The g_2 Spin Structure Function

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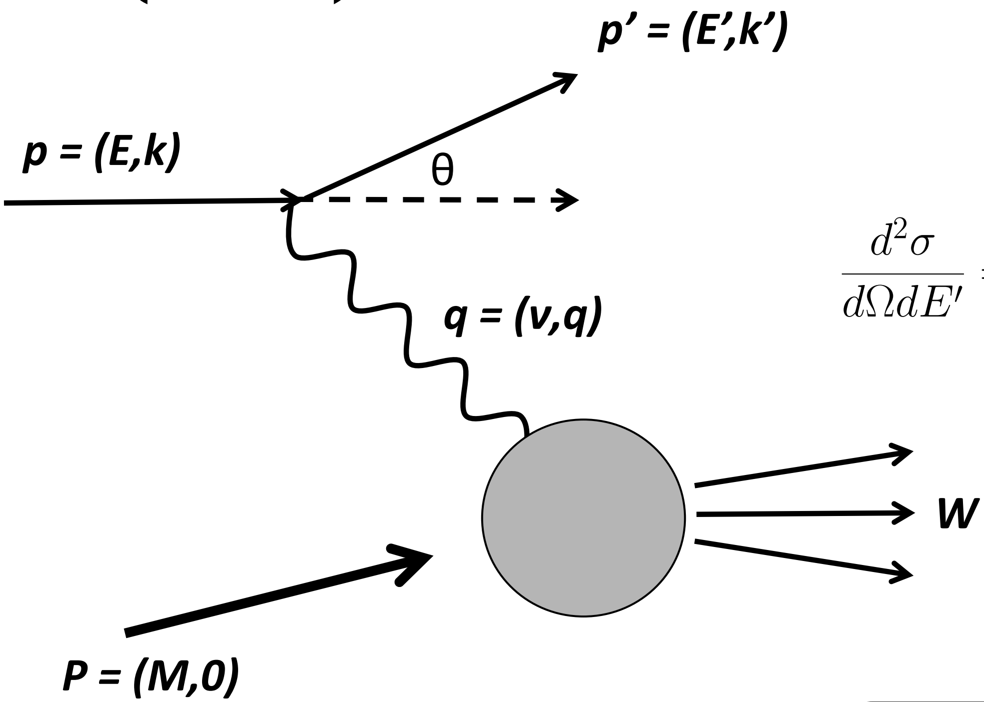
Inclusive Electron Scattering

N(e,e')X



To describe scattering from a nucleon requires structure functions:

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{Mott}[\frac{1}{\nu}\boxed{F_2}(x,Q^2) + \frac{2}{M}\boxed{F_1}(x,Q^2)\tan^2\frac{\theta}{2}]$$

Inclusive inelastic unpolarized cross section

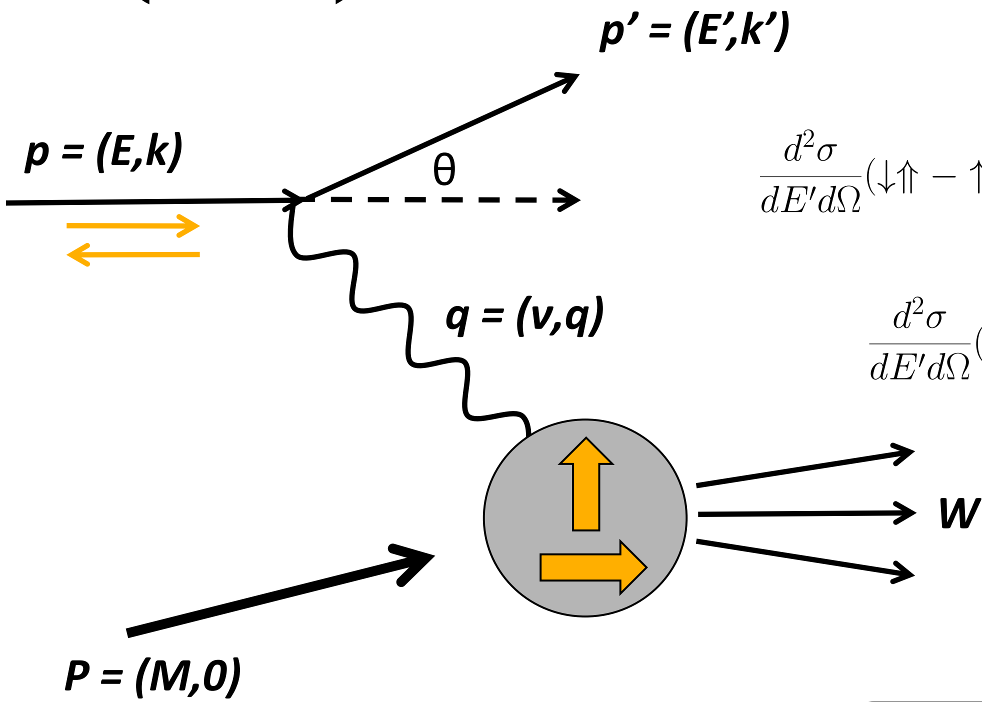
$$\vec{q} = \vec{k} - \vec{k'}$$

$$Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$$

$$x = \frac{Q^2}{2M\nu}$$

Inclusive Electron Scattering

N(e,e')X



*Inclusive **polarized** cross sections*

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E+E'\cos\theta)\boxed{g_1(x,Q^2)} - \frac{Q^2}{\nu}\boxed{g_2(x,Q^2)}\right]$$

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2\sin\theta}{MQ^2}\frac{E'^2}{\nu^2 E}\left[\nu\boxed{g_1(x,Q^2)} + 2E\boxed{g_2(x,Q^2)}\right]$$

g_1,g_2 are related to the spin distribution

$$\vec{q} = \vec{k} - \vec{k}'$$

$$Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$$

$$x = \frac{Q^2}{2M\nu}$$

Quark-Parton Model

• Infinite momentum frame: partons are point-like, non-interacting particles

• Structure functions can be written in terms of quark distribution functions:

$$F_1(x) = \frac{1}{2}\sum_f z_f^2[q_f(x) + \bar{q}_f(x)]$$

$$F_2(x) = 2xF_1(x)$$

$$g_1(x) = \frac{1}{2}\sum_f z_f^2[q_f(x) - \bar{q}_f(x)]$$

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$$g_1(x) = \frac{1}{2}\sum_f z_f^2[q_f(x) - \bar{q}_f(x)]$$

No simple interpretation for g_2

g_2 includes contributions from quark gluon interactions

What is g_2?

