Uncertainty on g_2 due to target spin misalignment

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This document is trying to calculate the uncertainty of g_2 caused by the angle deflection of the target polarization with respect to the beam direction.

For the polarized cross-section, the difference of it can be written as[1]:

$$\begin{aligned}
\Delta\sigma_{\parallel} &= -\frac{4\alpha^2 E'}{\nu E Q^2}[(E + E'cos\theta)g_1 - 2Mxg_2] \\
\Delta\sigma_{\perp} &= -\frac{4\alpha^2 E'^2}{\nu E Q^2}(g_1 + \frac{2ME}{\nu}g_2)
\end{aligned} \tag{1}$$

(1) can be recast as:

$$\begin{aligned}
\Delta\sigma_{\parallel} &= K_1(a_1 g_1 + a_2 g_2) \\
\Delta\sigma_{\perp} &= K_2(g_1 + b_2 g_2)
\end{aligned} \tag{2}$$

where:

$$\begin{aligned}
K_1 &= -\frac{4\alpha^2 E'}{\nu E Q^2} \\
K_2 &= E' K_1 \\
a_1 &= E + E'cos\theta \\
a_2 &= -2Mx \\
b_2 &= \frac{2ME}{\nu}
\end{aligned}$$

From (2) the g_2 can be shown as:

$$g_2 = \frac{1}{K_2 b_2}[\Delta\sigma_{\perp} - K_2 g_1] \tag{3}$$

Since we assume that there has a small angle deflection $\Delta\alpha$, the relationship between $\Delta\sigma_{\perp}$ and $\Delta\sigma$ will be:

$$\Delta\sigma_{\perp} - \Delta\sigma = (\sigma^{\overset{\uparrow}{\rightarrow}} - \sigma^{\overset{\nearrow}{\rightarrow}}) - (\sigma^{\overset{\downarrow}{\rightarrow}} - \sigma^{\overset{\swarrow}{\rightarrow}}) \tag{4}$$

Since we assume that

$$\begin{aligned} \sigma^{\overset{\nearrow}{\rightarrow}} &= \sigma^{\overset{\uparrow}{\rightarrow}} cos\Delta\alpha + \sigma^{\rightrightarrows} sin\Delta\alpha \\ \sigma^{\overset{\swarrow}{\rightarrow}} &= \sigma^{\overset{\downarrow}{\rightarrow}} cos\Delta\alpha + \sigma^{\leftrightarrows} sin\Delta\alpha \end{aligned} \tag{5}$$

Then:

$$\begin{aligned} \Delta g_2 &= \frac{1}{K_2 b_2}[(1 - cos\Delta\alpha)\Delta\sigma_{\perp} - sin\Delta\alpha\Delta\sigma_{\parallel}] \\ &= \frac{1}{K_2 b_2}(\frac{\Delta\alpha^2}{2}\Delta\sigma_{\perp} - \Delta\alpha\Delta\sigma_{\parallel}) \\ &\approx -\frac{1}{K_2 b_2}\Delta\alpha\Delta\sigma_{\parallel} \\ &= -\frac{K_1}{K_2 b_2}(a_1 g_1 + a_2 g_2)\Delta\alpha \end{aligned} \tag{6}$$

for small $\Delta\alpha\Delta\sigma_{\perp}$.
(6) $\Longrightarrow$

$$\frac{(\frac{dg_2}{g_2})}{d\alpha} = -\frac{K_1}{K_2 b_2}(a_1\frac{g_1}{g_2} + a_2) \tag{7}$$

In the virtual photon notation, the g_1 and g_2 have the relationship as[1]:

$$\begin{aligned} \sigma_{TT} &= \frac{4\pi^2\alpha}{MK}(g_1 - \gamma^2 g_2) \\ \sigma_{LT} &= \frac{4\pi^2\alpha}{MK}\gamma(g_1 + g_2) \end{aligned} \tag{8}$$

where $K = \nu(1 - x)$, $\gamma = \frac{2Mx}{Q} = \frac{Q}{\nu}$.
(8)$\Longrightarrow$

$$\begin{aligned} g_1 &= \frac{MK}{4\pi^2\alpha(1 + \gamma^2)}(\sigma_{TT} + \gamma\sigma_{LT}) \\ g_2 &= \frac{MK}{4\pi^2\alpha(1 + \gamma^2)\gamma}(\sigma_{LT} - \gamma\sigma_{TT}) \end{aligned} \tag{9}$$

References

[1] J.-P. Chen. Moments of spin structure functions: Sum rules and polarizabilities. *arxiv:1001.3898v1 [nucl-ex]*, 19:17, 2010.