Transversity: present and future

Alessandro Bacchetta

Outline

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- Overview of experimental possibilities

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- First extraction from SIDIS

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- New SIDIS data

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- Not much on TMD functions

Warning

In the talk, I'll make use of three notations for transversity:

h_1^q $\quad$ δq $\quad$ $\Delta_T q$

Overview of experimental possibilities

Where to observe it: SIDIS

Process	Experiment	Observable	Grade
$l\,p^{\uparrow} \to l\,\pi\,X$	Hermes, Compass, JLab, EIC	$h_1 \otimes H_1^{\perp}$	★★★★
		$h_1\,\tilde{H}$	★★
$l\,p^{\uparrow} \to l\,(\pi\pi)\,X$	Hermes, Compass, JLab@12GeV, EIC	$h_1\,H_1^{\sphericalangle}$	★★★
$l\,p^{\uparrow} \to l\,\Lambda\,X$	Hermes, Compass, JLab@12GeV, EIC	$h_1\,H_1$	★

Where to observe it: SIDIS

CAVEAT: grades reflect my personal opinion! It would be nice to discuss them together…

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$l\,p^{\uparrow} \to l\,\Lambda\,X$	Hermes, Compass, JLab@12GeV, EIC	$h_1\,H_1$	★

Talks by Miller, Bradamante, Grosse-Perdekamp, Jiang

Hadron-hadron scattering

	Process	Experiment	Observable	Grade
Doubly polarized	$p^{\uparrow}(p/\bar{p})^{\uparrow} \to l\bar{l}\,X$	Rhic	$h_1\,\bar{h}_1$	★
		JParc		★★★
		Pax		★★★★★
	$p^{\uparrow}(p/\bar{p})^{\uparrow} \to \pi X$	Rhic	$h_1\,h_1\,D_1$	★★
		JParc		★★★
		Pax		★★★★
Singly polarized	$(p/\bar{p}/\pi)p^{\uparrow} \to (\pi\pi)X$	Rhic, JParc Compass, Panda	$f_1 h_1\,H_1^{\sphericalangle}$	★★★
	$(p/\bar{p}/\pi)p^{\uparrow} \to \Lambda X$	Rhic, JParc Compass, Panda	$f_1 h_1\,H_1$	★
	$(\pi/\bar{p})p^{\uparrow} \to l\bar{l}\,X$	Compass, JParc, Panda	$h_1^{\perp} \otimes h_1$	★★
	$(p/\bar{p}/\pi)p^{\uparrow} \to j(j/\gamma)X$	Rhic, Compass, JParc, Panda	$h_1^{\perp} \otimes h_1$	★★
	$\to \pi(j/\gamma)X$		$f_1 \otimes h_1 \otimes H_1^{\perp}$	★
	$\to (\pi/j/\gamma)X$		$h_1^{\perp} \otimes h_1 \otimes D_1$	★

Hadron-hadron scattering

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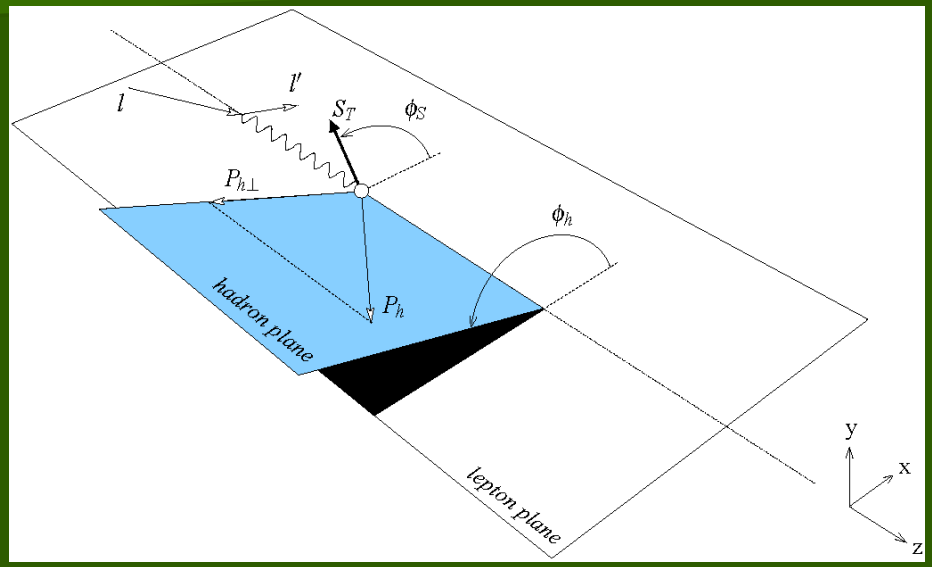
doubly polarized	$p^{\uparrow}(p/\bar{p})^{\uparrow} \to l\bar{l}\,X$	Rhic	$h_1\,\bar{h}_1$	★
		JParc		★★★
		Pax		★★★★★
	$p^{\uparrow}(p/\bar{p})^{\uparrow} \to \pi X$	Rhic	$h_1\,h_1\,D_1$	★★
		JParc		★★★
		Compass, Panda		★★★
Singly polarized	$(p/\bar{p}/\pi)p^{\uparrow} \to \Lambda X$	Rhic, JParc Compass, Panda	$f_1 h_1\,H_1$	★
	$(\pi/\bar{p})p^{\uparrow} \to l\bar{l}\,X$	Compass, JParc, Panda	$h_1^{\perp} \otimes h_1$	★★
	$(p/\bar{p}/\pi)p^{\uparrow} \to j(j/\gamma)X$	Rhic, Compass, JParc, Panda	$h_1^{\perp} \otimes h_1$	★★
	$\to \pi(j/\gamma)X$		$f_1 \otimes h_1 \otimes H_1^{\perp}$	★
	$\to (\pi/j/\gamma)X$		$h_1^{\perp} \otimes h_1 \otimes D_1$	★

Talks by Bland, Grosse-Perdekamp, Goto, Dalpiaz

First extraction from SIDIS

Transversity in SIDIS



$$A_{UT}^{\sin(\phi_h+\phi_S)}(x,y,z,P_{h\perp}^2) = \frac{\frac{(1-y)}{x^2y^2}\,\mathcal{C}\left[-\frac{\vec{k}_T\cdot\vec{P}_{h\perp}}{M_h\left|\vec{P}_{h\perp}\right|}\,h_1\,H_1^\perp\right]}{\frac{1-y+y^2/2}{x^2y^2}\,\mathcal{C}[f_1D_1]}$$

Convolution

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$$\mathcal{C}[f_1 D_1] = x \sum_q e_q^2 \int \mathrm{d}^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z) f_1^q(x, p_T^2) D_1^q(z, k_T^2)$$

Convolution

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Gaussian Ansatz

$$f_1(x,p_T^2) = f_1(x)\frac{1}{\pi\left\langle \vec{p}_T^{\,2}\right\rangle} e^{-\frac{\vec{p}_T^{\,2}}{\left\langle \vec{p}_T^{\,2}\right\rangle}} \qquad D_1(z,k_T^2) = \frac{D_1(z)}{z^2}\frac{1}{\pi\left\langle \vec{k}_T^{\,2}\right\rangle} e^{-\frac{\vec{k}_T^{\,2}}{\left\langle \vec{k}_T^{\,2}\right\rangle}}$$

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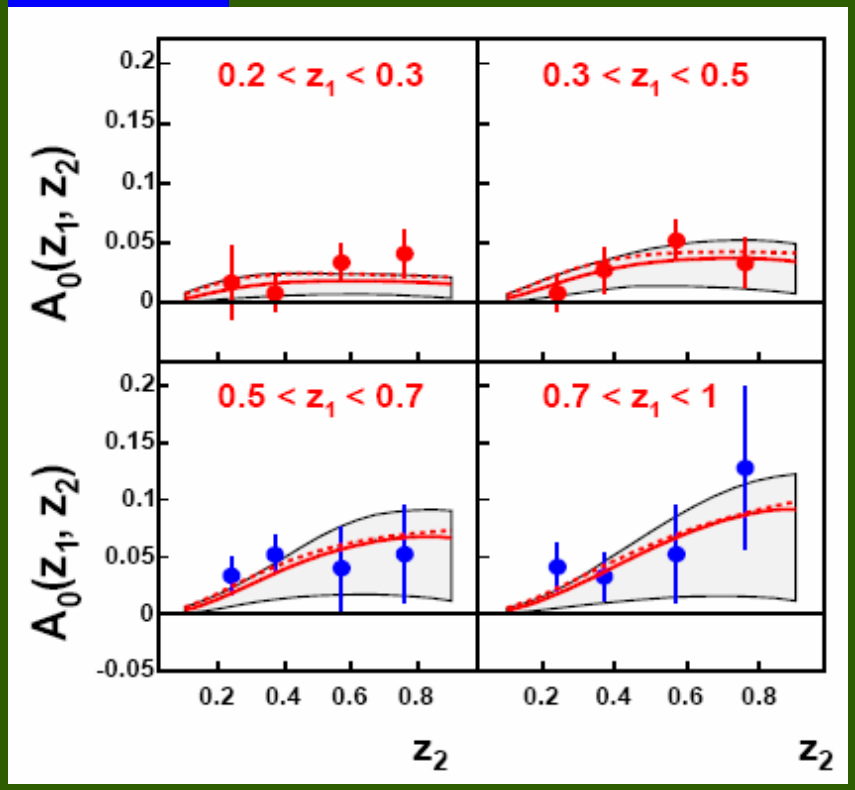
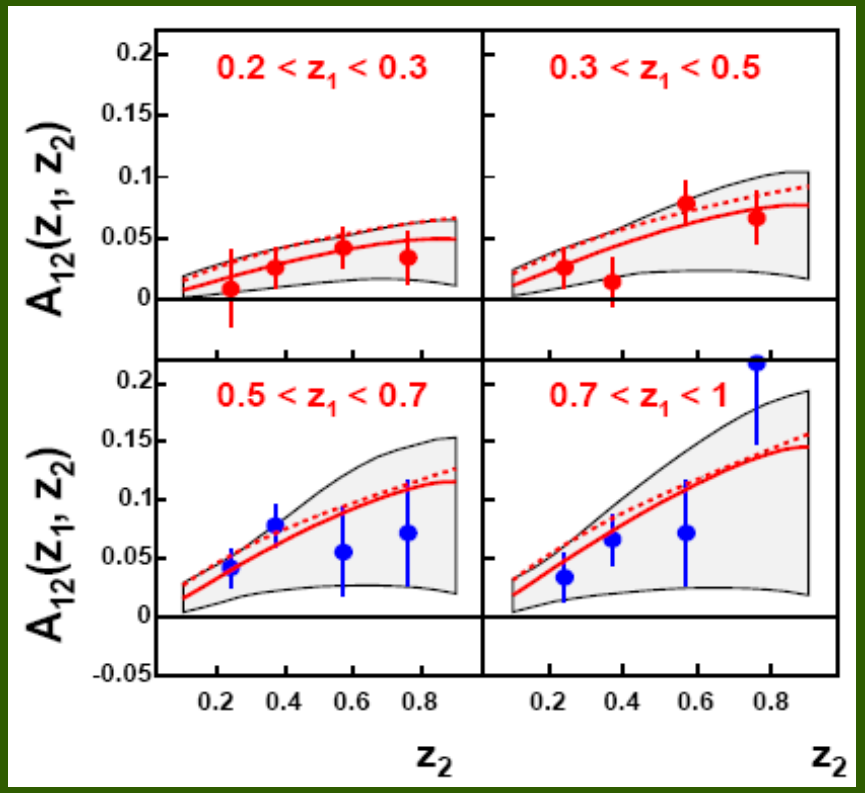
$$\mathcal{C}[f_1 D_1] = x\sum_q e_q^2 f_1^q(x) D_1^q(z) \frac{1}{\pi\left\langle \vec{P}_{h\perp}^{\,2}\right\rangle} e^{-\frac{\vec{P}_{h\perp}^{\,2}}{\left\langle \vec{P}_{h\perp}^{\,2}\right\rangle}}$$

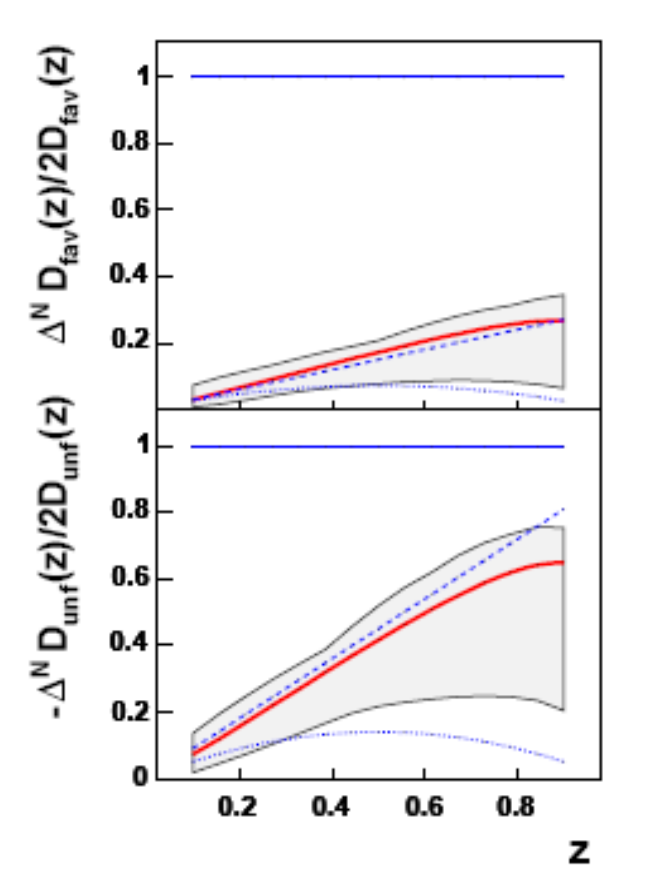
Data from BELLE





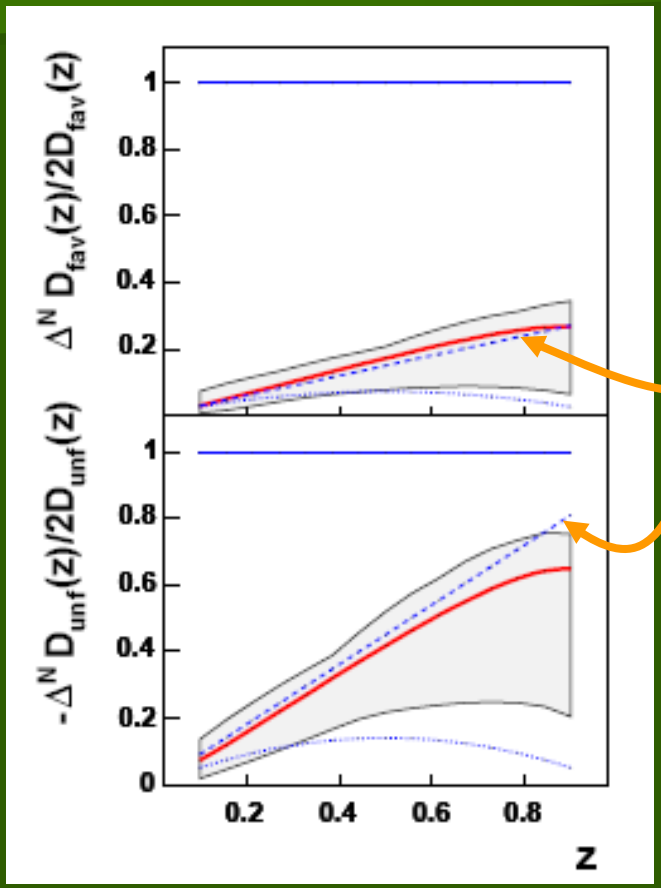
BELLE, PRL 96 (06)

Collins function

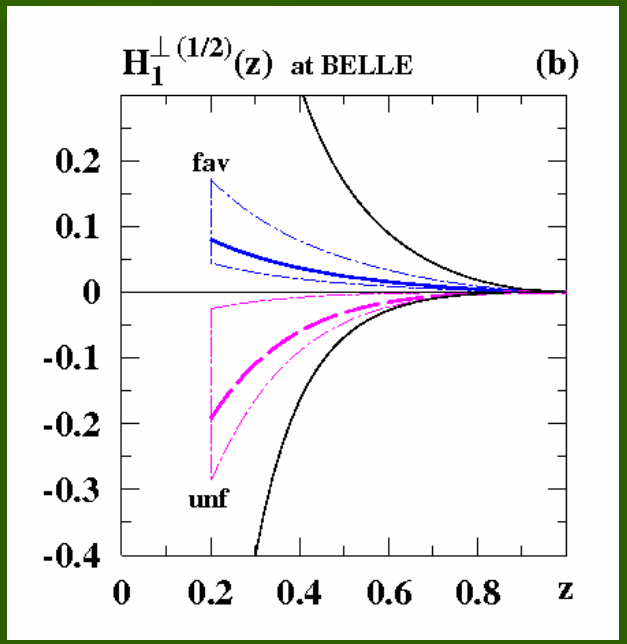


Anselmino et al., PRD 75 (07)

