SIDIS observables beyond leading twist

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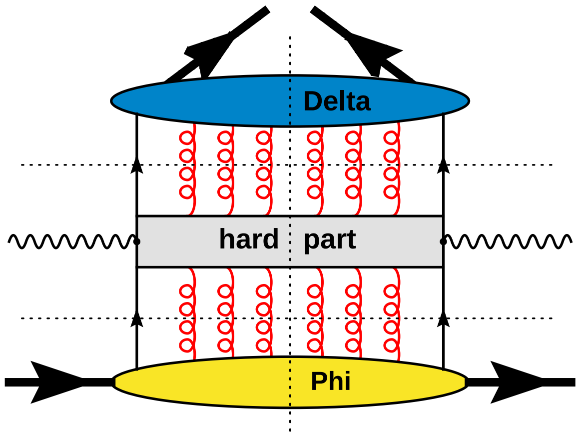
Partonic picture of semi-inclusive deep-inelastic scattering (SIDIS)



$$\sigma_{\text{SIDIS}} = \Phi \otimes (\text{hard}) \otimes \Delta$$

quark-quark correlators:

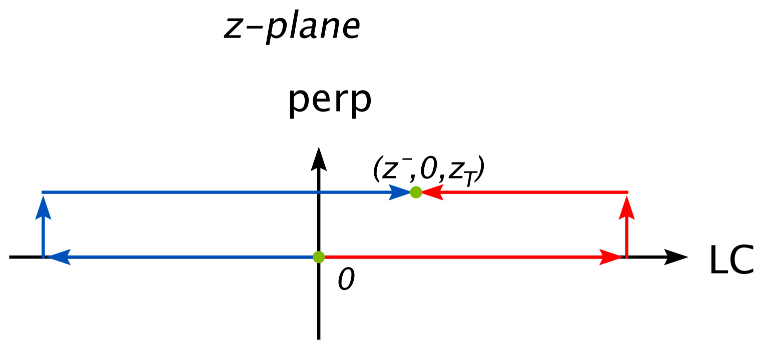
$$\Phi_{ij}(x,\vec{p}_T) = \int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3}\, \mathrm{e}^{ixP^+\xi^- - i\vec{p}_T\cdot\vec{\xi}_T} \langle P|\, \bar{\psi}_j(0)\, \mathcal{W}[0|\xi]\, \psi_i(\xi)\, |P\rangle\Big|_{\xi^+=0}$$

$$\Delta_{ij}(z,\vec{k}_T) = \mathcal{FT} \sum_X \langle 0|\mathcal{W}[\infty|\xi]\, \psi_i(\xi)\, |P_h;X\rangle\langle P_h;X|\, \bar{\psi}_j(0)\, \mathcal{W}[0|\infty]|0\rangle\Big|_{\xi^-=0}$$

- $\Phi \longrightarrow p_T$-dependent parton distributions.
- $\Delta \longrightarrow k_T$-dependent fragmentation functions.

- Color gauge invariance:

SIDIS, DY: p_T-dependence



Choice of the Wilson line: process dependent:

$$\mathcal{W}[0,z|\text{path}] = \mathscr{P} \exp\left\{-ig \int_0^z ds_\mu\, A^\mu(s)\right\}$$

Transverse Momentum Dependent (TMD) Parton Distributions

- Twist-2 TMD parton distributions, parameterization of matrix elements, $f = f(x, \vec{p}_T^2)$

$$\mathcal{FT}\left[\langle P,S|\bar{\psi}\gamma^+\mathcal{W}\psi|P,S\rangle\right] = f_1 - \frac{\epsilon_T^{ij}p_T^i S_T^j}{M}\underbrace{f_{1T}^\perp}_{\text{Sivers}}$$

$$\mathcal{FT}\left[\langle P,S|\bar{\psi}\gamma^+\gamma_5\mathcal{W}\psi|P,S\rangle\right] = \lambda g_{1L} + \frac{\vec{p}_T\cdot\vec{S}_T}{M}g_{1T}$$

$$\mathcal{FT}\left[\langle P,S|\bar{\psi}i\sigma^{i+}\gamma_5\mathcal{W}\psi|P,S\rangle\right] = \underbrace{S_T^j\left(\delta^{ij}h_{1T} + \frac{p_T^i p_T^j}{M^2}h_{1T}^\perp\right)}_{\to\text{transversity } h_1(x,\vec{p}_T^2)} + \lambda\frac{p_T^i}{M}h_{1L}^\perp + \frac{\epsilon_T^{ij}p_T^j}{M}\underbrace{h_1^\perp}_{\text{Boer-Mulders}}$$