(Parton Model Description)

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$$

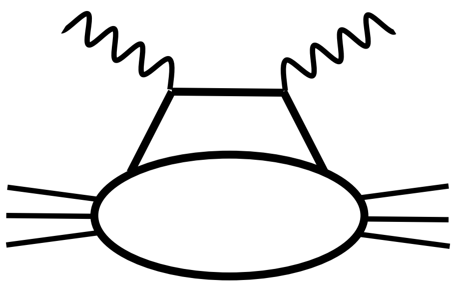
What is g_2?

$$g_2(x,Q^2) = \boxed{g_2^{WW}(x,Q^2)} + \bar{g}_2(x,Q^2)$$

Wandzura–Wilczek Relation:

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2)$$

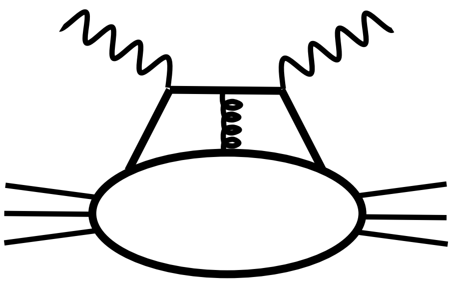
Leading twist-2 term



What is g_2?

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \boxed{\bar{g}_2(x,Q^2)}$$

$$\int_x^1 \frac{\partial}{\partial y}\left[\frac{m_q}{M}\boxed{h_T(y,Q^2)} + \boxed{\zeta(y,Q^2)}\right]\frac{dy}{y}$$



h_T: Arises from quark transverse polarization distribution

ζ: Arises from quark-gluon interactions (twist-3)

Measurements of g_2 and its Moments

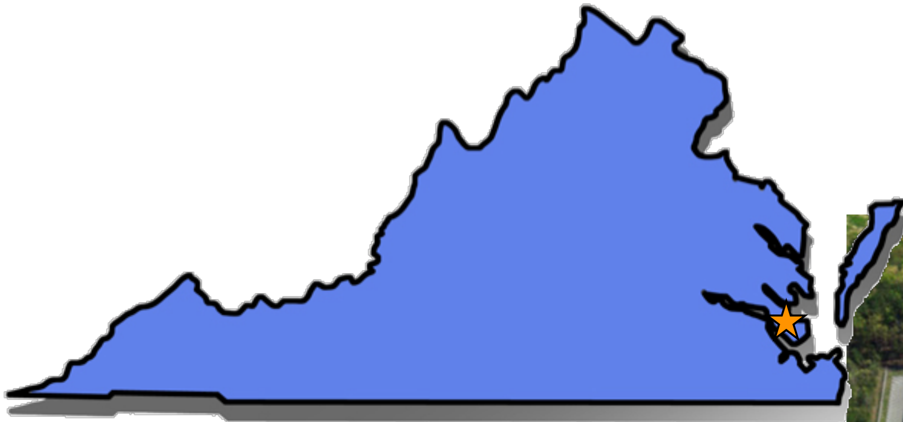
- Measurements of g_2 require a transversely polarized target – more difficult experimentally

- 0th moment (no x-weighting): Burkhardt-Cottingham Sum Rule
  - Valid at all Q^2

$$\int_0^1 g_2(x, Q^2)dx = 0$$

- 1st moment (x^2 weighting):
  - High Q^2 – d_2, twist-3 color polarizability, test of lattice QCD
  - Low Q^2 – spin polarizabilities, test of χPT

Measurements of g_2 and its Moments



Jefferson Lab

CEBAF

- High intensity electron accelerator based on CW SRF technology
- E_{max} = 6 GeV
- I_{max} = 200 μA
- Pol_{max} = 85%

Recently upgraded to 12 GeV

Measurements of g_2 and its Moments

- Prior to measurements at JLab, first dedicated experiment was SLAC E155x

- g_2 Measurements on the neutron at JLab:
  - E97-103: W>2 GeV, $Q^2 \approx 1$ GeV^2, $x \approx 0.2$, study higher twist (published)
  - E99-117: W>2 GeV, high Q^2 (3-5 GeV^2) (published)
  - E94-010: moments at low Q^2 (0.1-1 GeV^2) (published)
  - E97-110: moments at very low Q^2 (0.02-0.3 GeV^2) (analysis)
  - E01-012: moments at intermediate Q^2 (1-4 GeV^2) (published)
  - E06-014: moments at high Q^2 (2-6 GeV^2) (analysis)

