Transverse spin densities of the Nucleon in a light-front Constituent Quark Model

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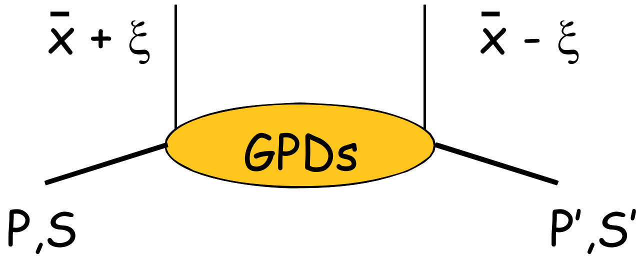
Outline

- ❖ GPDs in the light-cone wave function overlap representation
- ❖ Light-cone model for the valence quark contribution to nucleon wave function
- ❖ Spin densities for transverse and longitudinal polarization

Generalized Parton Distributions



$$\Phi^{[\Gamma]} = \langle P', S' \,|\int \frac{\mathrm{d}z^-}{2\pi} \bar{q}(-\frac{z}{2})\ \Gamma\ q(\frac{z}{2})\, e^{ixP^+z^-} |\, P, S\rangle$$

$$\gamma^+,\ \gamma^+\gamma_5,\ i\sigma^{i+}\gamma_5$$

- $P \neq P' \Rightarrow$ GPDs depend on two momentum fractions $\bar{x}$, ξ, and t

$$\bar{x} = \frac{(k+k')^+}{(P+P')^+} = \frac{\bar{k}^+}{\bar{P}^+}$$

average fraction of the longitudinal momentum carried by partons

$$\xi = \frac{(P-P')^+}{(P+P')^+} = -\frac{\Delta^+}{2\bar{P}^+}$$

skewness parameter

$$t = (P-P')^2$$

t-channel momentum transfer squared

Forward Limit

- Unpolarized GPDs $\Phi^{[\gamma^+]} \Longrightarrow H(\bar{x},\xi,t),\ E(\bar{x},\xi,t)$ — $H^q(x,0,0) = q(x)$
- Longitudinally Pol. GPDs $\Phi^{[\gamma^+\gamma_5]} \Longrightarrow \tilde{H}(\bar{x},\xi,t),\ \tilde{E}(\bar{x},\xi,t)$ — $\tilde{H}^q(x,0,0) = \Delta q(x)$
- Transversely Pol. GPDs $\Phi^{[i\sigma^{i+}\gamma_5]} \Longrightarrow H_T(\bar{x},\xi,t),\ E_T(\bar{x},\xi,t),\ \tilde{H}_T(\bar{x},\xi,t),\ \tilde{E}_T(\bar{x},\xi,t)$ — $H_T^q(x,0,0) = h_1^q(x)$

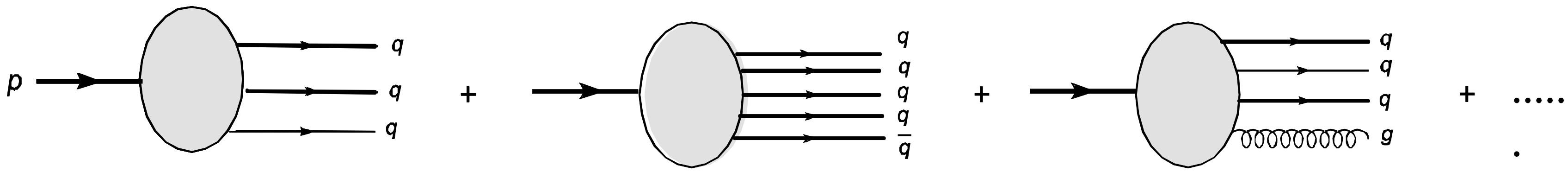
Light-Cone Fock Expansion

❖ complete relativistic many-particle basis constructed from the ground state

$$|N, \beta; k_1, k_2, \cdots, k_N\rangle = \Pi_i b^\dagger_{q_i}(w_i)\, \Pi_j d^\dagger_{q_j}(w_j)\, \Pi_l a^\dagger(w_l)\, |0\rangle$$

❖ Fock-space expansion of the nucleon bound state

$$|\Psi\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3qg}|qqqg\rangle + \cdots$$



$$|(P^+, \vec{P}_\perp), \lambda\rangle = \sum_{N,\beta} \int [dx]_N\, [d\vec{k}_\perp]_N\, \Psi^f_{\lambda,N,\beta}(x_i, \vec{k}_{\perp,i})\, |N, \beta; x_i P^+, x_i\vec{P}_\perp + \vec{k}_{\perp,i}, \lambda_i\rangle$$

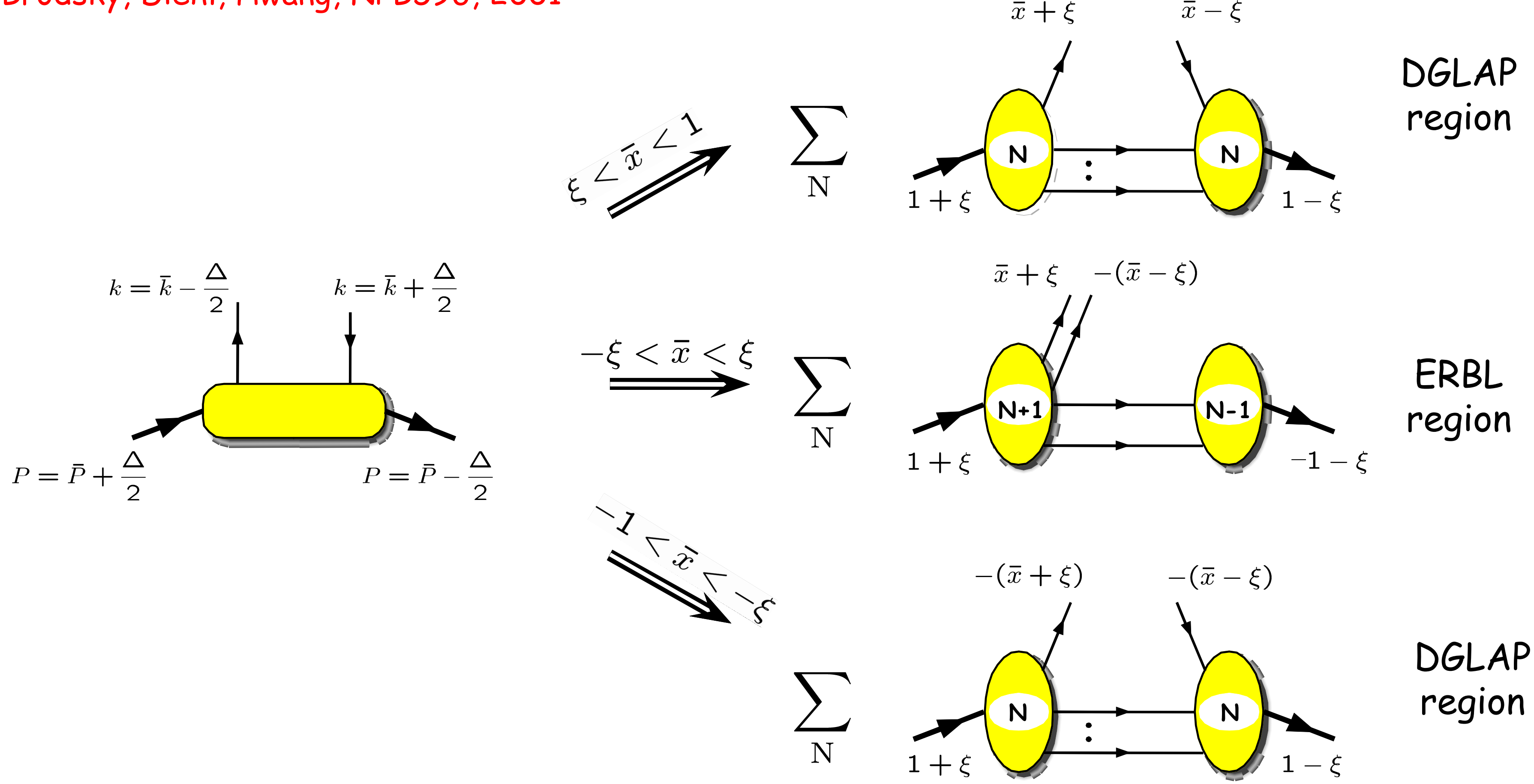
➢ internal variables: $x_i = \frac{p_i^+}{P^+}$ $\sum_{i=1}^{N} x_i = 1$ $\sum_{i=1}^{N} \vec{k}_{\perp\, i} = \vec{0}_\perp$