- *Time-reversal odd (T-odd)* PDFs $f_{1T}^\perp$, $h_1^\perp$ $\longrightarrow$ Consequence of the gauge link.
- *Higher twist* parton distributions $\longrightarrow$ matrix elements with $\gamma_\perp^i$, $\gamma_\perp^i\gamma_5$, $\mathbb{1}$, γ_5, $i\sigma^{+-}\gamma_5$, $i\sigma^{ij}\gamma_5$, e.g.

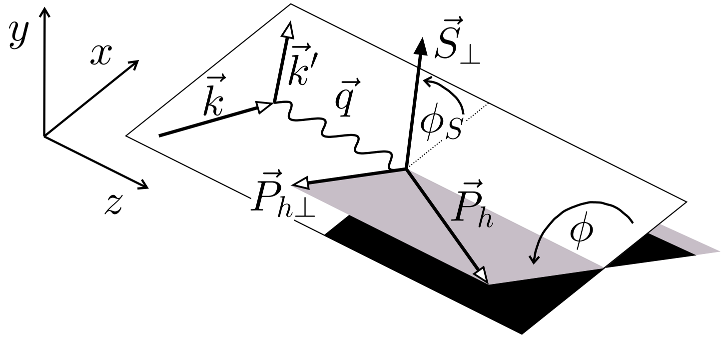
$$\mathcal{FT}\left[\langle P,S|\bar{\psi}\gamma_\perp^i\gamma_5\mathcal{W}\psi|P,S\rangle\right] = \frac{M}{P^+}\left(\underbrace{S_T^i\left(\delta^{ij}g_T' + \frac{p_T^i p_T^j}{M^2}g_T^\perp\right)}_{\to g_T(x,\vec{p}_T^2)} + \lambda\frac{p_T^i}{M}g_L^\perp + \frac{\epsilon_T^{ij}p_T^j}{M}g^\perp\right)$$

- 8 T-even, 8 T-odd twist-3 parton distributions.

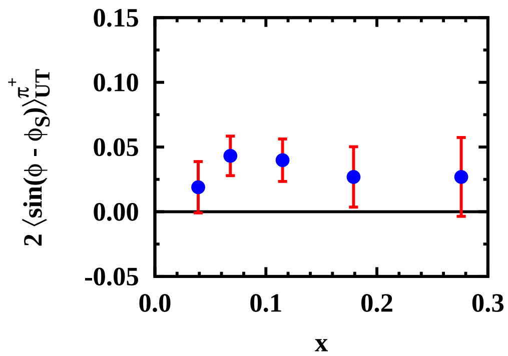
Single-Spin asymmetries $A = \frac{\sigma(\uparrow)-\sigma(\downarrow)}{\sigma(\uparrow)+\sigma(\downarrow)}$ in SIDIS

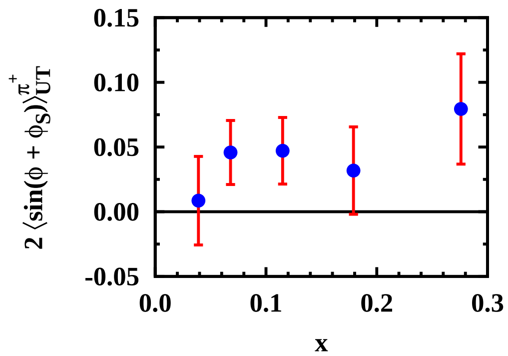
Twist-2 SSA for transversely pol. target:

$$\begin{aligned} A_{UT}^{\text{Sivers}} &\propto \sin(\phi_h - \phi_s)\, f_{1T}^{\perp} D_1 \\ A_{UT}^{\text{Collins}} &\propto \sin(\phi_h + \phi_s)\, h_1 H_1^{\perp}, \end{aligned}$$



