gmc_trans
a Monte Carlo generator for transverse-momentum-dependent distribution and fragmentation functions

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Outline

- Motivation & Basics
- Details of the MC generator
  - Gaussian Ansatz
  - Positivity limits
  - Event generation
- Some results
  - Tuning of transverse momentum dependence
  - (un)weighted Sivers and Collins amplitudes
- Implemented models
- Going beyond Collins and Sivers asymmetries

Motivation

- **Monte Carlo Simulations are a indispensable tool in modern nuclear and particle physics experiments**
- **various "physics generators" exists for the various fields (e.g.,** PYTHIA, LEPTO, AROMA **etc.)**
- **used for predictions, for the understanding of the experiment, and also for the "correction" of data (e.g., acceptance effects, background processes etc.)**
- **no generator was available for transverse-momentum dependence of distribution and fragmentation functions**

Initial goals for gmc_trans

- physics generator for SIDIS pion production
- include transverse-momentum dependence, in particular simulate Collins and Sivers effects
- be fast
- allow comparison of input model and reconstructed amplitudes
- to be used with standard HERMES Monte Carlo
- be extendable (e.g., open for new models)

Basic workings

- use cross section that can (almost) be calculated analytically
- start from 1-hadron SIDIS expressions of Mulders & Tangerman (Nucl.Phys.B461:197-237,1996)
- use Gaussian Ansatz for all transverse-momentum dependencies of DFs and FFs
- unpolarized DFs (as well as helicity distribution) and FFs from fits/parametrizations (e.g., Kretzer FFs etc.)
- "polarized" DFs and FFs either related to unpolarized ones (e.g., saturation of Soffer bound for transversity) or some parametrizations used

Caution: Details to follow!

SIDIS Cross Section incl. TMDs

$$d\sigma_{UT} \equiv d\sigma_{UT}^{\text{Collins}} \cdot \sin(\phi + \phi_S) + d\sigma_{UT}^{\text{Sivers}} \cdot \sin(\phi - \phi_S)$$

$$d\sigma_{UT}^{\text{Collins}}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} B(y) \sum_q e_q^2 \, \mathcal{I}\left[\left(\frac{k_T \cdot \hat{P}_{h\perp}}{M_h}\right) \cdot h_1^q H_1^{\perp q}\right]$$

$$d\sigma_{UT}^{\text{Sivers}}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \, \mathcal{I}\left[\left(\frac{p_T \cdot \hat{P}_{h\perp}}{M_N}\right) \cdot f_{1T}^{\perp q} D_1^q\right]$$

$$d\sigma_{UU}(x, y, z, \phi_S, P_{h\perp}) \equiv \frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \, \mathcal{I}\left[f_1^q D_1^q\right]$$

where

$$\mathcal{I}\left[\mathcal{W} \, f \, D\right] \equiv \int d^2 p_T d^2 k_T \, \delta^{(2)}\left(p_T - \frac{P_{h\perp}}{z} - k_T\right) \left[\mathcal{W} \, f(x, p_T) \, D(z, k_T)\right]$$

Gaussian Ansatz

- want to deconvolve convolution integral over transverse momenta
- easy Ansatz: Gaussian dependencies of DFs and FFs on intrinsic (quark) transverse momentum:

$$\mathcal{I}[f_1(x,\boldsymbol{p_T^2})D_1(z,z^2\boldsymbol{k_T^2})] = f_1(x)\cdot D_1(z)\cdot \frac{R^2}{\pi z^2}\cdot e^{-R^2\frac{P_{h\perp}^2}{z^2}}$$

with $\boldsymbol{f_1(x,p_T^2) = f_1(x)\frac{1}{\pi\langle p_T^2\rangle}e^{-\frac{p_T^2}{\langle p_T^2\rangle}}}$ $\frac{1}{R^2} \equiv \langle k_T^2\rangle + \langle p_T^2\rangle = \frac{\langle P_{h\perp}^2\rangle}{z^2}$

(similar: $D_1(z,z^2\boldsymbol{k_T^2})$)

Caution: different notations for intrinsic transverse momentum exist!

Positivity Constraints

- **DFs (FFs) have to fulfill various positivity constraints (resulting cross section must not be negative!)**
- **based on probability considerations can derive positivity limits for leading-twist functions: Bacchetta et al., Phys.Rev.Lett.85:712-715, 2000**

➡ **transversity: e.g., Soffer bound**

➡ **Sivers and Collins functions: e.g., loose bounds:**

$$\frac{|p_T|}{2M_N} f_{1T}^{\perp}(x, p_T^2) \equiv f_{1T}^{\perp(1/2)}(x, p_T^2) \leq \frac{1}{2} f_1(x, p_T^2)$$

$$\frac{|k_T|}{2M_h} H_1^{\perp}(z, z^2 k_T^2) \equiv H_1^{\perp(1/2)}(z, z^2 k_T^2) \leq \frac{1}{2} D_1(z, z^2 k_T^2)$$

Positivity and the Gaussian Ansatz

$$\frac{|\boldsymbol{p_T}|}{2M_N} f_{1T}^{\perp}(x, \boldsymbol{p_T}^2) \leq \frac{1}{2} f_1(x, \boldsymbol{p_T}^2)$$

with
$$f_1(x, p_T^2) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}$$

$$f_{1T}^{\perp}(x, p_T^2) = f_{1T}^{\perp}(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}$$

$$\Rightarrow \quad |p_T| f_{1T}^{\perp}(x) \leq M_N f_1(x)$$

Positivity and the Gaussian Ansatz

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$$f_{1T}^{\perp}(x, p_T^2) = f_{1T}^{\perp}(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}$$

$$\Rightarrow \quad |p_T| f_{1T}^{\perp}(x) \leq M_N f_1(x)$$

Problem for non-zero Sivers function!

Modify Gaussian width

$$f_{1T}^{\perp}(x, p_T^2) = f_{1T}^{\perp}(x) \frac{1}{(1-C)\pi\langle p_T^2\rangle} e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}}$$