Collins function

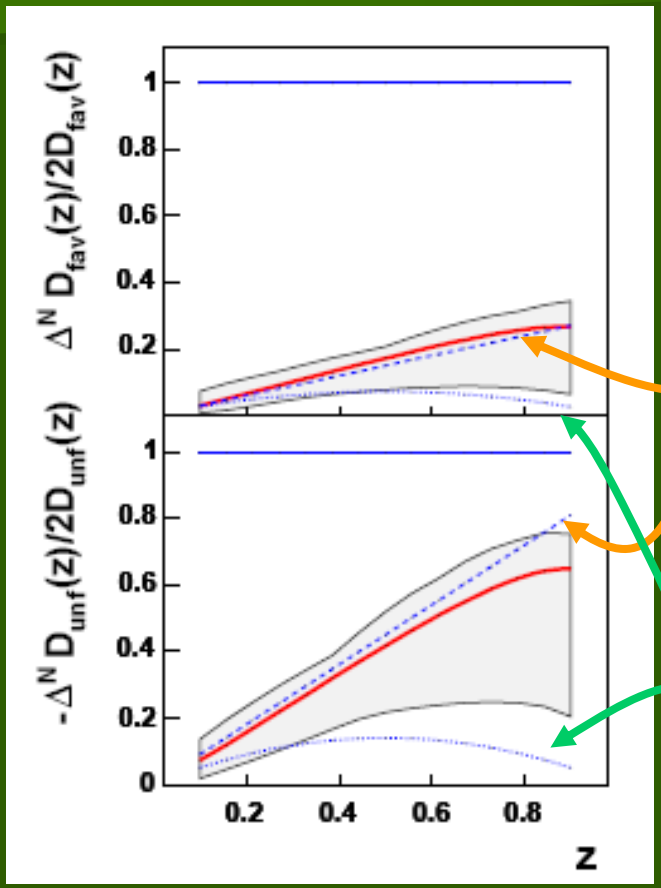


Anselmino et al., PRD 75 (07)

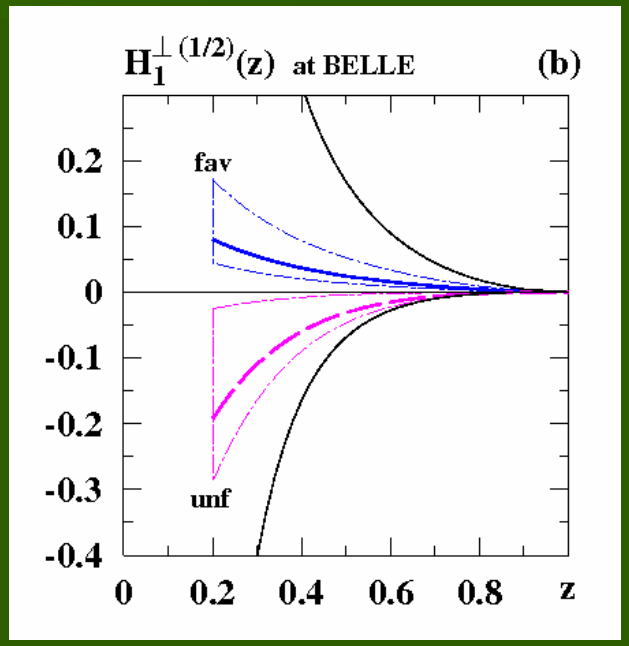


Efremov, Goeke, Schweitzer, PRD 73 (06)

Collins function



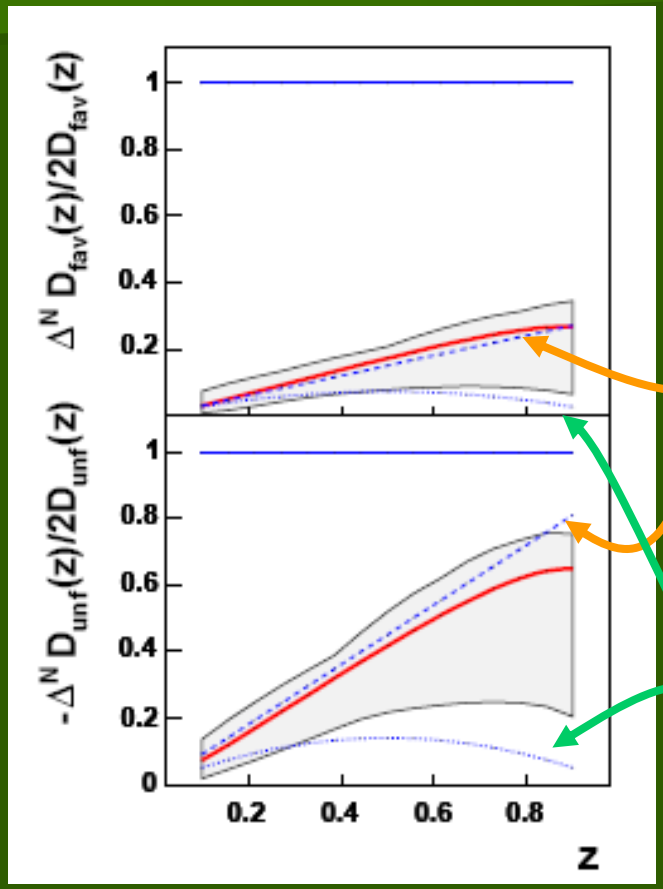
Anselmino et al., PRD 75 (07)



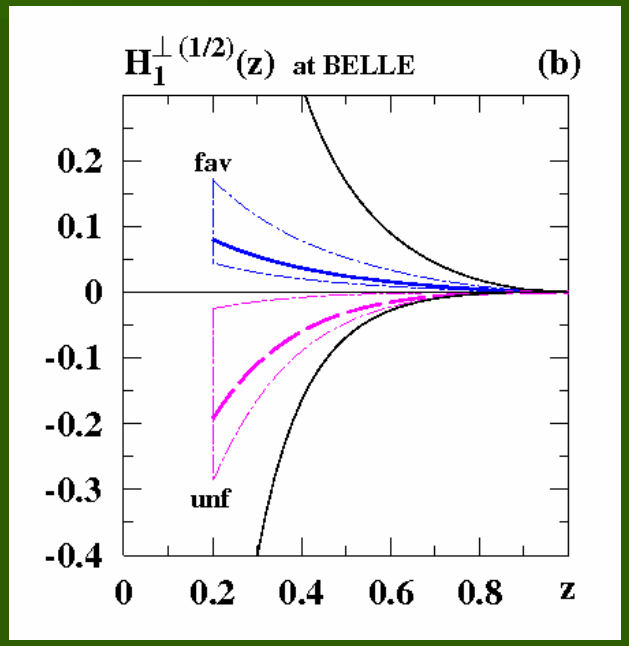
Efremov, Goeke, Schweitzer, PRD 73 (06)

Vogelsang, Yuan, PRD 73 (06)

Collins function



Anselmino et al., PRD 75 (07)

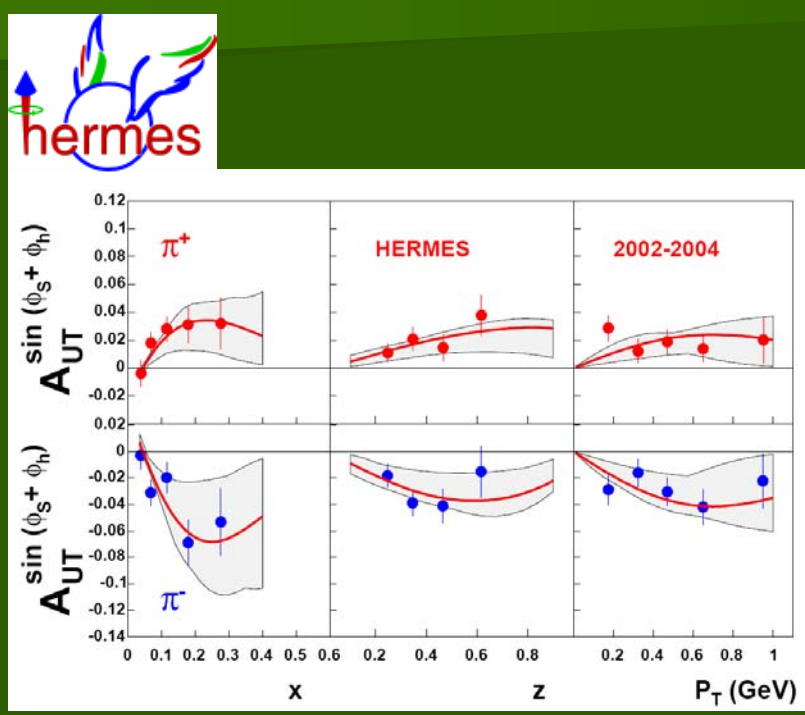


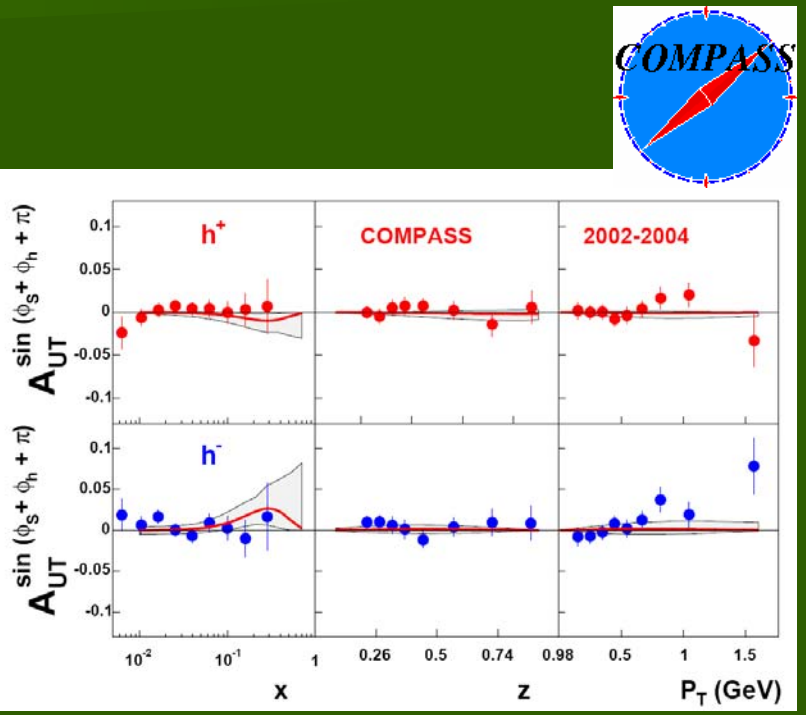
Efremov, Goeke, Schweitzer, PRD 73 (06)

Vogelsang, Yuan, PRD 73 (06)

Talks by Boglione, Efremov

Data from Hermes and Compass





PROTON

HERMES, hep-ex/0507013

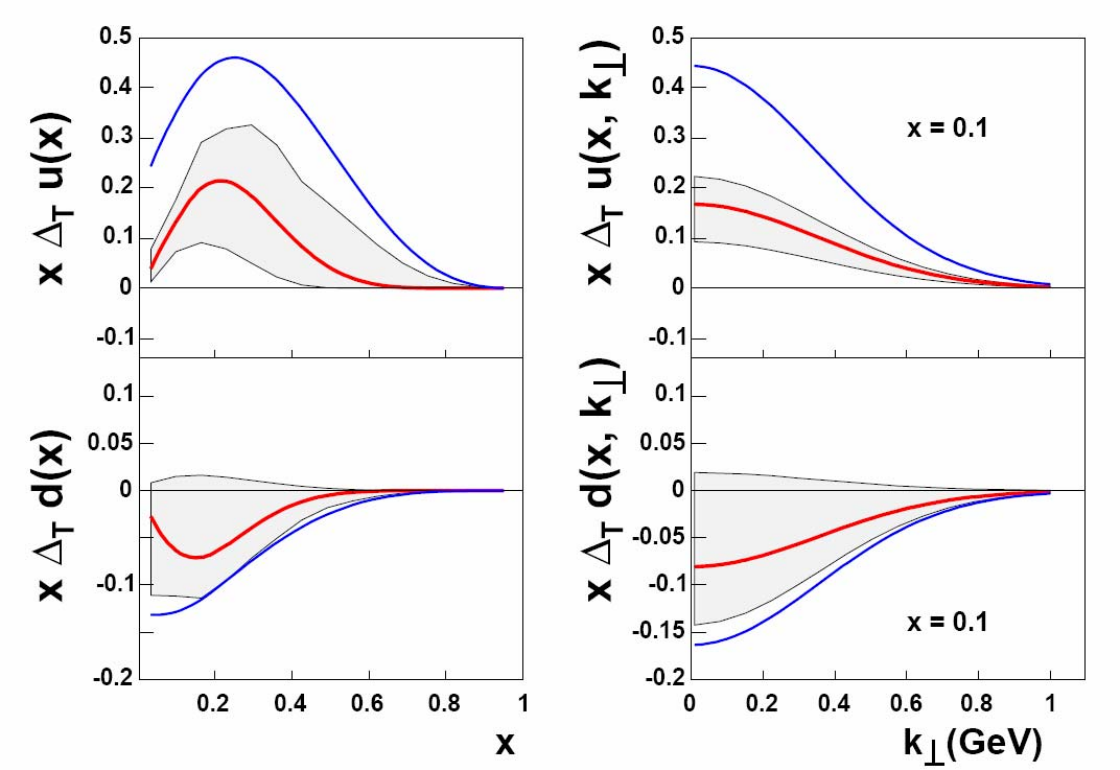
HERMES, PRL 94 (05)

DEUTERON

COMPASS, NPB 765 (07)

COMPASS, PRL 94 (05)

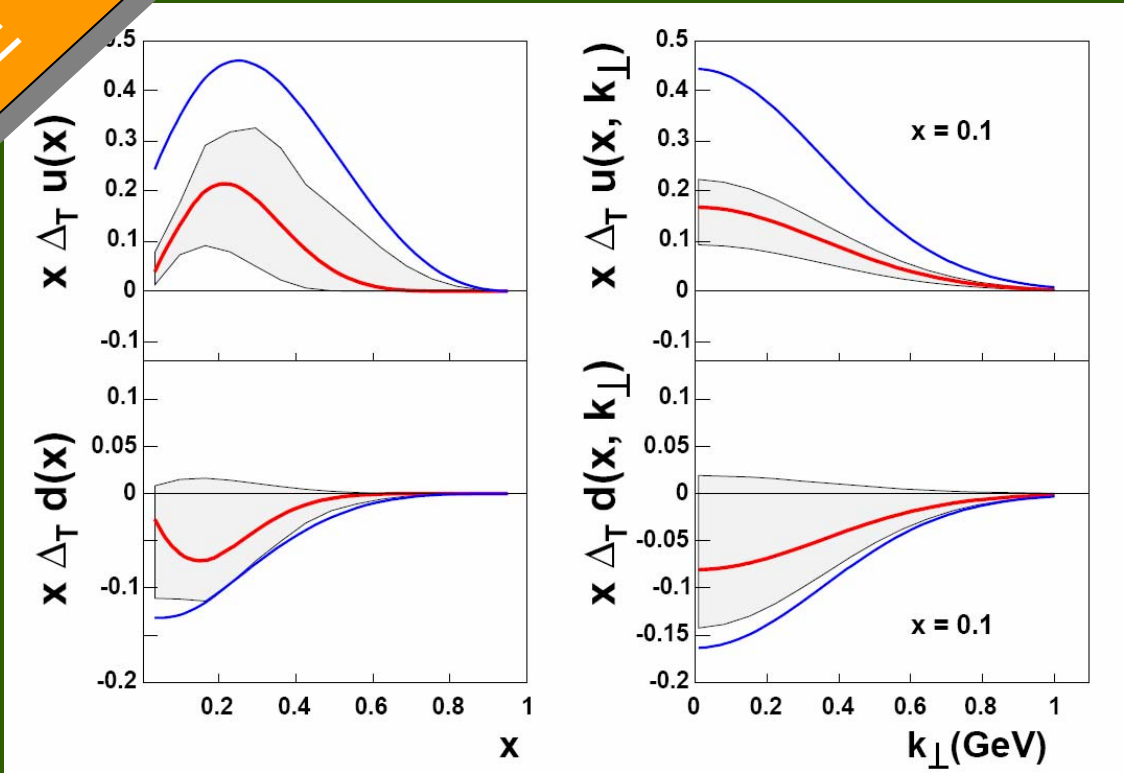
First extraction of transversity



Anselmino et al., PRD 75, 054032 (07)