Measurements of g_2 and its Moments

- g_2 Measurements on the proton at JLab

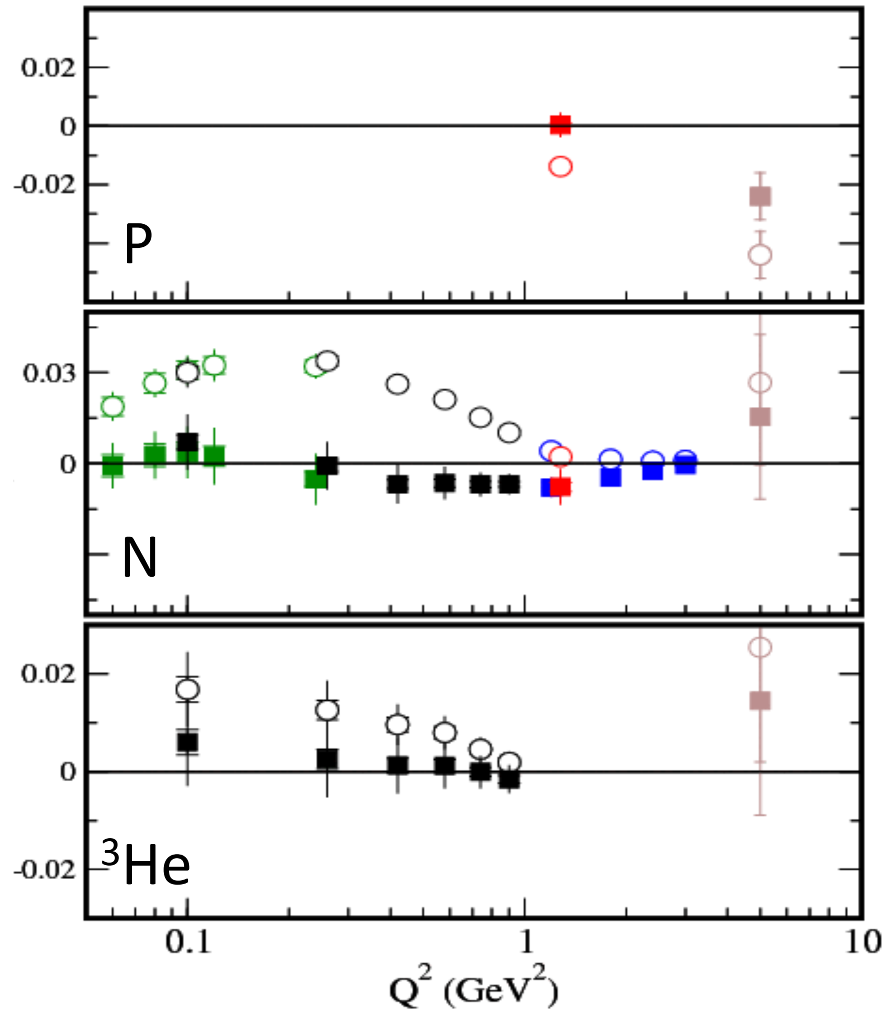
- Hall C:
  - RSS: moments at intermediate Q^2 (1-2 Gev^2) (published)
  - SANE: moments at high Q^2 (2-6 GeV^2) (analysis)

- Hall A:
  - E08-027 (g_2^p): moments at very low Q^2 (0.02-0.2 GeV^2) (analysis)

0^{th} Moment of g_2

$$\Gamma_2 = \int_0^1 g_2(x, Q^2)dx = 0$$

Brown: SLAC E155x
Red: Hall C RSS
Black: Hall A E94-010
Green: Hall A E97-110 (preliminary)
Blue: Hall A E01-012 (preliminary)



BC Sum = Measured + Low x + Elastic

Measured: open circles

Low x: unmeasured low-*x* part of the integral – assume leading twist behavior

Elastic: obtained from well known Form Factors

1st Moment: Spin Polarizabilities

- Generalized spin polarizabilities γ_0 and δ_{LT} are a benchmark test of χPT

- At low Q^2, generalized polarizabilities have been evaluated with χPT calculations
    - Difficulty is how to include the nucleon resonance contributions
    - γ_0 is sensitive to resonances, δ_{LT} is not

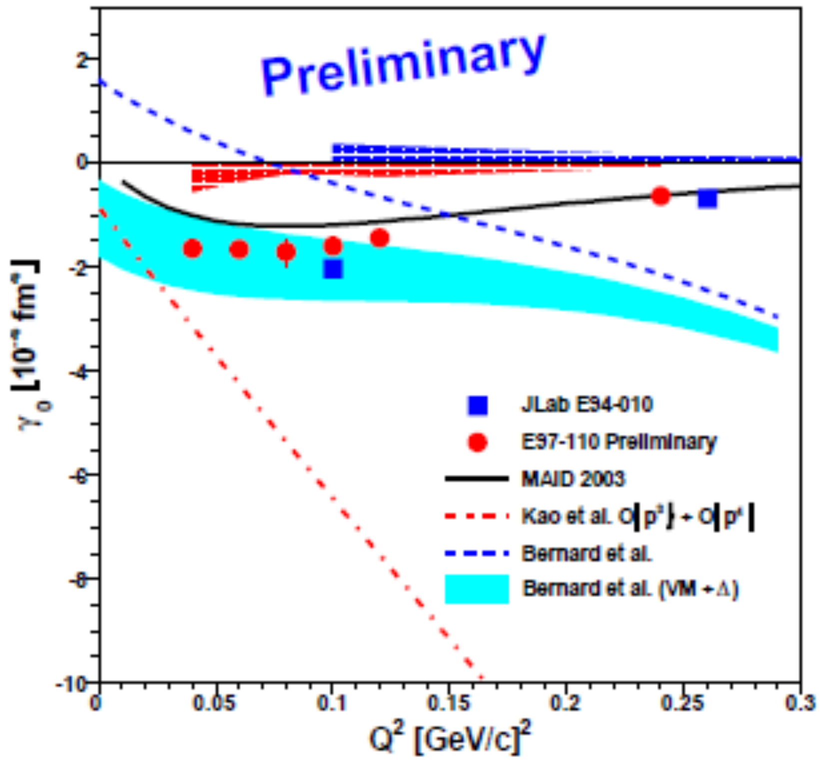
Spin Polarizabilities

(V. Sulkosky)

- Neutron results for γ_0

- RBχPT calculation (including resonance contributions) agrees with the experimental results

- Large discrepancy between data and HBχPT calculation (without explicit resonance contributions)

