Dilution Analysis for E08-027

Toby Badman

University of New Hampshire

tbadman@jlab.org

February 4, 2017

The E08-027 experiment measured inclusive electron scattering cross-section differences to calculate the proton spin structure function, g_2^p. Experimentally, we are unable to scatter off of a pure proton target so we instead look at scattering off of a solid $^{14}NH_3$ target submersed in a bath of LHe. The electron scattering off of the unpolarized ^{14}N, LHe and foil end caps of the target act to dilute our measured e-p scattering asymmetry. This requires an additional correction called the *dilution factor* to determine the real physics asymmetry. This tech note will summarize the method used to calculate the dilution factor for use in offline analysis.

1 Target Asymmetry

A well measured target asymmetry is needed to calculate electron scattering cross section differences. The asymmetry is expressed as the difference over the sum of the number of electrons in the positive (+) and negative (−) helicity states,

$$A_{phys} = \frac{N_+ - N_-}{N_+ + N_-} \tag{1}$$

where $N_{+/-}$ is the number of detected electrons in the positive/negative helicity states, respectively. We are only interested in detected electrons that have scattered off of polarized protons in the target material. In reality, some amount of the detected electrons in each helicity state will have scattered off of unpolarized 'background' material. The measured asymmetry is then diluted as

$$A_{meas} = \frac{(N_+ + 1/2 N_{bg}) - (N_- + 1/2 N_{bg})}{(N_+ + 1/2 N_{bg}) + (N_- + 1/2 N_{bg})} = \frac{N_+ - N_-}{N_+ + N_- + N_{bg}} \tag{2}$$

where N_{bg} are detected electrons that have scattered off of unpolarized target material. Any scattering off of unpolarized background material only appears in the denominator of equation (2), acting to dilute the final asymmetry. By introducing a correction factor of the form

$$f \equiv \frac{N_+ + N_-}{N_+ + N_- + N_{bg}} \tag{3}$$

the background contribution is removed, allowing to calculate the undiluted physics asymmetry,

$$\frac{1}{df} A_{meas} = \left(\frac{N_+ + N_- + N_{bg}}{N_+ + N_-} \right) \left(\frac{N_+ - N_-}{N_+ + N_- + N_{bg}} \right) = A_{phys} \tag{4}$$

which requires the calculation of the dilution factor as defined in equation (3). This is done using two separate methods. The first method uses dilution run data taken throughout E08-027, while the second method uses radiated cross section models. While tuning the radiated model to match our low Q^2 data was not ideal, we also did not have complete dilution run coverage for our kinematics. The final dilution factor results are a convolution of both methods.

2 Material Parameterization

To find the dilution factor we first need an expression for N_{bg} in terms of experimental conditions. For the g2p experiment a solid $^{14}NH_3$ target material submersed in a LHe bath between two Aluminum foil windows was used. The background charge normalized counts are then defined by $N_{bg} = N_N + N_{He} + N_{Al}$ since only electrons scattered from the spin polarized proton part of $^{14}NH_3$ contributes to the asymmetry.

Following closely the parameterization derivation of [1], we write N_x in detail as $N_x = AN_0\rho_x L_x \sigma_x / eM_x$, where N_0 is Avagadro's numbers, A is the experimental acceptance, ρ_x is the density of material x, M_x is the atomic weight, L_x is the thickness of the material and σ_x is the fully radiated cross section for material x. The charge normalized background counts can then be expressed as

$$N_{bg} = \frac{AN_0}{e}\left(\frac{\rho_A L_{tg}(pf)}{M_A}\sigma_N + \frac{\rho_{He} L_{tg}(1-pf)}{M_{He}}\sigma_{He} + \frac{\rho_{Al} L_{Al}}{M_{Al}}\sigma_{Al}\right) \tag{5}$$

where pf is the length fraction of the target cell that is filled with ammonia, called the *packing fraction*.