First extraction of transversity

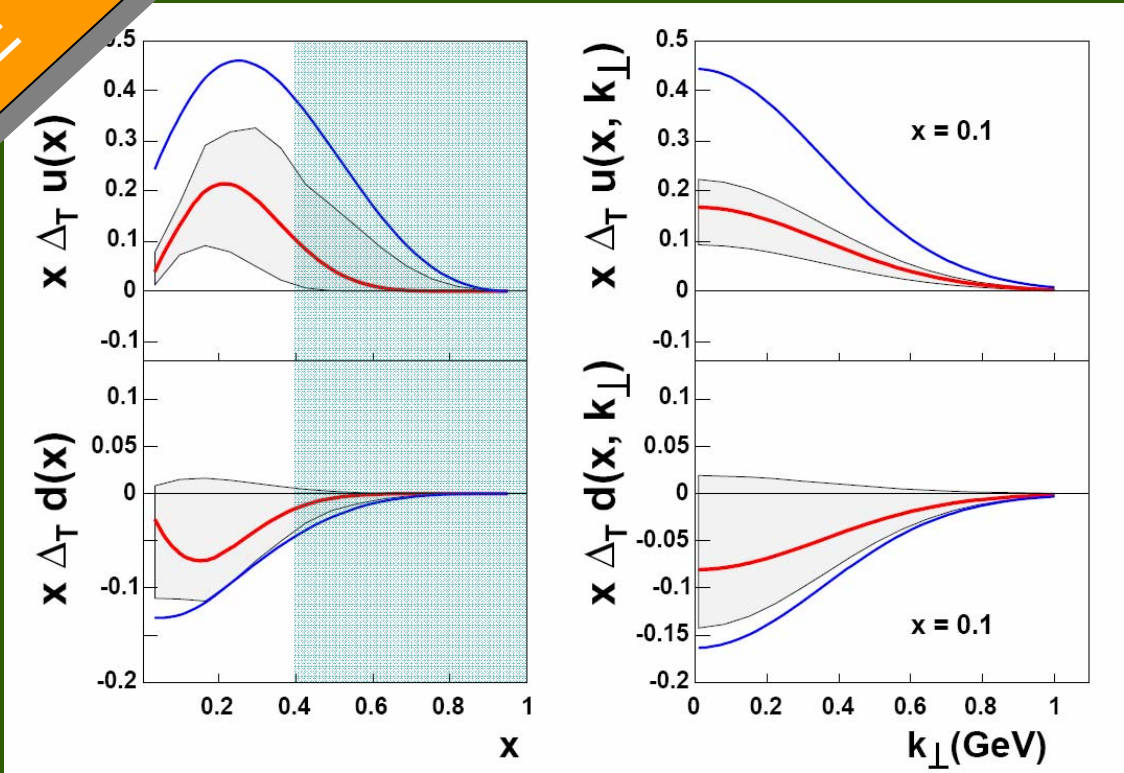
MILESTONE



Anselmino et al., PRD 75, 054032 (07)

First extraction of transversity

MILESTONE



Anselmino et al., PRD 75, 054032 (07)

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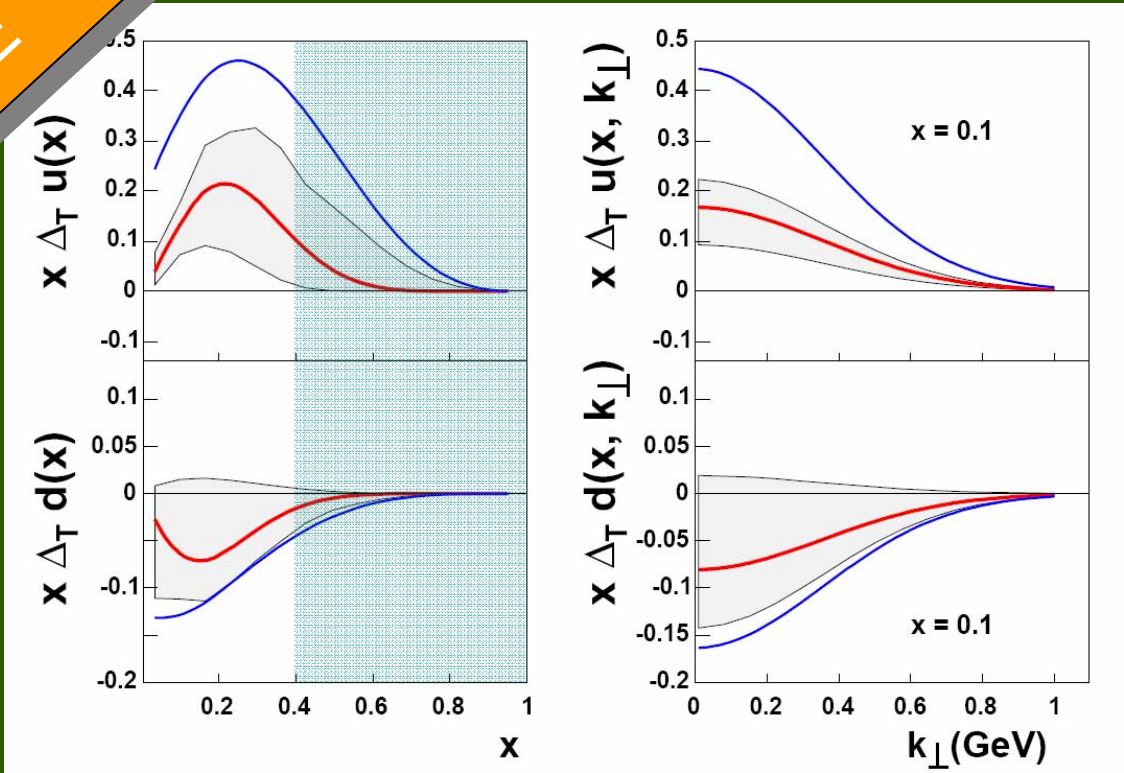
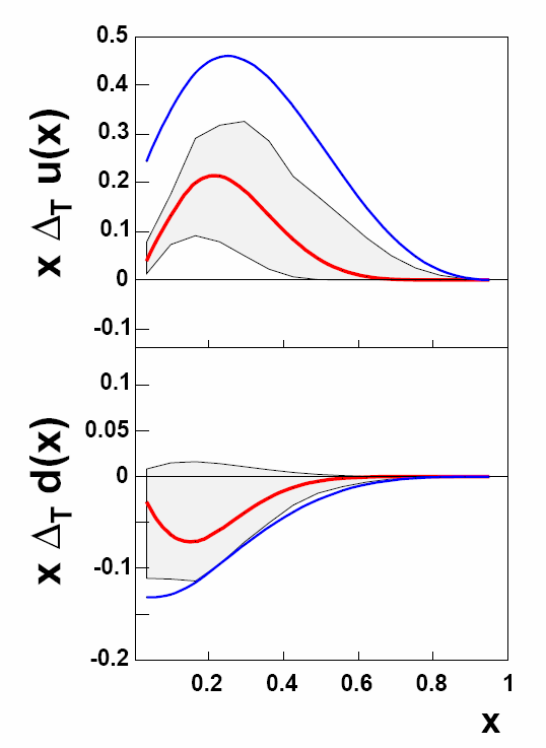
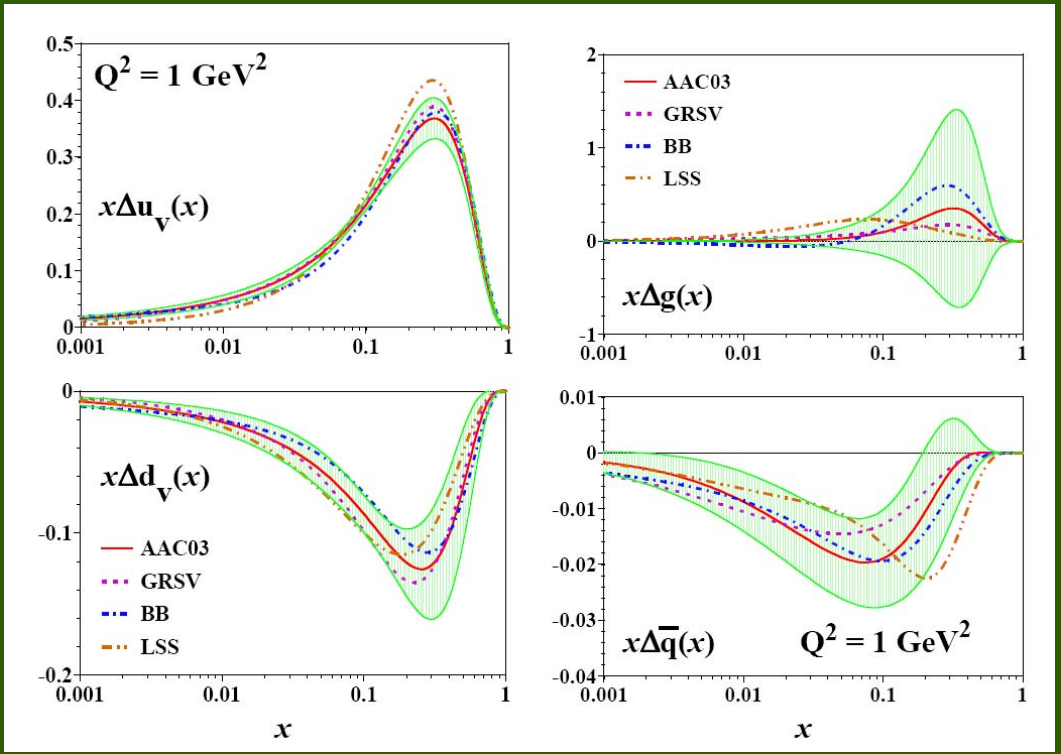


Anselmino et al., PRD 75, 054032 (07)

Talk by Boglione

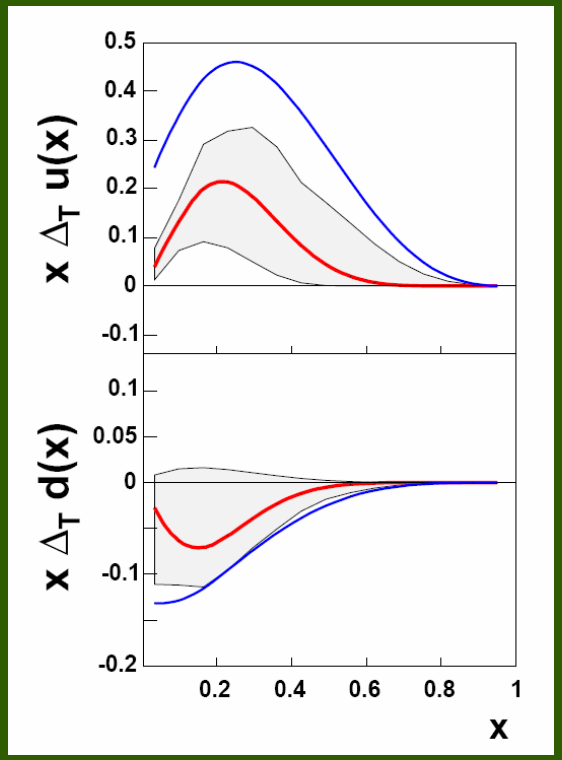
For comparison: helicity distribution

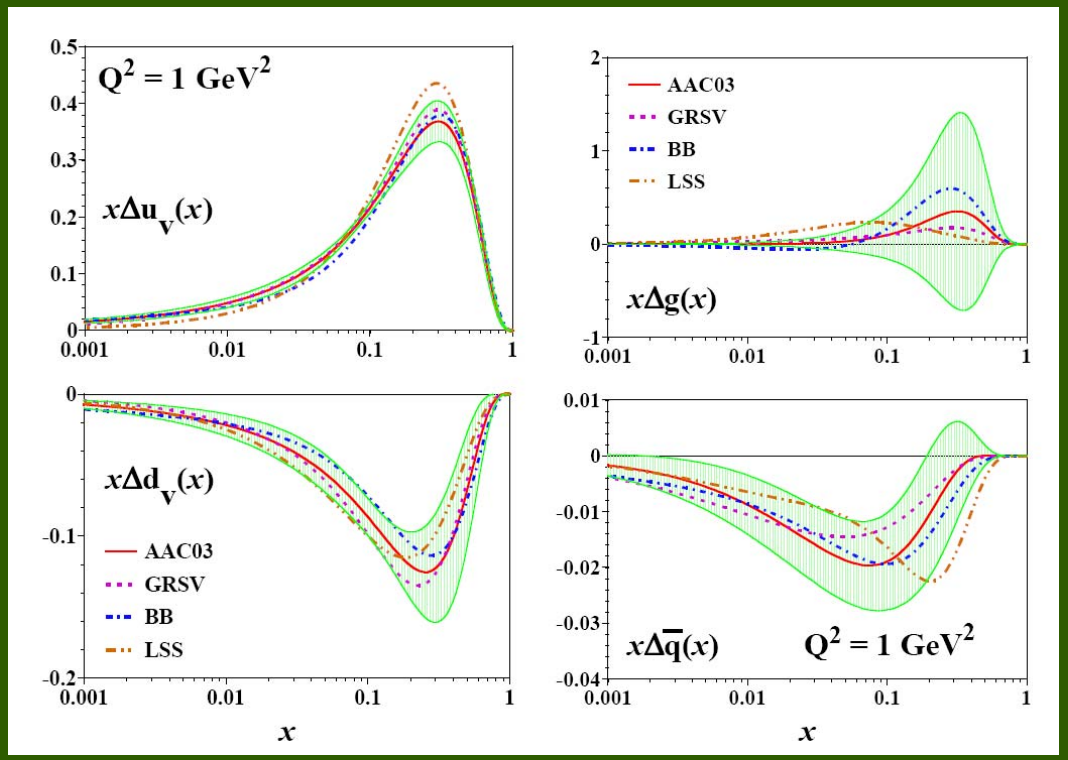




AAC, Hirai et al. PRD 69 (04)

For comparison: helicity distribution

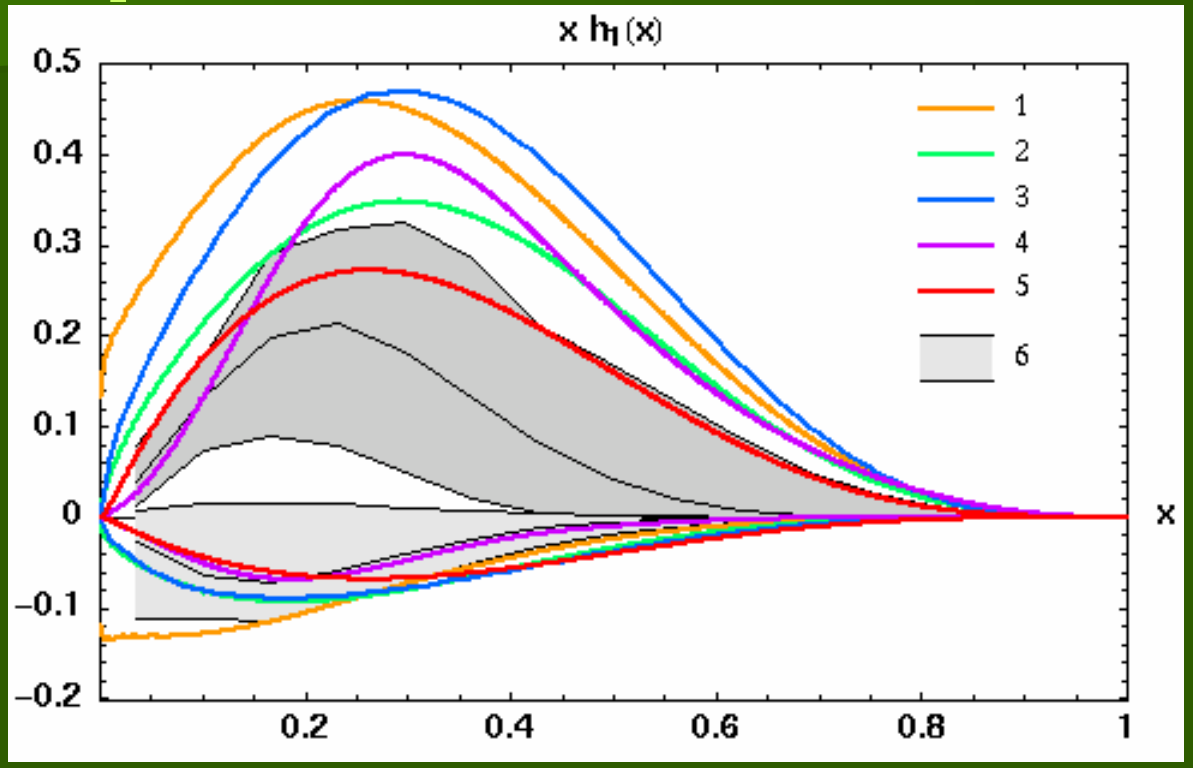




AAC, Hirai et al. PRD 69 (04)

NOTE: treatment of statistical uncertainty?

