Novel TMD distributions and their effects in the polarized or unpolarized Drell-Yan process

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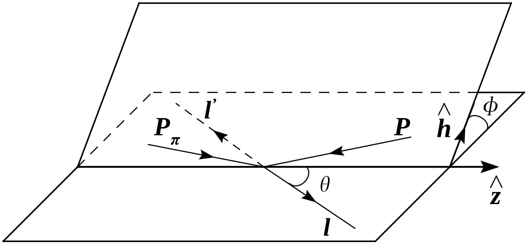
Boer-Mulders function and $\cos 2\phi$ anomaly in D-Y

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\sin2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos2\phi\right). \quad (1)$$

Kinematics



- PQCD predicts $1 - \mu - 2\nu = 0$ (Lum-Tung relation)
  $\mu \sim 1 \Rightarrow \nu = 0$
- Significant nonzero ν has been measured by NA10 and e165 Collaborations
- The asymmetry can be expressed in terms of Boer-Mulders functions (Boer)

$$\nu = 2\sum_{a,\hat{a}} e_a^2 \mathcal{F}\left[(2\hat{\mathbf{h}}\cdot\mathbf{p}_\perp\hat{\mathbf{h}}\cdot\mathbf{k}_\perp) - (\mathbf{p}_\perp\cdot\mathbf{k}_\perp)\frac{h_1^\perp \bar{h}_1^\perp}{M_1 M_2}\right] \Bigg/ \sum_{a,\bar{a}} e_a^2 \mathcal{F}[f_1 \bar{f}_1]$$

Parton probability interpretation

- Sivers function: Unpolarized parton distribution in the transversely polarized nucleon ← correlation between the nucleon transverse spin and the quark transverse momentum
- Boer-Mulders function: transversely polarized quark distribution in the unpolarized hadron ← correlation between the quark transverse spin and the quark transverse momentum

(a) (b)

Boer-Mulders function: experimental status

- While the Sivers function begins to be understood, still very little is known about the Boer-Mulders function from experiments, not measured.
- What measured in experiments is the asymmetry coefficient ν, by NA10 and e165 Collaboration in the πN Drell-Yan process (N=D or W), and currently by e866 Collaboration in the $pp(d)$ process
- More data is needed

proton case

The size and the sign of $h_1^{\perp,u}$ and $h_1^{\perp,d}$ are of great interests

Theoretical situation of valence Boer-Mulders functions:

- Bag model calculation $h_1^{\perp,u} = \frac{1}{2} h_1^{\perp,d}$ (Yuan)
- Axial-diquark model $h_1^{\perp,u} and h_1^{\perp,d}$ nearly different sign (Bacchetta,Schafer, Yang)
- Large N_c limit $h_1^{\perp,u} \approx h_1^{\perp,d}$ modulo $1/N_c$ (Pobylitsa)
- Lattice calculation $h_1^{\perp,u} \sim h_1^{\perp,d}$ (QCDSF)
- GPD approach $h_1^{\perp,u} \sim h_1^{\perp,d}$ (Buikardt)
- calculations presented in the talks by Gamberg and Prokudin.

Lattice calculation suggests significant size of valence Boer-Mulders functions

Are all Boer-Mulders functions alike? (Burkardt)

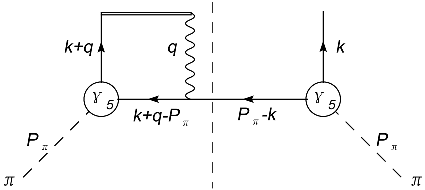
Pion case

- non-trivial transverse spin structure of pion. (Talk by Haegler)
- both isospin symmetry and model calculation predict (Lu, Ma)

$$h_{1\pi^+}^{\perp,u} = h_{1\pi^+}^{\perp,\bar{d}} = h_{1\pi^-}^{\perp,\bar{u}} = h_{1\pi^+}^{\perp,\mathrm{d}} = h_{1\pi}^{\perp}$$

- It seems that pion Boer-Mulders functions are alike.