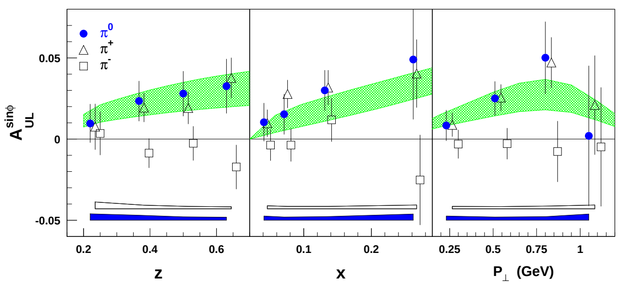
Experiments by HERMES (also by COMPASS):



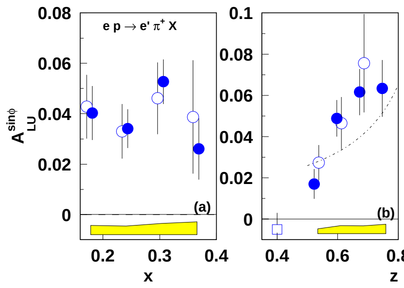


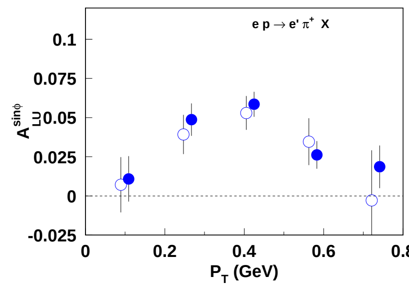
Twist-3 SSAs for longitudinally pol. target and beam:

$A_{UL}^{\sin\phi}$ measured by HERMES (2001):



$A_{LU}^{\sin\phi}$ measured by CLAS (2003):





$\Longrightarrow$ Twist-3 observables theoretically more complicated!

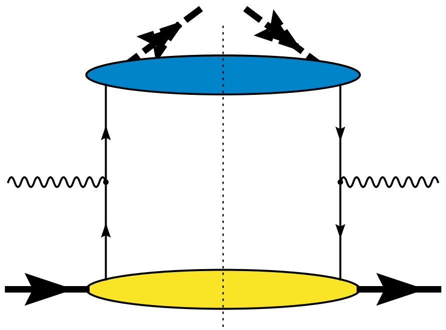
"Tree-level" formalism of twist-3 observables

(Mulders, Tangerman, 1996; Boer, Mulders, Pijlman, 2003)

How to describe subleading twist observables?

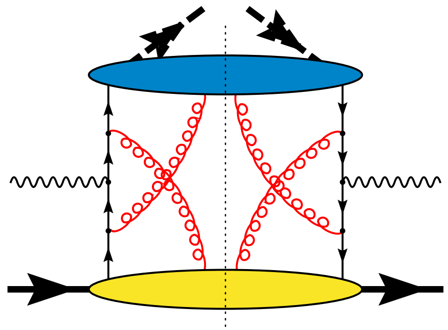
Contributions to twist-3 observables:

1. Twist-3 correlation functions:



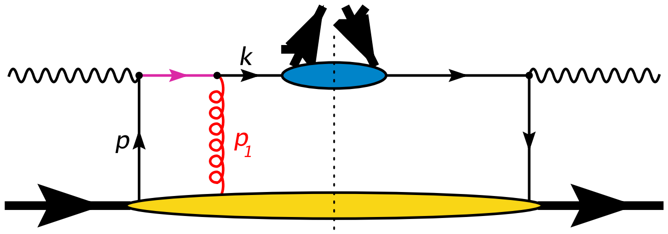
Contribution contains twist-3 PDFs/FFs

2. Genuine twist-3:



Quark-Gluon-Quark correlators

Treatment of qgq-Correlators:



$$\propto \int dp_1\, \mathcal{FT}\left(\langle P,S|\,\bar{\psi}(0)\gamma^\mu\Delta\gamma_\rho\frac{\not k-\not p_1+m}{(k-p_1)^2-m^2+i0}gA^\rho(\eta)\gamma^\nu\psi(\xi)\,|P,S\rangle\right)$$