Along with the $^{14}NH_3$ target, data was taken on various other targets to simulate background conditions. Ideally we would have used a pure nitrogen target to account for the ^{14}N background but such a target was not readily available. Instead a solid carbon target was used, and a nitrogen simulation was used to scale the resulting yield. In total three dilution run targets were used; a pure liquid helium target, referred to as the 'empty run', a liquid helium target with foil windows, referred to as the 'dummy run', and the carbon disk target, referred to as the 'carbon run'. Similar to the production background we can

parameterize each of these in terms of the contributing materials as follows

$$N_{empty} = \frac{AN_0}{e}\frac{\rho_{He}L_{tg}}{M_{He}}\sigma_{He} \tag{6}$$

$$N_{dummy} = \frac{AN_0}{e}\left(\frac{\rho_{He}L_{tg}}{M_{He}}\sigma_{He} + \frac{\rho_f L_f}{M_f}\sigma_f\right) \tag{7}$$

$$N_{carbon} = \frac{AN_0}{e}\left(\frac{\rho_C L_C}{M_C}\sigma_C + \frac{\rho_{He}(L_{tg}-L_C)}{M_{He}}\sigma_{He}\right) \tag{8}$$

We can solve equation (8) for σ_C and substitute all three parameterized dilution yields into equation (5) to find the charge normalized background counts in terms of measured dilution runs.

$$N_{bg} = a\frac{M_C\rho_A L_{tg} pf}{M_A\rho_C L_C}\left(N_{carbon} - \left(\frac{L_{tg}-L_C}{L_{tg}}\right)N_{empty}\right) + N_{dummy} - pfN_{empty} \tag{9}$$

where a is a scaling factor used to scale σ_C to σ_N. All of the quantities in equation (9) are universally or experimentally measured and can be found in Table 1 except for the scaling factor a and the packing fraction.

Parameter	Value
M_C	12.011 u
M_A	17.031 u
ρ_C	2.267 g/cm^3
ρ_A	0.817 g/cm^3
L_{tg}	2.8307 cm
L_C	0.1016(0.3175) cm

Table 1: Known background parameters, terms in parenthesis were used after run 5103.

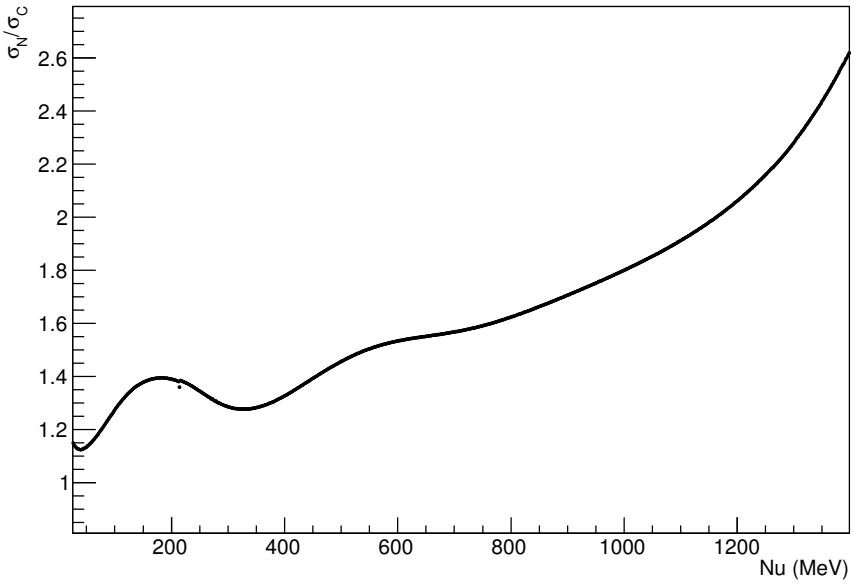


Figure 1: C12 to N14 scaling ratio using P.Bosted radiated cross section model.