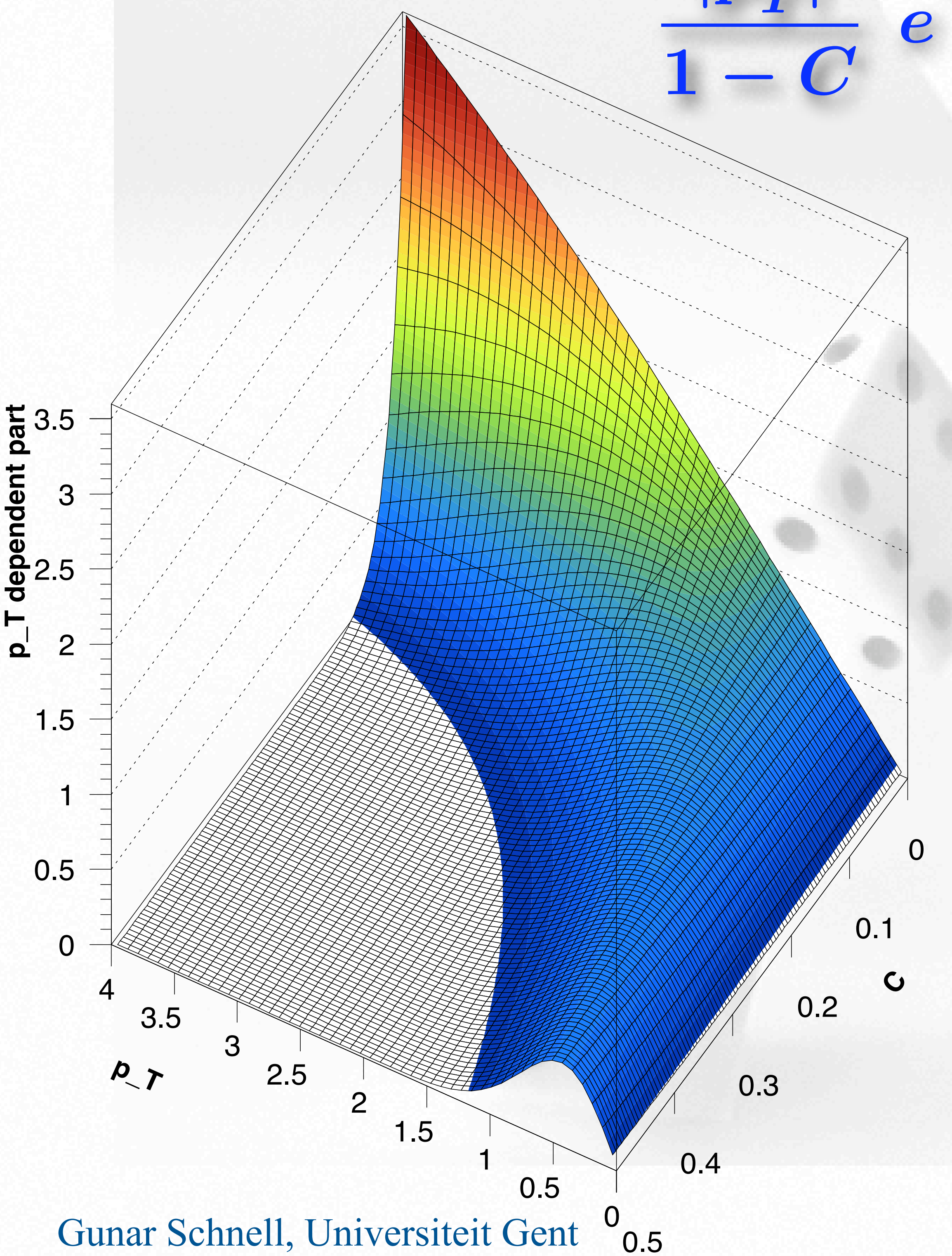
➡ **positivity limit:**

$$f_{1T}^{\perp}(x) \frac{|p_T|}{2M_N} \frac{1}{\pi(1-C)\langle p_T^2\rangle} e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}} \leq 1/2\, f_1(x) \frac{1}{\pi\langle p_T^2\rangle} e^{-\frac{p_T^2}{\langle p_T^2\rangle}}$$

$$\Rightarrow \frac{|p_T|}{1-C} e^{-\frac{C}{1-C}\frac{p_T^2}{\langle p_T^2\rangle}} \leq M_N \frac{f_1(x)}{f_{1T}^{\perp}(x)}$$

New positivity limit

$$\frac{|p_T|}{1-C}\, e^{-\frac{C}{1-C}\frac{p_T^2}{\langle p_T^2\rangle}} \leq M_N \frac{f_1(x)}{f_{1T}^{\perp}(x)}$$



Minimum at $p_T = \sqrt{\frac{\langle p_T^2\rangle}{2C}}$

thus $\frac{f_{1T}^{\perp}(x)}{f_1(x)} \leq M_N \sqrt{\frac{2eC(1-C)}{\langle p_T^2\rangle}}$

or $$\frac{f_{1T}^{\perp(1/2)}(x)}{f_1(x)} \leq \frac{1}{2}\sqrt{\frac{e\pi C}{2}}(1-C) \leq 0.4$$

SIDIS Cross Section incl. TMDs

$$\sum_q \frac{e_q^2}{4\pi} \frac{\alpha^2}{(MExyz)^2} \left[X_{UU} + |\mathrm{S}_T| X_{SIV} \sin(\phi_h - \phi_s) + |\mathrm{S}_T| X_{COL} \sin(\phi_h + \phi_s) \right]$$

using Gaussian Ansatz for transverse-momentum dependence of DFs and FFs:

$$X_{UU} = R^2 e^{-R^2 P_{h\perp}^2/z^2} \left(1 - y + \frac{y^2}{2}\right) f_1(x) \cdot D_1(z)$$

$$X_{COL} = + \frac{|P_{h\perp}|}{M_\pi z} \frac{(1-C)\langle k_T^2 \rangle}{\left[\langle p_T^2 \rangle + (1-C)\langle k_T^2 \rangle\right]^2} \exp\left[-\frac{P_{h\perp}^2/z^2}{\langle p_T^2 \rangle + (1-C)\langle k_T^2 \rangle}\right]$$

$$\times \quad (1-y) \cdot h_1(x) \cdot H_1^\perp(z)$$

$$X_{SIV} = - \frac{|P_{h\perp}|}{M_p z} \frac{(1-C')\langle p_T^2 \rangle}{\left[\langle k_T^2 \rangle + (1-C')\langle p_T^2 \rangle\right]^2} \exp\left[-\frac{P_{h\perp}^2/z^2}{\langle k_T^2 \rangle + (1-C')\langle p_T^2 \rangle}\right]$$

$$\times \quad \left(1 - y + \frac{y^2}{2}\right) f_{1T}^\perp(x) \cdot D_1(z)$$

Sivers (azimuthal) moments

use cross section expressions to evaluate azimuthal moments:

$$-\langle\sin(\phi-\phi_s)\rangle_{UT} = \frac{\sqrt{(1-C)\langle p_T^2\rangle}}{\sqrt{(1-C)\langle p_T^2\rangle+\langle k_T^2\rangle}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

$$-\langle\sin(\phi-\phi_s)\rangle_{UT} = \frac{M_N\sqrt{\pi}}{2\sqrt{(1-C)\langle p_T^2\rangle+\langle k_T^2\rangle}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

$$-\langle\frac{|P_{h\perp}|}{zM_N}\sin(\phi-\phi_s)\rangle_{UT} = \frac{2\sqrt{(1-C)\langle p_T^2\rangle}}{M_N\sqrt{\pi}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

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model-dependence on transverse momenta "swallowed" by p_T^2- moment of Sivers fct.: $f_{1T}^{\perp(1)}$

Sivers (azimuthal) moments

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$$-\langle\sin(\phi-\phi_s)\rangle_{UT} = \frac{\sqrt{(1-C)\langle p_T^2\rangle}}{\sqrt{(1-C)\langle p_T^2\rangle + \langle k_T^2\rangle}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