➢ $\Psi^f_{\lambda,N,\beta}(x_i, \vec{k}_{\perp,i})$: frame INdependent

➡ probability amplitude to find the N parton configuration with the complex of quantum number β in the nucleon with helicity λ

Light-Cone Wave Function Overlap Representation

Diehl, Feldmann, Jakob, Kroll, NPB596, 2001
Brodsky, Diehl, Hwang, NPB596, 2001

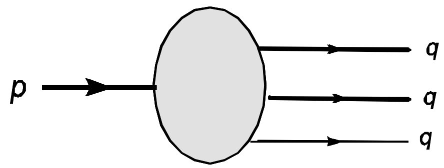


GPDs ~ $\Sigma_N \int dk_N\, \Psi_N(k_N)\, \Psi_N^*(k'_N)\, \delta(\ldots)$ interference of probability amplitudes

PDF ~ $\Sigma_N \int dk_N\, |\Psi_N(k_N)|^2\, \delta(\ldots)$ probability density

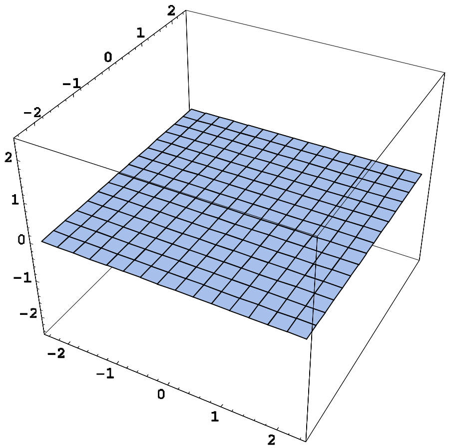
N=3 VALENCE QUARK ⇒ Light-cone Constituent quark model

$$| N\rangle \quad \simeq \quad \Psi^{N}_{(3q)} | N(qqq)\rangle + \cdots$$

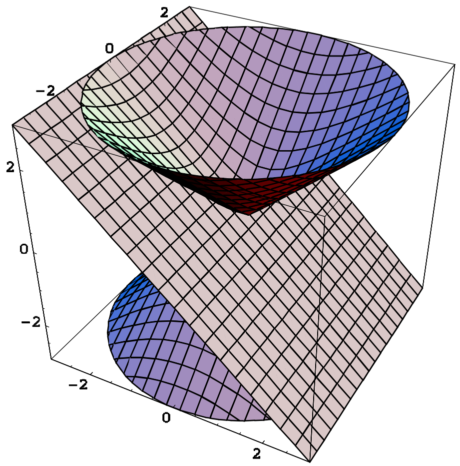


Valence quarks: Light-Cone CQM

Instant form:
x^0 time;
x^1, x^2, x^3 space



Light-front form:
x^+ time;
x^-, $x_\perp$ space



➢ Light-front eigenvalue equation

$$\mathcal{M}|\, M, j_f, \mu_f\rangle_f = [M_0 + \mathcal{V}]|\, M, j_f, \mu_f\rangle_f$$

$M_0 = \sum_i^N \sqrt{m_i^2 + \vec{k}_i^2}$: free mass operator $\mathcal{V}$: interaction operator

➢ Instant Form (canonical) eigenvalue equation

$$M|\, M, j_c, \mu_c\rangle_c = [M_0 + V]|\, M, j_c, \mu_c\rangle_c$$

$\mathcal{M} = \mathcal{R}^\dagger\, M\, \mathcal{R}$ ➡ $\mathcal{R} = \Pi_{i=1}^N\, R_M(\vec{k}_{\perp,i}, x_i, m_i)$: generalized Melosh rotations

⬇

$$\Psi_N^f = \langle \{x_i, \vec{k}_{\perp,i}, \lambda_i\}_N\, |M, j_f, \mu_f\rangle_f = \frac{2(2\pi)^3}{\sqrt{M_0}} \Pi_{i=1}^N \sqrt{\frac{E_i}{x_i}} \sum_{\{\lambda'_i\}} \langle \{\lambda_i\}\, |\mathcal{R}^\dagger|\, \{\lambda'_i\}\rangle \Psi_N^c$$

Boffi, B.P., Traini, '03

Model Calculation

❖ Instant-form wave function: $\Psi = \Phi^I \otimes \Phi^S \otimes \tilde{\Psi}(\{\vec{k}_i\})$

✓ spin and isospin component: SU(6) symmetric

✓ momentum-space component: s wave

$$\tilde{\Psi} = \frac{N}{(M_0^2 + \beta^2)^\gamma} \text{ with } M_0 = \sum_i \sqrt{k_i^2 + m_q^2}$$

Three free parameters: m_q, β, γ fitted to reproduce the magnetic moment of the proton and the axial coupling constant g_A

m_q = 263 MeV β = 607 MeV γ = 3.5

Schlumpf, PhD thesis, hep-ph/9211255

❖ Light-cone wavefunction

✓ breaking of SU(6) symmetry

✓ non-zero quark orbital angular momentum

Melosh Rotations

$$q_{LC}^{\uparrow} = w\,[(k^+ + m_q)\,q_I^{\uparrow} + (k_x + ik_y)\,q_I^{\downarrow}]$$

$$q_{LC}^{\downarrow} = w\,[-(k_x - ik_y)\,q_I^{\uparrow} + (k^+ + m_q)q_I^{\downarrow}]$$