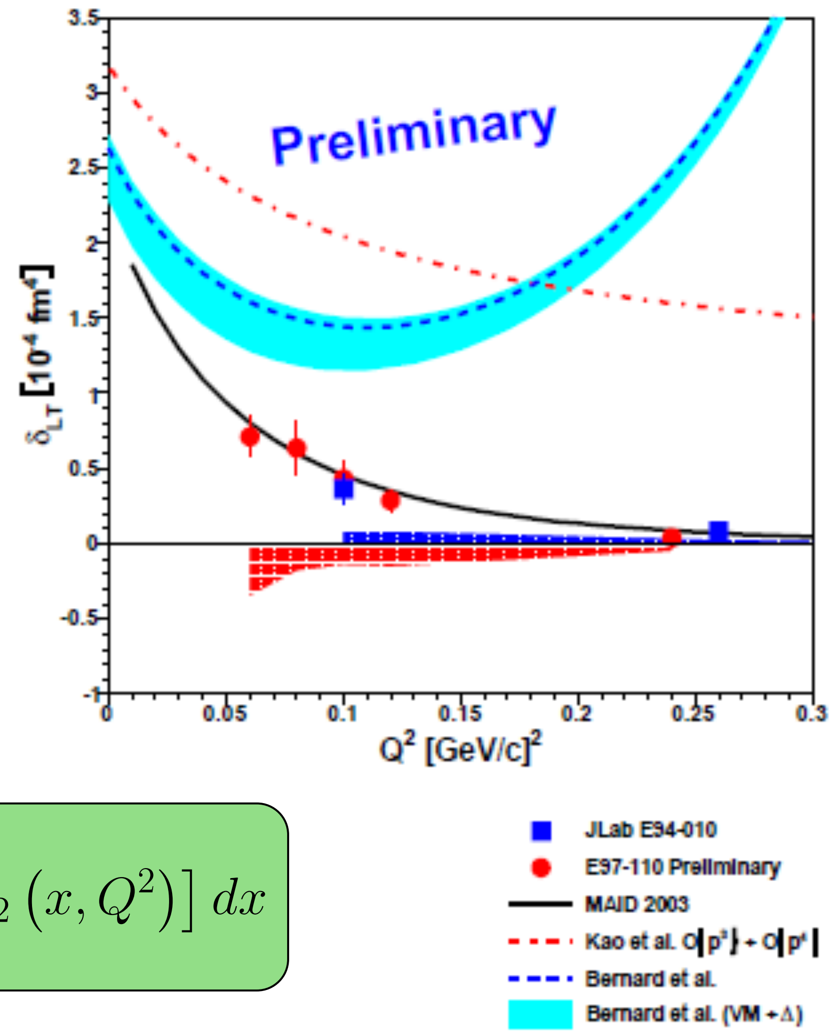


$$\gamma_0\left(Q^2\right) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1\left(x,Q^2\right) - \frac{4M^2}{Q^2}x^2 g_2\left(x,Q^2\right)\right]dx$$

Spin Polarizabilities

(V. Sulkosky)

- Neutron results for δ_{LT}
- δ_{LT} is seen as a more suitable testing ground – insensitive to Δ-resonance
- Data is in significant disagreement with χPT calculations
- MAID predictions are in good agreement with the results



$$\delta_{LT}\left(Q^2\right)=\frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1\left(x,Q^2\right)+g_2\left(x,Q^2\right)\right]dx$$

d_2 & Higher Twist

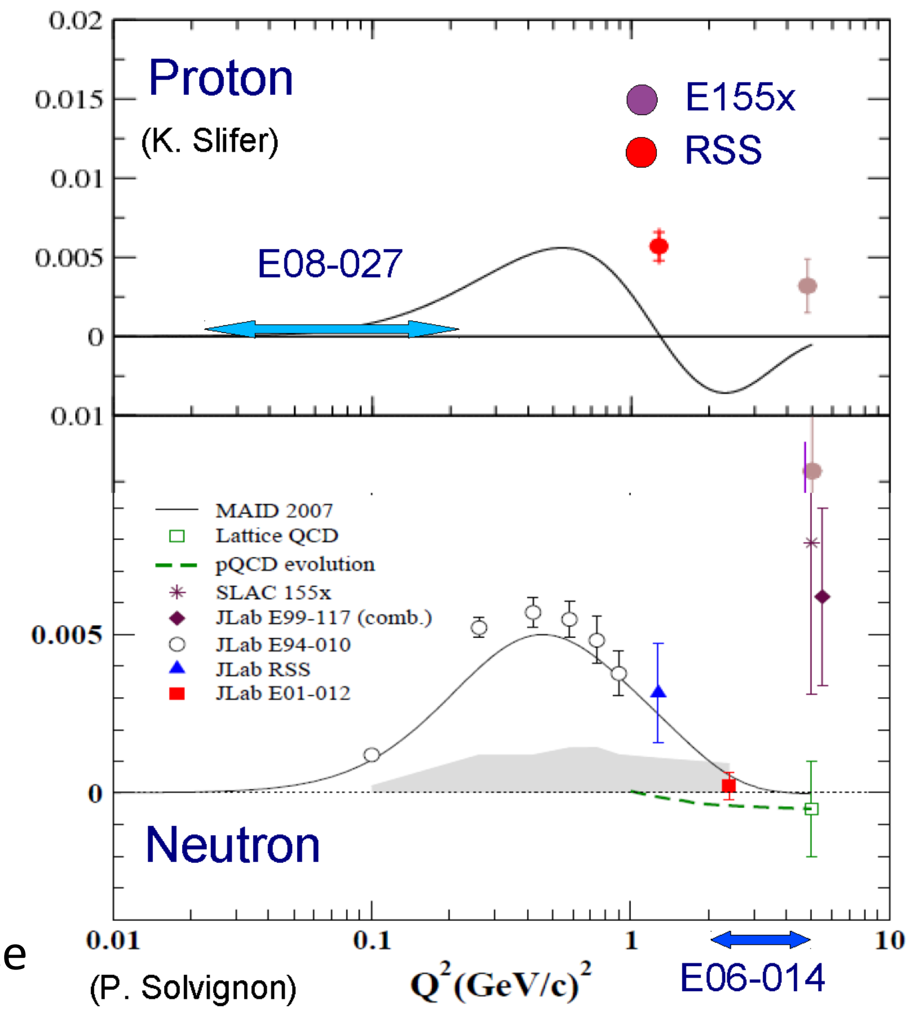
$$d_2(Q^2) = \int_0^1 dx x^2 \left(2g_1(x,Q^2) + 3g_2(x,Q^2)\right)$$

$$= 3\int_0^1 dx x^2 \left(g_2(x,Q^2) - g_2^{WW}(x,Q^2)\right)$$

- Doesn't contain any twist-2 contributions

- High Q^2: parton model with gluon exchange
- Low Q^2: ?