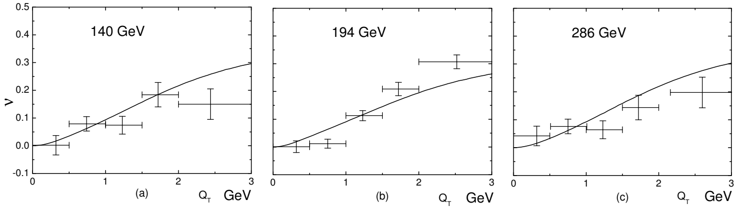
pion Boer-Mulders function in the spectator model



- model result:

$$h_{1\pi}^{\perp}(x, \mathbf{k}_{\perp}^{2}) = \frac{|e_1 e_2|}{4\pi} \frac{N_{\pi}^{2}(1-x)^{3} m M_{\pi}}{2(2\pi)^{3} L_{\pi}^{2}(\mathbf{k}_{\perp}^{2} + L_{\pi}^{2})^{3}}, \quad (2)$$

- describing the data (NA10 pi-N D-Y):



To test the flavor dependence of the Boer-Mulders functions of the proton (Lu, Ma, Schmidt)

- probe: pion Boer-Mulders functions
- Simple flavor dependence of pion Boer-Mulders functions
- Measuring the $\cos 2\phi$ asymmetry ν in the πp and πD Drell-Yan processes and take the ratio:

$$\hat{\nu}_p = a_{UU}\frac{\langle W\rangle_{\pi^- p}}{\langle 1\rangle_{\pi^- p}} = \frac{h_{1\pi}^{\perp(1)}(x_1)h_1^{\perp(1),u}(x_2)}{f_{1\pi}(x_1)f_1^u(x_2)},$$

$$\hat{\nu}_D = a_{UU}\frac{\langle W\rangle_{\pi^- D}}{\langle 1\rangle_{\pi^- D}} = \frac{h_{1\pi}^{\perp(1)}(x_1)(h_1^{\perp(1),u}(x_2)+h_1^{\perp(1),d}(x_2))}{f_{1\pi}(x_1)(f_1^u(x_2)+f_1^d(x_2))},$$

- Weighting function:

$$W = \frac{Q_T^2\cos 2\phi}{4M_AM_B}$$

- the ratio is determined by $h_1^{\perp(1),d}(x)/h_1^{\perp(1),u}(x)$, does not depend on the pion distribution functions

$$\frac{\hat{\nu}_D}{\hat{\nu}_p} = \frac{1 + \frac{h_1^{\perp(1),d}(x_2)}{h_1^{\perp(1),u}(x_2)}}{1 + \frac{f_1^d(x_2)}{f_1^u(x_2)}}. \tag{3}$$

- Is it possible to be measured at COMPASS?.
  - hadron beam program will begin
  - proton target will be available

numerical results of $\hat{\nu}_D$ and $\hat{\nu}_p$ from different sets of Boer-Mulders functions at COMPASS

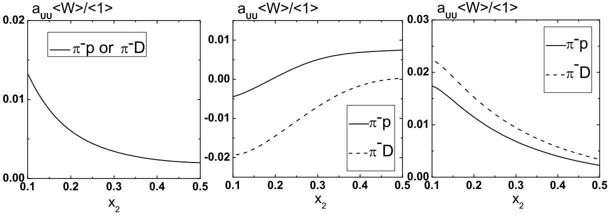


Figure: The weighted $\cos 2\phi$ asymmetries in unpolarized $\pi^- p$ (solid line) and $\pi^- D$ (dashed line) Drell-Yan processes. (a) The MIT Bag model result. (b) The axial-vector diquark model result. (c) The result from large-N_c.

Flavor dependence of valence Boer-Mulders function could be tested in the unpolarized πp and and πD D-Y process.

