Temperature of CH_2 foil under 2.2 GeV-150nA electron beam for g_2p experiment

Maxime Defurne - Ecole Centrale Paris

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1 Motivation

The g_2p experiment has as goal to determine the structure function g_2 of the proton at low Q^2 which is not measured yet. Thus we will be able to compare it with present QCD predictions about moments. In the case of the neutron, the second moment already differs with QCD. Moreover, we need it to explain the different results of the proton radius lately discovered.

In order to get accurate results, we need to calculate the acceptance of the spectrometer thanks to special measurement with special target. The best material would be the polyethylene. However its low thermal conductivity could make it melt down under the electron beam. The purpose of this article is to answer the question:

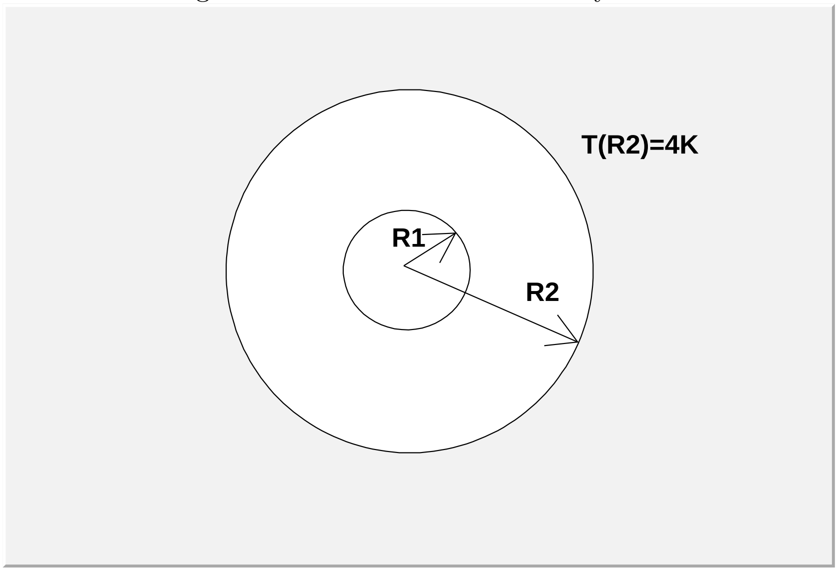
Could a CH_2 foil melt down under a 2.2GeV-150nA electron beam?

2 description of the system

We are going to calculate the maximal temperature of a CH_2 foil under a 2.2GeV-150nA electron beam. The CH_2 foil is cirucular with a radius of R_2 and a thickness e. This foil is in vacuum. We will only consider the thermal conduction as heat transfer. λ will be the thermal conductivity of polyethylene and ρ its density.

We will consider that the electron beam is also circular with a radius R_1. To calculate the energy deposit by electrons in CH_2, we will only take collisionning energy deposit. Indeed the radiative contribution can be negligible because of the thin thickness of the foil.

Figure 1: schema of the studied system



3 calculation of absorbed beam power

The formula to calculate the beam power absorbed by the foil is given by:

$$\Phi = \frac{dE}{dx} \times \rho \times i \times e \text{ } [\mathbf{1}]$$

with i the beam current.

We can also define an absorbed power density by:

$$P_v = \frac{\Phi}{\pi \times R_1^2 \times e}$$

4 Set of equations

We can define two domain on the CH_2 foil. The first one is the center of the foil with $r \in [0; R_1[$. In domain 1, the set of equations is given by:

$$\begin{cases} -\lambda \nabla T = P_v \text{ } \mathbf{(1)} \\ \text{in } r = 0, \text{ } \iint \overrightarrow{j} . \overrightarrow{dS} = 0 \text{ } \mathbf{(a)} \end{cases} \quad \text{with } \overrightarrow{j} = -\lambda \overrightarrow{\nabla} T \text{ , according to Fourier's law}$$

Because of the cylindrical symmetry, the Laplacian is given by $\nabla T = \frac{1}{r} \frac{d(r \frac{dT}{dr})}{dr}$.

For the second domain defined by $r \in [R_1; R_2]$, the equation and boundary conditions are:

$$\begin{cases} \nabla T = 0 \\ \text{In } r = R_1,\ \iint \overrightarrow{j}.\overrightarrow{dS} = \Phi\ \textbf{(2)} \\ \text{In } r = R_2,\ T(R_2) = 4K \end{cases}$$

As you can see, we have one boundary condition for domain 1 whereas there are two boundary conditions for domain 2. We are going to determine $T(R_1)$ on domain 2 to get the second boundary condition for domain 1.

