Fig.8, but included in Fig.9. They agree well when $\Delta E > 8MeV$ with ionization included.

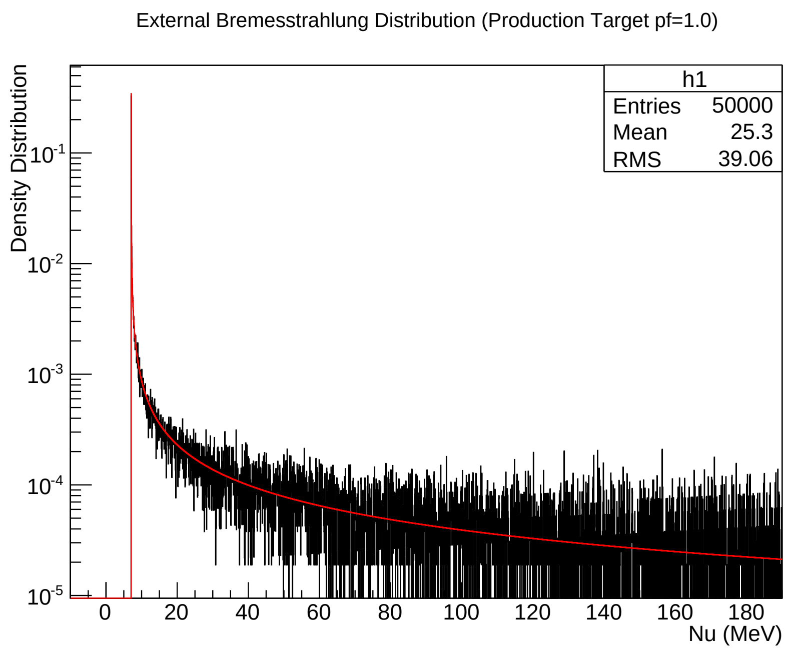

Figure 6: External straggling effect simulation, cross section weighted. Beam 2.253GeV, scattering 0.1008rad, production target pf=1. Red: Mo and Tsai model; Black: simulation.