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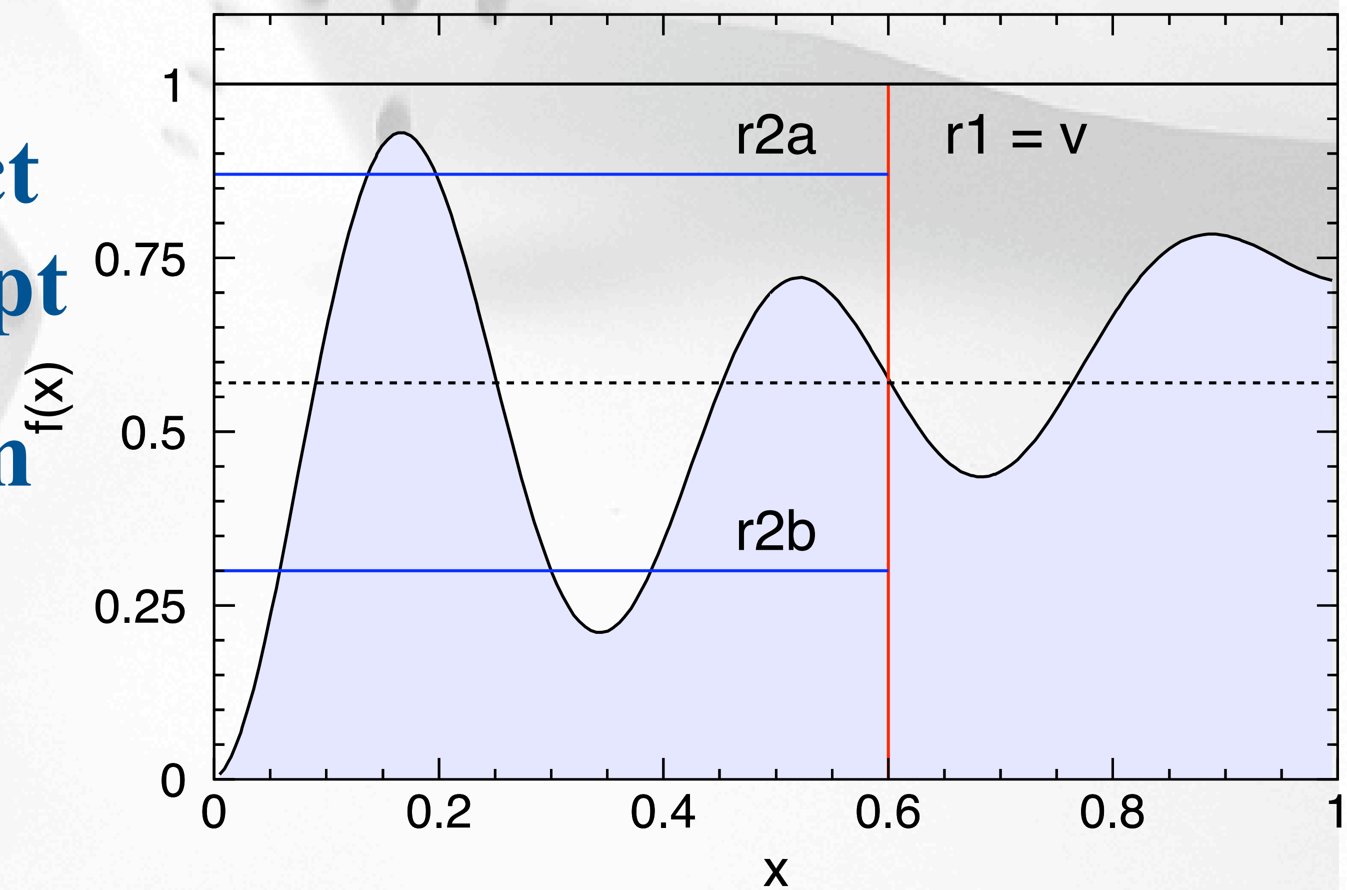
(similar for Collins moments)

Monte Carlo event generation

- need to generate events according to cross section:
  - throw flavor of struck quark according to integrated (unpolarized) cross section for each quark flavor
  - throw (x, Q^2, z) according to unpolarized cross section
  - throw pion's transverse momentum $P_{h\perp}^2$ according to Gaussian Ansatz
  - generate azimuthal angles (ϕ, ϕ_S) according to polarized cross section
- cross section should be positive automatically if positivity constraints on DFs and FFs are fulfilled, but better check again

Some (more) details

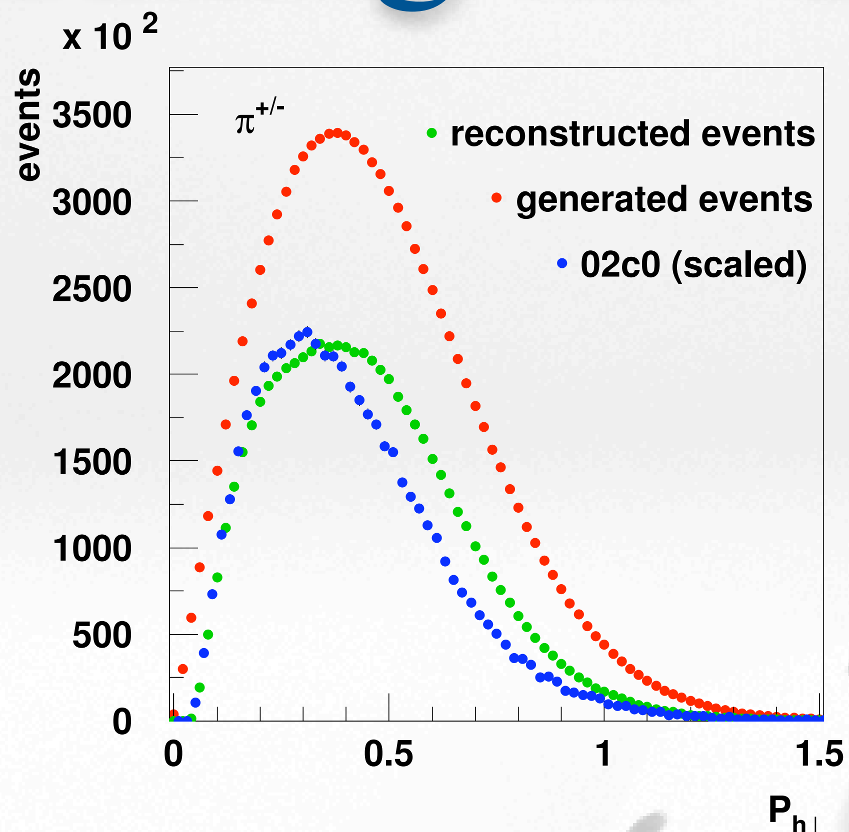
- **event generation by accept-or-reject method:**
  - **first throw flat in (x, Q^2, z), e.g., get value r1**
  - **second random number r2 determines accept/reject status:**
    r2 above curve: reject
    r2 below curve: accept