- High precision data at large Q^2 is necessary for a benchmark test of Lattice QCD predictions



g_2^p Experiment at JLab (E08-027)

- Will provide the first measurement of g_2 for the proton at low to moderate Q^2

- Will provide insight on several outstanding physics puzzles:
    - BC sum rule
        - Discrepancy suggested for high-Q^2 data
    - δ_{LT} polarizability
        - χPT calculations do not match data
    - Finite size effects:
        - Hydrogen hyperfine splitting: proton structure contributes to uncertainty
        - Proton charge radius: proton polarizability contributes to uncertainty

- Data was taken in Hall A in 2012 – analysis is currently underway

Finite Size Effects

Hyperfine Splitting of Hydrogen:

Splitting expressed in terms of Fermi Energy E_F:

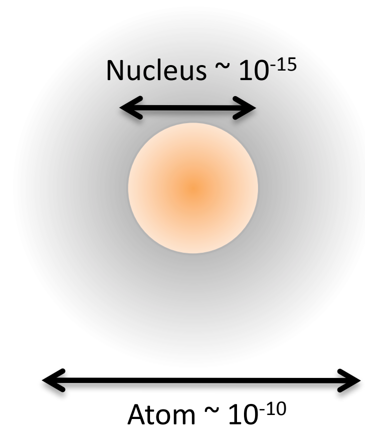
$$\Delta_E = (1+\delta)\, E_F$$

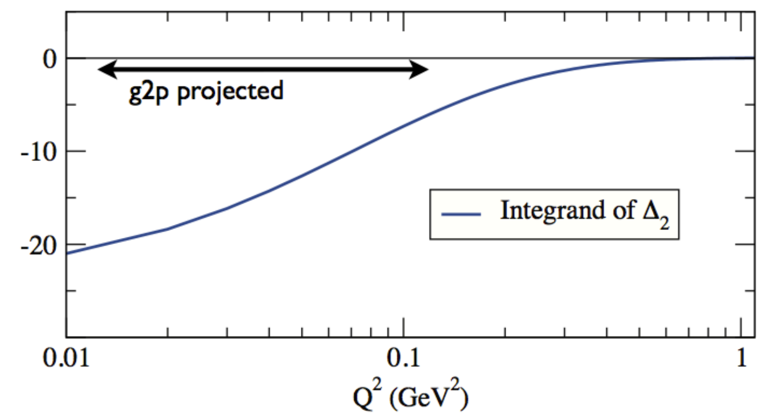
Where:

$$\delta = 1 + (\delta_{QED} + \delta_R + \delta_{small}) + \underline{\underline{\Delta_S}}$$

$$\underline{\underline{\Delta_S}} = \Delta_Z + \underline{\underline{\Delta_{pol}}}$$

$$\underline{\underline{\Delta_{pol}}} = \frac{\alpha m_e}{\pi g_p m_p} (\Delta_1 + \underline{\underline{\Delta_2}})$$





Dominated by low Q^2 g_2^p

Finite Size Effects

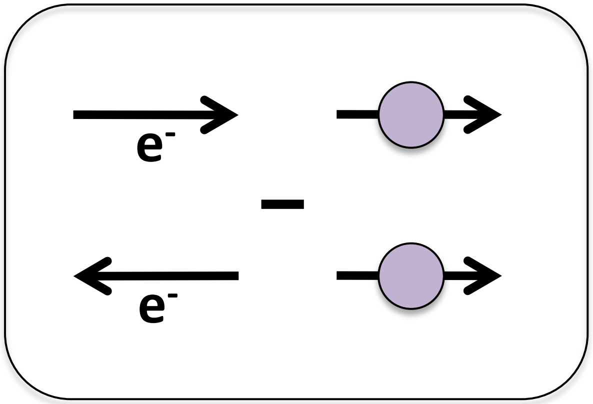
Proton Charge Radius:

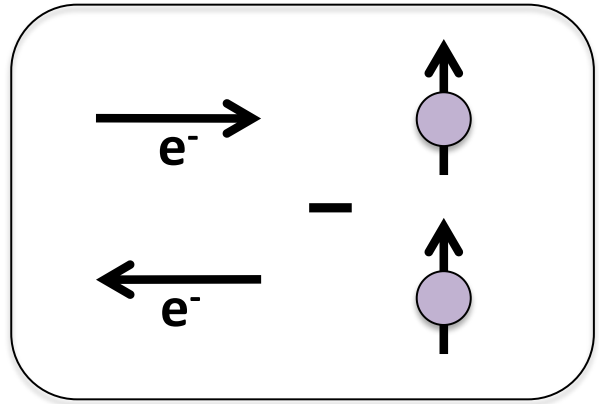
- Proton charge radius from μP disagrees with eP scattering result by ~7σ

$\langle R_p \rangle = 0.84184 \pm 0.00067$ fm	Lamb shift in muonic hydrogen
$\langle R_p \rangle = 0.897 \pm 0.018$ fm	World analysis of eP scattering
$\langle R_p \rangle = 0.8768 \pm 0.0069$ fm	CODATA world average

- Main uncertainties arise from the proton polarizability and different value of the Zemach radius