3 Scaling ^{12}C to ^{14}N

Before we can substitute the charge normalized counts into equation (5) we must find a relation between σ_C and σ_N. A very crude approximation is to consider each cross section as if it was made up solely of its constituent nucleons. In this scenario the carbon and nitrogen cross sections can be written as

$$\sigma_C = 6\sigma_H \tag{10}$$

$$\sigma_N = 7\sigma_H \tag{11}$$

$$\sigma_N = \frac{7}{6}\sigma_C \tag{12}$$

This is a good approximation in the deep inelastic region but breaks down in the resonances[2]. To find the scaling factor everywhere we simply replace the constant scaling factor with a fitting parameter

$$\sigma_N = a\sigma_C \tag{13}$$

and generate a radiated cross section ratio between carbon and nitrogen to find a, which can then be applied to the carbon yield bin by bin. Such a ratio for one Q^2 setting can be seen in Figure 1. The models are tuned to data at larger Q^2 settings and any variance in the accuracy is included in the final systematic uncertainty. The model uncertainty was found to be roughly 5% at all Q^2 settings[6]. It is important to note that, while theoretically the C12 to N14 ratio should approach 1.167 in the DIS region, the model ratio actually diverges quite significantly due to the differing radiation lengths and scattering angles of each material. This is discussed in more detail in Section 5.

4 Packing Fraction

The last piece needed before the background can be calculated is the fractional length of the target cell that is filled with $^{14}NH_3$. To find this we start with the definition of the charge normalized background counts given by equation (5), but with an added hydrogen component to produce the total production yield

$$N_{production} = \frac{AN_0}{e}\left(\frac{\rho_A L_{tg}(pf)}{M_A}(\sigma_N + 3\sigma_H) + \frac{\rho_{He} L_{tg}(1-pf)}{M_{He}}\sigma_{He} + \frac{\rho_{Al} L_{Al}}{M_{Al}}\sigma_{Al}\right). \tag{14}$$

Then we can substitute in our definition of σ_N given by equation (13) and use eqs. (6), (7), and (8) to find the charge normalized production counts in terms of the dilution run

Setting			Material ID	PF	Unc.
0^o	5T	2.254GeV	17	0.516	0.019
			18	0.581	0.019
90^o		2.254GeV	19	0.597	0.025
			20	0.610	0.028
		3.350GeV[1]	19	0.644	0.015
			20	0.544	0.011
	2.5T	2.254GeV	7	0.630	0.048
			8	0.658	0.051
		1.710GeV	7	0.770	0.059
			8	0.850	0.066

Table 2: Packing fractions for all target materials

normalized counts

$$N_{producton} = \left(\frac{M_C \rho_A L_{tg}(pf)}{M_A \rho_C L_C}\right)\left(N_{carbon} - \frac{L_{tg}-L_C}{L_{tg}} N_{empty}\right)\left(a + 3\frac{\sigma_H}{\sigma_C}\right) - (pf)N_{empty} + N_{dummy} \tag{15}$$

solving for the packing fraction gives

$$pf = \frac{N_{production} - N_{dummy}}{\left(\frac{M_C \rho_A L_{tg}}{M_A \rho_C L_C}\right)\left(N_{carbon} - \frac{L_{tg}-L_C}{L_{tg}} N_{empty}\right)\left(a + 3\frac{\sigma_H}{\sigma_C}\right) - N_{empty}}. \tag{16}$$

All of the terms in the above equation are given in Table 1 except for the cross section ratio σ_H/σ_C which is found using simulation, in an identical method to the σ_N/σ_C ratio found in Section 3.

Since the packing fraction should be kinematically independent we can use a constant carbon to nitrogen scaling ratio of 1.167 and perform a linear fit in the DIS region where this is a good approximation[2, 3]. Figure 2 shows an example fit for one Q^2 setting. This

[1] The 3.350GeV setting has yield drift problems so the recorded packing fraction is only a preliminary estimate based off averaging drifting yields.

[2] 1.157GeV data is still being analyzed.