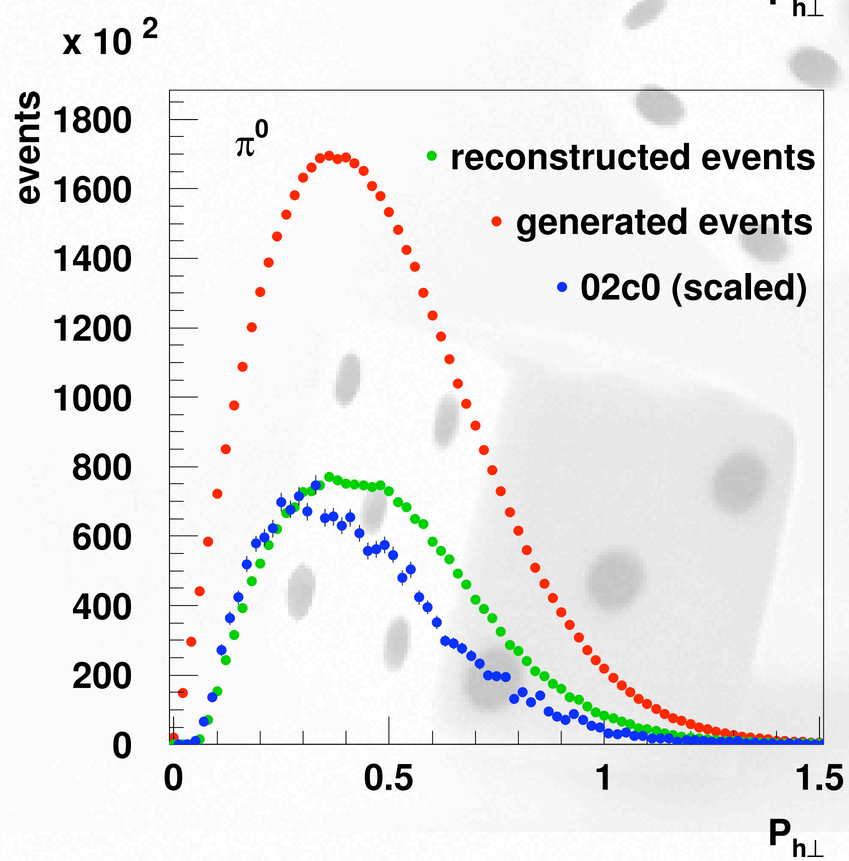


- **have to know maximum of f(x), i.e., of the cross section (checked at beginning)**
- **can gain some speed by not throwing flat in, e.g., Q^2; but according to global behaviour**

Some Results

Tuning the Gaussians in gmc_trans





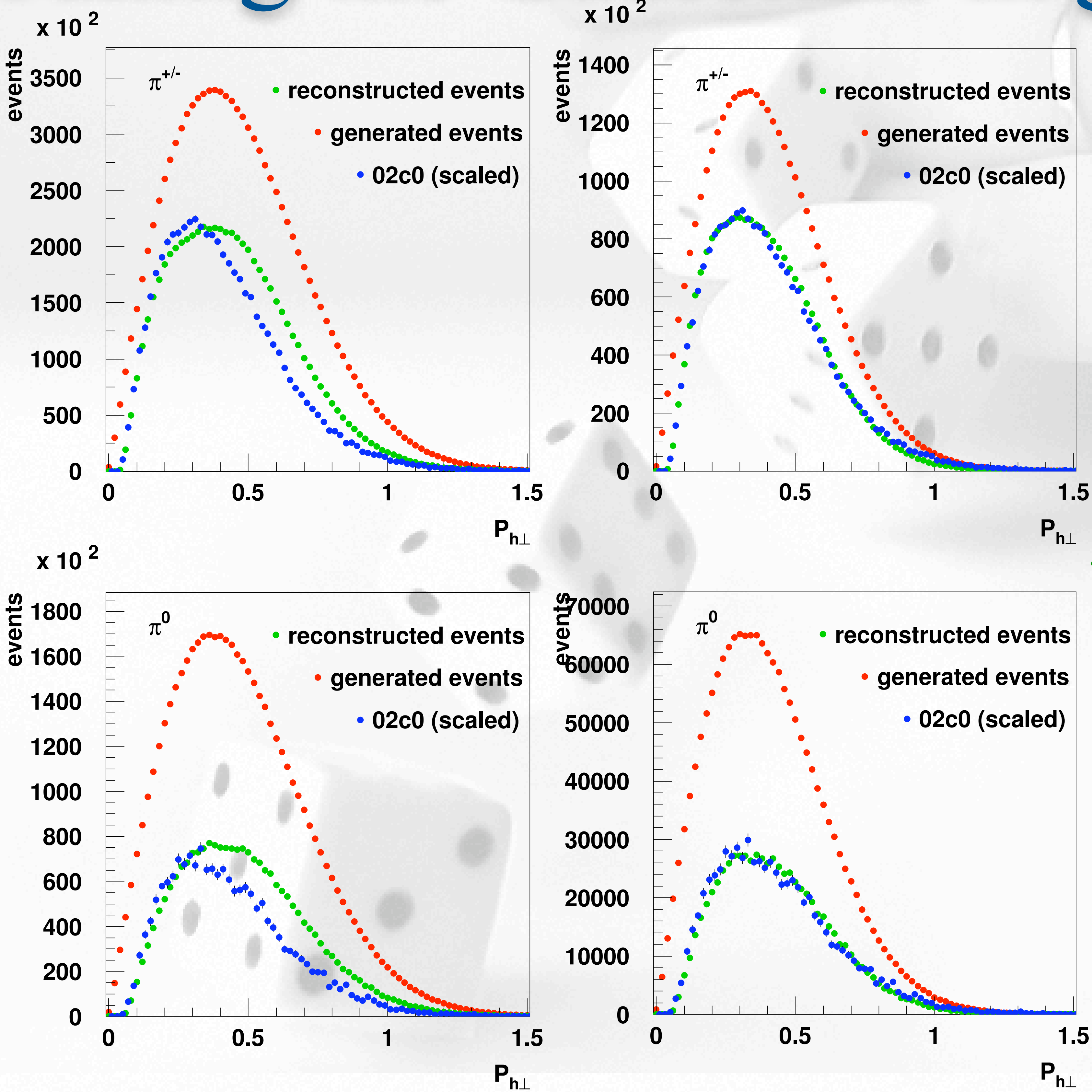
- **constant Gaussian widths, i.e., no dependence on x or z:**