Boer-Mulders functions of sea quarks in the nucleon (Lu,Ma,Schmidt)

- small effect, but calculable
- Using meson-baryon fluctuation model – a model for the nucleon sea (Brodsky,Ma)

$$q^{in}(x) \quad = \quad \int_x^1 \frac{dy}{y} f_{M/BM}(y) q_M\left(\frac{x}{y}\right), \qquad (4)$$

a convolution form can be applied to calculate $h_1^{\perp,\bar{u}}$ and $h_1^{\perp,\bar{d}}$, as follows:

$$h_1^{\perp,\bar{u}}(x,\mathbf{k}_\perp^2) = \int_x^1 \frac{dy}{y} f_{\pi^-/\Delta^{++}\pi^-}(y) h_{1\pi^-}^{\perp,\bar{u}}\left(\frac{x}{y},\mathbf{k}_\perp^2\right), \qquad (5)$$

$$h_1^{\perp,\bar{d}}(x,\mathbf{k}_\perp^2) = \int_x^1 \frac{dy}{y} f_{\pi^+/n\pi^+}(y) h_{1\pi^+}^{\perp,\bar{d}}\left(\frac{x}{y},\mathbf{k}_\perp^2\right). \qquad (6)$$

Input: $h_{1\pi}^\perp$

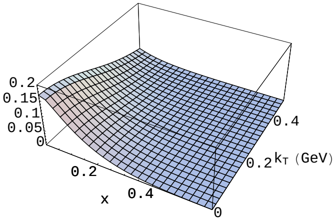


Figure: The Boer-Mulders functions of $\bar{u}$ quark $h_1^{\perp,\bar{u}}(x,\mathbf{k}_\perp^2)$ (left column) and $\bar{d}$ quark $h_1^{\perp,\bar{d}}(x,\mathbf{k}_\perp^2)$ (right column) inside the proton as two-dimensional densities.

- same sign
- size is determined by the probability to find the $\Delta^{++}\pi^-$ and $n\pi^+$ configurations inside the proton

$h^{\perp,\bar{u}}$ and $h^{\perp,\bar{d}}$ has been applied to calculate the $\cos 2\phi$ asymmetry ν in the unpolarized pp and pD Drell-Yan process.

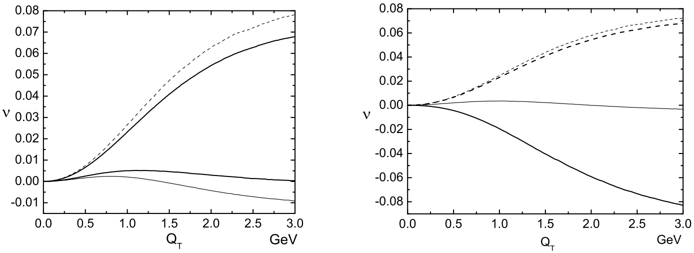


Figure: The $\cos 2\phi$ asymmetries ν_p (thick lines) and ν_D (thin lines) for the E866 experiment, as functions of Q_T, Leading order differential cross-section for the double transversely polarized Drell-Yan process (Boer):

$$\frac{d\sigma^{(2)}(h_1^\uparrow h_2^\uparrow \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha_{em}^2}{3Q^2} \sum_q \Bigg\{ \cdots$$

$$+\frac{A_1(y)}{2} |\mathbf{S}_{1T}| \, |\mathbf{S}_{2T}| \, \cos(2\phi - \phi_{S_1} - \phi_{S_2})$$

$$\times \mathcal{F}\left[\hat{\mathbf{h}}\cdot\mathbf{p}_T \; \hat{\mathbf{h}}\cdot\mathbf{k}_T \, \frac{f_{1T}^{\perp q} f_{1T}^{\perp \bar{q}} - g_{1T}^q g_{1T}^{\bar{q}}}{M_1 M_2} \right] \frac{A_1(y)}{2} |\mathbf{S}_{1T}| \, |\mathbf{S}_{2T}|$$