$1/Q$-expansion of the propagator:

- A^+ ($A_T^i(x^-=\infty)$)-gluons $\longrightarrow$ Gauge Link
- Transverse $A_T^i(x)$-gluons $\longrightarrow$ Twist-3 $\longrightarrow$ color g.i. qgq-corr. Φ_A^i

- Parameterization of qgq-correlator $\Phi_A^i \propto \langle P|\,\bar{\psi}\,(\int \mathcal{W}\, F^{+i}\, \mathcal{W})\,\psi\,|P\rangle$ in terms of *tilde*-functions, e.g.

$$\mathrm{Tr}\left[\Phi_{A,i} i\sigma^{i+}\gamma_5\right] = 2Mx\left((\tilde{h}_L + i\tilde{e}_L) + \frac{\vec{p}_T\cdot\vec{S}_T}{M}(\tilde{h}_T + i\tilde{e}_T)\right),\ \ldots$$

- Connection between *tilde*-functions and PDFs/FFs via QCD-equation of motion, e.g.

$$\tilde{h}_L = h_L + \frac{\vec{p}_T^{\,2}}{2M^2}\frac{h_{1L}^\perp}{x} - \frac{m}{M}\frac{g_{1L}}{x} \quad ; \quad \tilde{g}_T = g_T - \frac{\vec{p}_T^{\,2}}{2M^2}\frac{g_{1T}}{x} - \frac{m}{M}\frac{h_1}{x},\ \ldots$$

- Decomposition of SIDIS-cross section into 8 Twist-2 and 8 Twist-3 structure functions

Twist-2: $F_{UU},\ F_{UU}^{\cos 2\phi_h},F_{UL}^{\sin 2\phi_h},\ F_{LL},\ F_{UT}^{\sin(\phi_h-\phi_S)},\ F_{UT}^{\sin(\phi_h+\phi_S)},\ F_{UT}^{\sin(3\phi_h-\phi_S)},\ F_{LT}^{\cos(\phi_h-\phi_S)}$

Twist-3: $F_{UU}^{\cos\phi_h},\ F_{LU}^{\sin\phi_h},\ F_{UL}^{\sin\phi_h},\ F_{LL}^{\cos\phi_h},\ F_{UT}^{\sin\phi_S},\ F_{UT}^{\sin(2\phi_h-\phi_S)},\ F_{LT}^{\cos\phi_h},\ F_{LT}^{\cos(2\phi_h-\phi_S)}$

- Structure functions in terms of PDFs and FFs, e.g. twist-3 *beam-spin asymmetry*
(Mulders, Tangerman, 1996; Boer, Mulders, 1998; Bacchetta, Mulders, Pijlman, 2004; Bacchetta, Diehl, Goeke, Metz, Mulders, M.S., 2006)

$$F_{LU}^{\sin\phi_h} \quad\propto\quad \frac{M}{Q}\left[e\otimes H_1^\perp + f_1\otimes\tilde{G}^\perp + g^\perp\otimes D_1 + h_1^\perp\otimes\tilde{E}\right],\ \text{and many more...}$$

- Semi-inclusive jet production: jet single-spin asymmetries (Bacchetta, Mulders, Pijlman, 2004)

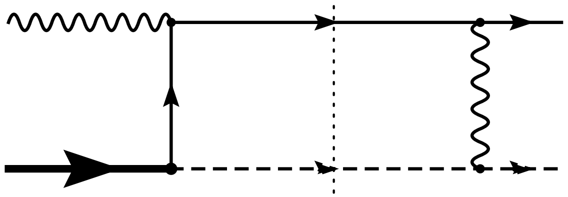
$$A_{LU,\,\mathrm{jet}}^{\sin\phi_h} \propto \frac{M}{Q}\frac{g^{\perp(1)}(x)}{f_1(x)} \quad ; \quad A_{UL,\,\mathrm{jet}}^{\sin\phi_h} \propto \frac{M}{Q}\frac{f_L^{\perp(1)}(x)}{f_1(x)}$$