Experimental Technique





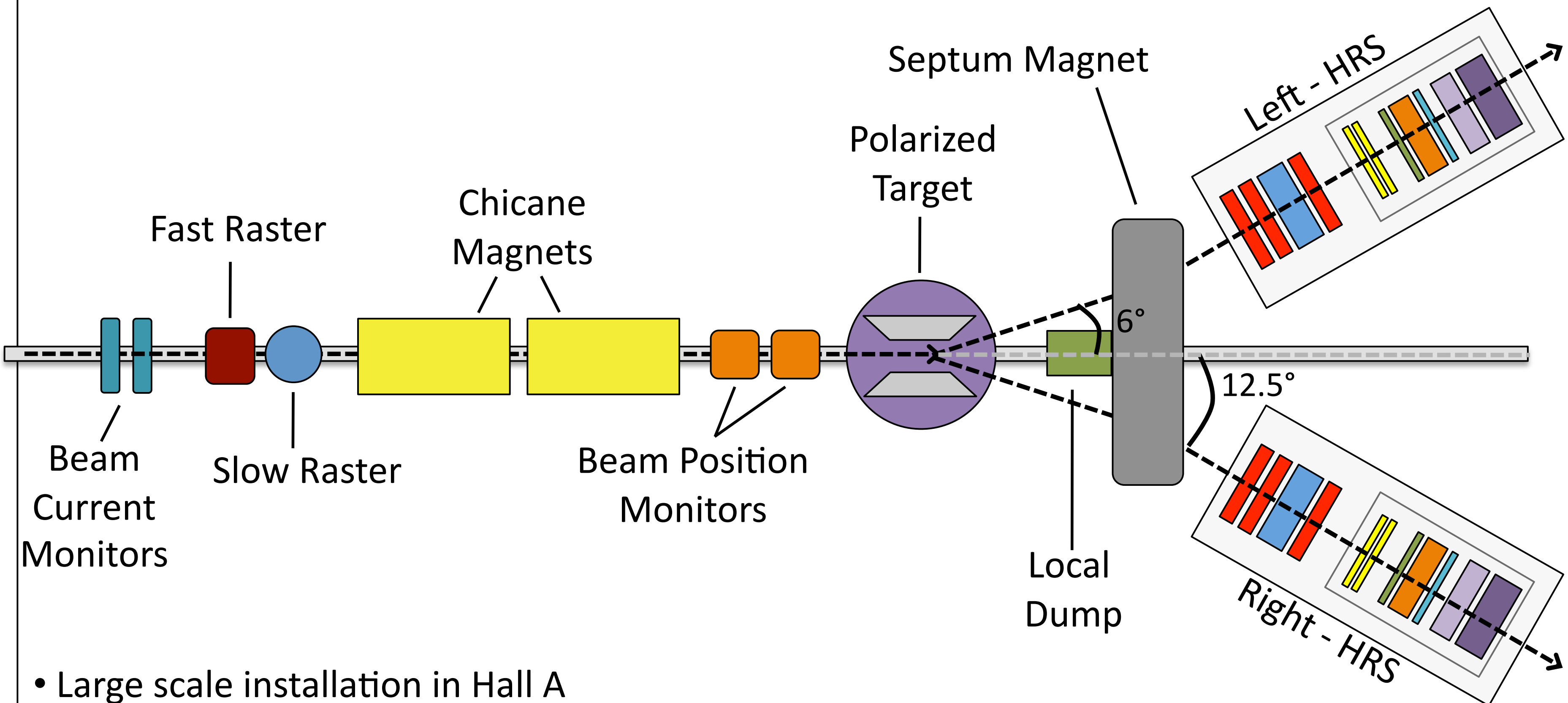
$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E + E'\cos\theta)g_1(x,Q^2) - \frac{Q^2}{\nu}g_2(x,Q^2)\right]$$

$\Delta\sigma_{||}$ measured during EG4 experiment in Hall B: will extract g_1^p at low Q^2

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2\sin\theta}{MQ^2}\frac{E'^2}{\nu^2 E}\left[\nu g_1(x,Q^2) + 2E g_2(x,Q^2)\right]$$

$\Delta\sigma_{\perp}$ obtained from g_2^p experiment

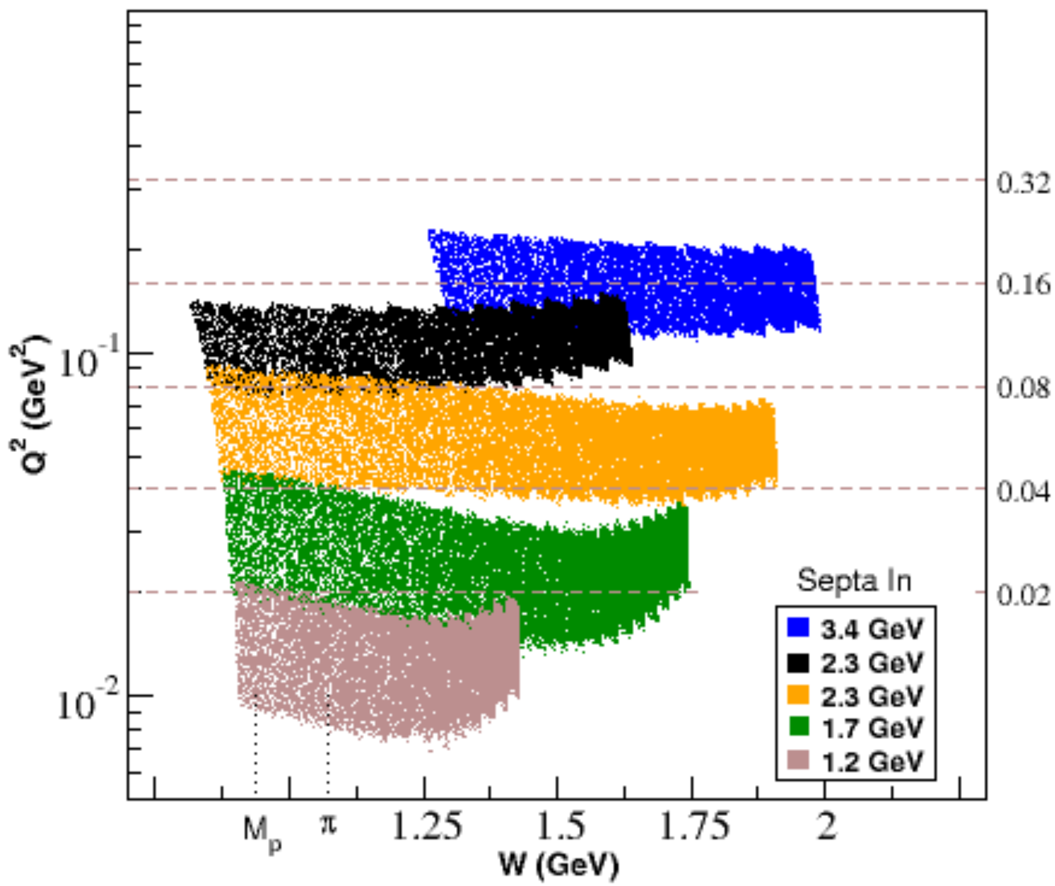
Experimental Setup



- Large scale installation in Hall A
  - DNP NH_3 target with 2.5/5 T magnetic field (longitudinal and transverse configurations)
  - New beamline diagnostics for low current (<100 nA) running
  - Chicane and septum magnets
  - Local dump