$$\times \cos(\phi - \phi_{S_1}) \, \cos(\phi - \phi_{S_2}) \, \mathcal{F}\left[\mathbf{p}_T\cdot\mathbf{k}_T \, \frac{f_{1T}^{\perp q} f_{1T}^{\perp \bar{q}}}{M_1 M_2} \right]$$

$$-\frac{A_1(y)}{2} |\mathbf{S}_{1T}| \, |\mathbf{S}_{2T}| \, \sin(\phi - \phi_{S_1}) \, \sin(\phi - \phi_{S_2}) \, \mathcal{F}\left[\mathbf{p}_T\cdot\mathbf{k}_T \, \frac{g_{1T}^q g_{1T}^{\bar{q}}}{M_1 M_2} \right]$$

Weighted asymmetry

one integrates the differential cross section with a proper weighting function $W(Q_T, \phi, \phi_{S_1}, \phi_{S_2})$:

$$\begin{aligned} &\langle W(Q_T, \phi, \phi_{S_1}, \phi_{S_2})\rangle \\ = &\int d\phi d\phi_{S_1} d\mathbf{q}_T^2 \frac{d\sigma(h_1 h_2 \to l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} \\ &\times W(Q_T, \phi, \phi_{S_1}, \phi_{S_2}). \end{aligned} \tag{7}$$

DSA contributed by Sivers function:

$$\left(\frac{A(y)\alpha_{em}^2}{3Q^2}\right)^{-1} \cdot \left\langle \frac{Q_T^2}{M_p^2}\,(W_C + 3W_S)\right\rangle_{TT} = -8\pi^2 \sum_q e_q^2 f_{1T}^{\perp(1)q}(x_1) f_{1T}^{\perp(1)\bar{q}}(x_2), \quad (8)$$

W_C and W_S defined as:

$$W_C = \cos(\phi - \phi_{S_1})\cos(\phi - \phi_{S_2}), \quad (9)$$
$$W_S = \sin(\phi - \phi_{S_1})\sin(\phi - \phi_{S_2}). \quad (10)$$

DSA contributed by Sivers function:

$$A^f_{TT} = \frac{\left\langle \frac{Q_T^2}{M^2}(W_C + 3W_S) \right\rangle_{TT}}{\langle 1 \rangle_{UU}} = -\frac{2\sum_q e_q^2 f_{1T}^{\perp(1)q}(x_1) f_{1T}^{\perp(1)\bar{q}}(x_2)}{\sum_q e_q^2 f_1^q(x_1) f_1^{\bar{q}}(x_2)}.$$

available parametrizations for Sivers functions

- Anselmino et. al.

$$-\frac{2|\mathbf{p}_T|}{M_N}\, f_{1T}^{\perp q}(x,\mathbf{p}_T^2) = 2\, N_q(x)\, f_{q/p}(x)\, g(\mathbf{p}_T^2)\, h(\mathbf{p}_T^2)\,,$$

- Collins et. al.

$$f_{1T}^{\perp u}(x,\mathbf{p}_T^2) = -f_{1T}^{\perp d}(x,\mathbf{p}_T^2) = f_{1T}^{\perp(1)}(x)e^{-\mathbf{p}_T^2/\langle\mathbf{p}_T^2\rangle}$$

$$xf_{1T}^{\perp(1)u}(x) = Ax^b(1-x)^5$$

- Vogelsang-Yuan

$$q_T^{(1/2)}(x) \equiv \int d^2\mathbf{p_T}\frac{|\mathbf{p_T}|}{M_N} f_{1T}^{\perp q}(x,\mathbf{p}_T^2)\ . \qquad (11)$$

$$\frac{u_T^{(1/2)}(x)}{u(x)} = S_u x(1-x)\ , \qquad \frac{d_T^{(1/2)}(x)}{u(x)} = S_d x(1-x)\ ,$$

