TMD DGLAP Phenomenology

Federico A. Ceccopieri

Università di Parma, Italia

In collaboration with Luca Trentadue

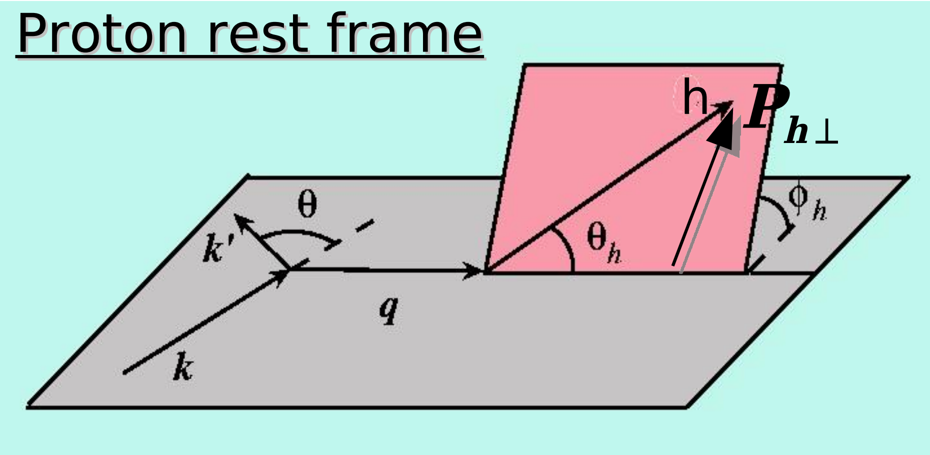
Outline:

- Unpolarized NLO SIDIS vs. EMC'91 data;
- Perturbative approach to TMD distributions;
- TMD DGLAP evolution equations;
- Evolution properties;
- Discussion of initial condition dependences;
- TMD DGLAP vs. EMC'91 data.

Idea: use pQCD to pin down TMD distributions properties.

· Unpolarized deep inelastic semi-inclusive reaction:

$$l(k)+H(P)\rightarrow l(k')+H'(h)+X$$

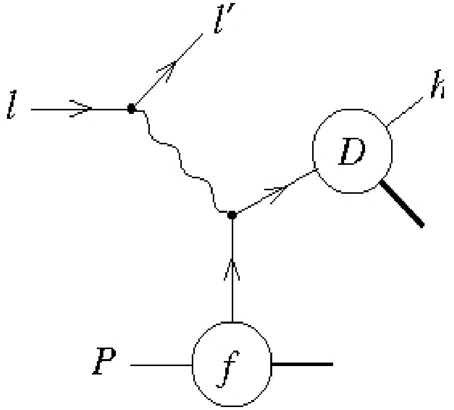


Invariant set:

$$x_B=\frac{Q^2}{2\,P\cdot q} \qquad z_h=\frac{P\cdot h}{P\cdot q}$$

$$Q^2=-q^2=-(k-k')^2$$

SIDIS QCD-factorization in the current fragmentation region:

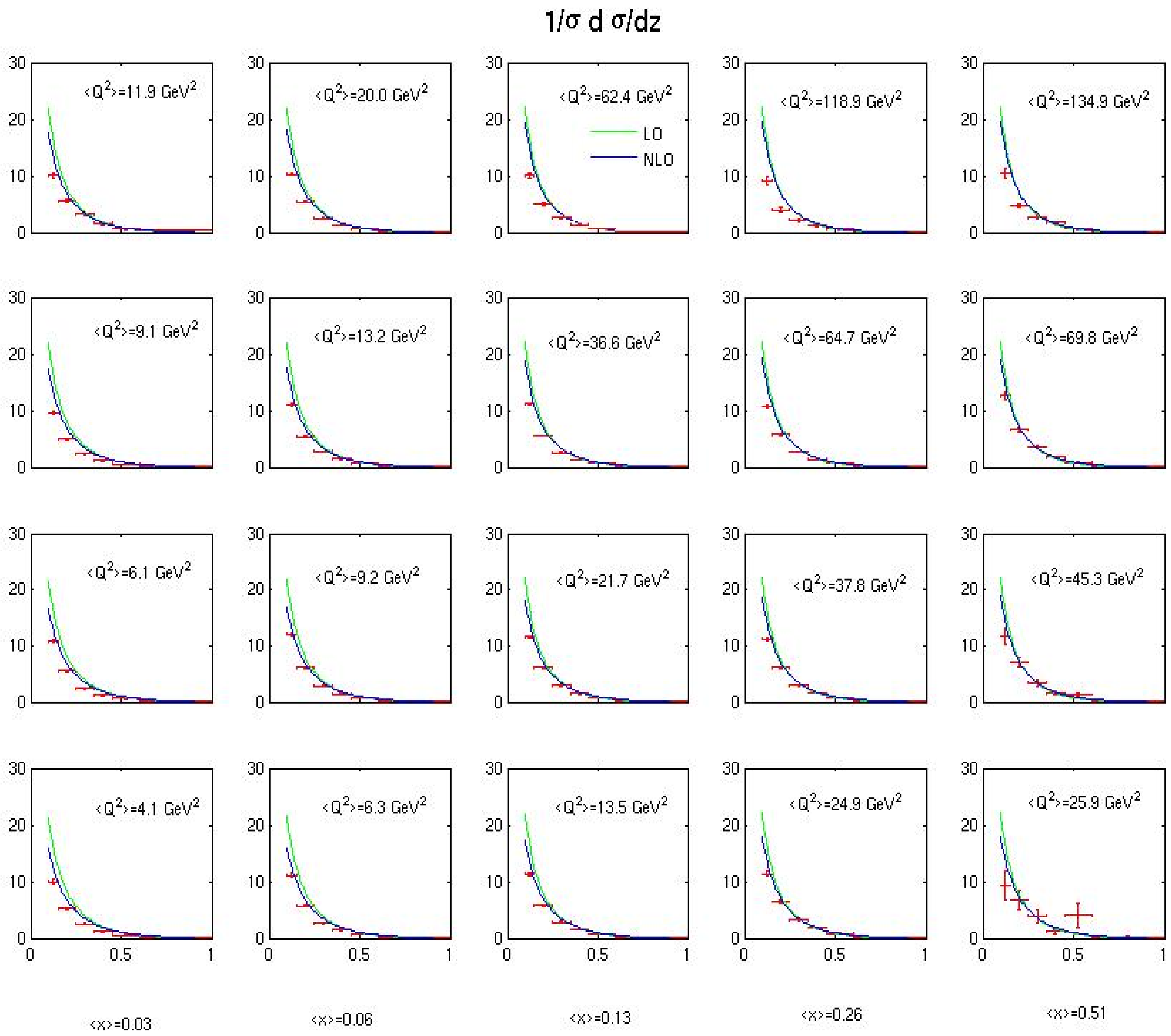


$$\sigma_C\approx\int_{x_B}^{1}\int_{z_h}^{1}\frac{dx'}{x'}\frac{dz'}{z'}F_P^i(x',Q^2)\,\hat{\sigma}_{ij}\left(\frac{x_B}{x'},\frac{z_h}{z'},Q^2\right)D_h^j(z',Q^2)$$

Standard approach : $P_{h\perp}$ integrated over

SIDIS longitudinal cross-sections for $h^+ + h^-$ production:

[**]



PDF: MRST LO '01

FF: Kretzer '00

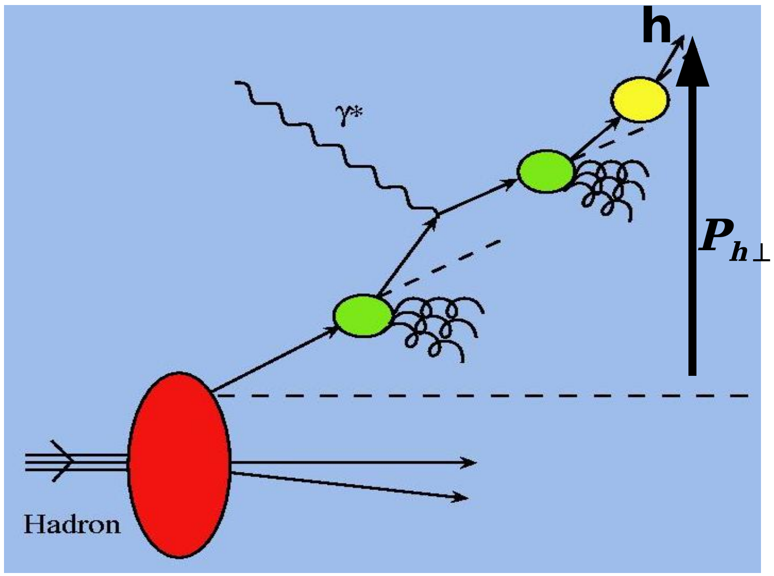
NLO* needed. Large angle parton emissions