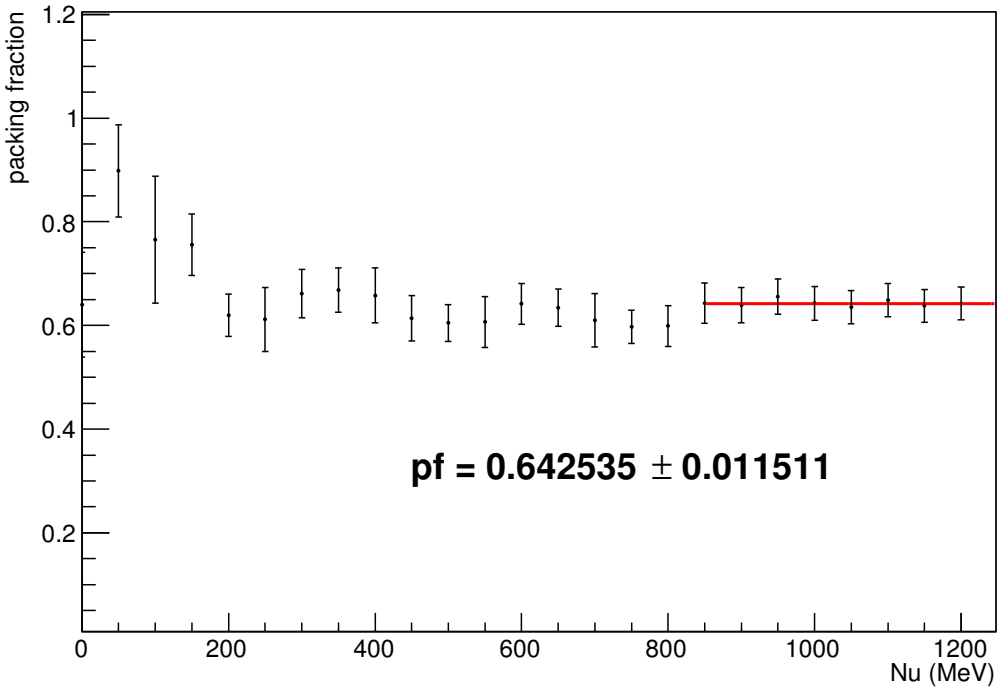


Figure 2: Packing fraction with linear fit at large ν.

analysis was done each time a production target was removed. A list of packing fractions for each material is shown in Table 2.

5 Radiation Length and Scattering Angle Corrections

An important correction factor that has been overlooked to this point is the radiative length and scattering angle dependence of each dilution run. Equation (9) is found by substituting the dilution run cross-sections into the parameterized definition for the production background. Each of the dilution runs uses different target materials, which results in different average scattering angles and radiative lengths. Because of this, each type of dilution run is not directly comparable. Instead we must introduce a scaling factor to match the radiation

Table 3: Material Thicknesses

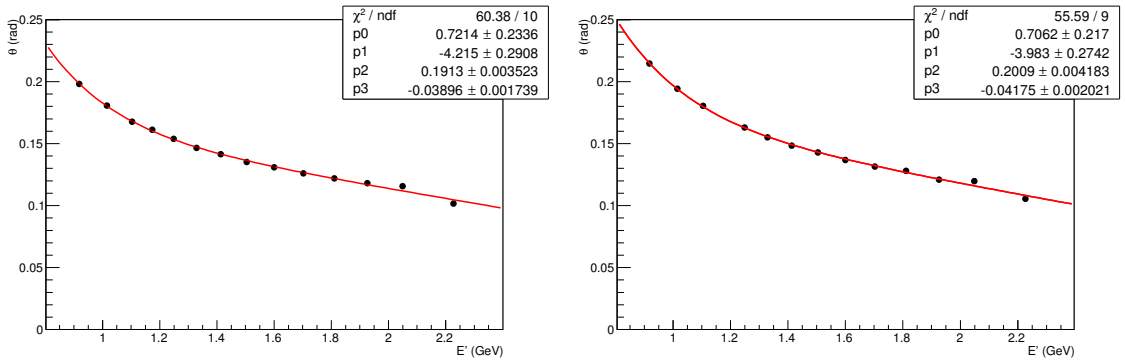
Run	Material	Density(g/cm³)	Thickness (cm)	Rad. Length	Rad. Thickness
Empty	He4	0.145	3.7045	94.3224	0.00569
Carbon	C12	2.267	0.1016	42.6969	0.00539
Carbon	He4	0.145	3.6029	94.3224	0.00554
Production	N14	0.817	1.5549	40.8721	0.03108
Production	He4	0.145	2.146	94.3224	0.0033
Production	Al27	2.7	0.0036	24.0112	0.0004

length and scattering angle of each dilution cross section to their corresponding production run conditions.

