Central Scattering Angle Measurement

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May 23th, 2014

1 Introduction

Scattering angle is the angle between the direction of a scattered electron and the direction of electron beam. Scattering angle measurement contains two parts: central scattering angle θ_0 and spectrometer (optics) reconstruction (target angles). The relation between scattering angle (θ) and θ_0 and target angles (θ_{tg} and ϕ_{tg}) is

$$\theta = acos\frac{cos(\theta_0) - \phi_{tg}sin(\theta_0)}{\sqrt{1+\theta_{tg}^2+\varphi_{tg}^2}} \quad (1)$$

In this document, we will focus on the measurement of the central scattering angle θ_0.

Spectrometer central angle (θ_0) is the angle between the line connecting target center and sieve slit center and the ideal beam line. There are two methods to measure θ_0: survey and pointing measurement. These two methods will be discussed in detail in Section 2 and 3.

2 Survey

A survey measures the position of sieve slit and target center. The uncentainties of the survey measurements are shown in tabel. 1

The uncertainties from these measurements lead to the uncertainty of the suvery angle as 0.7 mr. The values and uncertainties of center angles for LHRS and RHRS are listed in Table. 2.

sieve x,y	0.5mm
sieve z	1mm
target z	1.5mm

Table 1: Uncertainties of survey measurements and target position determination

Arm	Survey values (rad)
LHRS	0.1007 ± 0.0007
RHRS	0.1009 ± 0.0007

Table 2: Survey results of central scattering angle

3 Pointing measurement

Pointing measurement is a method to determine the central scattering angle using elastic scattering. The equation for elastic scattering off a target of mass M is

$$E^{'} = \frac{E - E_{loss}}{1 + \frac{E - E_{loss}}{M}(1 - \cos\theta)} - E'_{loss} \tag{2}$$

Here E, $E^{'}$, θ and E_{loss} represent beam energy, scattered electron energy, scattering angle, and energy loss respectively. Since M is well known, by accurately measuring E and $E^{'}$, θ could be calculated.

The uncertainty of this calculation will be reduced by using the difference of $E^{'}$ between two nuclei, as shown in Eqn. 3. $\triangle E^{'}$ can be determined with higher accuracy than $E^{'}$. In most cases, these two nuclei are in the same target, such as CH_2 foil or watercell target, where E_{loss}'s cancel out, and also reduce the uncertainty. The uncertainty estimation of this formula is derived in Appendix A. From the derivation, the uncertainty of the scattering energy difference needs to be around 4.5×10^{-5}GeV to achieve the angle relative uncertainty goal of 0.5

$$\triangle E^{'} = E_1^{'} - E_2^{'} = \frac{E - E_{1loss}}{1 + \frac{E - E_{1loss}}{M1}(1 - \cos\theta)} - \frac{E - E_{2loss}}{1 + \frac{E - E_{2loss}}{M2}(1 - \cos\theta)} - (E'_{1loss} - E'_{2loss}) \tag{3}$$

Before looking into the calculation, a special situation of g2p experiment has been investigated. The electron beam in g2p has an offset from nominal beamline, which is changing frequently over the period of the experiment.

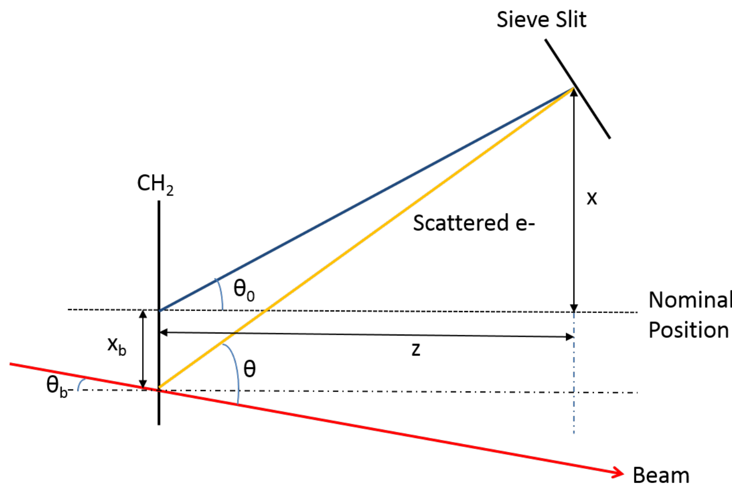

Figure 1: Schematic of the angle measurement on CH_2 foil target.

The beam position has an uncertainty of about 1.5mm, which is an order larger than the standard situations in the previous experiments in Hall A, JLab.

g2p experiment took pointing data with beam energy 2.254GeV. In order to have two nuclei, carbon foil in Liquid Helium and CH_2 foil targets were used. The investigation on CH_2 will be shown in detail here.