Test for PDF and FF to be used later

NLO < LO at small z, required by data

[*] Furmanski, Petronzio, *Z.Phys.* **C11,** 293, (1982)
[**]European Muon Collaboration *Z.Phys.* **C52**, 361, (1991)

Sources of transverse momentum in $l+P \rightarrow l+h+X$:



- Intrinsic distribution: $\boldsymbol{k}_{\perp}$
- Radiative: $\boldsymbol{q}_{T}$
- Intrinsic fragmentation: $\boldsymbol{p}_{\perp}$

Detected hadron transverse momentum: $\boldsymbol{P}_{h\perp} = \boldsymbol{p}_{h\perp} + z_h \boldsymbol{k}_{h\perp}$

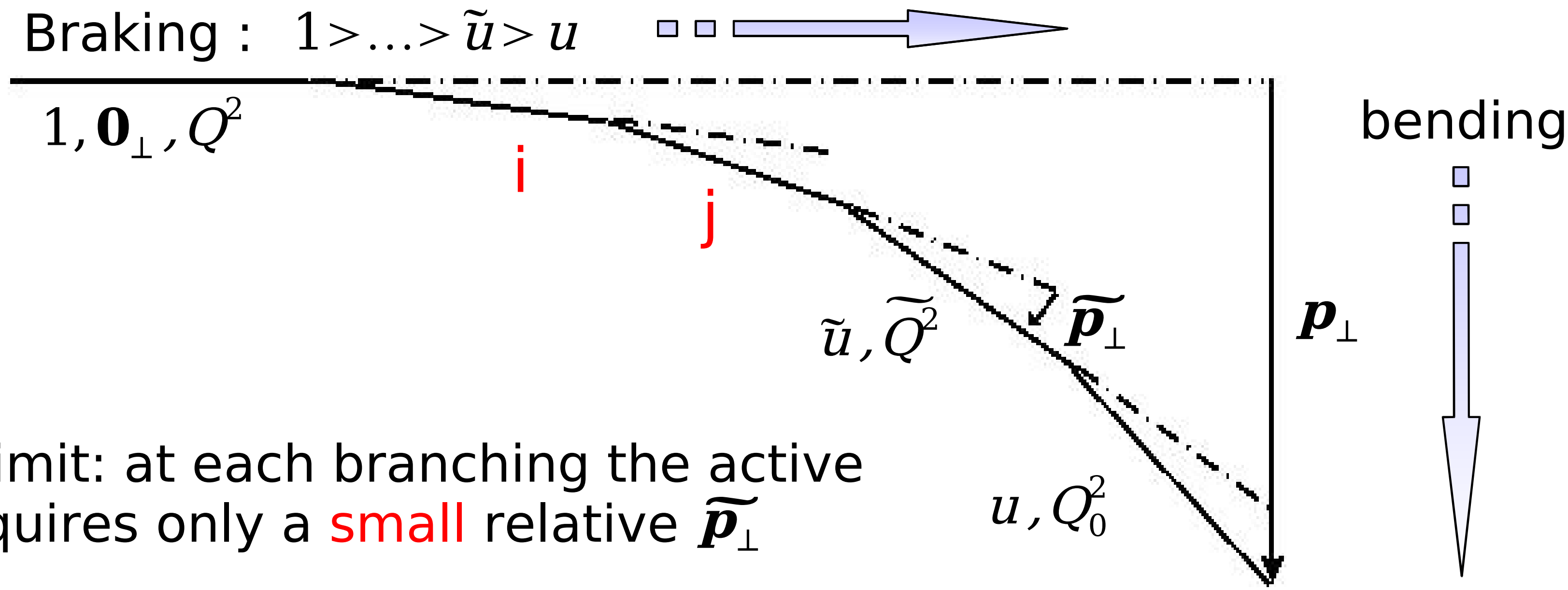
But $\dfrac{d^5 \sigma^{lp \rightarrow lhX}}{dx_B \, dQ^2 \, dz_h \, d^2 \boldsymbol{P}_{h\perp}}$ depends on $\boldsymbol{q}_T$!