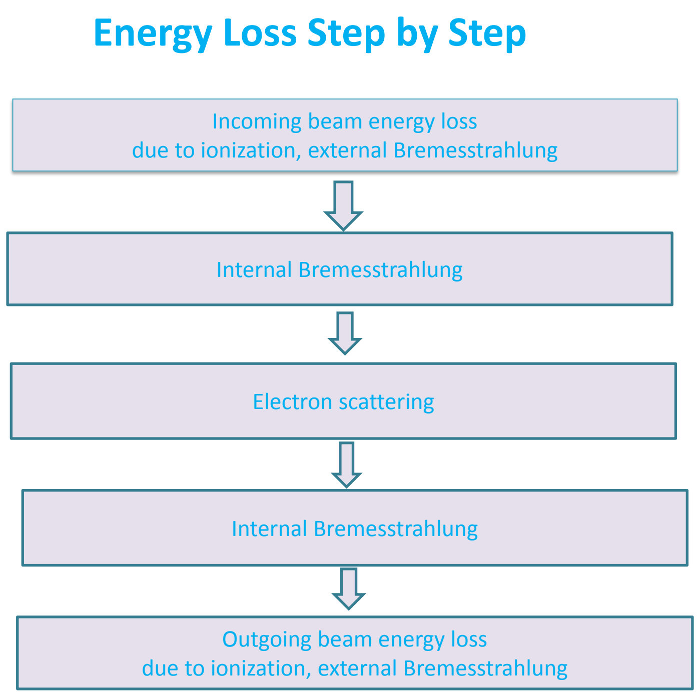

Figure 7: Energy loss simulation process

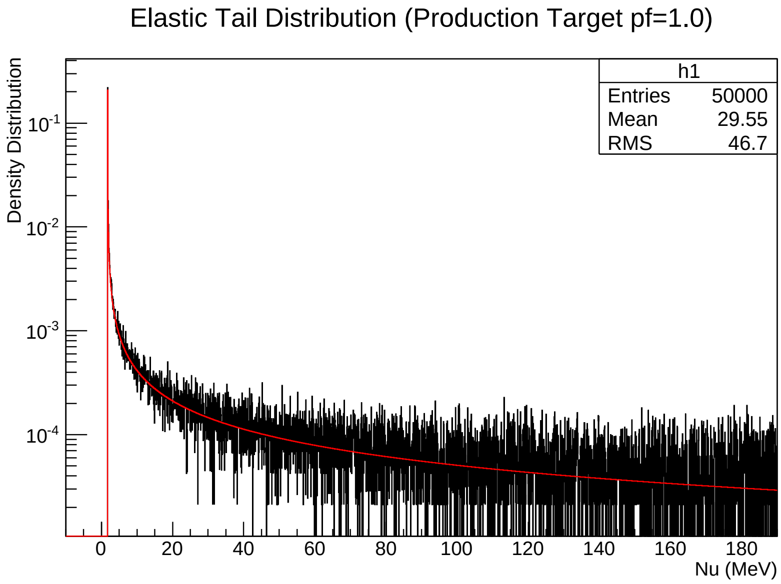

Figure 8: Elastic tail simulation (include vacuum, vertex, internal bremesstrahlung, external bremesstrahlung), cross section weighted. Beam 2.253GeV, scattering 0.1008rad, production target pf=1. Red: Mo and Tsai model; Black: simulation.

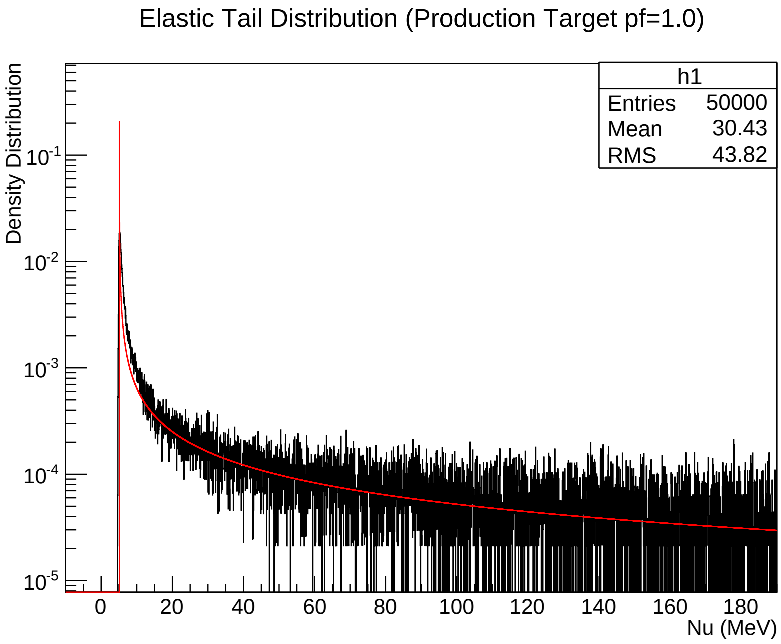

Figure 9: Elastic tail simulation (include ionization, vacuum, vertex, internal bremesstrahlung, external bremesstrahlung. Beam 2.253GeV, scattering 0.1008rad, production target pf=1. Red: Mo and Tsai model; Black: simulation.

5.2 Quasi-Elastic Tail Simualtion vs Model

The radiative tail associated with this quasi-elastic scattering is given in the following formula [5]:

$$\delta_{qt} = (\frac{R\Delta E}{E_s})^{b(t_b+t_r)}(\frac{\Delta E}{E_p})^{b(t_a+t_r)}[1 - \frac{\xi/\Delta E}{1-b(t_a+t_b+2t_r)}]\widetilde{\sigma_q}(E_s,E_p)$$

$$+\int_{E_{smin}}^{E_s-R\Delta E}\widetilde{\sigma_q}(E_s^{'},E_p)(\frac{E_s-E_s^{'}}{E_p}R)^{b(t_a+t_r)}(\frac{E_s-E_s^{'}}{E_s})^{b(t_b+t_r)}[\frac{b(t_b+t_r)}{E_s-E_s^{'}}\phi(\frac{E_s-E_s^{'}}{E_s})+\frac{\xi}{2(E_s-E_s^{'})^2}]dE_s^{'}$$

$$+\int_{E_p+\Delta E}^{E_{pmax}}\widetilde{\sigma_q}(E_s,E_p^{'})(\frac{E_p^{'}-E_p}{E_p^{'}})^{b(t_a+t_r)}(\frac{(E_P^{'}-E_p)R}{E_s})^{b(t_b+t_r)}[\frac{b(t_b+t_r)}{E_p-E_p^{'}}\phi(\frac{E_p^{'}-E_p}{E_p^{'}})+\frac{\xi}{2(E_p^{'}-E_p)^2}]dE_p^{'}$$

where t_a and t_b are total path length in units of radiation length of the electron in the target before and after the scattering.