g_2^p Experiment at JLab

First data on g_2 for proton at low Q^2



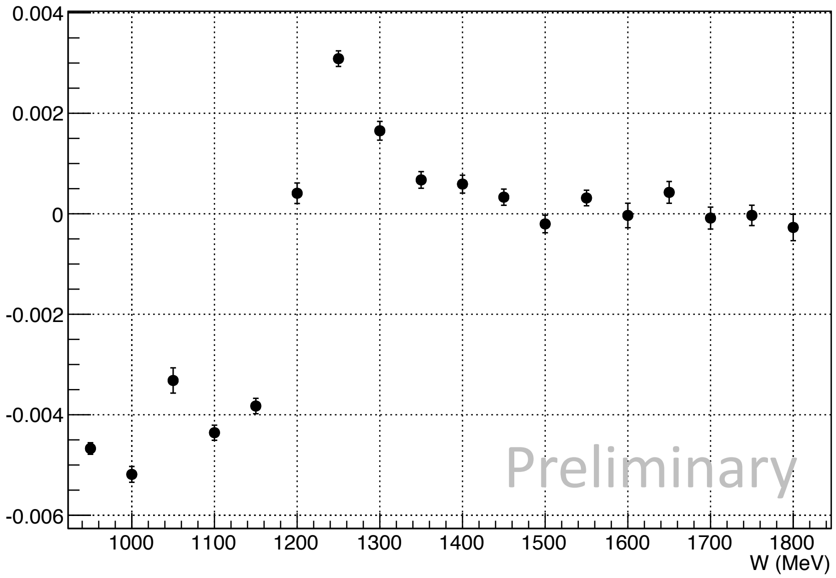
$W < 2$ GeV

$0.02 < Q^2 < 0.2$ GeV^2

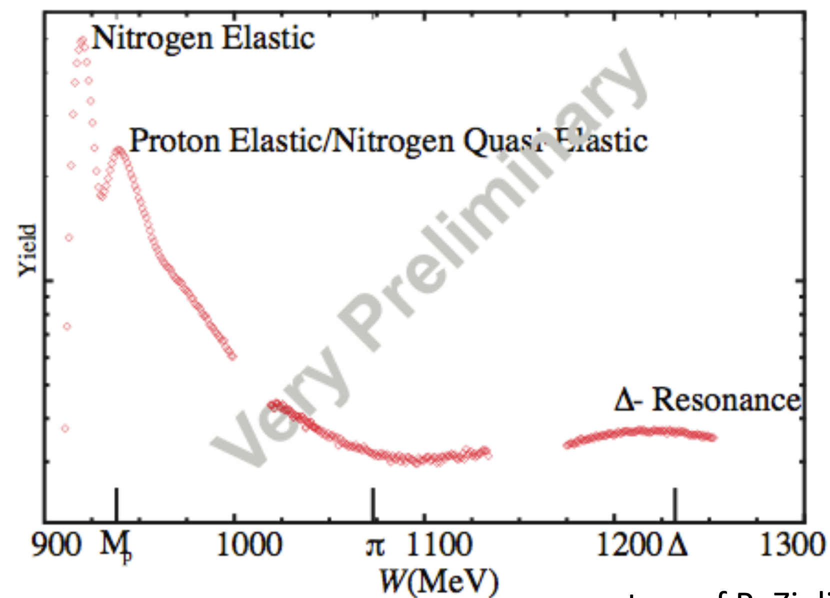
Beam Energy (GeV)	Target Field (T)
2.2	2.5
1.7	2.5
1.1	2.5
2.2	5.0
3.3	5.0

Preliminary Results

Asymmetry



Yield



courtesy of R. Zielinski

$$A_{\perp} = \left(\frac{1}{P_b P_t}\right) \frac{Y_+ - Y_-}{Y_+ + Y_-}$$

$$Y_{\pm} = \frac{N_{\pm}}{Q_{\pm} L T_{\pm}}$$

Summary

- g_2^p experiment will provide first precision measurement for proton at low Q^2
  $0.02 < Q^2 < 0.2$ GeV^2

- Will provide insight on several outstanding physics puzzles
  - BC Sum Rule: Violation suggested for proton at large Q^2 (SLAC E155x)
  - Longitudinal-transverse spin polarizability: benchmark test of χPT, discrepancy seen for neutron data
  - Hydrogen hyperfine splitting: correction for proton structure contributes to uncertainty
  - Proton charge radius: contributions to uncertainty include proton polarizability