Approaches: - hard parton emissions $O(\alpha_s)$ Altarelli,Martinelli

- resummation at small $|\boldsymbol{P}_{h\perp}|$ Nadolsky et &

The idea behind perturbative TMD DGLAP.

Consider the decay of a coloured off-shell parton:



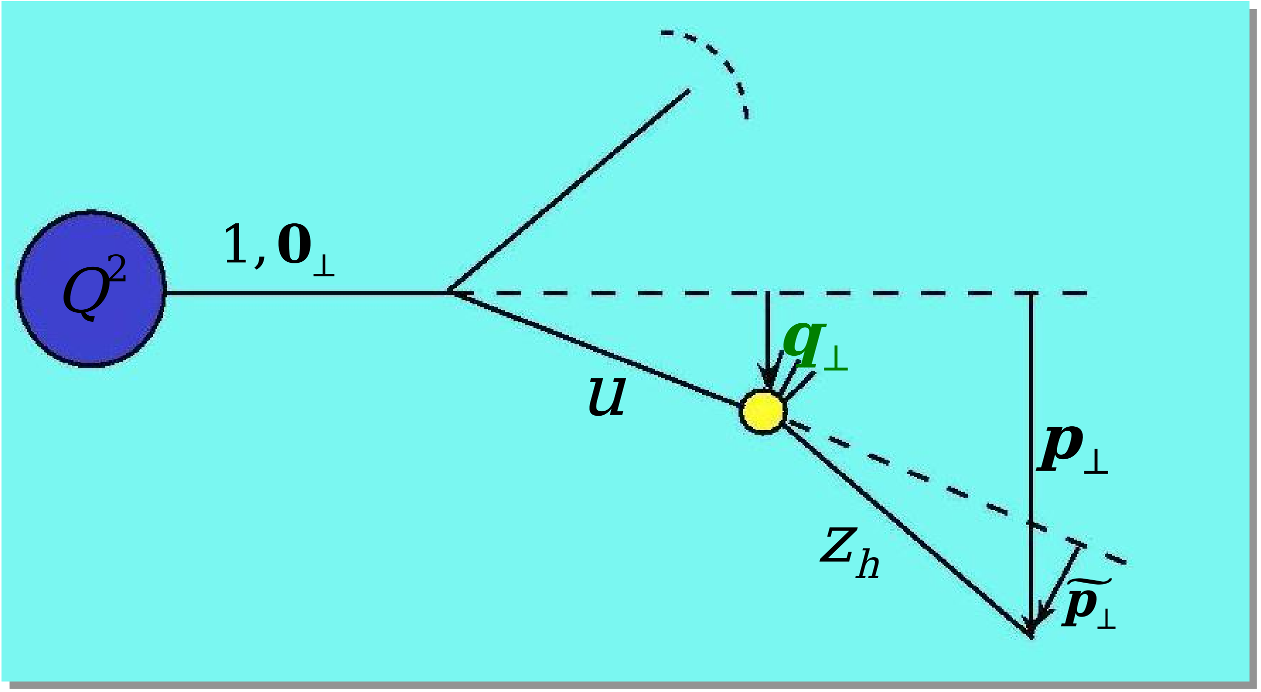
- Collinear limit: at each branching the active parton acquires only a small relative $\widetilde{\boldsymbol{p}}_\perp$

- Strongly ordered $\widetilde{\boldsymbol{p}}_\perp^{\,2}$ chains give leading logarithmic contributions, which are then resummed by DGLAP equations

The radiative process generates an appreciable $\boldsymbol{p}_\perp$ even in the collinear limit!

N.B. Collinear is i→j branching, not initial→final

- Time-like TMD DGLAP evolution equation



Branching kinematics:

$$\widetilde{\boldsymbol{p}}_\perp = \boldsymbol{p}_\perp - \frac{z_h}{u}\boldsymbol{q}_\perp$$