$$\langle p_T \rangle = 0.44$$

$$\langle K_T \rangle = 0.44$$

- **tune to data integrated over whole kinematic range**

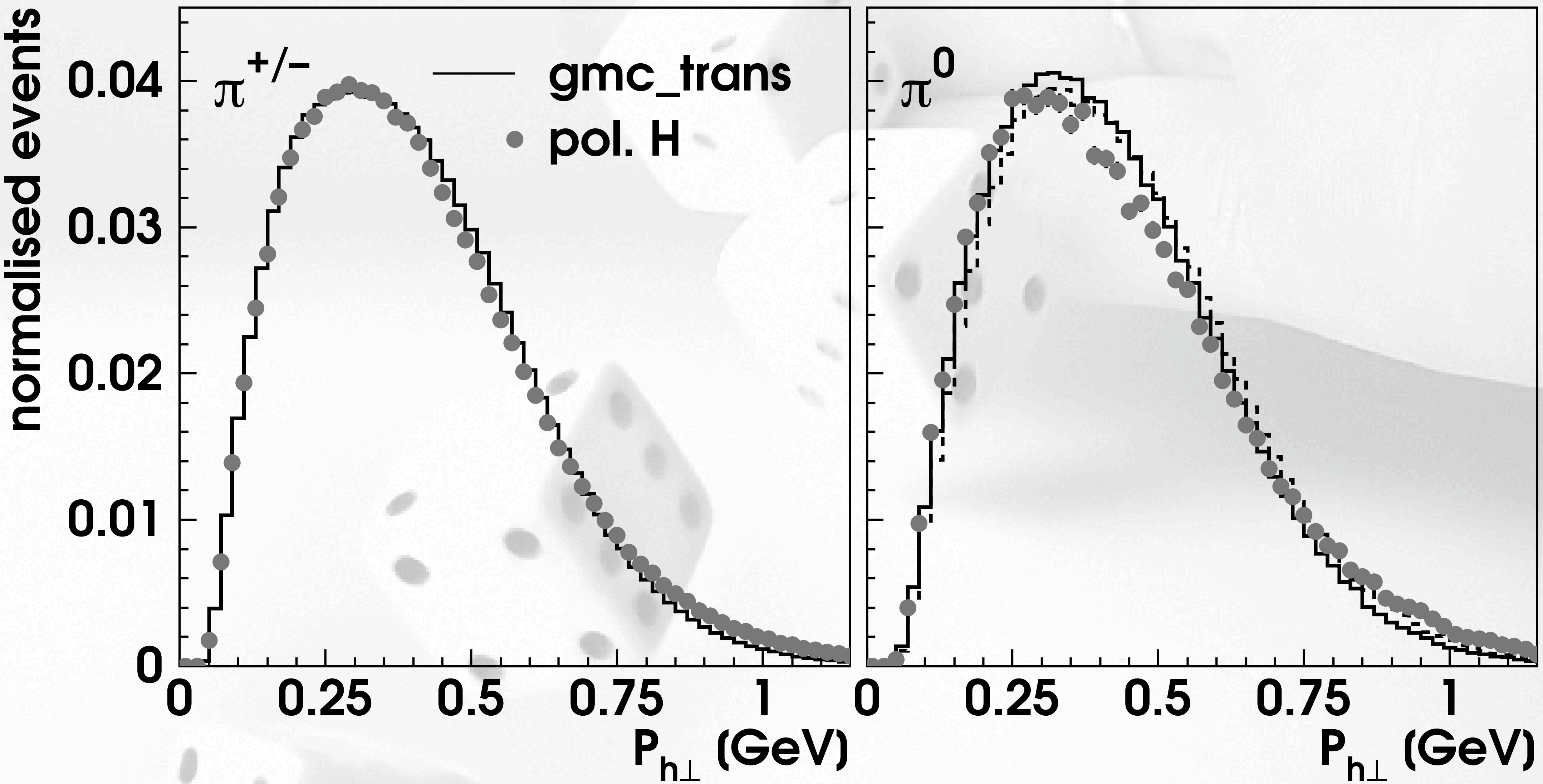
Tuning the Gaussians in gmc_trans



$$\langle p_T \rangle = 0.38$$

$$\langle K_T \rangle = 0.38$$

Comparison Data-MC: $P_{h\perp}$



$\langle p_T^2 \rangle = \langle K_T^2 \rangle = 0.18\,\text{GeV}^2 \;(\langle|\vec{p}_T|\rangle = \langle|\vec{K}_T|\rangle = 0.38\,\text{GeV})$

where: $\langle K_T^2 \rangle = z^2 \langle k_T^2 \rangle$

Tuning the Gaussians in gmc_trans

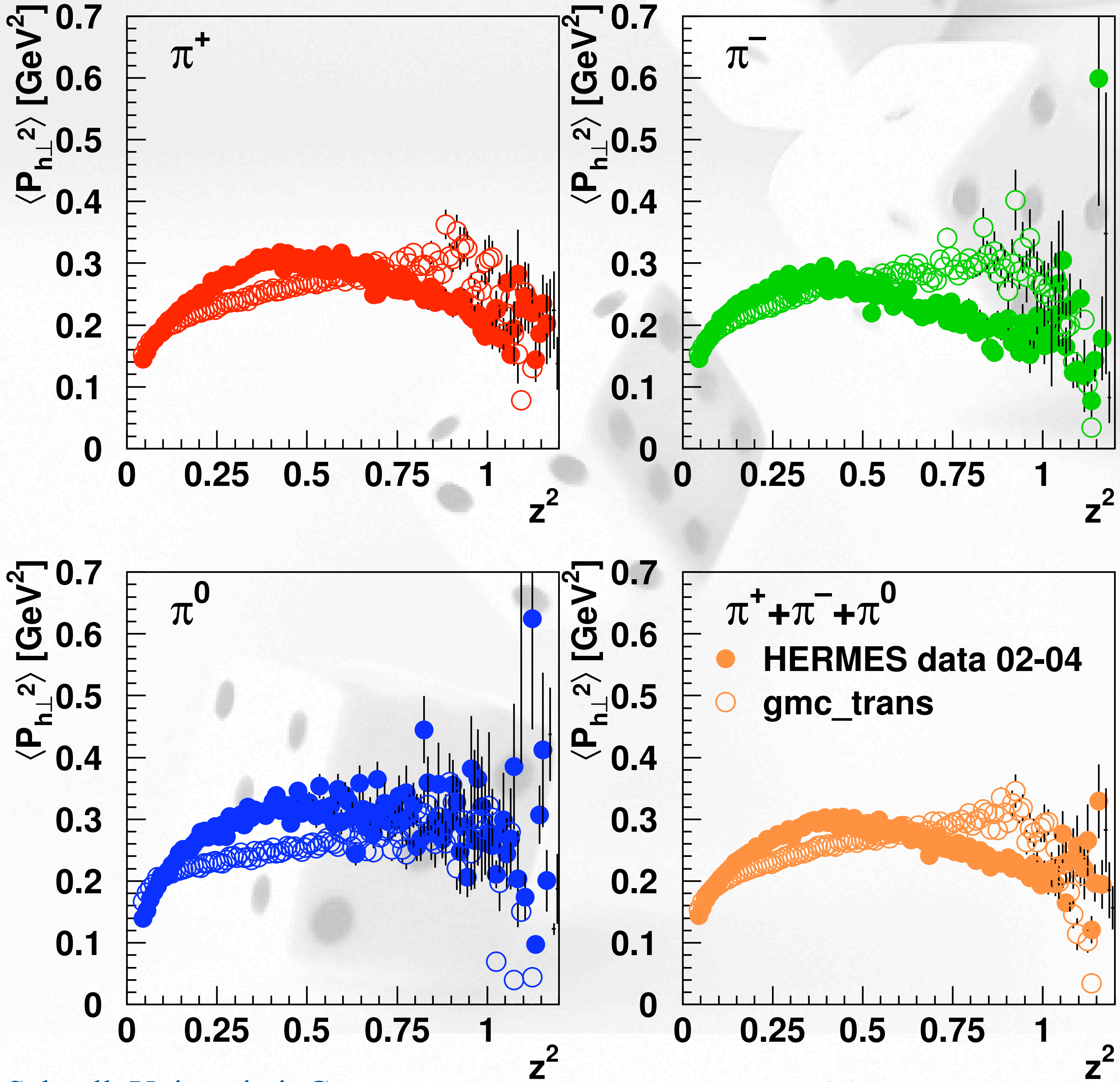
in general: $\langle P_{h\perp}^2(x,z)\rangle = z^2 \langle p_T^2(x)\rangle + \langle K_T^2(z)\rangle$

so far: $\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2\rangle$

constant!