Comparison with some models



[1] Soffer et al. PRD 65 (02)

[2] Korotkov et al. EPJC 18 (01)

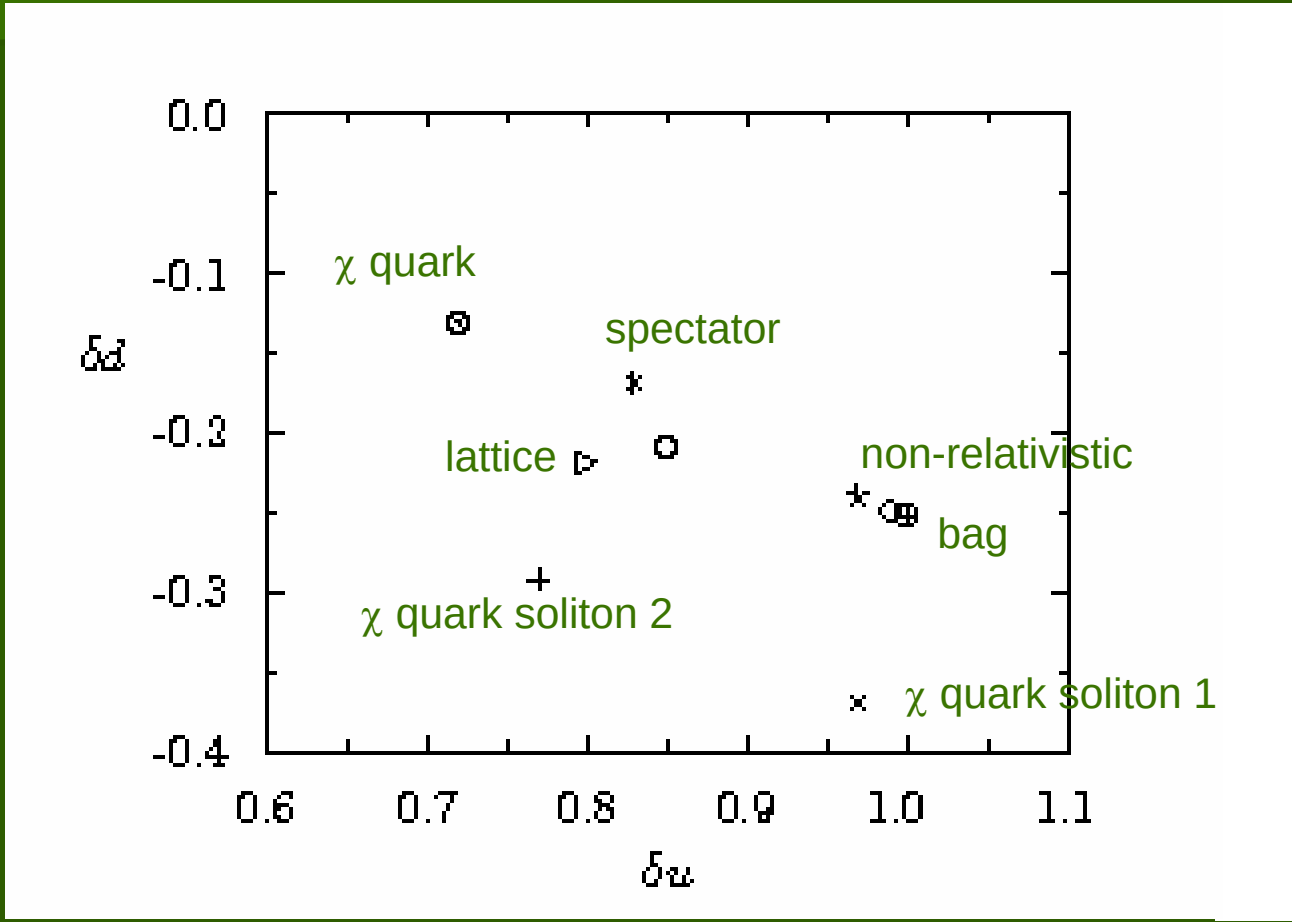
[3] Schweitzer et al., PRD 64 (01)

[4] Wakamatsu, PLB 509 (01)

[5] Pasquini et al., PRD 72 (05)

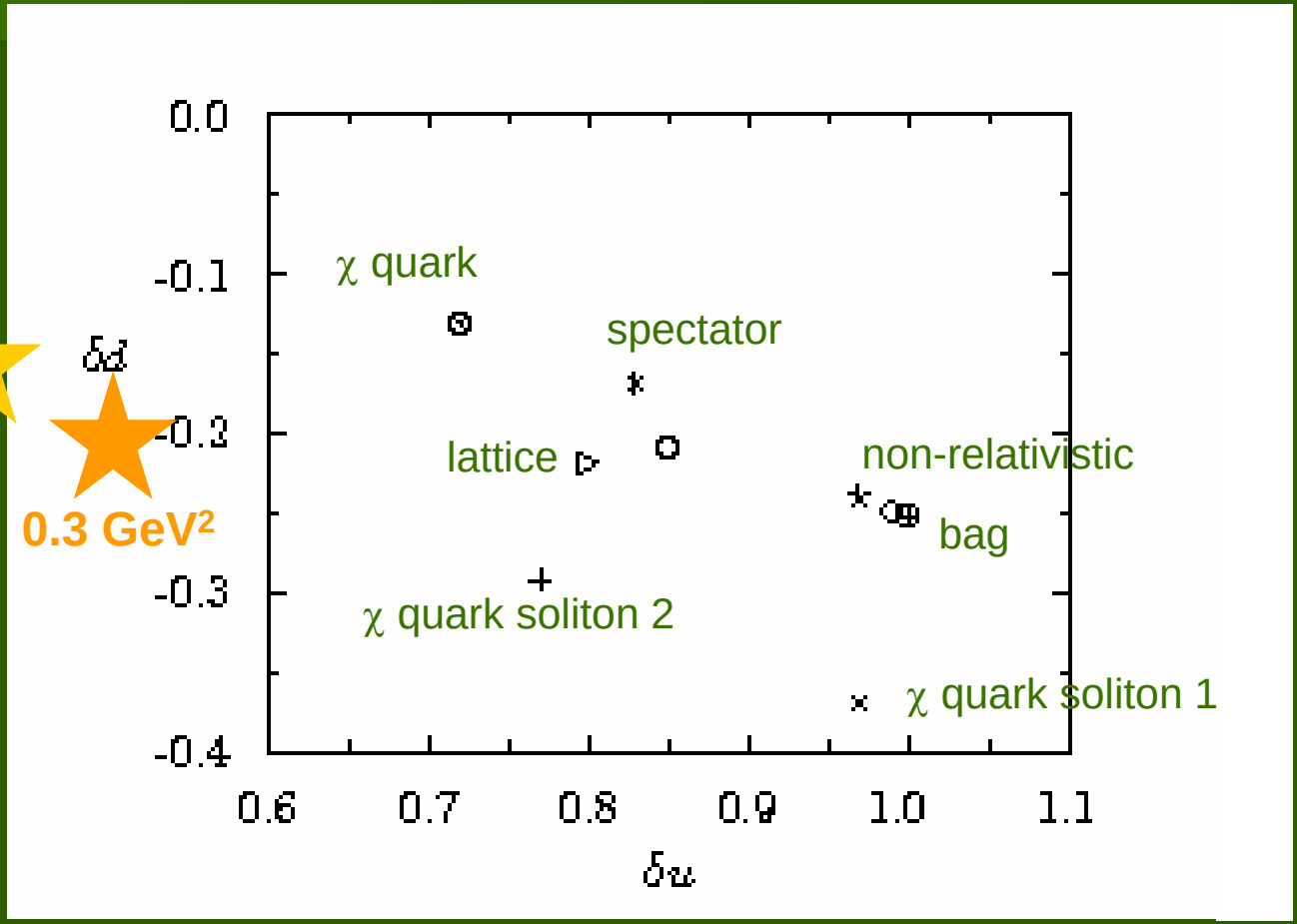
[6] Anselmino et al., PRD 75 (07)

Comparison of tensor charges



Barone, Drago, Ratcliffe, PR 359 (2002)

Comparison of tensor charges



Wakamatsu, 0705.2917[hep-ph]

Barone, Drago, Ratcliffe, PR 359 (2002)

Limits of the extraction

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- Evolution equations for unintegrated distribution function

  see e.g. Ceccopieri, Trentadue, PLB 636 (06)

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Talk by Boer

Limits of the extraction

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  Talk by Boer

- Gaussian transverse-momentum Ansatz

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- Gaussian transverse-momentum Ansatz
- No sea quarks taken into consideration

Limits of the extraction

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  see e.g. Ceccopieri, Trentadue, PLB 636 (06)

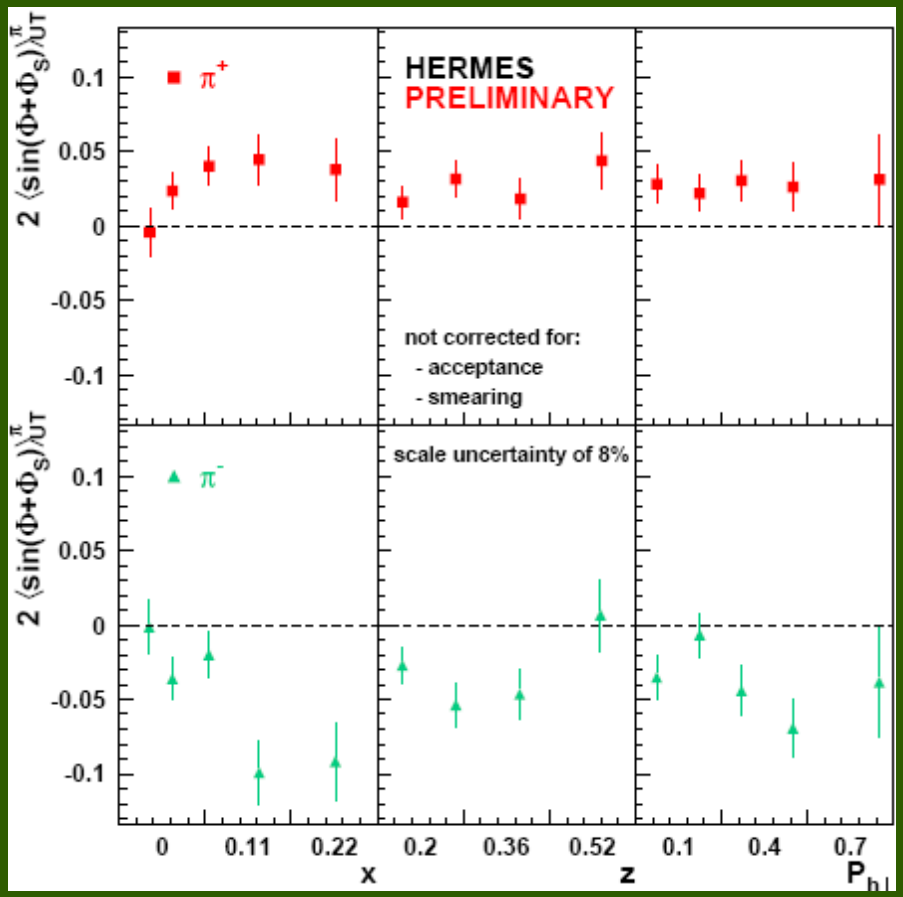
Talk by Boer

- Gaussian transverse-momentum Ansatz
- No sea quarks taken into consideration
- Limited *x*-range