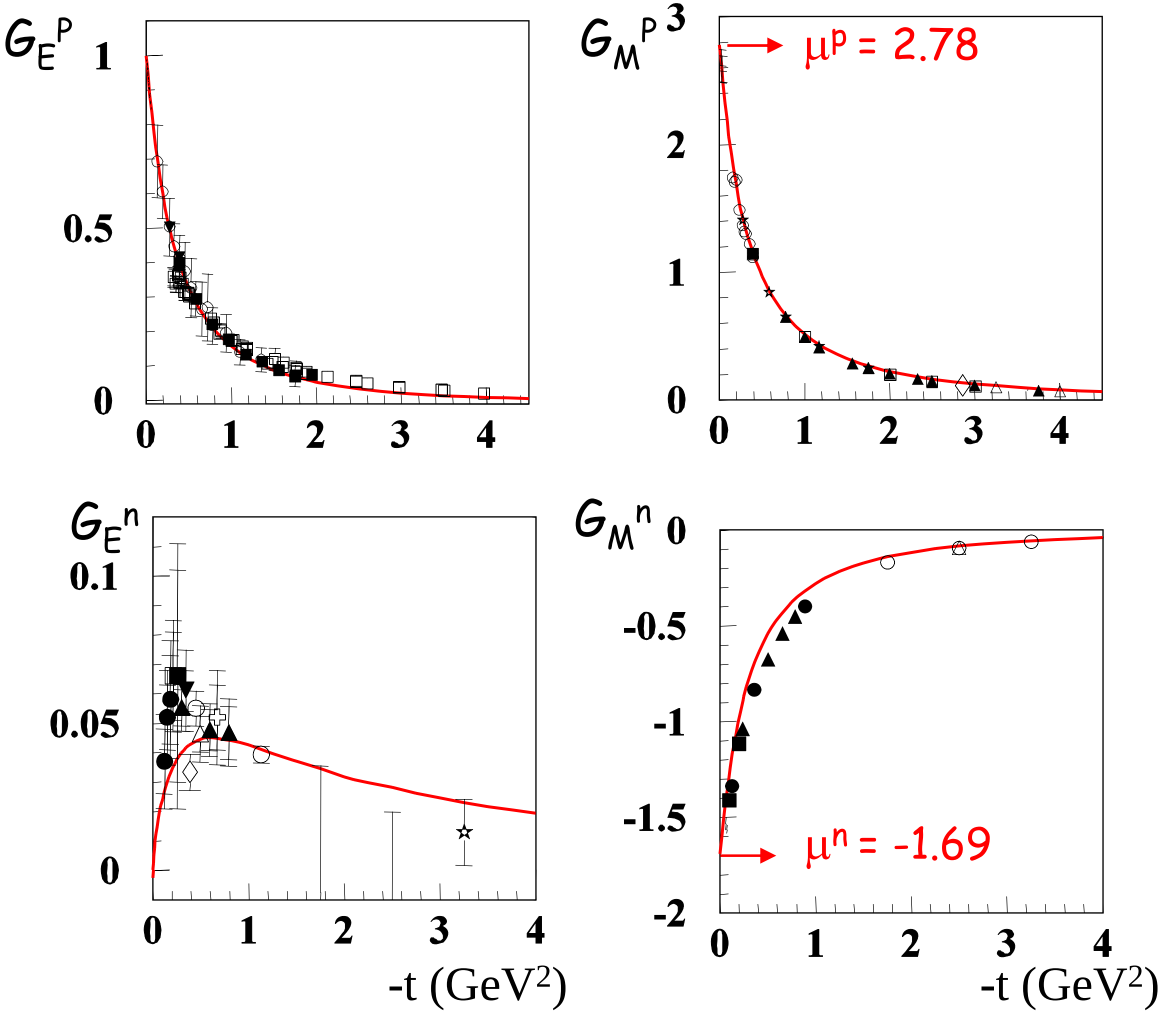
$$(w = [(k^+ + m_q)^2 + k_\perp^2]^{-1/2})$$

The boost to infinite momentum frame (Melosh Rotations) introduces a non trivial spin structure and a correlation between quark spin and quark orbital angular momentum

Nucleon Electromagnetic Form Factors

$$\int \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t) \qquad \int \mathrm{d}x\, E^q(x,\xi,t) = F_2^q(t)$$

$$G_E(t) = F_1(t) + \frac{t}{4M^2} F_2(t) \qquad G_M(t) = F_1(t) + F_2(t)$$



Exp. data
from the compilation of Friedrich and Walcher, EPJA17, 2005

Light cone wave function overlap representation of Parton Distributions

$$f_1^q(x) = \left(2\delta_{\tau_q 1/2} + \delta_{\tau_q -1/2}\right)\int [dx]_3[d\vec{k}_\perp]_3\,\delta(x-x_3)|\tilde{\psi}_\uparrow(\{x_i\},\{\vec{k}_{\perp,i}\}|^2$$

$$h_1^q(x) = \left(\frac{4}{3}\delta_{\tau_q 1/2} - \frac{1}{3}\delta_{\tau_q -1/2}\right)\int [dx]_3[d\vec{k}_\perp]_3\,\delta(x-x_3)|\tilde{\psi}_\uparrow(\{x_i\},\{\vec{k}_{\perp,i}\}|^2\mathcal{M}_T$$

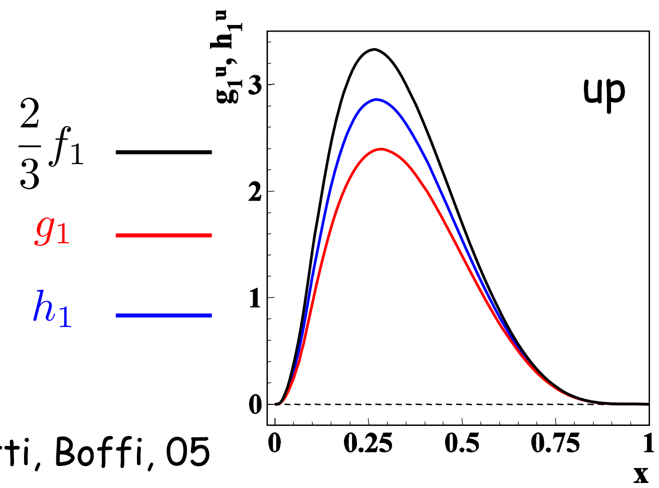
$$g_1^q(x) = \left(\frac{4}{3}\delta_{\tau_q 1/2} - \frac{1}{3}\delta_{\tau_q -1/2}\right)\int [dx]_3[d\vec{k}_\perp]_3\,\delta(x-x_3)|\tilde{\psi}_\uparrow(\{x_i\},\{\vec{k}_{\perp,i}\}|^2\mathcal{M}$$