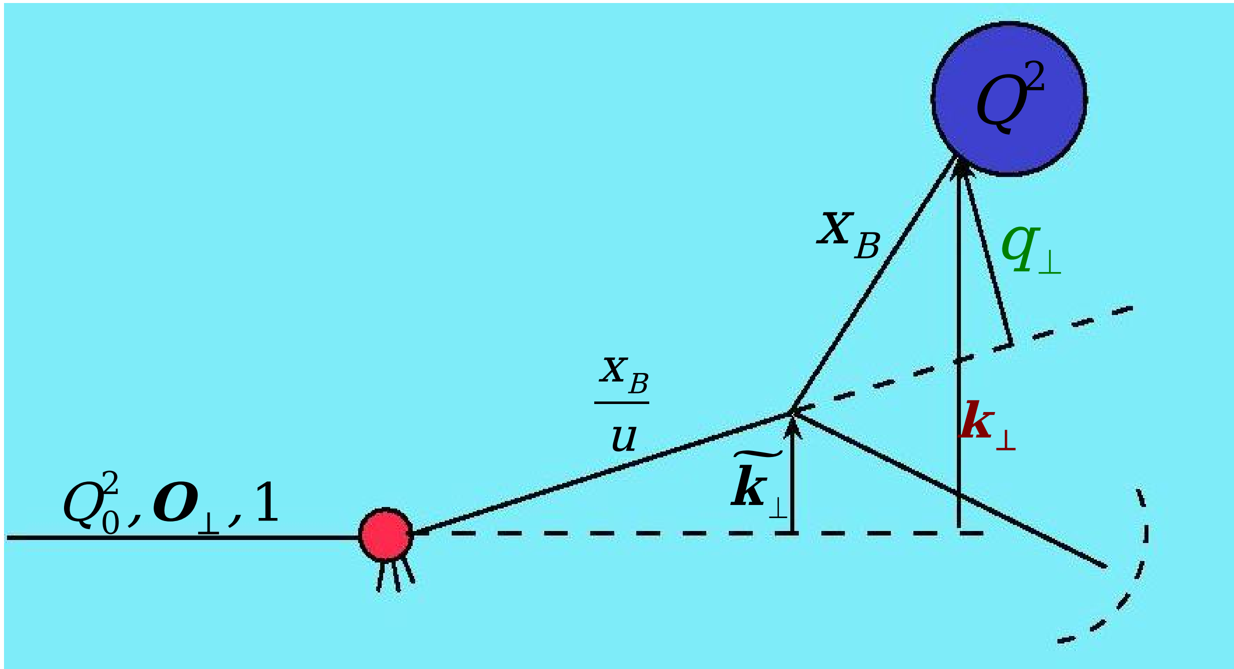
$$u(1-u)Q^2 = \boldsymbol{q}_\perp^2$$

$$Q^2 \frac{\partial D_a^b(Q^2, z_h, \boldsymbol{p}_\perp)}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{du}{u} \int \frac{d^2\boldsymbol{q}_\perp}{\pi} \delta[u(1-u)Q^2 - \boldsymbol{q}_\perp^2] P_a^c(u) D_c^b\left(Q^2, \frac{z_h}{u}, \boldsymbol{p}_\perp - \frac{z_h}{u}\boldsymbol{q}_\perp\right)$$

A.Bassetto, M. Ciafaloni, G.Marchesini, *Nucl. Phys.* **B163**, (1980)

- Space-like TMD DGLAP evolution equation



Branching kinematics:

$$\tilde{\boldsymbol{k}}_\perp = \frac{k_\perp - q_\perp}{u}$$

$$(1-u)\,Q^2 = \boldsymbol{q}_\perp^2$$

$$Q^2 \frac{\partial F_a^b(Q^2, x_B, \boldsymbol{k}_\perp)}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} \int \frac{d^2 \boldsymbol{q}_\perp}{\pi} \delta[(1-u)\,Q^2 - \boldsymbol{q}_\perp^2]\, P_a^c(u)\, F_c^b\left(Q^2, \frac{x_B}{u}, \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}\right)$$

F.A.C. and L.Trentadue, *Phys. Lett.* **B636**, (2006)

◉ Numerical solutions by a brute force method on a $(x, k_\perp^2)$ grid

◉ Simulation of the full light quark-gluon mixing matrix

➥ factorized initial condition: $F(x_B, Q_0^2, \boldsymbol{k}_\perp) = F(x_B, Q_0^2) \dfrac{e^{-\boldsymbol{k}_\perp^2 / < \boldsymbol{k}_{\perp,0}^2 >}}{\pi < \boldsymbol{k}_{\perp,0}^2 >}$ PDF: MRST LO '01

$$D(z_h, Q_0^2, \boldsymbol{p}_\perp) = D(z_h, Q_0^2) \frac{e^{-\boldsymbol{p}_\perp^2 / < \boldsymbol{p}_{\perp,0}^2 >}}{\pi < \boldsymbol{p}_{\perp,0}^2 >}$$ FF: Kretzer '00