New SIDIS data

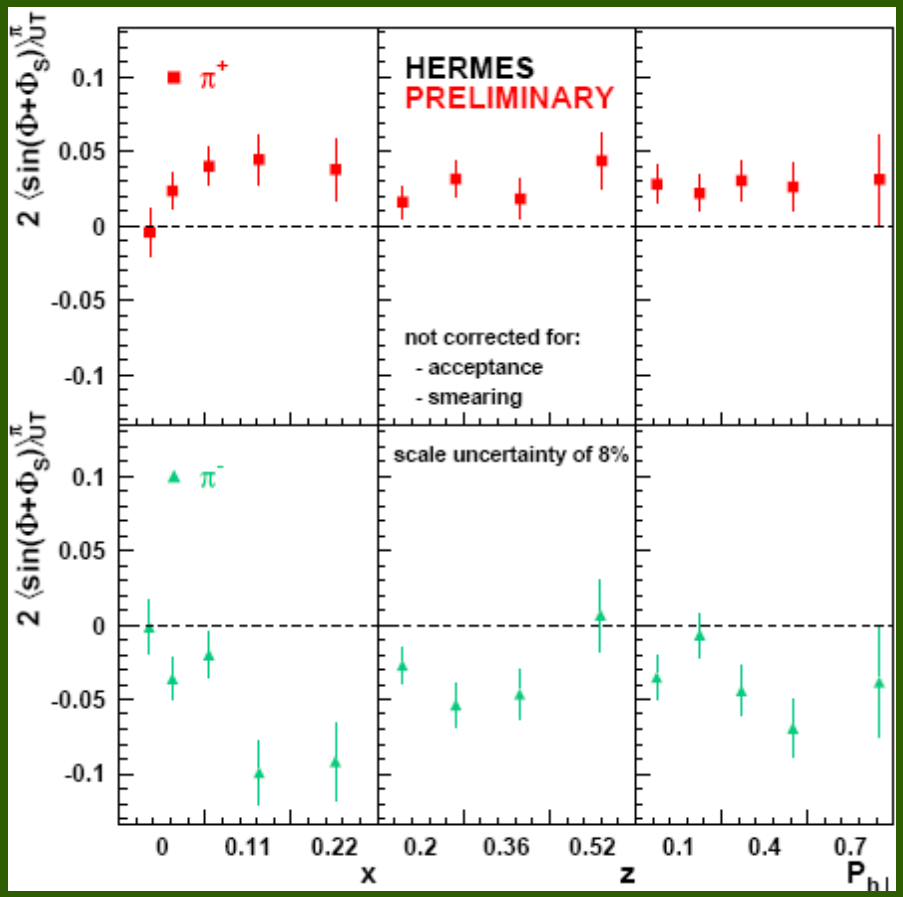
Recent data from HERMES

OLD

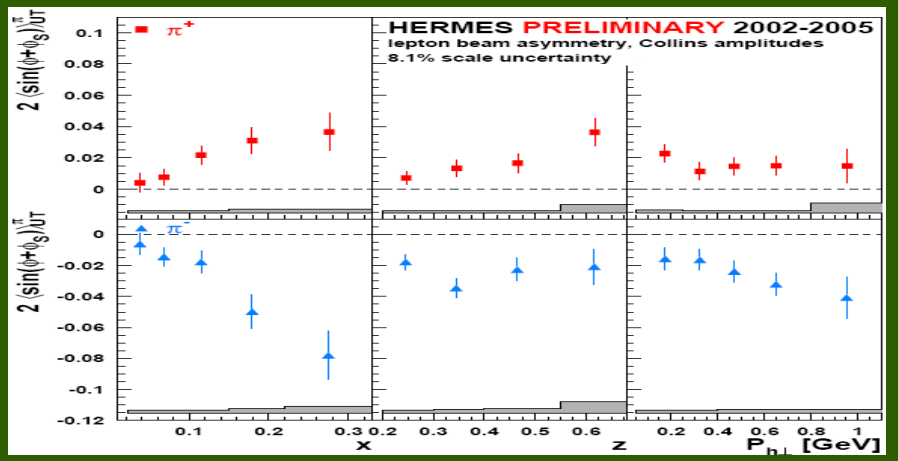


Recent data from HERMES

OLD



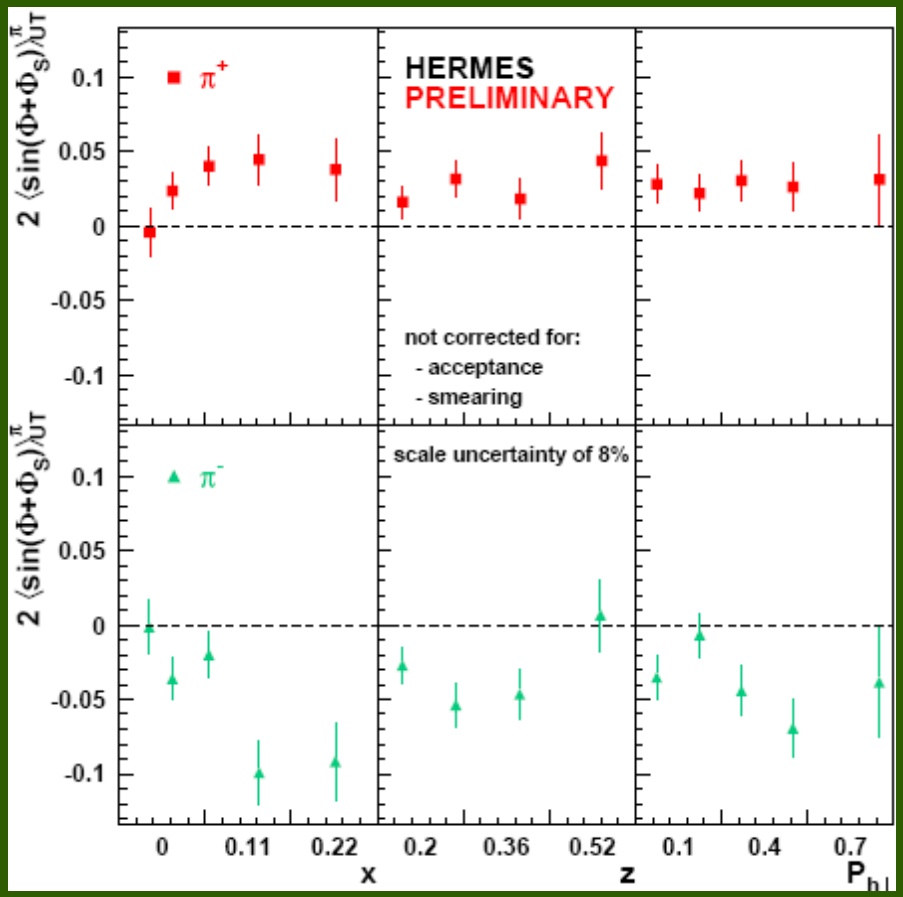
NEW



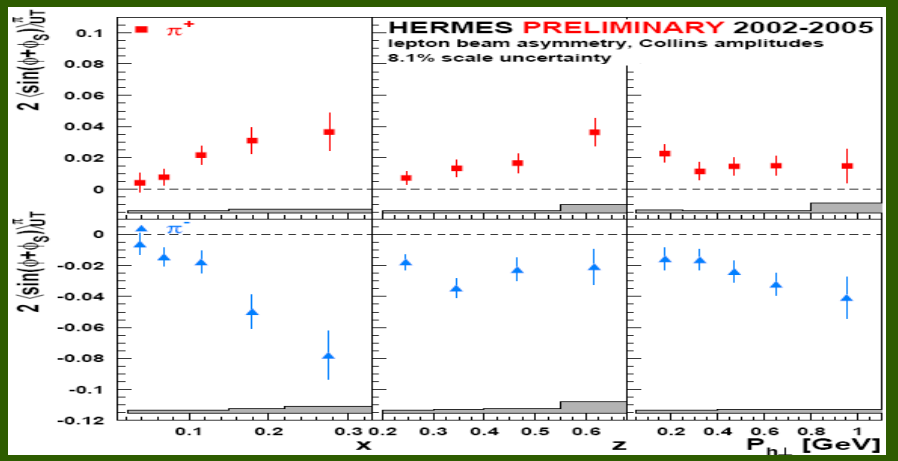
M. Diefenthaler, talk at DIS07

Recent data from HERMES

OLD



NEW

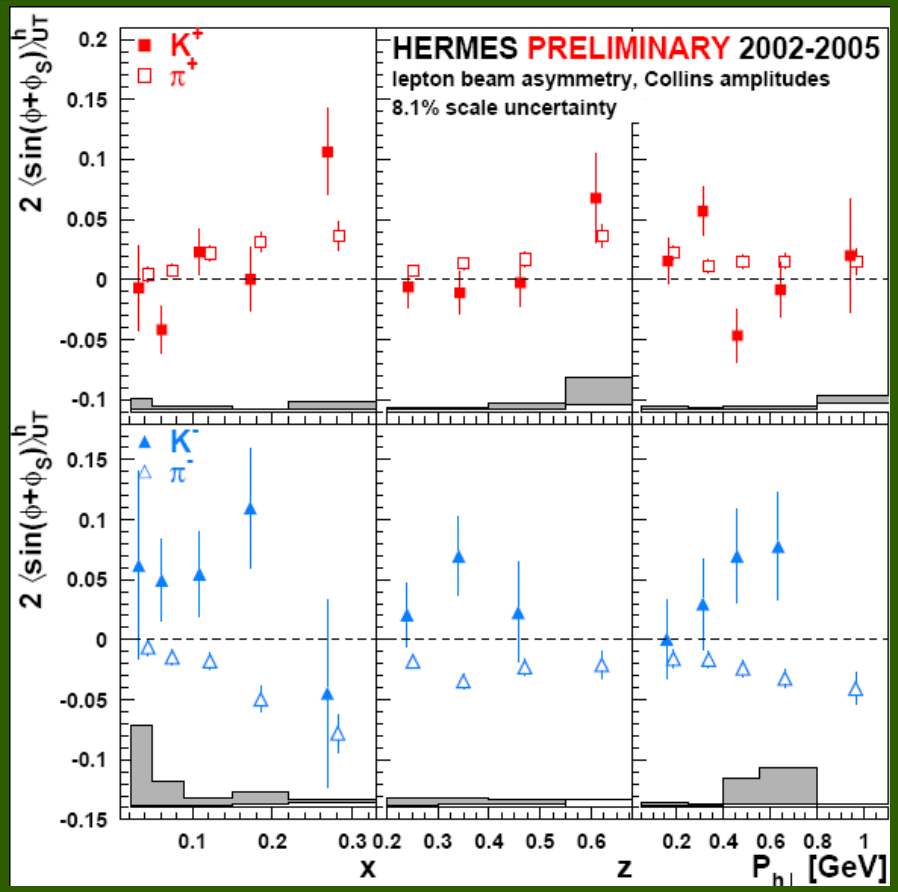


M. Diefenthaler, talk at DIS07

Talk by Miller

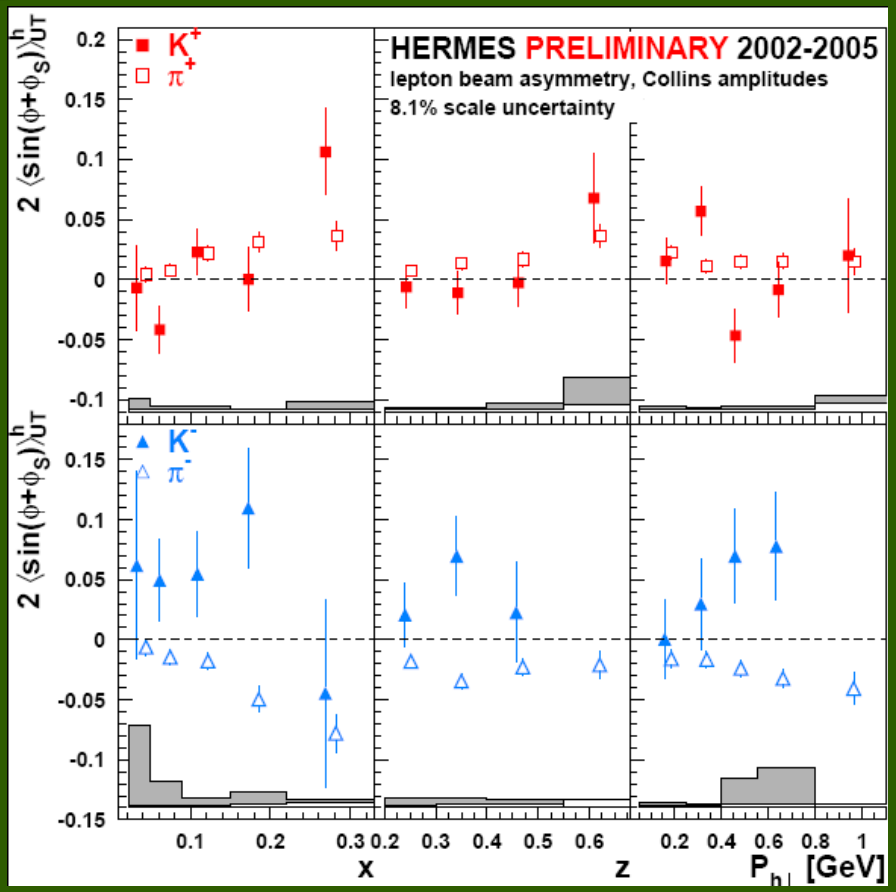
Recent data from HERMES

KAONS

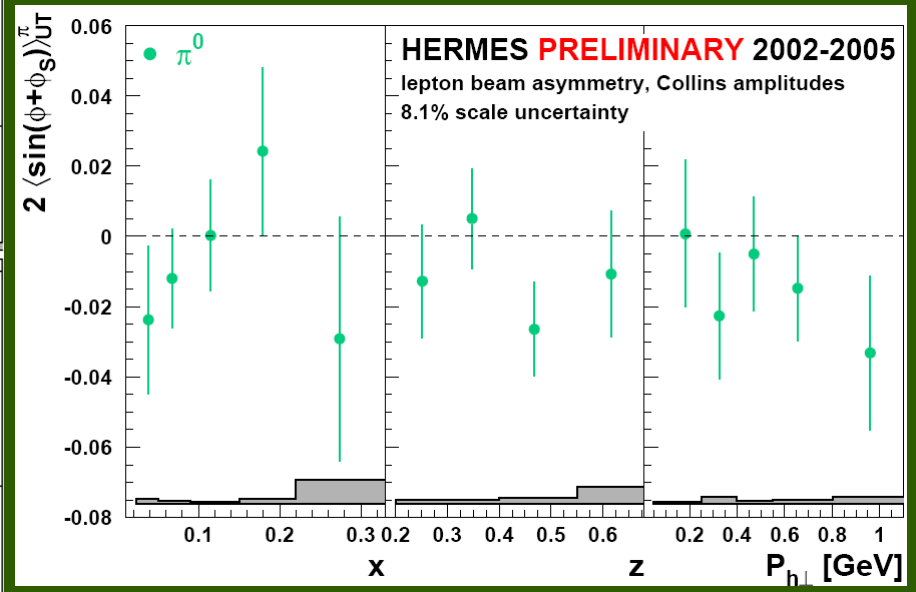


Recent data from HERMES

KAONS



π^0



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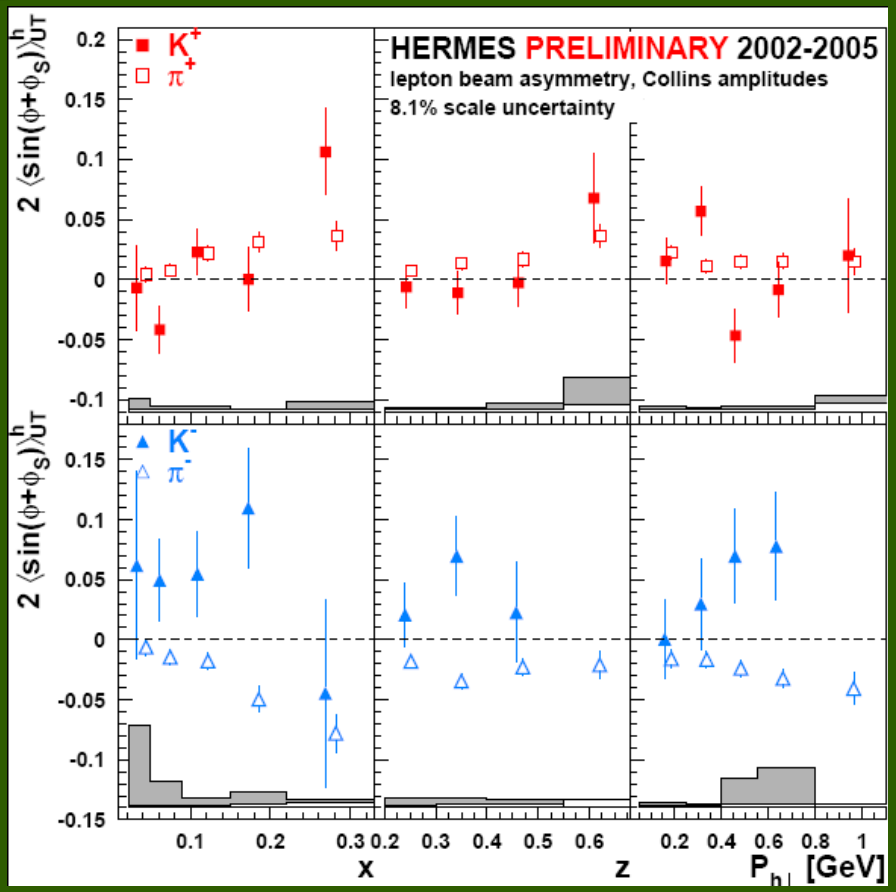
KAONS



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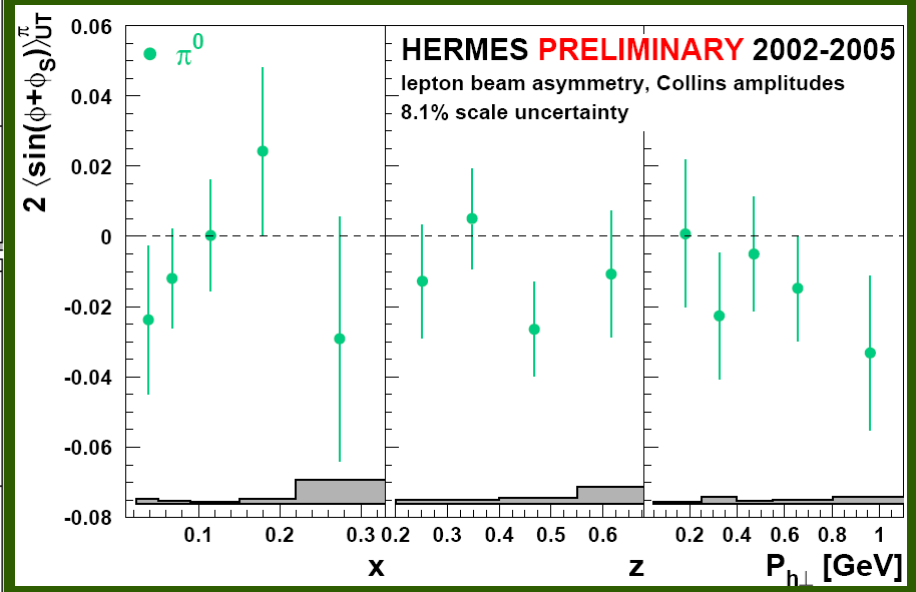
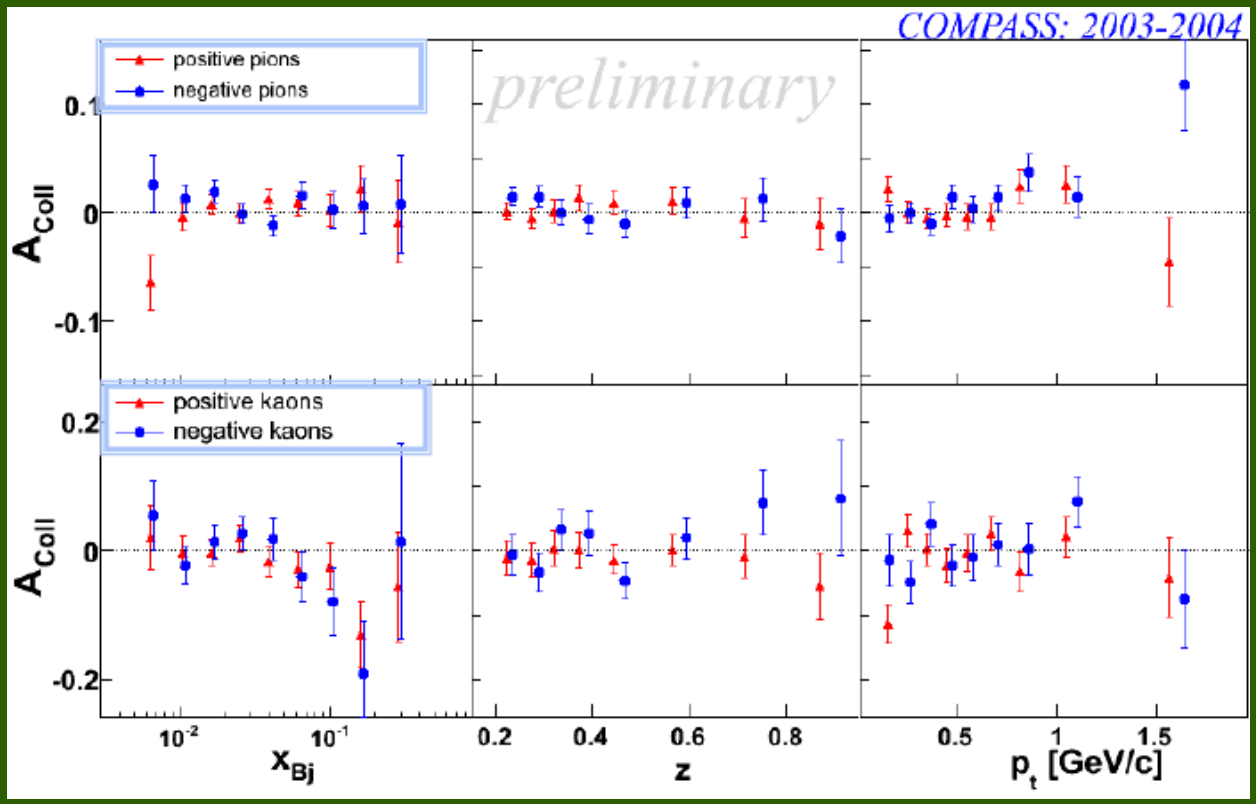