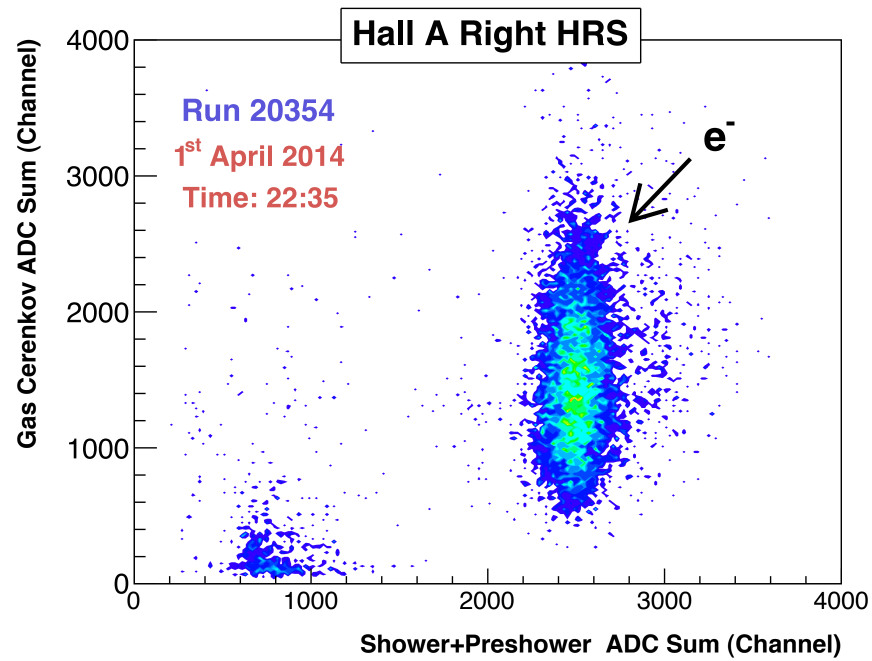
Future Experiments

- Upcoming measurements at JLab in the 12 GeV era

- Hall A
  - E12-06-122: *A1n* in valence quark region (8.8 and 6.6 GeV)

- Hall B
  - E12-06-109: longitudinal spin structure of the nucleon

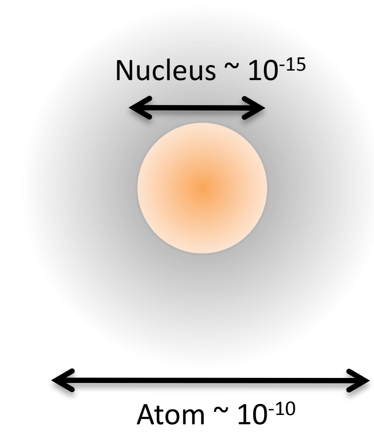
- Hall C
  - E12-06-110: *A1n* in valence quark region (11 GeV)
  - E12-06-121: g_2^n and d_2^n at high Q^2



Backup

Finite Size Effects

Hyperfine Splitting of Hydrogen:



Splitting expressed in terms of Fermi Energy E_F:

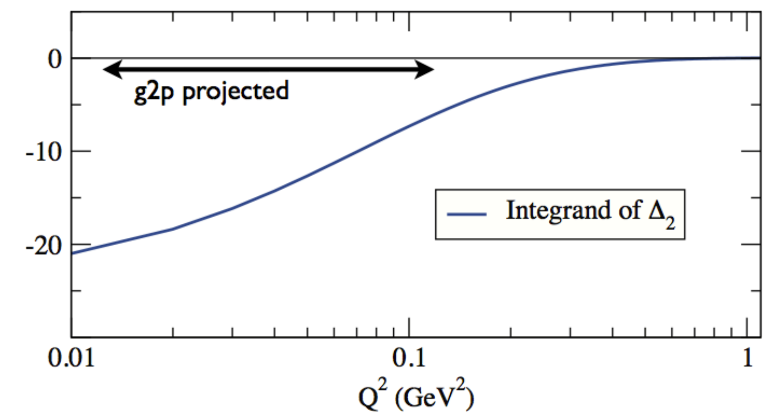
$$\Delta_E = (1+\delta)\, E_F$$

Where:

$$\delta = 1 + (\delta_{QED} + \delta_R + \delta_{small}) + \Delta_S$$

$$\Delta_S = \Delta_Z + \Delta_{pol}$$

$$\Delta_{pol} = \frac{\alpha m_e}{\pi g_p m_p} (\Delta_1 + \Delta_2)$$



Dominated by low Q^2 g_2^p

Finite Size Effects

Δ_S depends on ground state and excited properties:

$$\Delta_S = \Delta_Z + \Delta_{pol}$$

Determined from elastic scattering:

$$\Delta_Z = -2\alpha m_e r_Z (1 + \delta_Z^{rad})$$

Involves contributions where the proton is excited:

$$\Delta_{pol} = \frac{\alpha m_e}{\pi g_p m_p}(\Delta_1 + \Delta_2)$$

Depends only on the g_2 structure function

Involves the Pauli form factor and g_1 structure function

$$\Delta_2 = -24 m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2)$$

$$B_2(Q^2) = \int_0^{x_{th}} dx \beta_2(\tau) g_2(x, Q^2)$$

$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}$$

$$\tau = \nu^2/Q^2 \qquad x^{th} = \text{pion production threshold}$$

Error Budget

Systematic Error Budget for Polarized Cross Section Difference

Source	%
Cross Section	5-7
$P_b P_t$	4-5
Radiative Corrections	3
Parallel Contribution	< 1
Total	**7-9**

Error Budget

Experimental Observables:

$$A_{raw} = \frac{\frac{N^+}{LT^+Q^+} - \frac{N^-}{LT^-Q^-}}{\frac{N^+}{LT^+Q^+} + \frac{N^-}{LT^-Q^-}} \longrightarrow A_{\perp}^{exp} = \frac{A_{\perp}^{raw}}{f P_t P_b}$$

Source	%
Target Polarization	3-4
Beam Polarization	2-3
Dilution Factor/Packing Fraction	~1

Error Budget

Experimental Observables:

$$\sigma_0^{raw} = \frac{d\sigma^{raw}}{d\Omega dE'} = \frac{ps_1 N}{N_{in}\rho L T \epsilon_{det}} \frac{1}{\Delta\Omega \Delta E' \Delta Z}$$

$$\sigma_0^{exp} = \sigma_0^{raw} - \sigma^{unpol}$$

Source	%
Acceptance/Optics	~3
Dilution Factor/Packing Fraction	~1
Density	2-3
Beam Charge	1-2
Position & Angle Determination	2-4
Detector Efficiencies	~1
Background (pions)	< 1
Radiative Corrections	1-4