The ratio is found by generating two different cross sections for each background material. One cross section uses the conditions of the dilution run material, while the second cross section uses the conditions of the same material in the production run. Then, by taking the ratio of the two cross sections, we find a scaling factor that can then be applied to the dilution yields.

The first and simplest condition to consider is the radiative length change. During the carbon dilution run we used 0.1016cm of carbon which has a radiation thickness of $\rho L/\chi_0 = 0.00539$ where the density of carbon is $2.267\,\mathrm{g/cm^3}$ and χ_0 is the material dependent radiation length. The nitrogen target has a thickness of $pf \times L_{tg} \approx 1.55\,\mathrm{cm}$ which gives a radiation length of 0.03108. So by generating a carbon cross section and radiating using each of these two radiation lengths we find the appropriate scaling factor to then be applied to the dilution yields. Table 3 contains the physical lengths, densities and radiation lengths of each material used in generating these ratios.

The second condition to be considered is the scattering angle dependance of each target material. To correct for different scattering angles we need a good understanding of how the scattering angle changes as a function of E' for each material. First, we plot the cen-



(a) Production target (b) Carbon target

Figure 3: Fit to central scattering angle for one kinematic setting.

tral scattering angle vs. HRS momentum for each run and perform a weighted average of overlapping bins to find θ_{sc} vs. HRS P_0 for each target type. The data is then fit with an exponential of the form $f(x) = e^{p_0+p_1 x} + p_2 x + p_3$[4], and the four target dependent fit parameters are saved to a text file. Figure 3 shows this fit being applied to a Carbon run and a Production run for comparison. Note that the difference in average scattering angles between runs is small but the cross section is highly sensitive to this ratio.

Once we have the fit parameters for each target type we can again generate two cross section models for each material, one using the fit parameters for the dilution run of interest, and the second using the parameters of the production run we are scaling to. Then, by taking the ratio of these two models, we find the scattering angle correction to be applied to the dilution run yields.

It is worthwhile to note that the two scaling processes that have been described in this section were done simultaneously. Although it is more straightforward to explain each process separately, in reality the change in scattering angle will have an impact on the radiative effects, and vise versa. So each model must be generated with both scattering angle and radiative length dependence. As expected, the He4 correction was typically small ($< 1\%$)

due to the very small correction in radiation length, while the C12 correction was much larger because of the scattering angle difference between the Carbon and Production run types.

6 Dilution Factor Uncertainty

With all of the components we can now find an expression for the dilution factor.

$$f = 1 - \frac{N_{bg}}{N_{production}} \tag{17}$$

Using the definition of N_{bg} given by equation (9)

$$f = 1 - \frac{a(E,E')\frac{M_C \rho_A L_{tg} pf}{M_A \rho_C L_C}\left(N_{carbon} - \left(\frac{L_{tg}-L_C}{L_{tg}}\right)N_{empty}\right) + N_{dummy} - pfN_{empty}}{N_{production}} \tag{18}$$

where each charge normalized count now has the proper scattering angle and radiation length scaling factor applied to it as addressed in Section 5. The dilution factor is then calculated at each Q^2 setting bin by bin, and can then be applied to the measured asymmetry. The dilution factor for all settings is shown in Appendix A. For analysis purposes the dilution was calculated in 1 MeV bins across the entire kinematic range for each setting. To reduce the statistical uncertainties of the final result 50 MeV bins were used and a statistically weighted average of the dilution in each bin was calculated using

$$f = \frac{\sum\limits_n \left(\frac{f_n}{\delta_n^2}\right)}{\sum\limits_n \left(\frac{1}{\delta_n^2}\right)} \tag{19}$$

where δ_n is the uncertainty in each dilution factor. Looking at equation (18) it is easy to see that the propagation of the statistical uncertainty is very tedious. Each normalized count

has a statistical uncertainty of $1/\sqrt{N}$ associated with it. It was found that the statistical uncertainty at each setting was negligible with large enough bin sizes.