Tuning the Gaussians in gmc_trans

so far: $\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2\rangle$



$\langle p_T\rangle = 0.38$

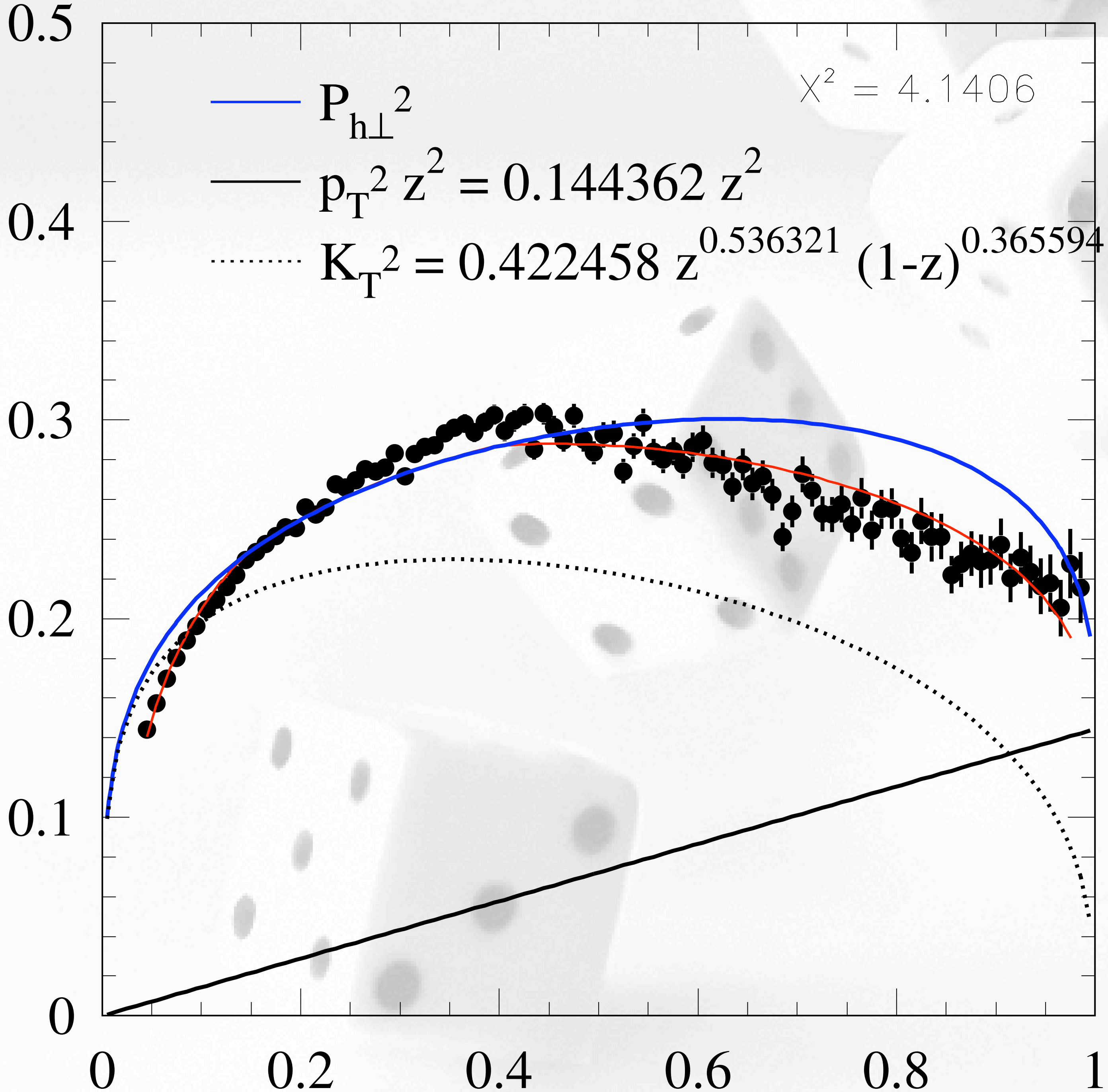
$\langle K_T\rangle = 0.38$

$\langle p_T^2\rangle \simeq 0.185$

$\langle K_T^2\rangle \simeq 0.185$

Tuning the Gaussians in gmc_trans

now: $\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2(z)\rangle$



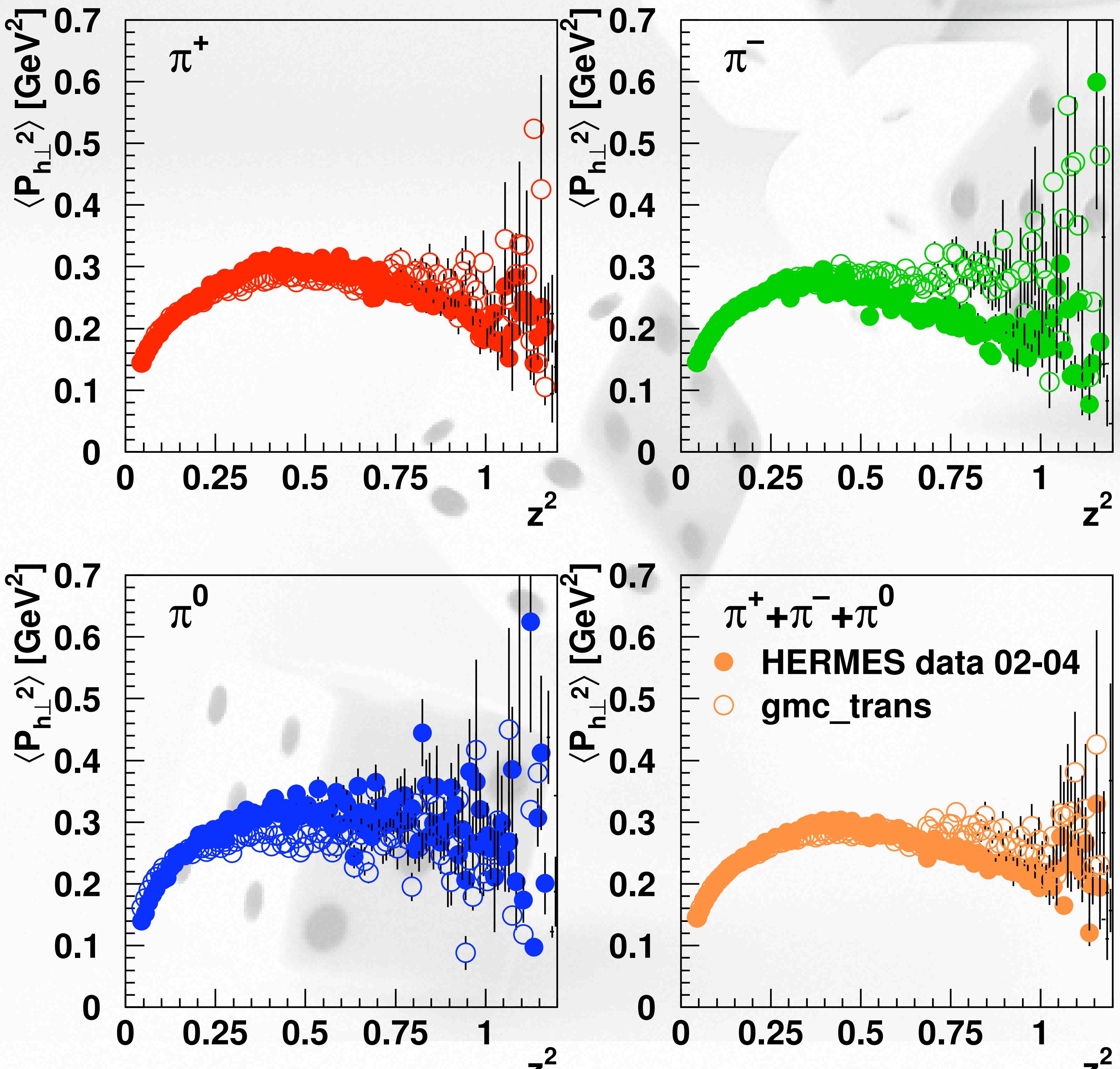
z-dependent!