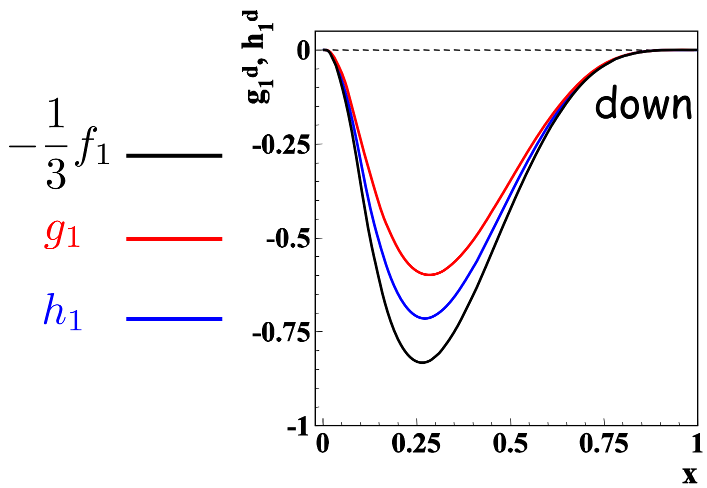
- Melosh rotations: relativistic effects due to the quark transverse motion

$$\mathcal{M}_T = \frac{(m+x_3M_0)^2}{(m+x_3M_0)^2+\vec{k}^2_{\perp,3}} \qquad \mathcal{M} = \frac{(m+x_3M_0)^2-\vec{k}^2_{\perp,3}}{(m+x_3M_0)^2+\vec{k}^2_{\perp,3}}$$

$$2h_1^u(x) = g_1^u(x) + \frac{2}{3}f_1^u(x) \qquad 2h_1^d(x) = g_1^d(x) - \frac{1}{3}f_1^d(x)$$

- Non relativistic limit ($k_\perp \to 0$) : $\mathcal{M}_T = \mathcal{M} = I \rightarrow h_1^q(x) = g_1^q(x)$





B.P., Pincetti, Boffi, 05

Nucleon Spin densities

❖ Fourier transform of Unpolarized GPD at ξ = 0: distribution of quarks with longitudinal momentum fraction x and transverse location $b_\perp$ from the nucleon centre of momentum

$$\rho(x, b_\perp) = \int d^2\Delta_\perp \, e^{i\Delta_\perp \cdot b_\perp} \, H(x, \xi = 0, \Delta_\perp^2) = H(x, b_\perp^2)$$

Burkardt, 2003

❖ Fourier transform of Tensor GPDs at ξ = 0: distributions in the transverse plane of transversely polarized quarks in a transversely polarized nucleon

Diehl, Haegler, 2005

transversity basis

$$|\, P,\, S_\perp = (1,0)\,\rangle = \frac{1}{\sqrt{2}}[|\, P,\, +\rangle + |\, P,\, -\rangle]$$

projector on the transverse quark spin $s_\perp$

$$\frac{1}{2}\bar{q}\,[\gamma^+ \,-\, s^j\, i\,\sigma^{+j}\gamma_5]\, q$$

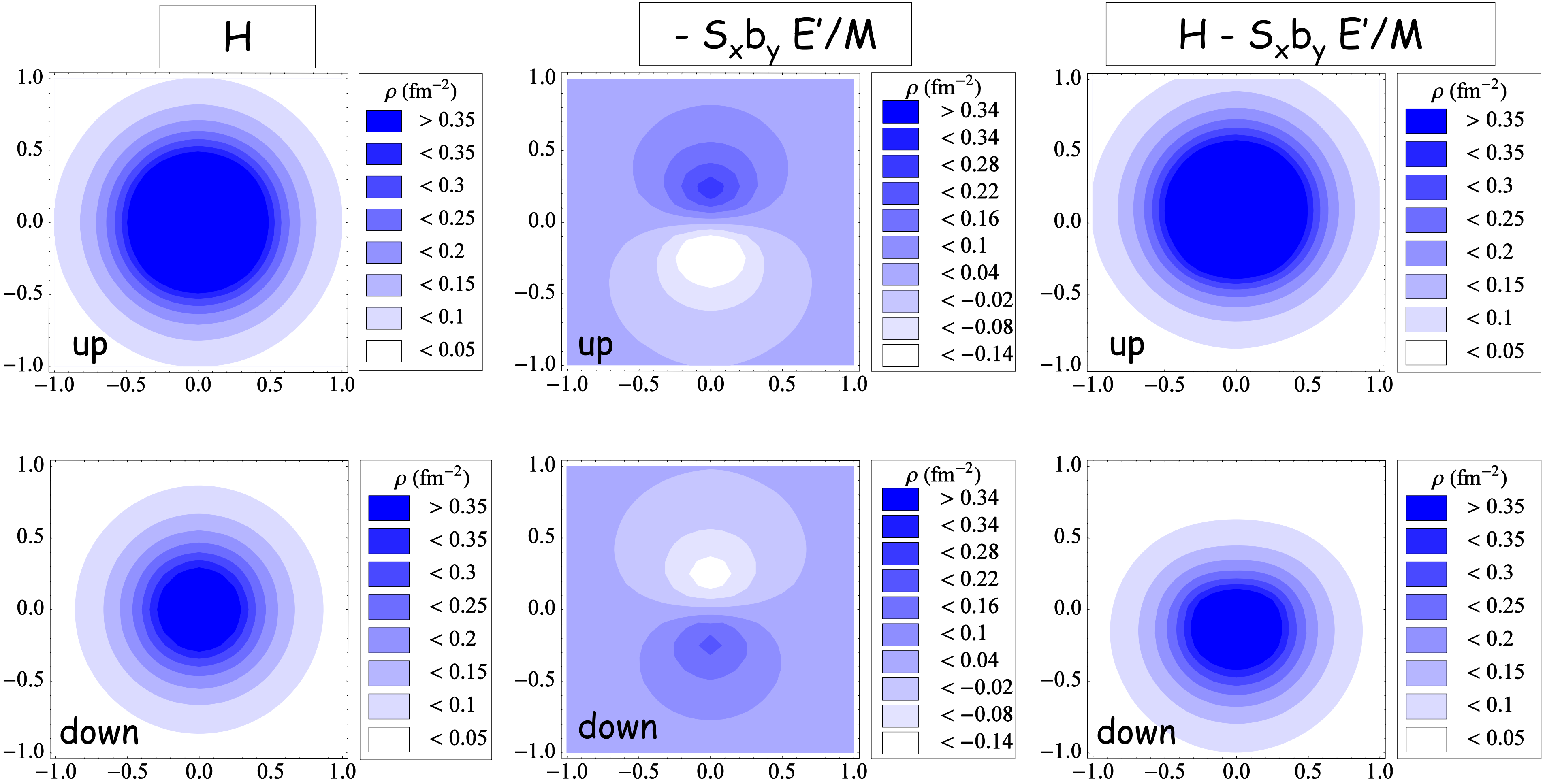
$$\rho(x, b_\perp, s_\perp, S_\perp) = \frac{1}{2}\Big[H(x, b_\perp^2) \,+\, s^i\, S^i \left(H_T(x, b_\perp^2) - \frac{1}{M^2}\Delta_b \tilde{H}_T(x, b_\perp^2)\right) \quad \text{monopole}$$

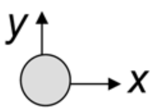
$$+ \frac{b^j \epsilon^{ji}}{M}\left(S^i\, E'(x, b_\perp^2) \,+\, s^i\, \left(E'_T(x, b_\perp^2) + 2\tilde{H}'_T(x, b_\perp^2)\right)\right) \quad \text{dipole}$$

$$+ s^i\,(2b^i b^j - b^2\delta_{ij})\, S^j\, \frac{1}{M^2}\, \tilde{H}''_T(x, b_\perp^2)\Big] \quad \text{quadrupole}$$