Longitudinal jet-SSAs in the scalar diquark-spectator-model:

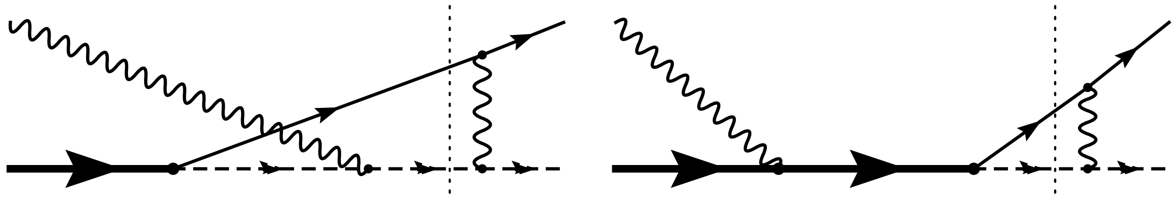
(Afanasev, Carlson, 2003, 2006; Metz, M.S., 2004)

- *Left hand side:* Direct calculation of jet asymmetries $A_{LU} \propto g^{\perp(1)}(x)$, $A_{UL} \propto f_L^{\perp(1)}(x)$

Rescattering effect:

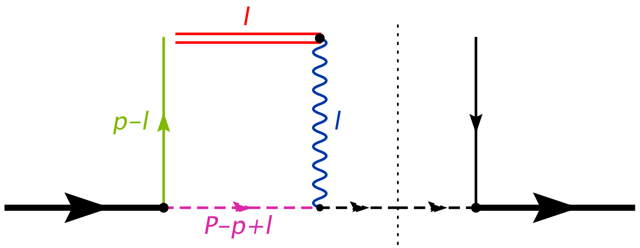


Other diagrams containing final state interactions:



$\Longrightarrow$ Result: Non-vanishing, *finite* asymmetries $\boxed{A_{UL} \neq 0}$, $\boxed{A_{LU} \neq 0}$.

- *Right hand side:* T-odd twist-3 parton distributions in the scalar diquark model
(Gamberg, Hwang, Metz, M.S., 2006)



$$+\text{ h.c.} \quad \boxed{g^{\perp} \propto \int \frac{d^4 l}{(2\pi)^4} \frac{n\cdot(2P-2p+l)\left[\epsilon_T^{ij} p_T^j \left(P^+ l^- - P^- l^+\right) + \epsilon_T^{ij} l_T^j \left(P^- p^+ - P^+ p^-\right)\right]}{[(l\cdot n)+i0]\left[l^2 - i0\right]\left[(P-p+l)^2 - m_s^2 - i0\right]\left[(p-l)^2 - m_q^2 - i0\right]}}$$

For $n = [1^-, 0^+, \vec{0}_T]$ on the light-cone $\longrightarrow$ *Divergence*!

- Regularization: "non lightlike" Wilson lines: $n = [n^-, n^+, \vec{0}_T],\ \left|\frac{n^+}{n^-}\right| \ll 1$

$$g^{\perp}(x, \vec{p}_T^{\,2}, n) \propto \frac{1-x}{x} \ln\left(\frac{n^2}{2(n\cdot P)^2}\right) + \text{finite} + \mathcal{O}\left(\left|\frac{n^+}{n^-}\right|\right)$$

- Shows LC-divergence explicitly
  $\longrightarrow$ same divergence for *all* T-odd twist-3 PDFs, also in quark-target-model.

- Finite Box-graph contributions for twist-2 T-odd PDFs $f_{1T}^{\perp}$, $h_1^{\perp}$.

- Regularization procedure: "Tree-level" predictions $(A_{LU} = \text{fin.}) \propto (g^{\perp} \to \infty)$, $(A_{UL} = \text{fin.}) \propto (f_L^{\perp} \to \infty) \Rightarrow$ Modification!?