5 Determination of $T(R_1)$

As $\nabla T = 0$ in domain 2, we have $\Delta T = R\Phi$ with R the thermal resistivity and $\Delta T = T(R_1) - T(R_2)$. Let's calculate R!
As

$$\nabla T = 0$$

we have

$$\frac{dT}{dr} = \frac{A}{r}$$

Now we can determine A by integrating between R_1 and R_2

$$T(R_2) - T(R_1) = A\ln(\frac{R_2}{R_1}) \Longrightarrow A = \frac{T(R_2) - T(R_1)}{\ln(R_2/R_1)}$$

Then, thanks to **(2)** and $\overrightarrow{\nabla} T = \frac{dT}{dr}\overrightarrow{e_r}$:

$$T(R_1) = T(R_2) + \frac{\Phi}{2\pi\lambda e}\ln(\frac{R_2}{R_1})\ \textbf{(b)}$$

6 Determination of the maximal temperature

The maximal temperature is in $r = 0$. Now, thanks to the previous section, we have two boundary conditions which allow us to solve **(1)**. After a first integration, we get for $r \in [0, R_1[$:

$$\frac{dT}{dr} = -\frac{P_v}{2\lambda}r + \frac{B}{r}$$

Because of **(a)**, B must be equal to 0. Then, after a second integration, we obtain:

$$T(r) = \frac{P_v}{4\lambda}r^2 + C$$

And now we use **(b)** and the expression of P_v to get:

$$\forall r \in [0, R_1[,\ T(r) = \frac{\Phi}{4\pi\lambda R_1^2 e}(R_1^2 - r^2) + \frac{\Phi}{2\pi\lambda e}\ln(R_2/R_1) + T(R_2)$$

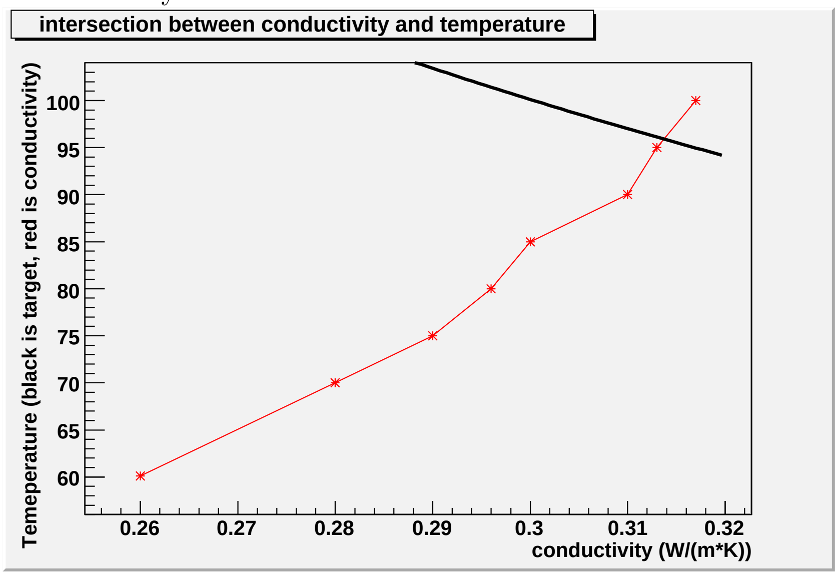
And so

$$T(0) = \frac{\Phi}{4\pi\lambda e}(2\ln(\frac{R_2}{R_1}) + 1) + T(R_2)$$

7 Thermal conductivity under 100K

The thermal conductivity is pretty constant for temperature higher than 100K. But, below 100K, it drops dramatically as you can see on the following graph.[**2**][**3**]
To determine the good temperature of the Target center, we just have to plot the intersection of it with the previous graph. The temperature in steady state is around 96K for the Nylon. Polyethylene has a better conductivity than Nylon. So the highest temperature will be below 96K.

Figure 2: Intersection between Target temperature and graph conductivity-temperature of Nylon



Finally we can conclude that the CH_2 foil will not melt down under the beam.

8 Numerical data

Polyethylene and beam data

$$R_1 = 0.1\ mm$$
$$R_2 = 1.25\ cm$$
$$e = 0.5\ mm$$
$$\rho = 0.89\ g \times cm^{-3}$$

Absorbed beam power

$$\frac{dE}{dx} = 2.53\ MeV \times cm^2 \times g^{-1}\ \textbf{[1]}$$
$$i = 150\ nA$$
$$\Phi = 17\ mW$$

References

[1] *http://physics.nist.gov/PhysRefData/Star/Text/ESTAR.html, consulted on 03/20/2011*

[2] *private conversation with David Meekins*

[3] *http://www.qdusa.com/techsupport/thermalCalculator.html, consulted on 04/04/2011*