There are two primary sources of systematic uncertainty in the dilution factor. The first is the uncertainty in the model used for radiation length and scattering angle scaling. The second is in the determination of the scattering angle itself. The scattering angle is known to 0.1 mRad[5], so a fit to the central value of the scattering angle will have some variance associated with it. To find the uncertainty due to the scattering angle as described in Section 5 two fits were done for each target material. One using the average scattering angle minus 0.1 mRad, the other using the average scattering angle plus 0.1 mRad. A cross section was generated using each of these fits and the ratio between them indicated how sensitive the cross section was to the scattering angle. It was found that the variance was always less than 5% so a total systematic uncertainty of 5% was included in the scattering angle calculation.

To find the systematic uncertainty in the generated models a detailed study of model tuning on preexisting data at different Q^2 settings was done[6]. The total scaling factor required to minimize the χ^2 for each data set was used as the systematic uncertainty in the model. The model uncertainty ranged from 5% to 10% depending on the target.

The systematic uncertainty was then propagated in the standard way through the dilution factor. It was found that the systematics were the dominating uncertainty in the dilution factor, final values with uncertainties are shown in Appendix A.

References

[1] O. Rondon, RSS Tech. Note #2005-03 (2006) https://userweb.jlab.org/~jones/tajima_rss/plots/2007/sep25/dfpf_oscar_report.pdf.

[2] Y. Prok, Ph.D Thesis, The University of Virginia, http://twist.phys.virginia.edu/Theses/prok_thesis.pdf.

[3] R. Fersch, Ph.D Thesis, The College of William and Mary, https://www.jlab.org/Hall-B/general/thesis/Fersch_thesis.pdf.

[4] J. Zhang, *Recalculate the scattered angle shifted due to target field using HRSMC v0.93*, https://hallaweb.jlab.org/dvcslog/g2p/47

[5] M. Huang, E08-027 Scattering Angle Tech. Note, http://hallaweb.jlab.org/experiment/g2p/collaborators/mhuang/technote/pointingSummary.pdf

[6] R. Zielinski, E08-027 Radiated Model Tech. Note, https://userweb.jlab.org/~rbziel/Unpol_Rad_Final.pdf.