❖ First moments of ρ: transverse spin probability densities in impact parameter space

Unpolarized quarks in a transversely pol. nucleon





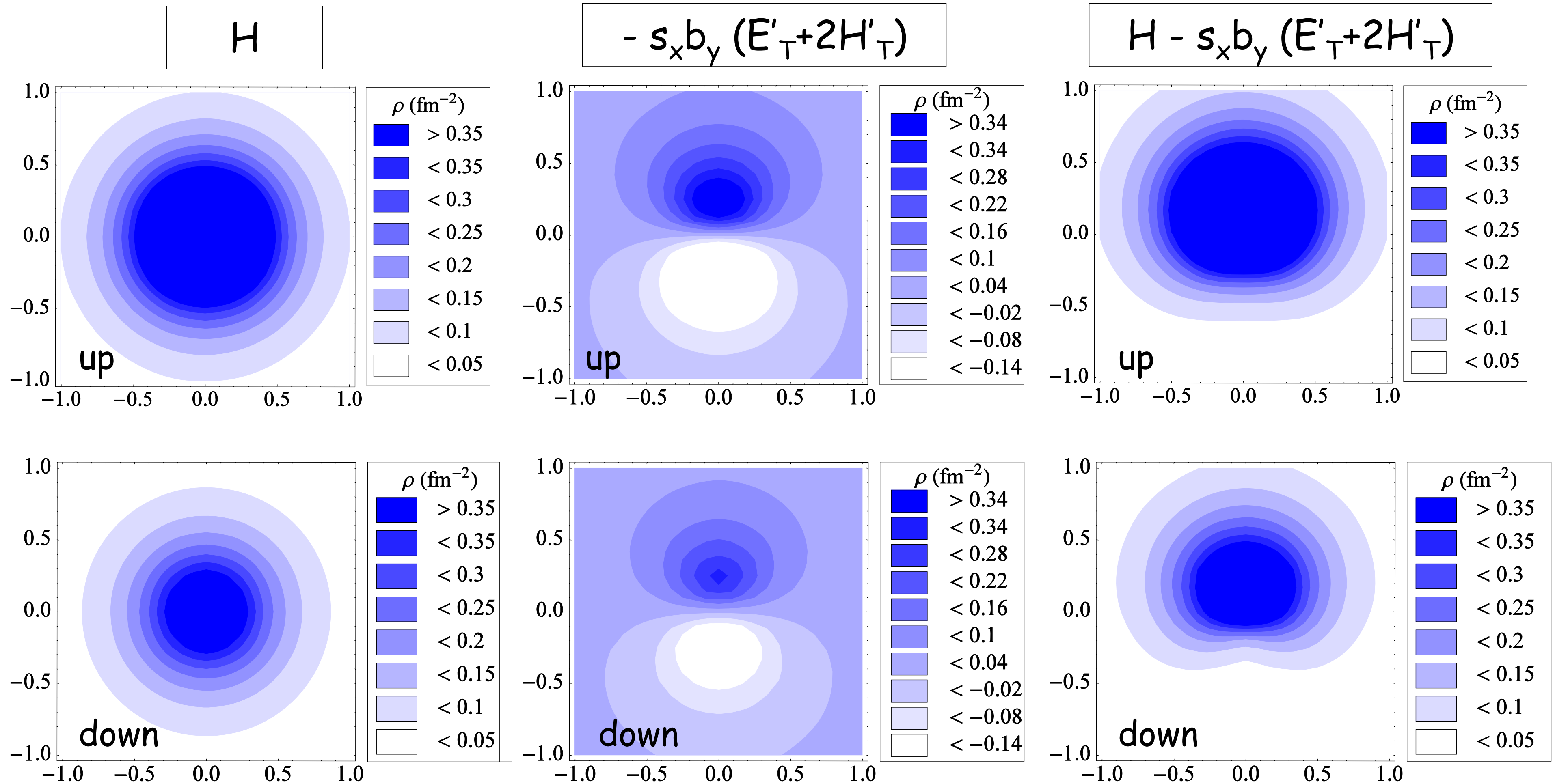
⊥ flavor dipole moment $d_y^q = \frac{\kappa^q}{2M}$

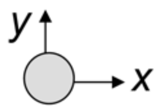
$$\kappa_u^p = 1.86,\ \kappa_d^p = -1.57 \quad \Rightarrow \quad |d_y^q| \sim 0.1 - 0.2\,\mathrm{fm}$$

opposite shift of up and down quark → opposite sign for Sivers function of up and down quark as confirmed by HERMES data (Phys. Rev. Lett. 94 (2005))

B.P., Boffi, hep-ph/0705.4345

Transversely pol. quarks in a unpolarized nucleon



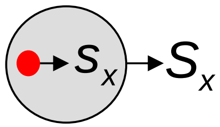


⊥ spin-flavor dipole moment: $\kappa_T^u = 3.98 \quad \kappa_T^d = 2.60$

⇓

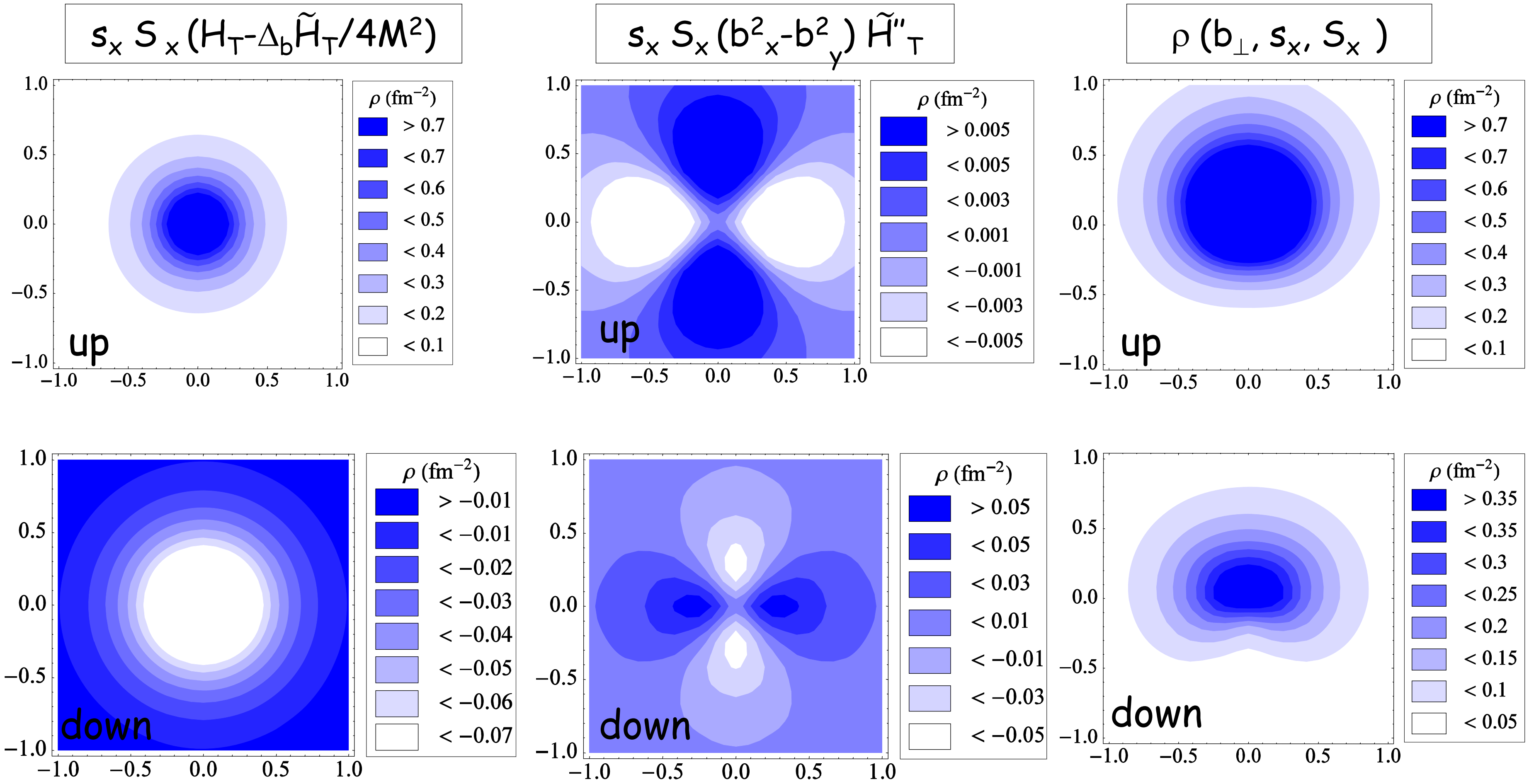
sizeable Boer-Mulders effect
consistent with lattice results (QCDSF/UKQCD Coll., hep-lat/0612032)