M. Diefenthaler, talk at DIS07

Talk by Miller

Recent data from COMPASS

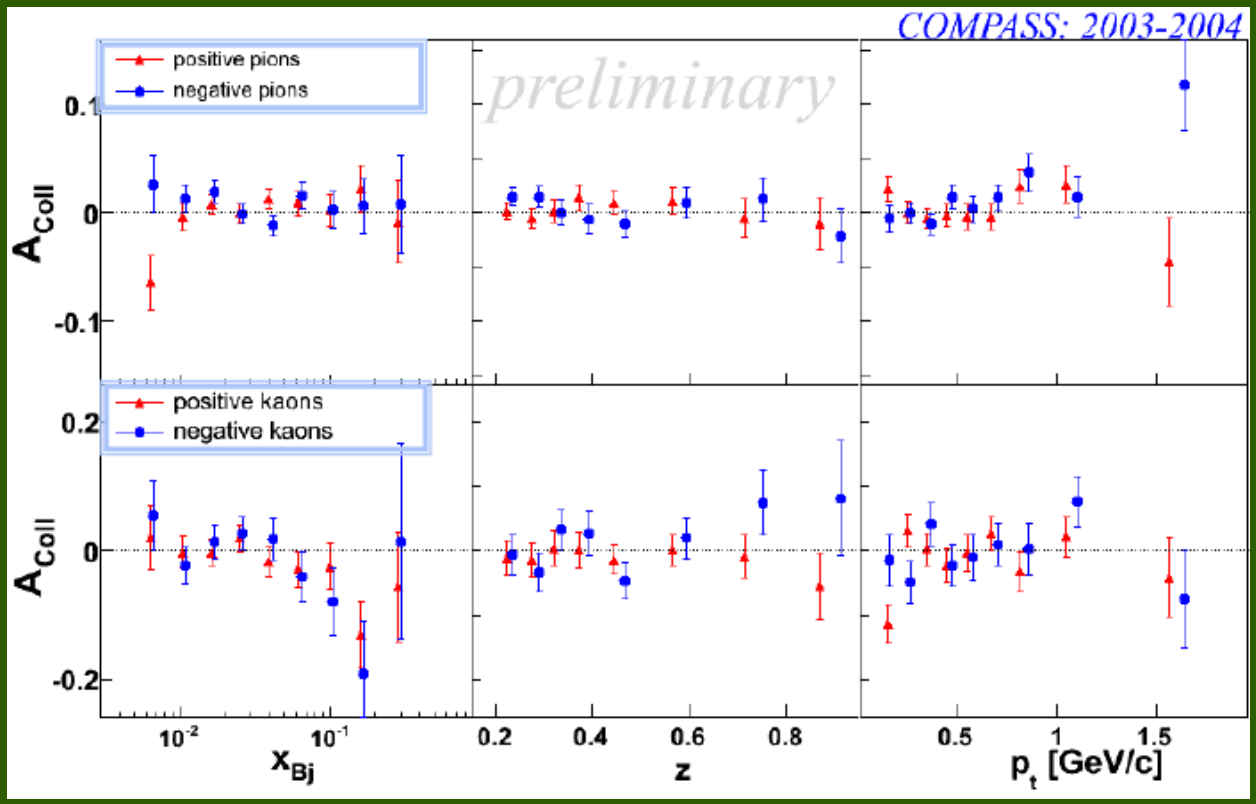
IDENTIFIED CHARGED HADRONS



A. Bressan, talk at DIS07

Recent data from COMPASS

IDENTIFIED CHARGED HADRONS

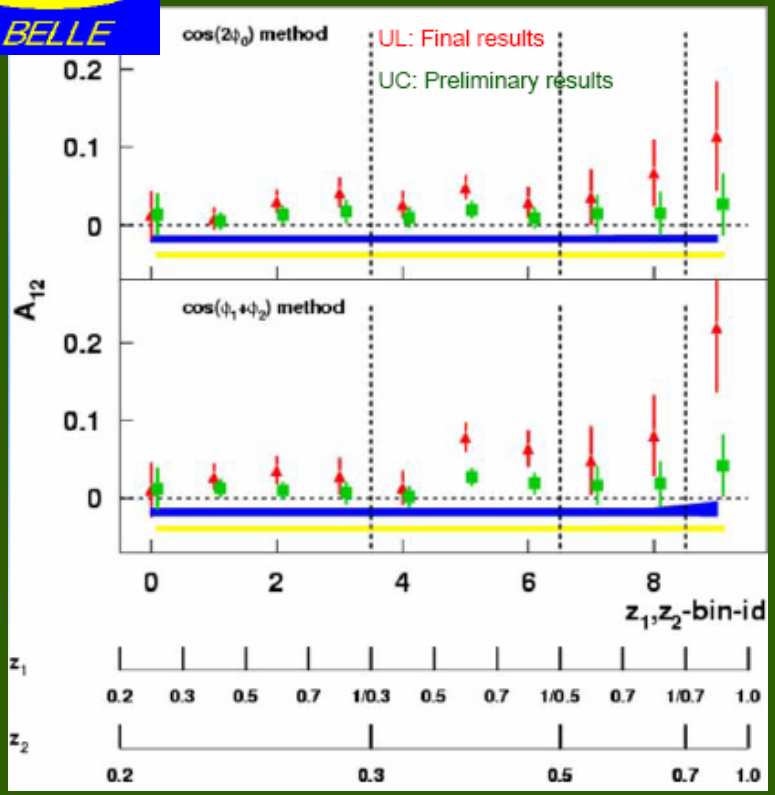


A. Bressan, talk at DIS07

Talk by Bradamante

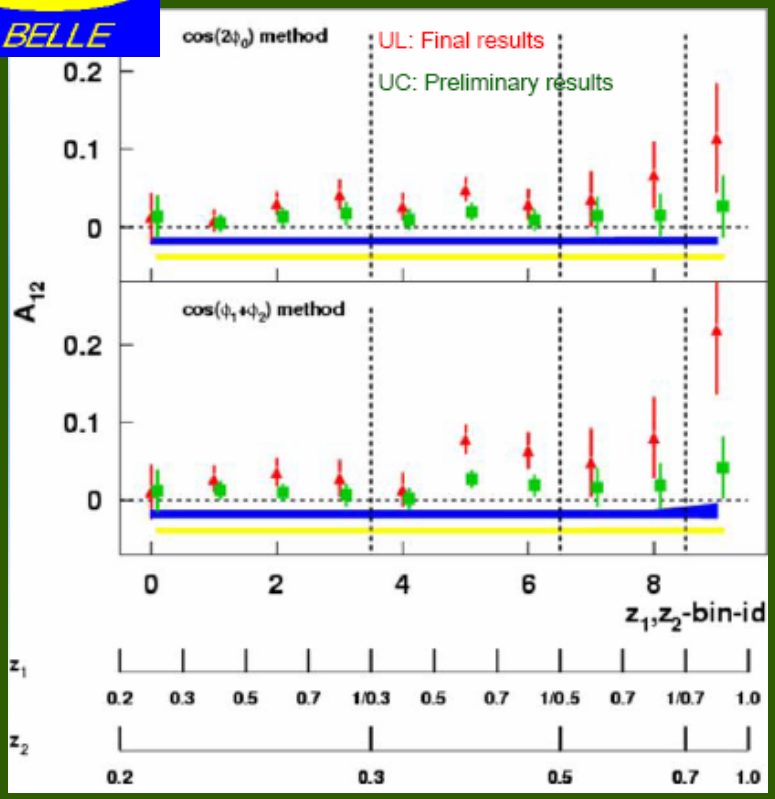
Recent data from BELLE

OLD

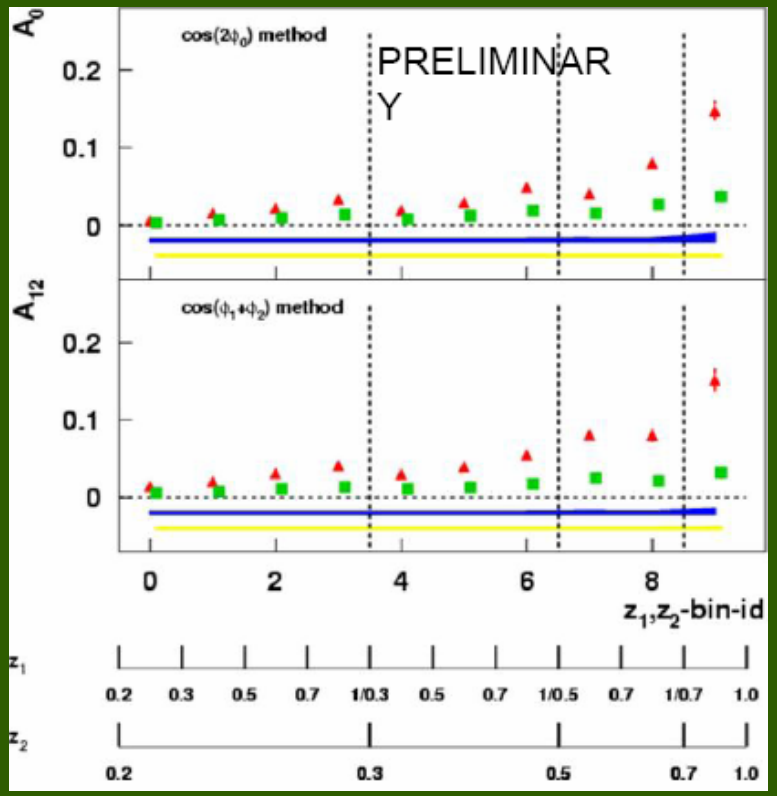


Recent data from BELLE

OLD



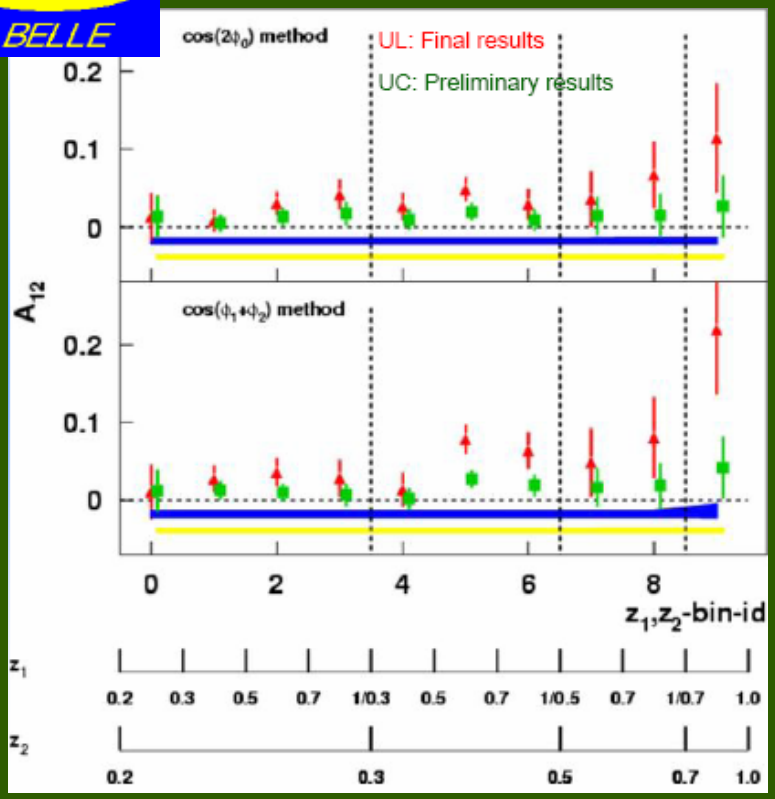
NEW



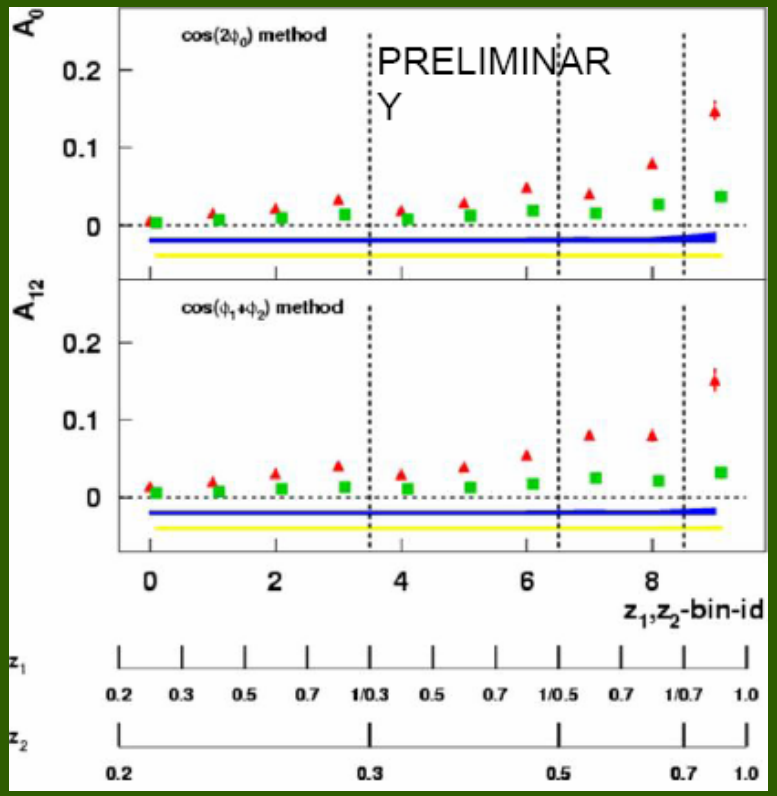
R. Seidl, talk at DIS07

Recent data from BELLE

OLD



NEW



R. Seidl, talk at DIS07

Talk by R. Seidl

Future data from SIDIS

- COMPASS proton data
- JLab HallA neutron data

- JLab@12GeV data

Other observables

Transversity and dihadron FF

Efremov, Mankiewicz, Tornquist, PLB 284 (92)
Collins, Heppelmann, Ladinsky, NPB 420 (94)
Jaffe, Jin, Tang, PRL 80 (98)

$$A_{UT}^{\sin(\phi_S+\phi_R)}(x,y,z,\theta,M_{\pi\pi}^2) = -\frac{\frac{(1-y)}{x^2y^2}\, x\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle}(z,\theta,M_{\pi\pi}^2)}{\frac{1-y+y^2/2}{x^2y^2}\, x\sum_q e_q^2\, f_1(x)\, D_1(z,\theta,M_{\pi\pi}^2)}$$

Theoretical advantages

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- Collinear factorization

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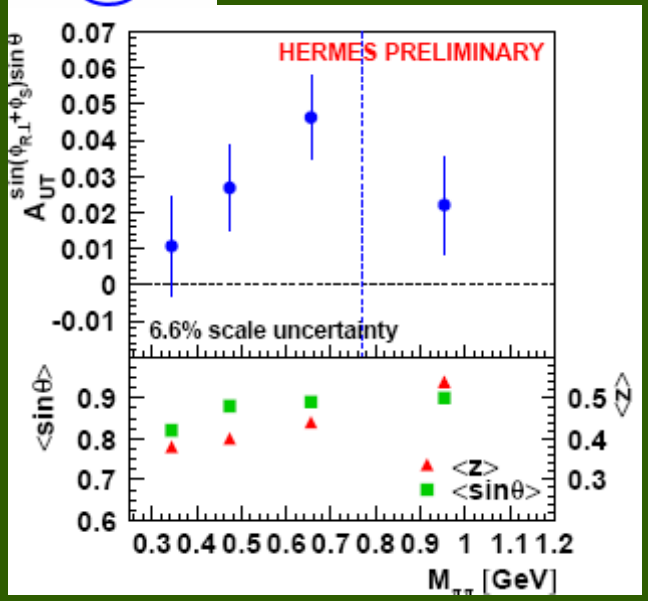
- No convolution in transverse momentum

Theoretical advantages

- Collinear factorization
- No convolution in transverse momentum
- Evolution equations known

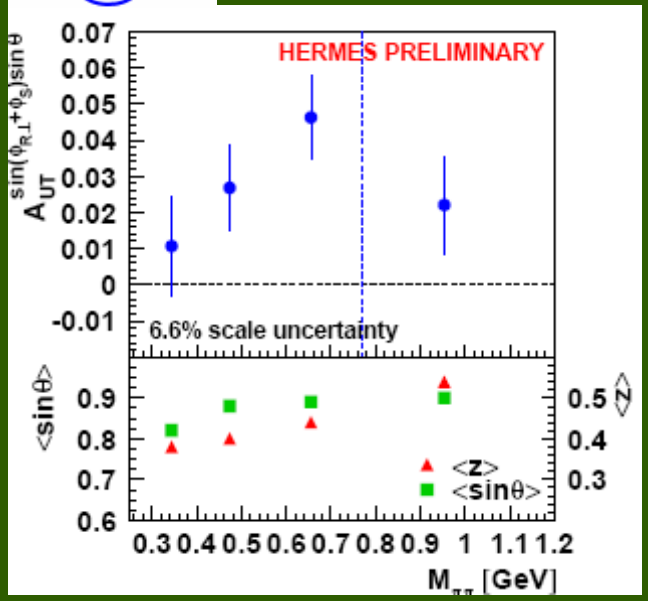
  Ceccopieri, Radici, Bacchetta, PLB 650 (07)