$$R = \frac{M_T + 2E_s sin^2\frac{1}{2\theta}}{M_T - 2E_s sin^2\frac{1}{2\theta}}, \phi(v) = 1 - v - \frac{3}{4}v^2, \widetilde{\sigma}(E_s,E_p) = \widetilde{F}q^2\sigma_q$$

$$\widetilde{F}q^2 = 1 + 0.5772bT + \frac{2\alpha}{\pi}[-\frac{14}{9}+\frac{13}{12}\ln\frac{-q^2}{m^2}] - \frac{\alpha}{2\pi}\ln^2(\frac{E_s}{E_p}) + \frac{\alpha}{\pi}[\frac{1}{6}\pi^2 - \phi(cos^2\frac{\theta}{2})]$$

The simulated quasi-elastic tail is show in Fig.9. The green, red, black are raw cross section, calculated cross section, simulated cross section respectively.

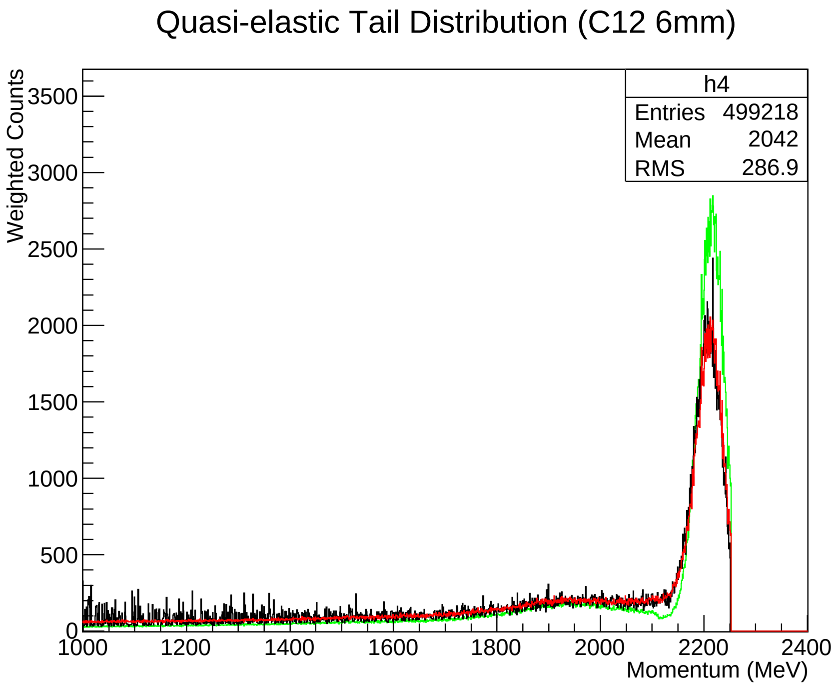

Figure 10: Quasi-elastic tail simulation (include ionization, vacuum, vertex, internal bremesstrahlung, external bremesstrahlung. Beam 2.253GeV, scattering 0.107rad, carbon target. Red: Stein model; Black: simulation; Green: Born XS

6 Conclusion

The radiation models are carefully examined and the simulated distributions are tested to match very well with the radiated model prediction.

References

[1] cern geant4 webpage is available at *http://geant4.cern.ch/G4UsersDocuments/UsersGuides/PhysicsReferenceManual/html/node41.html*

[2] lassila K.Lassila-Perini, L.Urbán, *Energy loss in thin layers in GEANT*, Nucl.Inst.Meth. A362(1995) 416.

[3] L. W. Mo, Y. S. Tsai, *Radiative Corrections to Elastic and Inelastic ep and μp scattering*, Rev.Mod.Phys.41(1969)205.

[4] Y. S. Tsai, *Pair Production and Brehmsstrahlung of Charged Leptons*, Rev.Mod.Phys.46(1974)815.

[5] S. Stein, *Electron scattering at* 4° *with energies of 4.5-20 Gev*, Phys.Rev.D.12.1884.