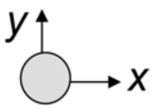
Transversely pol. quarks in a transversely pol. nucleon



$$\rho(b_\perp, s_x, S_x) = \int dx \, \frac{1}{2} \Big[H(x, b_\perp^2) + s_x S_x \Big(H_T(x, b_\perp^2) - \frac{1}{M^2} \Delta_b \tilde{H}_T(x, b_\perp^2) \Big)$$

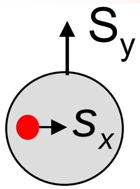
$$- \frac{b^y}{M} \Big(S_x E'(x, b_\perp^2) + s_x \big(E'_T(x, b_\perp^2) + 2\tilde{H}'_T(x, b_\perp^2) \big) \Big) + s_x (b_x^2 - b_y^2) S_x \frac{1}{M^2} \tilde{H}''_T(x, b_\perp^2) \Big]$$



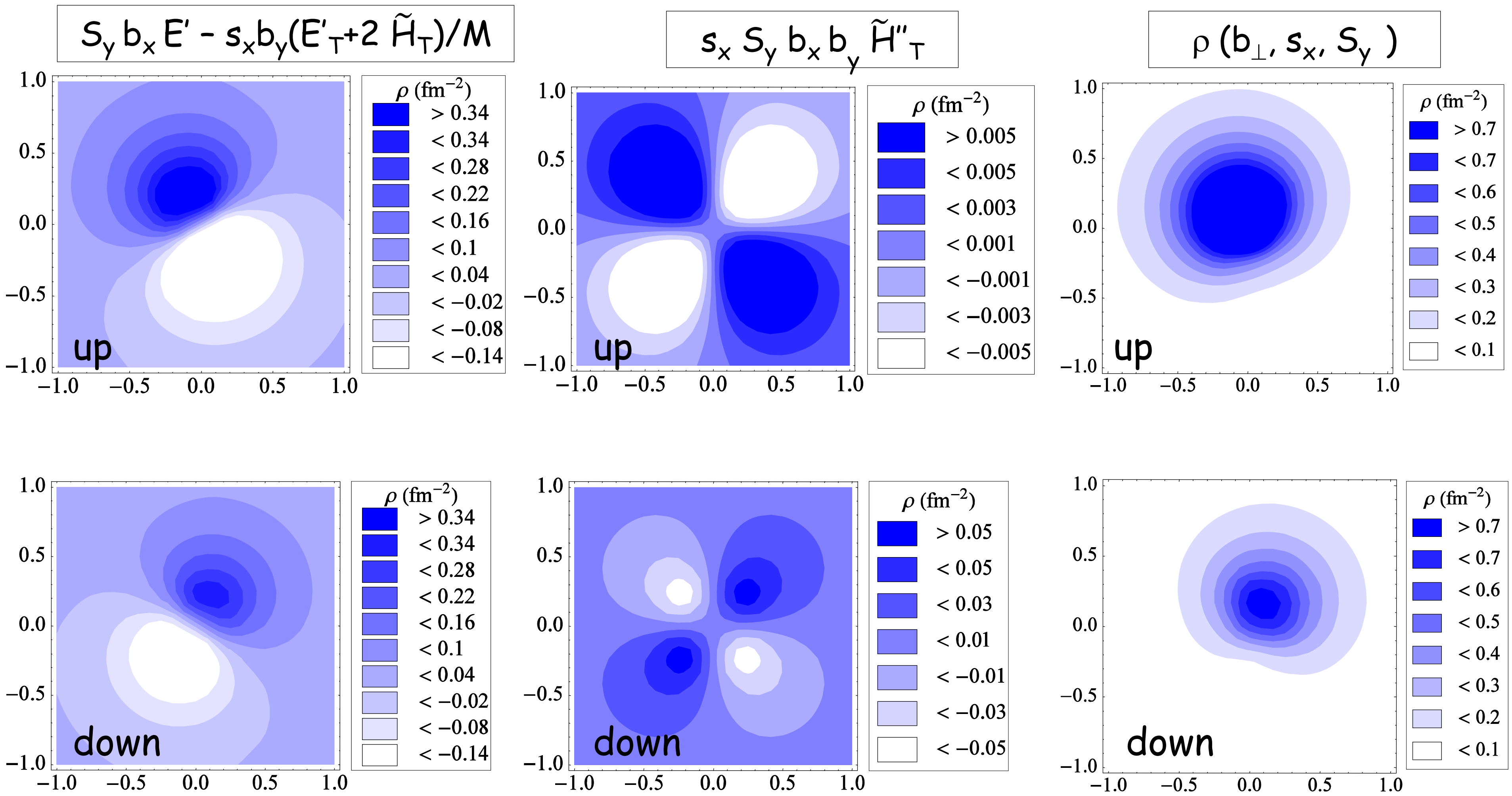


average quadrupole distortion { -0.04 up; 0.07 down

Transversely pol. quarks in a transversely pol. nucleon



$$\rho(b_\perp, s_x, S_y) = \int \mathrm{d}x \, \frac{1}{2} \Big[H(x, b_\perp^2) + \frac{1}{M} \Big(b_x \, S_y \, E'(x, b_\perp^2) - b_y \, s_x \, \big(E'_T(x, b_\perp^2) + 2\tilde{H}'_T(x, b_\perp^2) \big) \Big) + 2 b_x b_y \, s_x \, S_y \frac{1}{M^2} \tilde{H}''_T(x, b_\perp^2) \Big]$$