Preliminary data

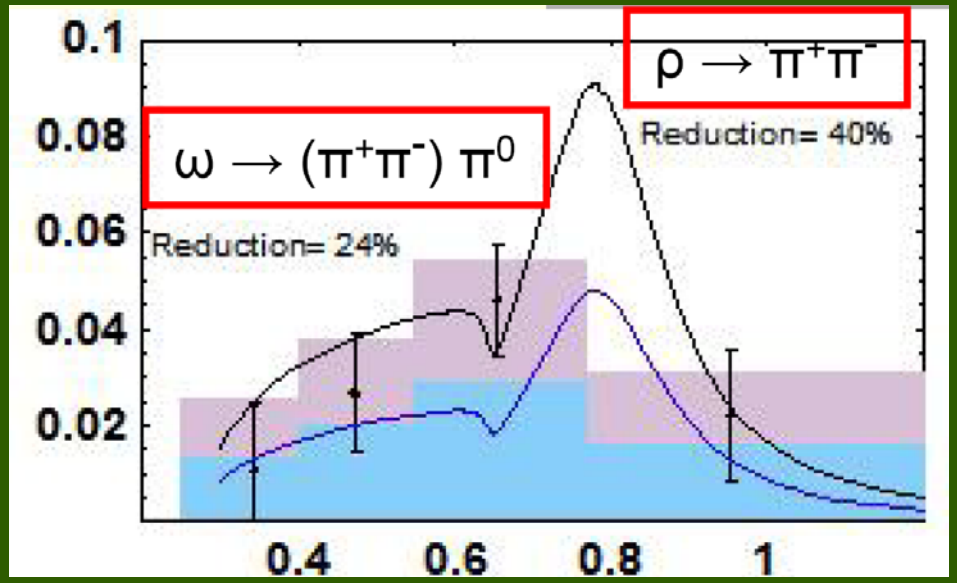


P. van der Nat, hep-ex/0512019

Preliminary data

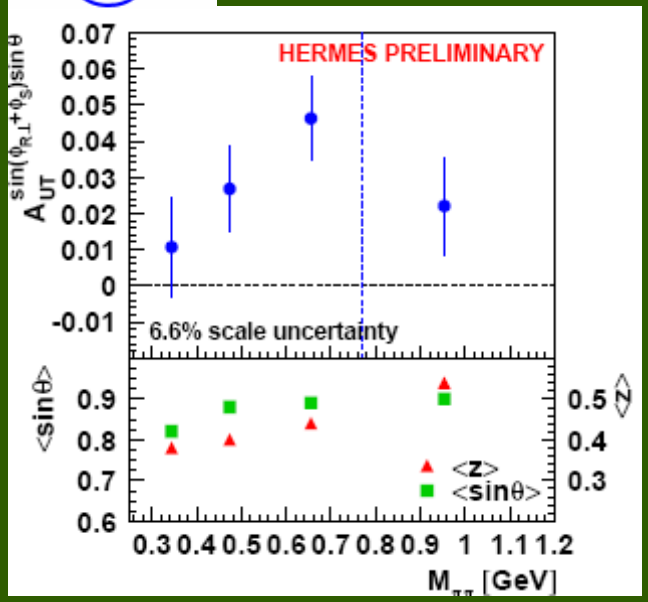


P. van der Nat, hep-ex/0512019

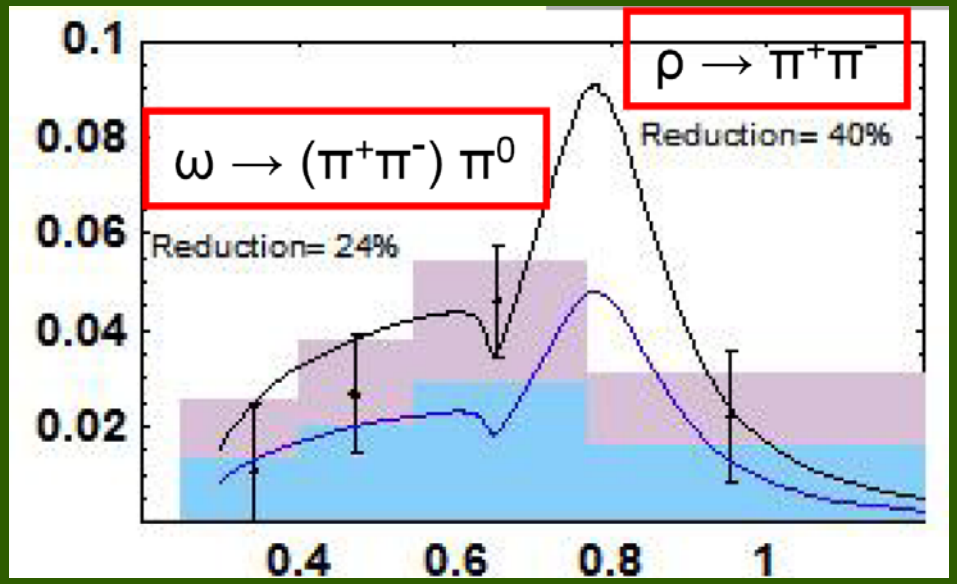


Based on Bacchetta, Radici, PRD 74 (06)

Preliminary data



P. van der Nat, hep-ex/0512019



Based on Bacchetta, Radici, PRD 74 (06)

Talk by M. Radici

Transversity and twist-3 FF

Jaffe, Ji, PRL 71 (93)
Mulders, Tangerman, NPB 461 (96)

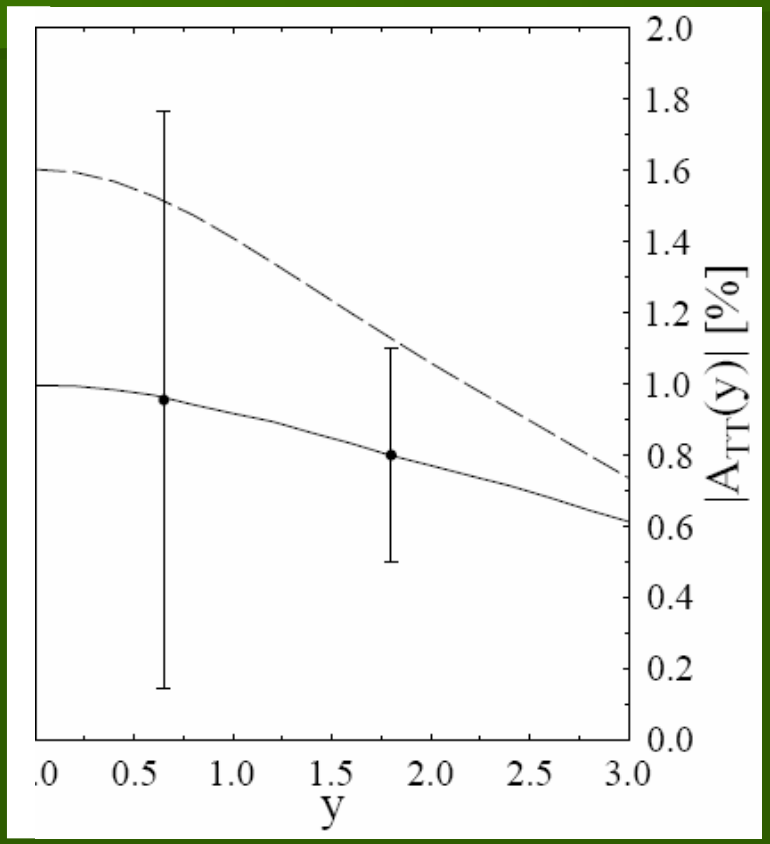
$$A_{UT}^{\sin\phi_S}(x,y,z) = -\frac{\dfrac{(2-y)\sqrt{1-y}}{x^2 y^2}\dfrac{2M_h}{Q}\, x\sum_q e_q^2\, h_1^q(x)\dfrac{\tilde{H}^q(z)}{z}}{\dfrac{1-y+y^2/2}{x^2 y^2}\, x\sum_q e_q^2\, f_1(x) D_1(z)}$$

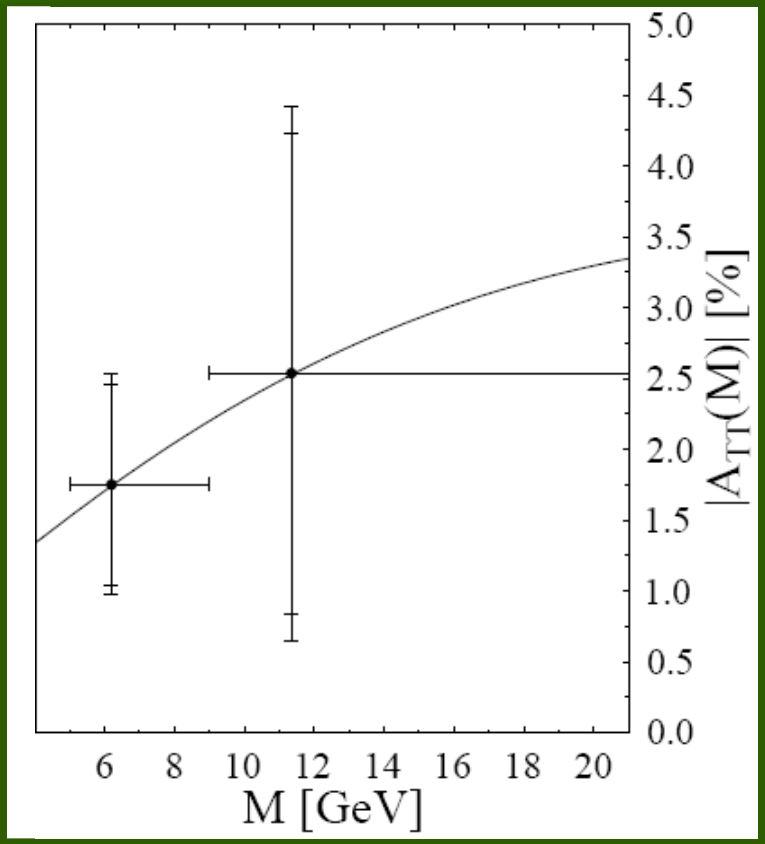
Transversity in Drell-Yan

Ralston, Soper, NPB 152 (79)

$$A_{TT}(x_1, x_2, y) = \frac{\frac{y(1-y)}{x_1^2 x_2^2} \sum_q e_q^2 \, x_1 h_1^q(x_1) \, x_2 h_1^{\bar{q}}(x_2)}{\frac{(1/2 - y + y^2)}{x_1^2 x_2^2} \sum_q e_q^2 \, x_1 f_1^q(x_1) \, x_2 f_1^{\bar{q}}(x_2)}$$

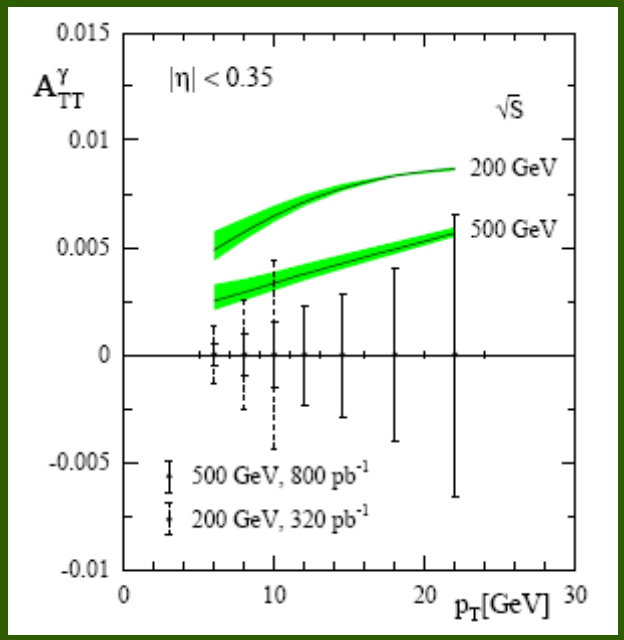
Drell-Yan at RHIC





Martin, Schäfer, Stratmann, Vogelsang, PRD60 (99)

Jet and photon production at RHIC



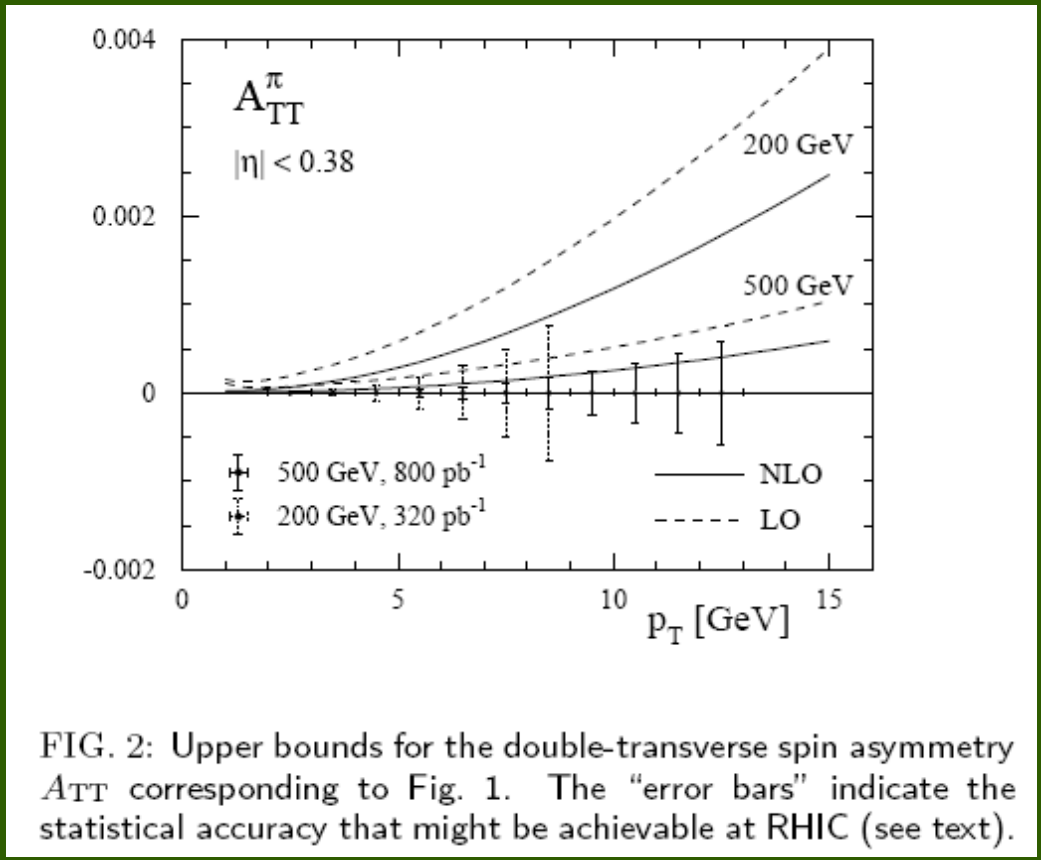
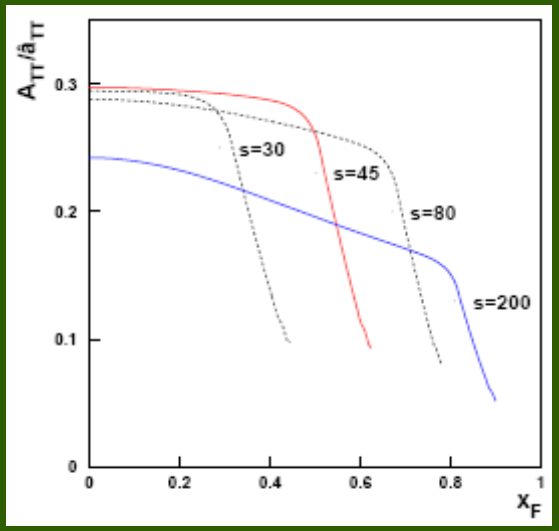


FIG. 2: Upper bounds for the double-transverse spin asymmetry A_{TT} corresponding to Fig. 1. The "error bars" indicate the statistical accuracy that might be achievable at RHIC (see text).

Soffer, Stratmann, Vogelsang, PRD65 (02)
Mukherjee, Stratmann, Vogelsang, PRD72 (05)

Drell-Yan at PAX



PAX, hep-ex/0505054

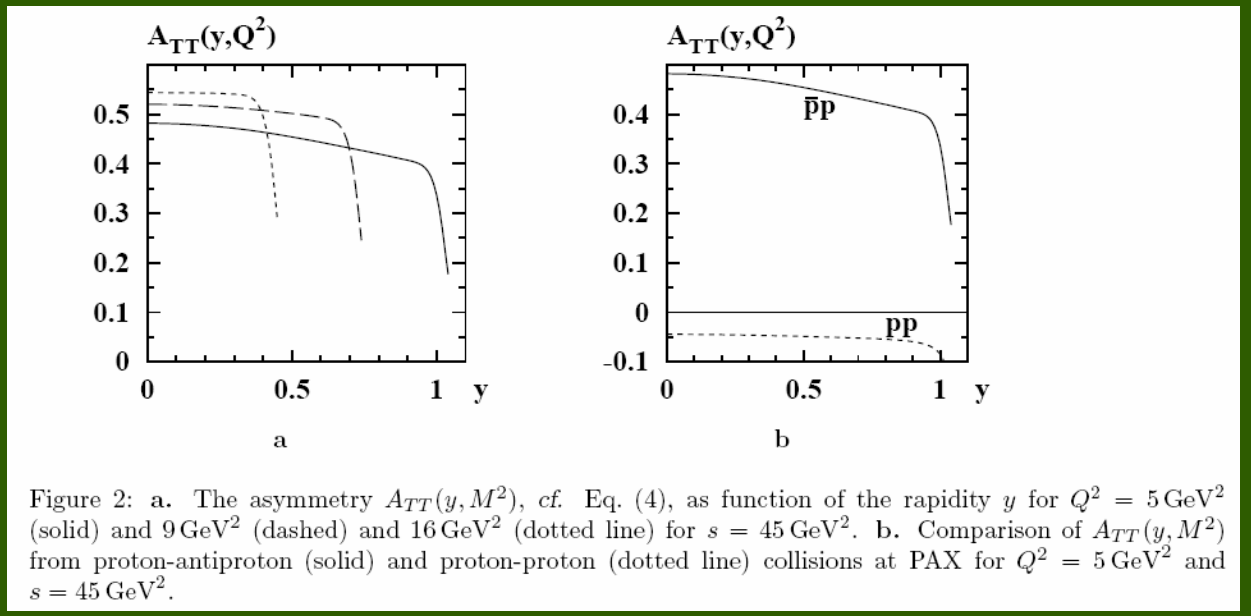


Figure 2: **a.** The asymmetry $A_{TT}(y, M^2)$, *cf.* Eq. (4), as function of the rapidity y for $Q^2 = 5\,\mathrm{GeV}^2$ (solid) and $9\,\mathrm{GeV}^2$ (dashed) and $16\,\mathrm{GeV}^2$ (dotted line) for $s = 45\,\mathrm{GeV}^2$. **b.** Comparison of $A_{TT}(y, M^2)$ from proton-antiproton (solid) and proton-proton (dotted line) collisions at PAX for $Q^2 = 5\,\mathrm{GeV}^2$ and $s = 45\,\mathrm{GeV}^2$.