tuned to HERMES data in acceptance

"Hashi set"

Tuning the Gaussians in gmc_trans

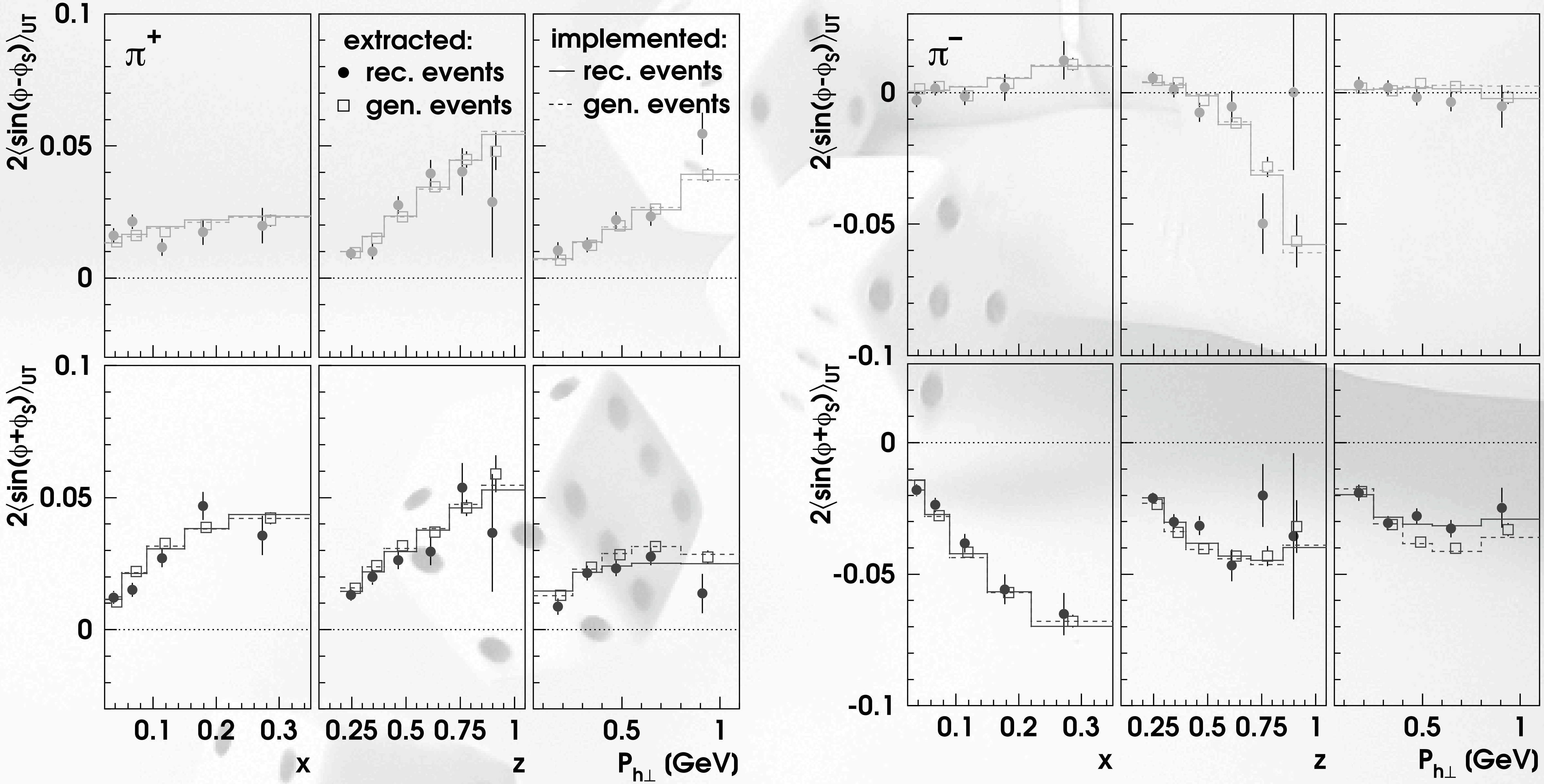
$$\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2(z)\rangle$$



z-dependent!

"Hashi set"