Charge density of partons in the transverse plane

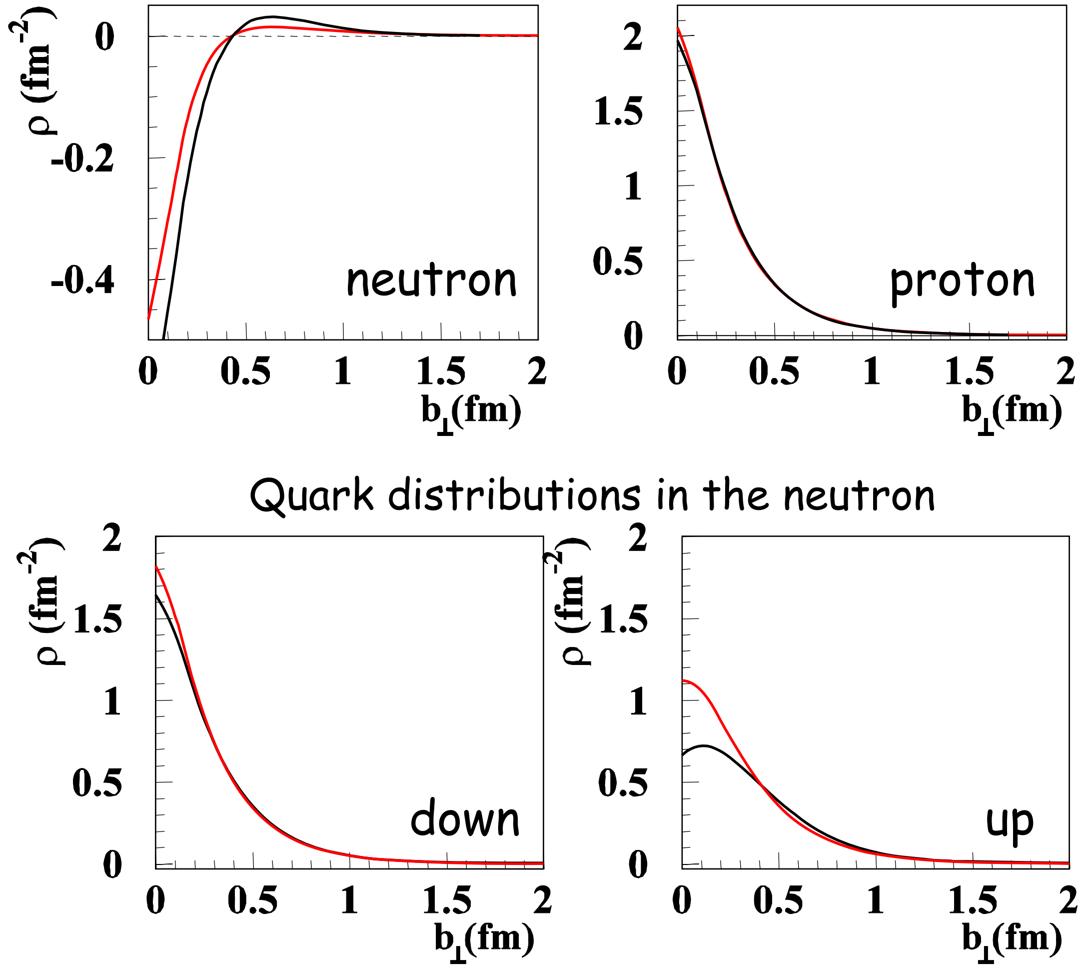
❖ Interpretation of two-dimensional Fourier Transform of F_1 as charge distribution in impact parameter space

$$\rho^q(b_\perp) = \int \mathrm{d}^2\Delta_\perp\, e^{i\Delta_\perp\cdot b_\perp} \int \mathrm{d}x\, H^q(x,\xi=0,\Delta_\perp^2) = \int \mathrm{d}^2\Delta_\perp\, e^{i\Delta_\perp\cdot b_\perp}\, F_1^q(\Delta_\perp^2)$$

G.A. Miller, nucl-th/0705.2409

fit to exp. form factor by Kelly, PRC70 (2004)