Efremov, Goeke, Schweitzer, EPJ C35 (04)

J/ψ production at PAX

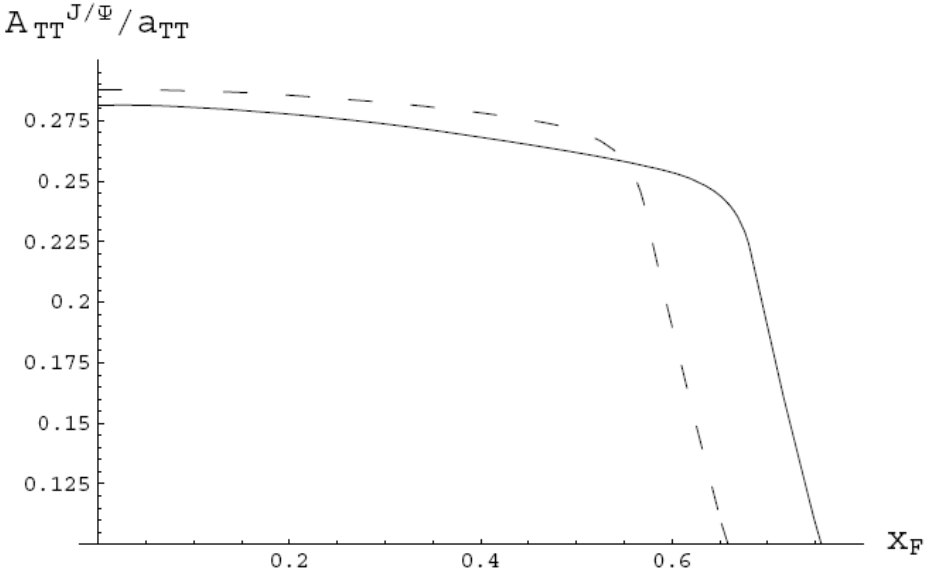


Figure 2: The double transverse spin asymmetry $\tilde{A}_{TT}^{J/\psi}$ for J/ψ production in $p\bar{p}$ collisions, as a function of x_F at $M = 3$ GeV/c^2 (solid curve: $s = 45$ GeV^2; dashed curve: $s = 30$ GeV^2).

Anselmino, Barone, Drago, Nikolaev, PLB594 (04)

Jet and photon production at PAX

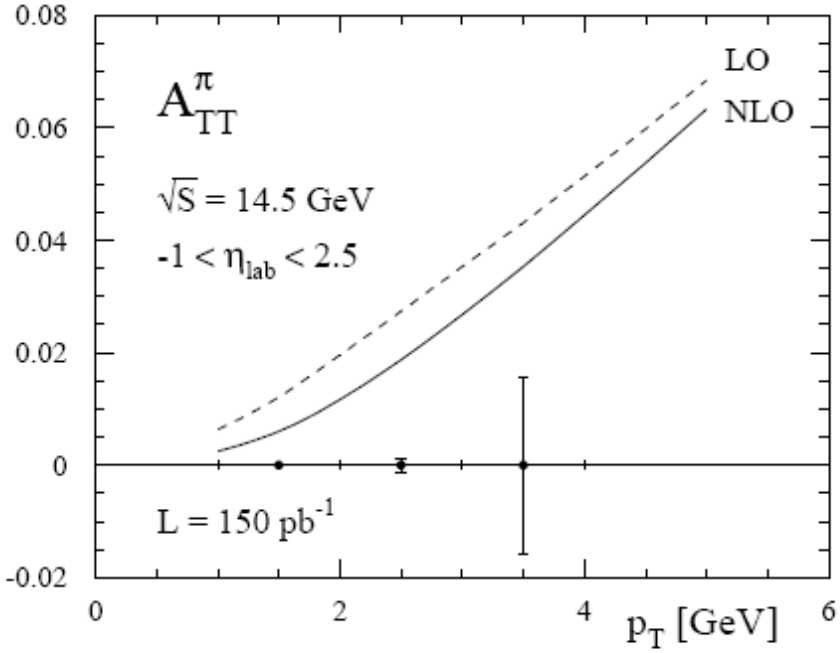
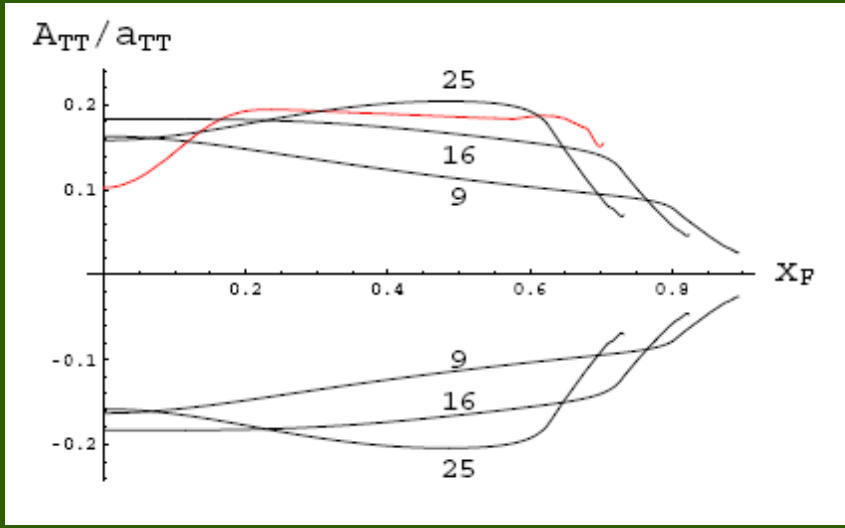


FIG. 4: Upper bounds for the transverse double-spin asymmetry A_{TT} for single-inclusive neutral-pion production in LO and NLO at GSI-FAIR. The "error bars" indicate the expected statistical accuracy for bins in p_T (see text).

Mukherjee, Stratmann, Vogelsang, PRD72 (05)

Drell-Yan at JPARC



Contalbrigo, Drago, Lenisa, hep-ph/0607143

A few other issues

Evolution equations

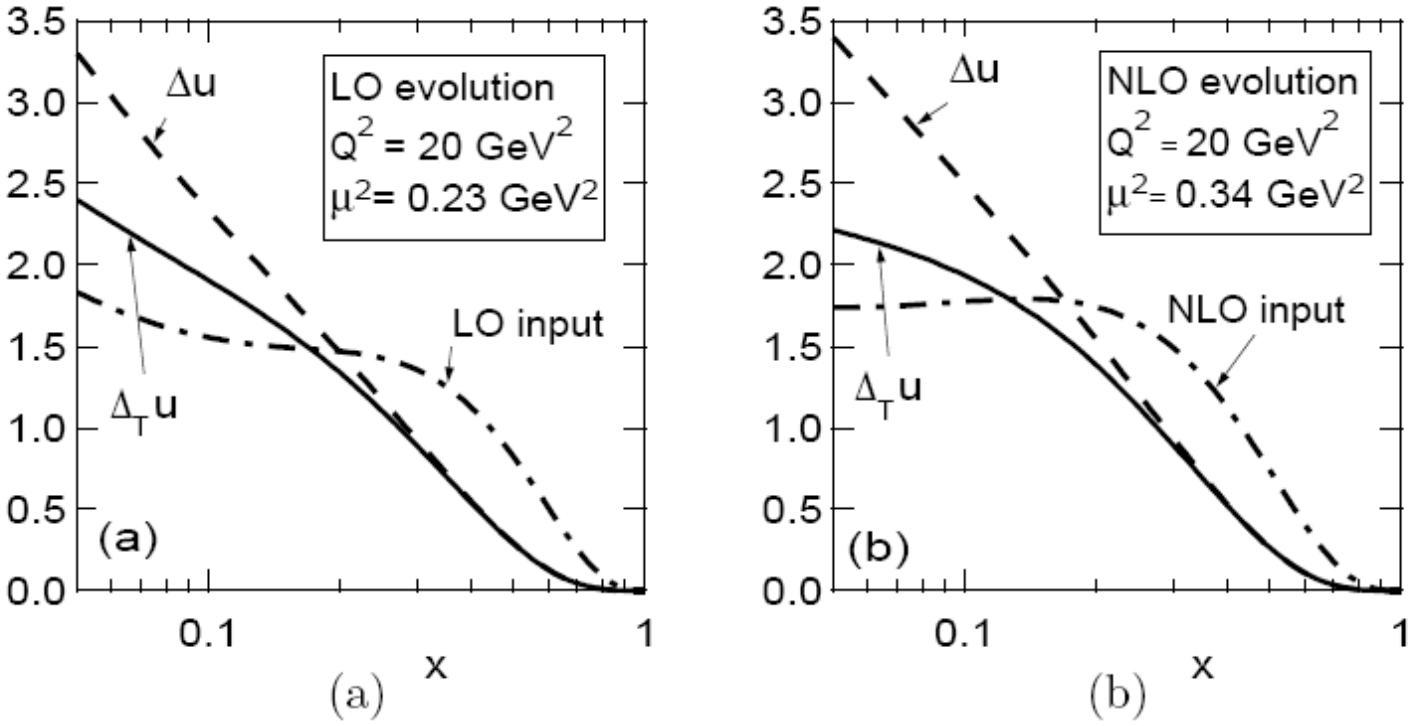


Fig. 19. Comparison of the Q^2-evolution of $\Delta_T u(x, Q^2)$ and $\Delta u(x, Q^2)$ at (a) LO and (b) NLO, from [72].

Barone, Drago, Ratcliffe, PR 359 (2002)

Hayashigaki, Kanazawa, Koike, PRD56 (97)

Transverse momentum dependence

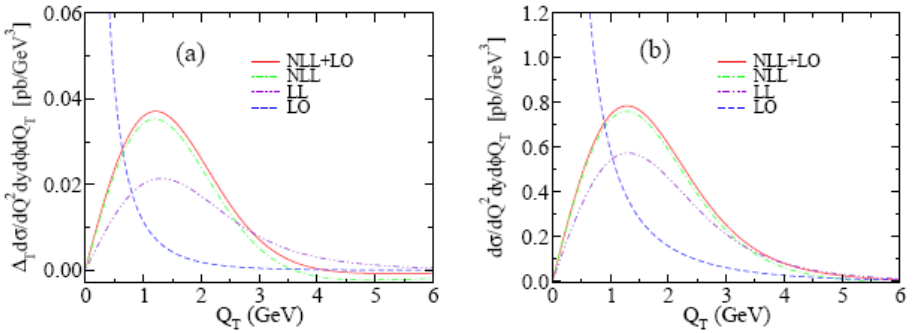


Figure 1: The spin-dependent and spin-averaged differential cross sections for tDY: (a) $\Delta_T d\sigma/dQ^2 dQ_T dy d\phi$ and (b) $d\sigma/dQ^2 dQ_T dy d\phi$, as a function of Q_T at RHIC kinematics, $\sqrt{S} = 200$ GeV, $Q = 5$ GeV, $y = 2$ and $\phi = 0$, with $g_{NP} = 0.5$ GeV^2.

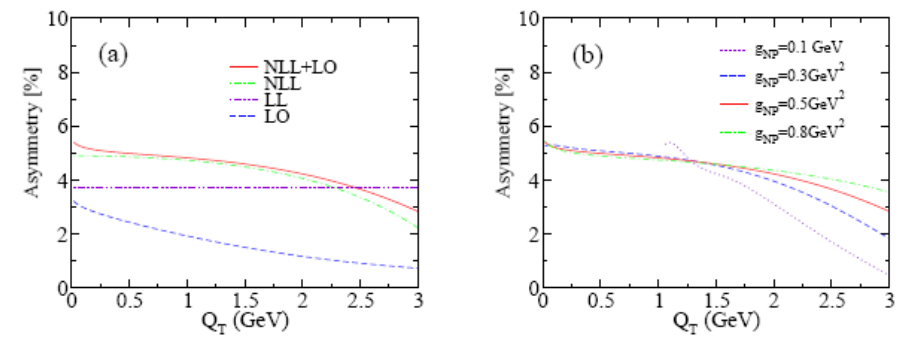


Figure 2: The asymmetries $\mathscr{A}_{TT}(Q_T)$ at RHIC kinematics, $\sqrt{S} = 200$ GeV, $Q = 5$ GeV, $y = 2$ and $\phi = 0$: (a) $\mathscr{A}_{TT}(Q_T)$ obtained from each curve in Fig. 1. (b) The NLL+LO $\mathscr{A}_{TT}(Q_T)$ of (20) with (22) using various values for g_{NP}.

Kawamura, Kodaira, Tanaka, hep-ph/0703079

Transversity “sum rule”

http://www.ts.infn.it/eventi/transversitySR/

$$\frac{1}{2} = \frac{1}{2}\sum_{a=q,\bar{q},g}\int dx\, g_1^a(x) + L_z^q + L_z^g$$

$$\frac{1}{2} = \frac{1}{2}\sum_{a=q,\bar{q}}\int dx\, h_1^a(x) + L_T^q + L_T^g$$

Bakker, Leader, Trueman, PRD 70 (04)

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Bakker, Leader, Trueman, PRD 70 (04)

CRITICISM: sum rule should be derived from hadronic matrix elements of conserved quark-gluon operators. There is no such operator for transversity.

A possible compromise

There is another sum rule, derived in the standard way

$$\frac{1}{2} = \frac{1}{2} \sum_{a=q,\bar{q},g} \int dx\, g_T^a(x) + L_T^q + L_T^g$$

O. Teryaev, B. Pire and J. Soffer, hep-ph/9806502
P.G. Ratcliffe, hep-ph/9811348

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O. Teryaev, B. Pire and J. Soffer, hep-ph/9806502
P.G. Ratcliffe, hep-ph/9811348

These sum rule could correspond to BLT sum rule if

$$\sum_{a=q,\bar{q},g} g_T^a(x) = \sum_{a=q,\bar{q}} h_1^a(x)$$

What kind of approximation is this? Free quarks? Wandzura--Wilczek like?

Gluon transversity (linearity)

Artru, Mekhfi, ZPC 45 (90)

Gluon transversity (linearity)

Artru, Mekhfi, ZPC 45 (90)

- Requires linearly polarized spin-1 target

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Artru, Mekhfi, ZPC 45 (90)

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- Gluon linearity in the photon: dihadron photoproduction

Gluon transversity (linearity)

Artru, Mekhfi, ZPC 45 (90)

- Requires linearly polarized spin-1 target
- Gluon linearity in the photon: dihadron photoproduction
- Gluon linearity in the deuteron: DIS with linearly polarized deuteron

Conclusions

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- With present data, there is already work for the next few years

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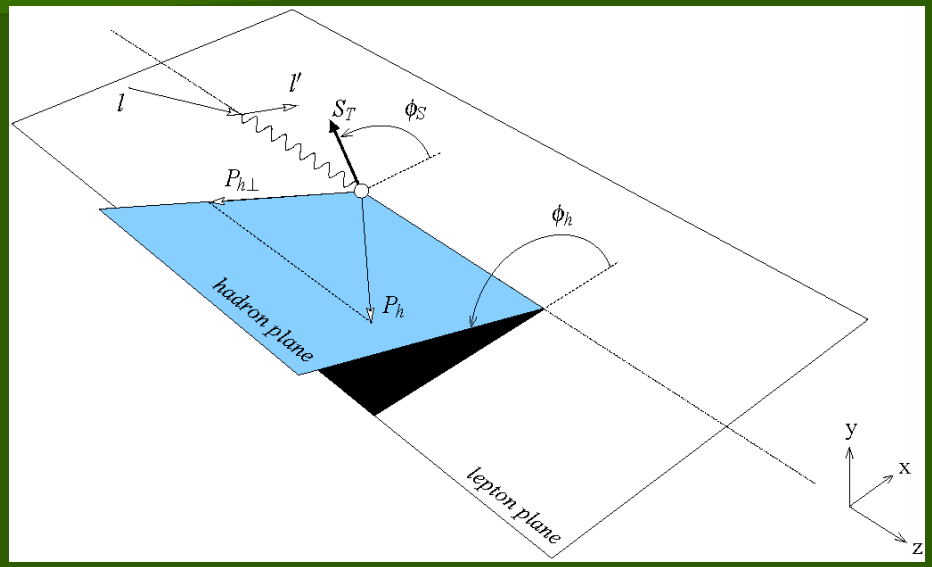
- It’s the beginning of a transverse era!

Transversity in SIDIS



$$d\sigma^{\uparrow} - d\sigma^{\downarrow} \sim \ldots + \frac{(1-y)}{x^2 y^2} \sin(\phi_h + \phi_S)\, \mathcal{C}\left[-\frac{\vec{k}_T \cdot \vec{P}_{h\perp}}{M_h \left|\vec{P}_{h\perp}\right|} h_1 \; H_1^{\perp} \right] + \ldots$$

$$d\sigma^{\uparrow} + d\sigma^{\downarrow} \sim \frac{1-y+y^2/2}{x^2 y^2} \mathcal{C}[f_1 D_1] + \cos 2\phi_h \ldots + \cos \phi_h \ldots$$