- Factorization theorem for twist-2 observables: (Ji, Ma, Yuan, 2004)

$$\frac{d\sigma_{UU}}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} \propto \int d^2p_T\, d^2k_T \left(\int d^2l_T\ S(\vec{l}_T)\delta^{(2)}(\vec{p}_T - \frac{\vec{P}_{h\perp}}{z_h} - \vec{k}_T + \vec{l}_T)\right) f_1(x_B, \vec{p}_T^{\,2})\, D_1(z_h, \vec{k}_T^{\,2}) + \ldots$$

Soft factor $S(\vec{l}_T)$ due to soft gluon radiation $\longrightarrow$ modifies δ-function.

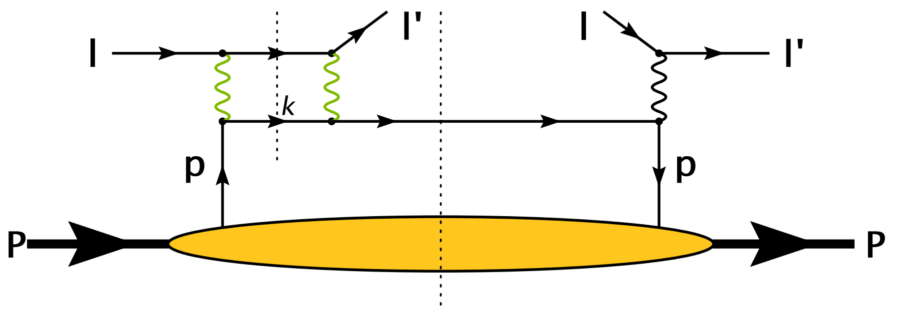
PDFs/FFs: "non light-like" Wilson lines $\longrightarrow$ "non-light-likeness" parameter $\zeta = \sqrt{\frac{2(P\cdot n)^2}{n^2}}$

- Generalization of "all-order factorization" for twist-3 observables possible?

Two-Photon Exchange (DIS)

- Inclusive DIS:
  Four structure functions in the Bjorken limit, F_1, F_2, g_1, g_2
  Single-spin asymmetries in One-Photon Exchange forbidden, T-invariance (Christ, Lee, 1966)
  $\longrightarrow$ beyond One-Photon Exchange: Correlation $\sim \epsilon^{\mu\nu\rho\sigma} l_\mu l'_\nu P_\rho S_\sigma$ allowed.

- Parton model calculation (Metz, MS, Goeke, 2006):



1) collinear parton model: $p^+ = xP^+$, no p_T
2) photons couple to the *same* parton.
3) SSA $\rightarrow$ imaginary part of box diagram

- SSA for transversely polarized lepton: leading twist-PDF f_1, IR-finite

$$\mathrm{SSA} \sim \alpha_{\mathrm{em}}^3 m \sin(\phi_s) \sum_q e_q^3 x f_1^{(q)}(x)$$

proportional to lepton mass, e^- : $\mathrm{SSA} \sim 10^{-7} - 10^{-6}$ $\longrightarrow$ DIS with myons at COMPASS?

- SSA for transversely polarized nucleon: twist-3, IR-divergences

$$\text{SSA} \sim \alpha_{\text{em}}^3 M \sin(\phi_s) \left((1-y)^2 \ln\left(\frac{Q^2}{\lambda^2}\right) + y(2-y)\ln y + y(1-y) \right) \sum_q e_q^3 x g_T^{(q)}(x)$$

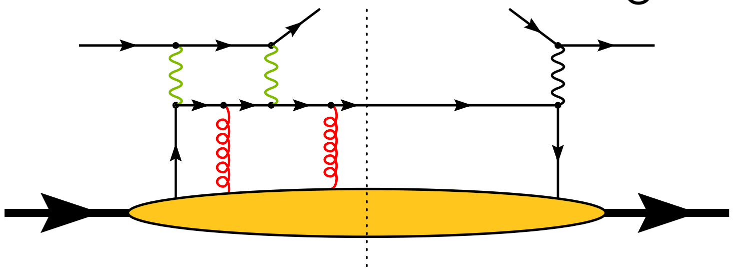
proportional to *nucleon* mass.

- How to *cancel* the IR-divergence?