B.P., Boffi, hep-ph/0705.4345

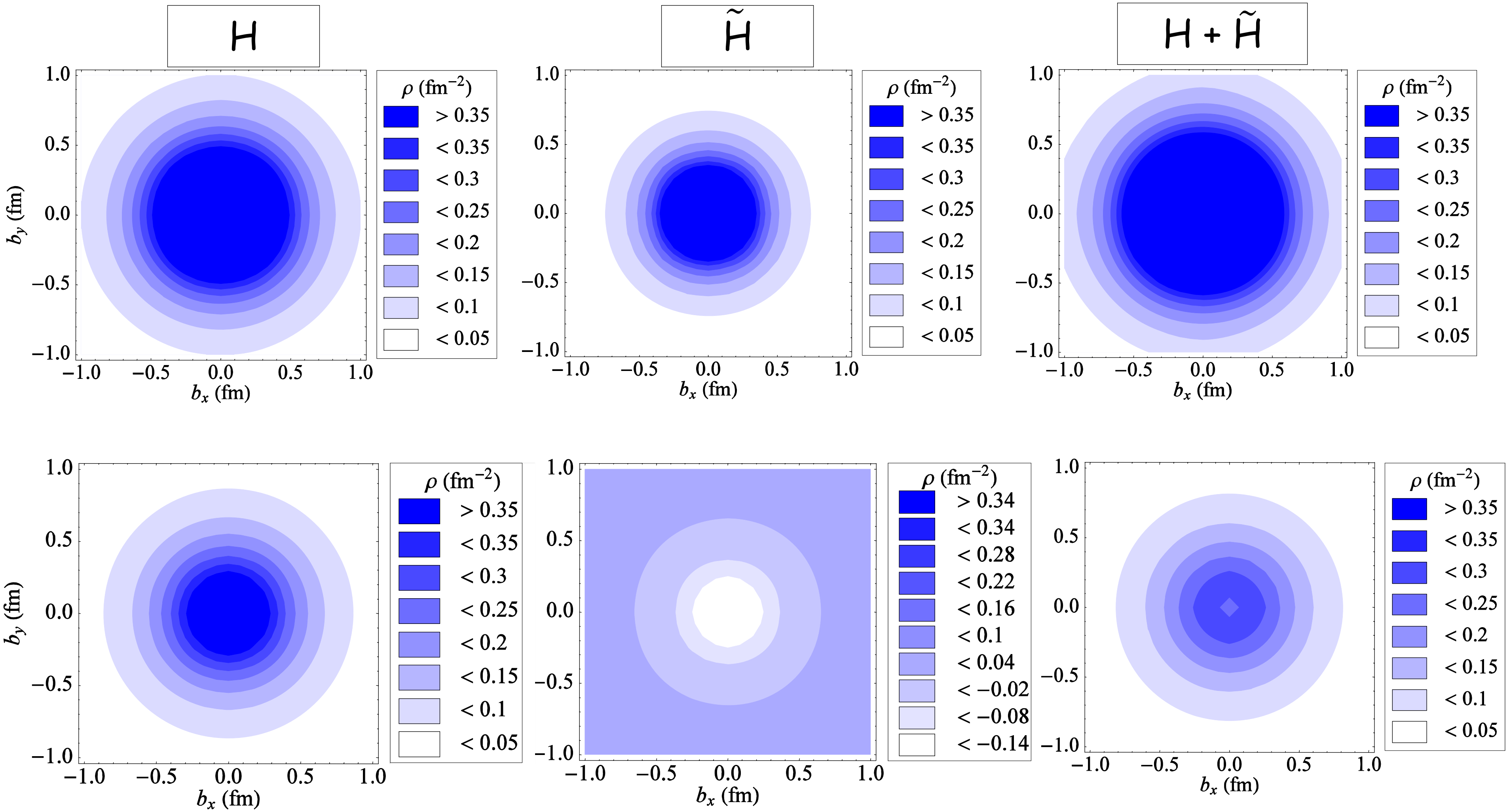


Longitudinally pol. quarks in a longitudinally pol. nucleon

➢ Probability density in impact parameter space of longitudinally polarized quarks in a longitudinally polarized nucleon

Burkardt, 2003

$$\int \mathrm{d}x\, \rho(x, b_\perp, \lambda, \Lambda) = \int \mathrm{d}x\, \frac{1}{2}\left[H(x,b^2) + \lambda\,\Lambda \tilde{H}(x,b^2)\right]$$

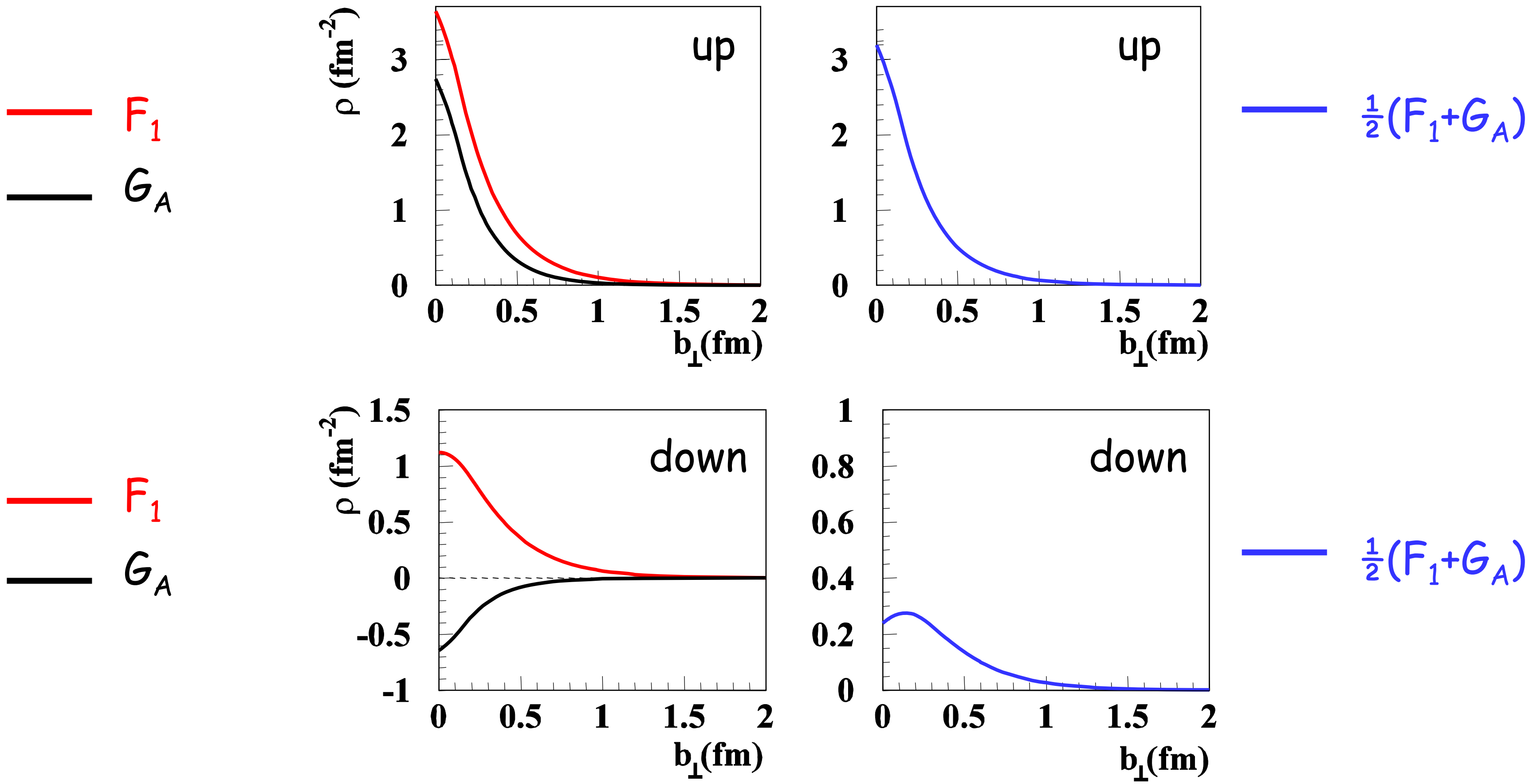


Helicity density of partons in the transverse plane

❖ Interpretation of two-dimensional Fourier Transform of $F_1 \pm G_A$ as helicity distribution in impact parameter space

$$\rho^q(b_\perp, \lambda, \Lambda) = \int d^2\Delta_\perp \, e^{i\Delta_\perp \cdot b_\perp} \int dx \, \frac{1}{2}[H^q(x, \xi = 0, \Delta_\perp^2) + \lambda\Lambda \tilde{H}(x, \xi = 0, \Delta_\perp^2)]$$

$$= \int d^2\Delta_\perp \, e^{i\Delta_\perp \cdot b_\perp} \, \frac{1}{2}[F_1^q(\Delta_\perp^2) + \lambda\,\Lambda\, G_A^q(\Delta_\perp^2)]$$



Summary

- Relativistic effects due to Melosh rotations in LCWF introduce a non trivial spin structure and correlations between quark spin and quark orbital angular momentum

⇓

relation for f_1, g_1 and h_1 for valence quark contribution at the hadronic scale

$$2h_1^u(x) = g_1^u(x) + \frac{2}{3}f_1^u(x) \qquad 2h_1^d(x) = g_1^d(x) - \frac{1}{3}f_1^d(x)$$

- Significant distortions for transverse spin densities

Unpolarized quarks in a transversely pol. nucleon
⇒ opposite sign for Sivers function of up and down quark as seen by HERMES

Transversely pol. quarks in an unpolarized nucleon
⇒ sizeable Boer-Mulder function effect with the same sign for up and down quark, as seen by lattice results

Transversely pol. quarks in a transversely pol. nucleon
⇒ correlation between quark orbital angular momentum and spin of the nucleon

- Charge and helicity densities of the nucleon show unexpected distributions for up and down quark distributions