A Dilution at all settings

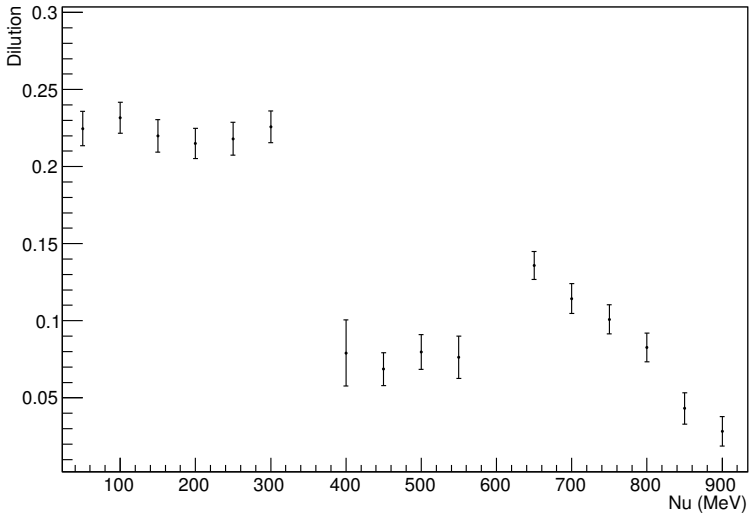


Figure 4: 1.710 GeV 2.5T Transverse

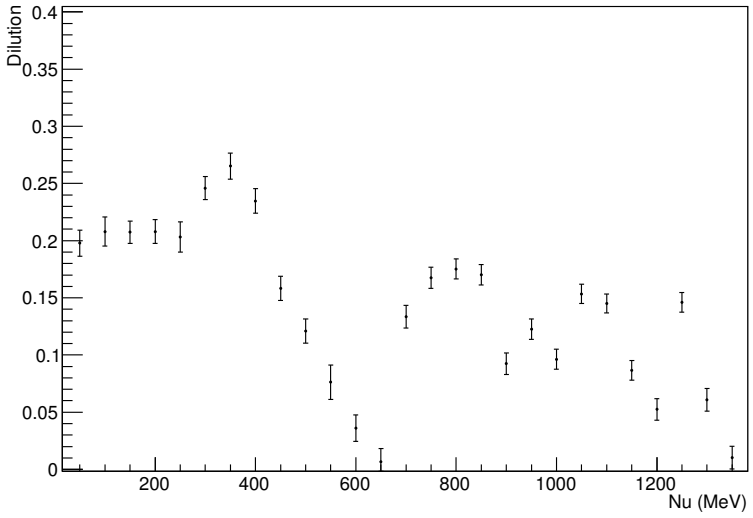


Figure 5: 2.254 GeV 2.5T Transverse

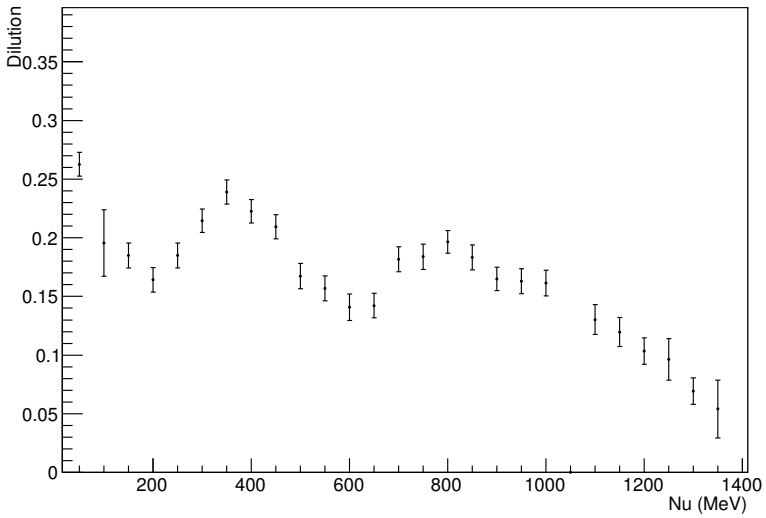


Figure 6: 2.254 GeV 5T Transverse

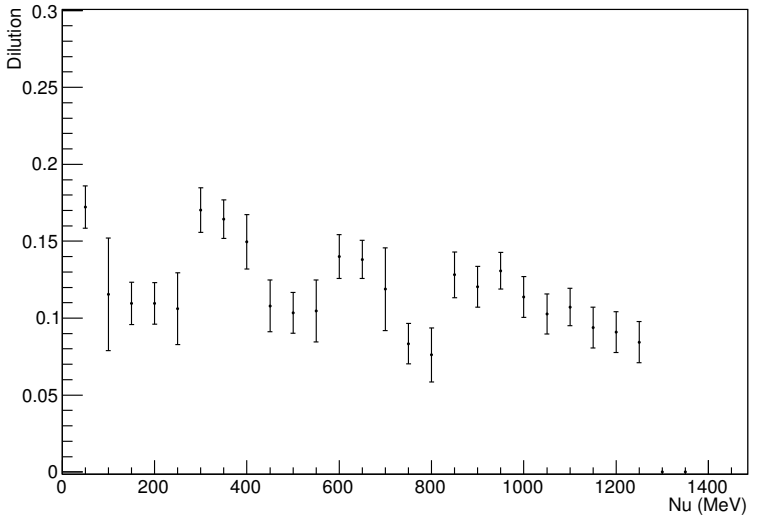


Figure 7: 2.254 GeV 5T Longitudinal