1) Include transverse parton momentum $p_T \longrightarrow$ TMD parton distributions

$$\text{SSA} \sim \alpha_{\text{em}}^3 M \sin(\phi_s) \ln\left(\frac{Q^2}{\lambda^2}\right) \sum_q e_q^3 \int d^2 p_T \underbrace{\left(x g_T - \frac{\vec{p}_T^{\,2}}{2M^2} g_{1T} - \frac{m_q}{M} h_1 \right)}_{= x\tilde{g}_T(x, \vec{p}_T^{\,2})} + \text{finite}$$

2) tilde function from Quark-Gluon-Quark correlator $\longrightarrow$ might cancel IR-divergence (?)



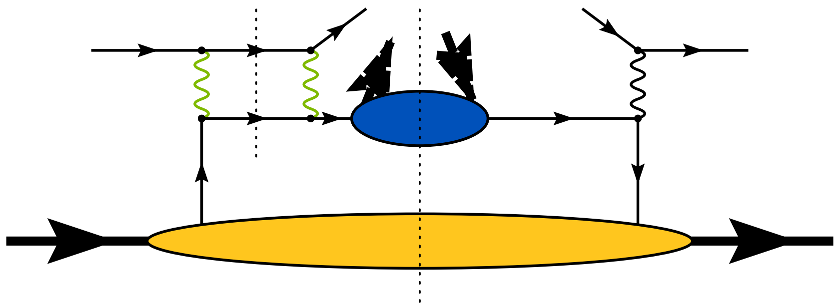
3) if not sufficient $\longrightarrow$ coupling to *different* parton necessary?

Still more work to do...

Two-Photon Exchange (SIDIS)

Two Photon Exchange effects also in Semi-Inclusive DIS?

- Inclusion of fragmentation:



- TPE enables $\sin(2\phi)$ beam spin asymmetry $\longrightarrow$ *Boer-Mulders* effect, *leading twist* SSA

$$A_{LU}^{\sin(2\phi)} \propto \alpha_{\rm em}^3 \lambda_e \sin(2\phi) \sum_q e_q^3 \left[h_1^{\perp,(q)} \otimes H_1^{\perp,(q)}\right]$$

$\longrightarrow$ Contribution *absent* for One Photon Exchange, IR-finite!

- Single-spin asymmetries for target polarization: transverse and longitudinal

$\longrightarrow$ e.m. corrections to twist-3 asymmetries $A_{UT}^{\sin(2\phi-\phi_S)}$, $A_{UT}^{\sin(\phi_S)}$ and $A_{UL}^{\sin(\phi)}$
$\longrightarrow$ IR-divergent asymmetries $\longrightarrow$ QCD-equ. of motion: SSA in terms of tilde functions
$\longrightarrow$ Inclusion of add. gluon exchanges cancels IR-divergences?

Summary & Conclusions

✘ Analysis of subleading twist observables in SIDIS:

"Tree-level" formalism: Central objects: Quark-Quark Correlators $\Phi_{ij}(x, \vec{p}_T)$ und $\Delta_{ij}(z, \vec{k}_T)$
⟶ Elemination of qgq-correlations (tilde functions) via QCD-equation of motion
⟶ complete determination of the twist-3 observables in terms (higher twist) PDFs and FFs

Calculation of twist-3 jet asymmetries A_{UL} and A_{LU} in a scalar diquark-spectator-model.
⟶(IR and collinear) finite results, Light-cone divergences for twist-3 T-odd PDFs
⟶ Modification of existing twist-3 predictions needed.

✘ Two-Photon Exchange:

Inclusive DIS: Transverse Spin asymmetries allowed beyond One-Photon Exchange.
⟶ collinear parton model: finite result for transversely pol. lepton, IR-divergent result for transverse pol. target.
⟶ inclusion of QGQ-correlations might cancel IR-divergences.

SIDIS: TPE generates new observables
⟶ $\sin(2\phi)$ beam-spin asymmetry: finite SSA, Boer-Mulders effect.
⟶ Target-Spin asymmetries: contributions of TPE to twist-3 oberservables only, IR-divergent.