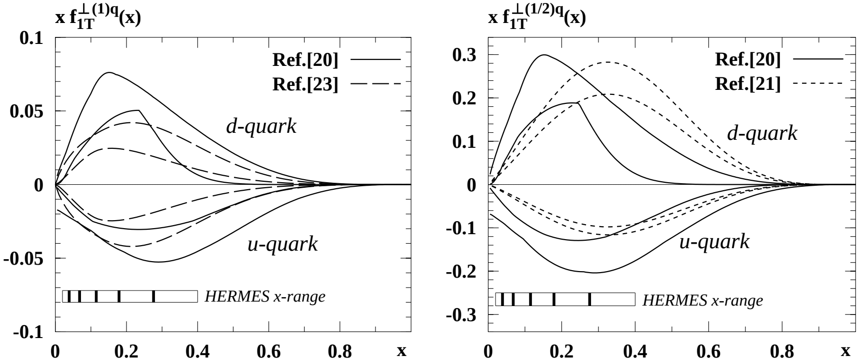
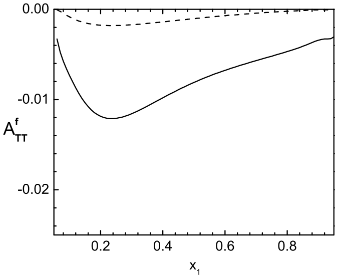


Figure: The first and 1/2-transverse moments of the Sivers quark distribution functions.

DSA contributed by Sivers functions in $p^{\uparrow}\bar{p}^{\uparrow} \rightarrow l^{+}l^{-}X$ process at GSI ($\sqrt{s} = 45$ GeV, $Q^2 = 2.5$ GeV):



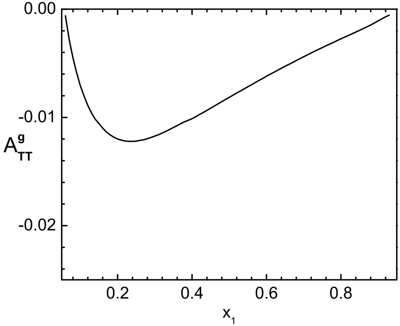
- solid line: from Sivers functions fitted by Anselmino et.al.
- dashed line: from Sivers functions fitted by Collins et.al.
- one percent asymmetry is found. Provide alternative constraint on Sivers functions

- DSA contributed by $g_{1T}(x, \mathbf{p}_T^2)$

$$A_{TT}^{g} = \frac{\left\langle \frac{Q_T^2}{M^2}(3W_C + W_S) \right\rangle_{TT}}{\langle 1 \rangle_{UU}} = -2\frac{\sum_q e_q^2 g_{1T}^{(1)q}(x_1) g_{1T}^{(1)\bar{q}}(x_2)}{\sum_q e_q^2 f_1^q(x_1) f_1^{\bar{q}}(x_2)},$$

- $g_{1T}(x, \mathbf{p}_T^2)$ also contributes in double LT spin asymmetry in SIDIS (Kotzinian, Mulders & Kotzinian, Parsamyan, Prokudin)

DSA contributed by $g_{1T}(x, \mathbf{p}_T^2)$ in $p^\uparrow \bar{p}^\uparrow \to l^+ l^- X$ process at GSI ($\sqrt{s} = 45$ GeV, $Q^2 = 2.5$ GeV):



$g_{1T}(x, \mathbf{p}_T^2)$ from the combination of Lorentz invariance relation and the Wandura-Wilzeck approximation(Kotzinian, Parsamyan, Prokudin)

Summary

- Measure the $\cos 2\phi$ asymmetry in the unpolarized $\pi^- p$ and $\pi^- D$ Drell-Yan processes to test the flavor dependence of Boer-Mulders function in the valence region.(COMPASS with pion beam?)
- First calculation on the sea quarks Boer-Mulders functions.
- Measurements of the DSA in the proton-antiproton Drell-Yan process can provide new information on the Sivers function, especially its size. (Also for g_{1T})