Generated vs. extracted amplitudes



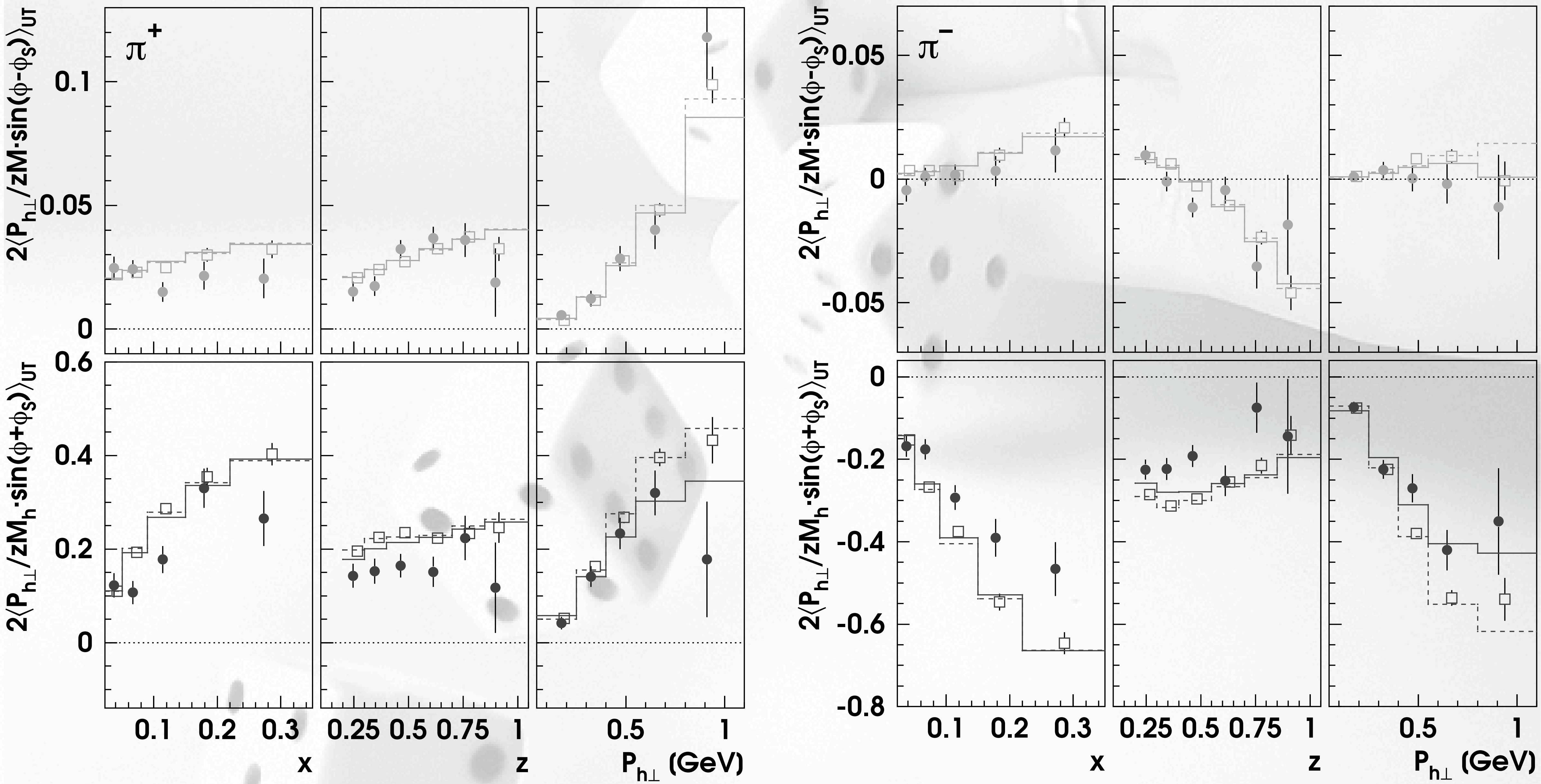
$$\delta u(x) = 0.7 \cdot \Delta u(x) \qquad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x) \qquad H_{1,\text{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\text{fav}}(z)$$

$$\delta d(x) = 0.7 \cdot \Delta d(x) \qquad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x) \qquad H_{1,\text{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\text{dis}}(z)$$

$$\delta q(x) = 0.3 \cdot \Delta q(x) \qquad f_{1T}^{\perp q}(x) = 0.0 \qquad q = \bar{u}, d, s, \bar{s}$$

$$C_S = C_C = 0.25$$

Comparison for weighted moments



Not so good news for weighted moments

Where to go from here?

Models for integrated DFs and FF

- **"usual PDFs"** f_1, g_1 **from PEPSI library**
- D_1 **from KKP or Kretzer**
- h_1 **can be**
  - $= g_1$
  - **Soffer bound**
  - **Leader parametrization**

Models for Sivers and Collins Fcts

- **Sivers function** $f_{1T}^{\perp}$
  - $f_{1T}^{\perp}(x) \sim f_1(x)$
  - $f_{1T}^{\perp}(x) \sim g_1(x)$
  - Boglione-Mulders parametrization
  - $f_{1T}^{\perp(1)}(x) \sim f_1(x)$
- **Collins function:** $H_1^{\perp}$
  - $H_1^{\perp}(z) \sim D_1(z)$
  - Boglione-Mulders
  - Leader
  - $H_1^{\perp(1)}(z) \sim D_1(z)$

Models for Sivers and Collins Fcts

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  - $H_1^{\perp}(z) \sim D_1(z)$
  - Boglione-Mulders
  - Leader
  - $H_1^{\perp(1)}(z) \sim D_1(z)$

obviously want more (new) fits/parametrizations

Beyond Collins and Sivers

- certainly would like to model all TMDs, e.g., Boer-Mulders function, to get full cross section
- even go to subleading-twist, e.g., Cahn effect
- first attempts to implement those have been made
- leading twist -- “straight forward” (just a few more convolution integrals)
- subleading twist -- “hmmmm...”
  - biggest problem there: positivity limits don’t exist on DF and FF level

Status of unpolarized cross section

- almost implemented:
  - Boer-Mulders effect
  - Cahn effect
    - can adjust Gaussian width, kinematic dependencies and normalization
    - generated values (cross-section and unweighted moments) are available for end-user, however, weighted moments not available

Status of polarized cross section

- **hardly implemented:**
  - **twist-3 AUT** $\sin\phi_S$ **term (involves transversity)**
    - **can partially adjust Gaussian width (involves 2 terms - only the one involving the Collins function is adjustable), kinematic dependencies and normalization**
    - **generated values: cross-section is available for end-user, however, moments are not available**

$\sin\phi_S$ - term in A_UT

$$-\mathcal{I}\left[\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left(xh_T H_1^{\perp} - xh_T^{\perp} H_1^{\perp} - \frac{M_h}{M} g_{1T}\frac{\tilde{G}^{\perp}}{z} + \frac{M_h}{M} f_{1T}^{\perp}\frac{\tilde{D}^{\perp}}{z}\right)\right]$$

$$-h_1 + \tilde{h}_T - \frac{p_T^2}{2M^2x} h_{1T}^{\perp}$$

$$h_1 - \tilde{h}_T^{\perp} - \frac{p_T^2}{2M^2x} h_{1T}^{\perp}$$

$$\left[-2h_1 + (\tilde{h}_T - \tilde{h}_T^{\perp})\right]$$

$\sin\phi_S$ - term in A_UT

$$2(2-y)\sqrt{1-y}\,\frac{M}{Q}\,\mathcal{I}\left[\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[2h_1+x(\tilde{h}_T^{\perp}-\tilde{h}_T)\right]H_1^{\perp}-\frac{M_h}{zM}\,h_1\tilde{H}\right]$$

convolution integral and
DF & FF known/implemented

twist-3 FF
(Koike?)
(turned off in
gmc_trans)

Cahn Effect

- similar to twist-3 AUT term
- reduced to product of f1 and D1
- neglect any interaction-dependent terms
- get f1 and D1 from PDF/FF library
- however,
  - need additional scaling factor, and
  - need different Gaussian width than normal f1 and D1

The Twist-3 Problems

- need scaling factors, even though Mulders&Tangerman etc. would suggest a normalization that is fixed by involved PDFs and FFs
- same is true for Gaussian widths: need different ones than for "normal" f1 and D1 (or transversity and Collins FF)
- therefore: intrinsicly inconsistent treatment -- failure of Gaussian Ansatz?
- in general: encountered severe positivity violations (in particular with Cahn effect)

Summary

- gmc_trans is a working MC generator for TMDs in SIDIS (pion production)
- based on Gaussian Ansatz for transverse momentum dependencies
- Collins and Sivers effect implemented
- z-dependence of Gaussian widths tuned to HERMES data
- implementation of other (partially subleading-twist) terms not straight-forward

Outlook / Wishlist

- finish Boer-Mulders implemention
- implement newest results from fits and model calculations on transversity, Sivers & Collins functions
- implement Kaons and neutron target
  - ➡ comparison with HERMES and COMPASS data possible
- add radiative corrections (RADGEN)
- solve twist-3 problems
- gmc_trans for 2-hadron production