Figure 1 shows the schematic of central angle measurement with CH_2 foil target. The electron beam deviates from nominal beamline by x_b, and scatters on the target with incoming angle θ_b. The distance of the sieve slit center from the target center is x and z in Hall coordinate system. The scattering angle of electron is θ, while the central scattering angle is θ_0. This figure shows that the pointing calculation will determine θ, and the relation between θ and θ_0 need to be determined to obtain θ_0.

A Monte Carlo simulation was developed to find out the uncertainty of this relation. Besides the uncertainty of x_b 1.5mm as mentioned above, uncertainties of other variables were used as shown in Table 1. Events are generated under gaussian distribution around the reaction point. Sieve slit center position was also smeared by both the survey position uncertainty and sieve hole dimension. Then two relation options, $\theta - \theta_0$ and θ/θ_0, are formed as in Figure 2. The uncertainties of these relations are 1.8mr and 18.3mr, respectively. They are both larger than the uncertainty of the survey results.

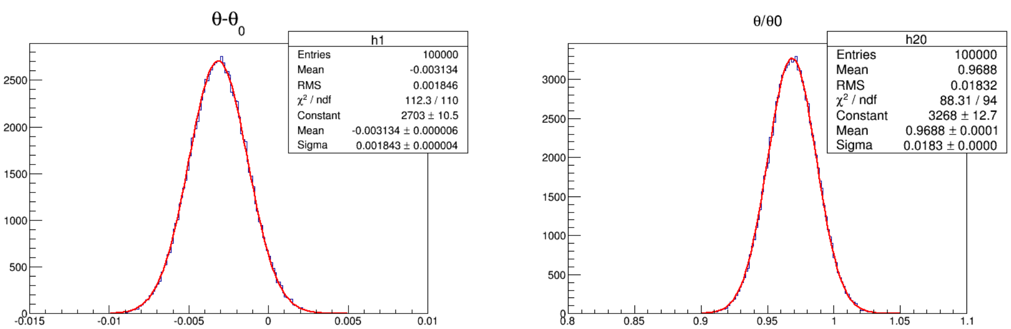

Figure 2: Uncertainty of $\theta - \theta_0$ (left) and θ/θ_0 (right) from MC simulation

In addition, the uncertainty of beam incoming angle, which is around 1.5mr, also contributes directly to the uncertainty of the relation of θ and θ_0. Together with the uncertainty described above, the total uncertainty of $\theta - \theta_0$ from beam is 2.3mr.

Considering the Carbon foil in LHe situation, the liquid helium is about 4.2cm long. The uncertainties of beam position and incoming angle is similar to the ones with CH_2 target. The target center of LHe is different from Carbon foil, and cross section needs to be weighted to obtain the extended LHe target center. These two facts introduce more uncertainties into the calculation. Therefore, this measurement results in larger uncertainty than the CH_2 foil target.

All in all, the uncertainty caused by beam uncertainties results in larger uncertainty than the survey results.

4 Conclusion

In the current level of beam position and incoming angle uncertainties, survey provides more accurate results of the central scattering angle. The results are show in Table 2.

5 Appendix

A Error propagation of Equation 3

In g2p experiment, the pointing runs have Carbon foil sitting in LHe. M1=M(carbon)=11.1849 GeV, M2=M(He4)=3.7274 GeV. Define x≡ $1-\cos\theta$,y≡ $\triangle E'$. Input these into Eqn. 3, we get

$$x=\frac{-7.46}{E}+\frac{3.73}{y}-\frac{3.73}{yE^2}\sqrt{E^2\left(y^2-4.00yE+E^2\right)} \tag{4}$$

Write Eqn. 4 as $x=f\left(E,y\right)$, then the derivative of this equation is $dx=\frac{\partial f}{\partial E}dE+\frac{\partial f}{\partial y}dy$.

Thus, the derivative of Eqn. 4 is

$$\begin{aligned} dx &= \sin\theta d\theta \\ &= \frac{3.73y^2E^2-7.46yE^3+7.46yE\sqrt{E^2\left(y^2-4.00yE+E^2\right)}}{yE^3\sqrt{E^2\left(y^2-4.00yE+E^2\right)}}dE \\ &+ \frac{-7.46yE^3+3.73E^4-3.73E^2\sqrt{E^2\left(y^2-4.00yE+E^2\right)}}{y^2E^2\sqrt{E^2\left(y^2-4.00yE+E^2\right)}}dy \end{aligned} \tag{5}$$

Assume $\theta=6^\circ$ to give an idea of the value of y (we will obtain this value from optics dp calibration in the pointing calculation), and in the case of E=2.254 GeV,

$$\sin\theta d\theta=-4.9\times10^{-3}dE+1.11d(\triangle E') \tag{6}$$

If $d\theta/\theta=0.5\%$ is desirable, $\sin\theta d\theta=5\times10^{-5}$, the contribution from dE term is negaligible, then $d\left(\triangle E'\right)=4.5\times10^{-5}$ GeV

References

[1] N. Liyanage, K. Saenboonruang, R. Michaels, *Q^2 measurement for PREX*, *PREX Technote*, 2011