➥ x,z,flavour independence: $< \boldsymbol{k}_{\perp,0}^2 > = 0.25\, GeV^2$ $< \boldsymbol{p}_{\perp,0}^2 > = 0.20\, GeV^2$ [*]

➥ Factorization scale: $Q_0^2 = 5\, GeV^2$ (EMC cut)

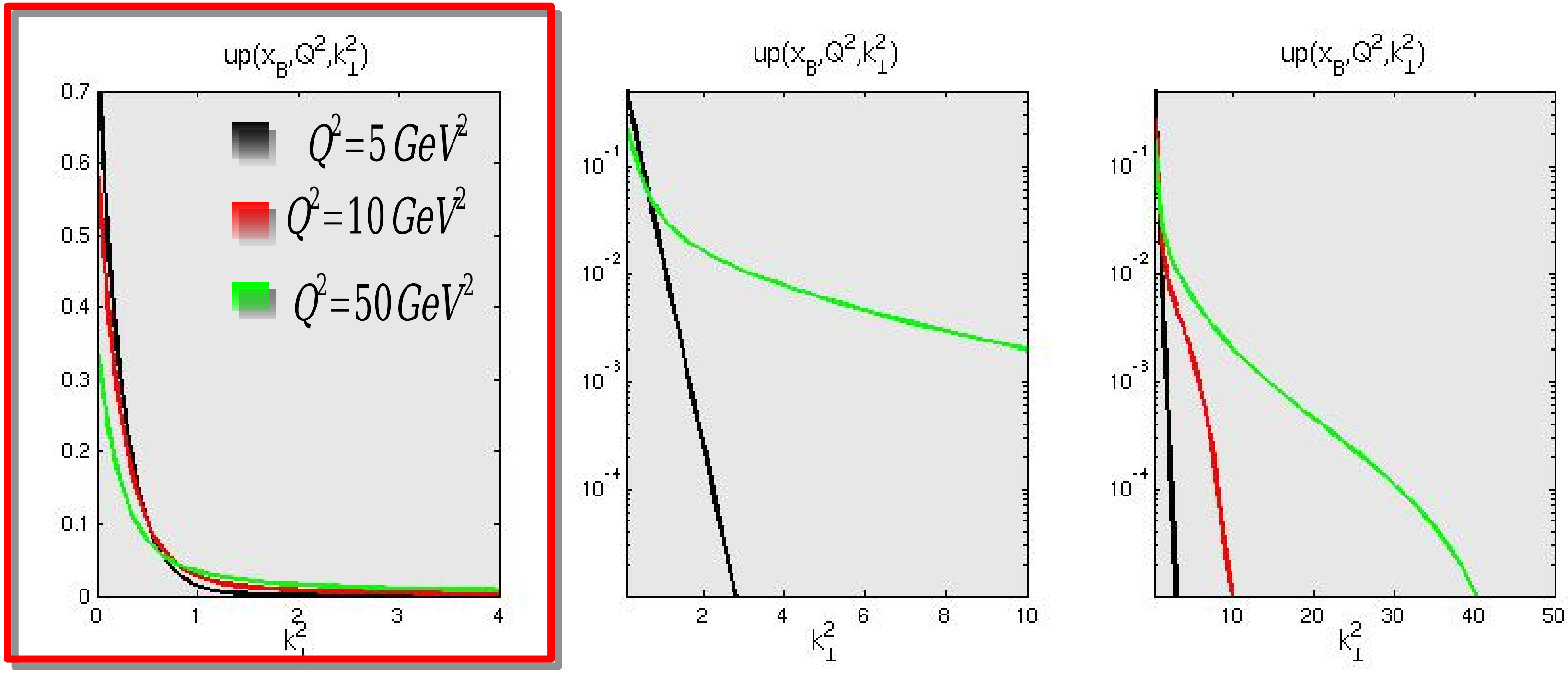
➥ normalization condition: $\int d^2 \boldsymbol{k_T} F(x_B, Q^2, \boldsymbol{k_T}) = F(x_B, Q^2), \ \forall Q^2$

$$\int d^2 \boldsymbol{p_T} D(z_h, Q^2, \boldsymbol{k_T}) = D(z_h, Q^2), \ \forall Q^2$$

➥ Leading log running of the strong coupling $\alpha_s(Q^2)$

[*] M. Anselmino & al. , *Phys. Rev.* **D71**, 074006 (2005)

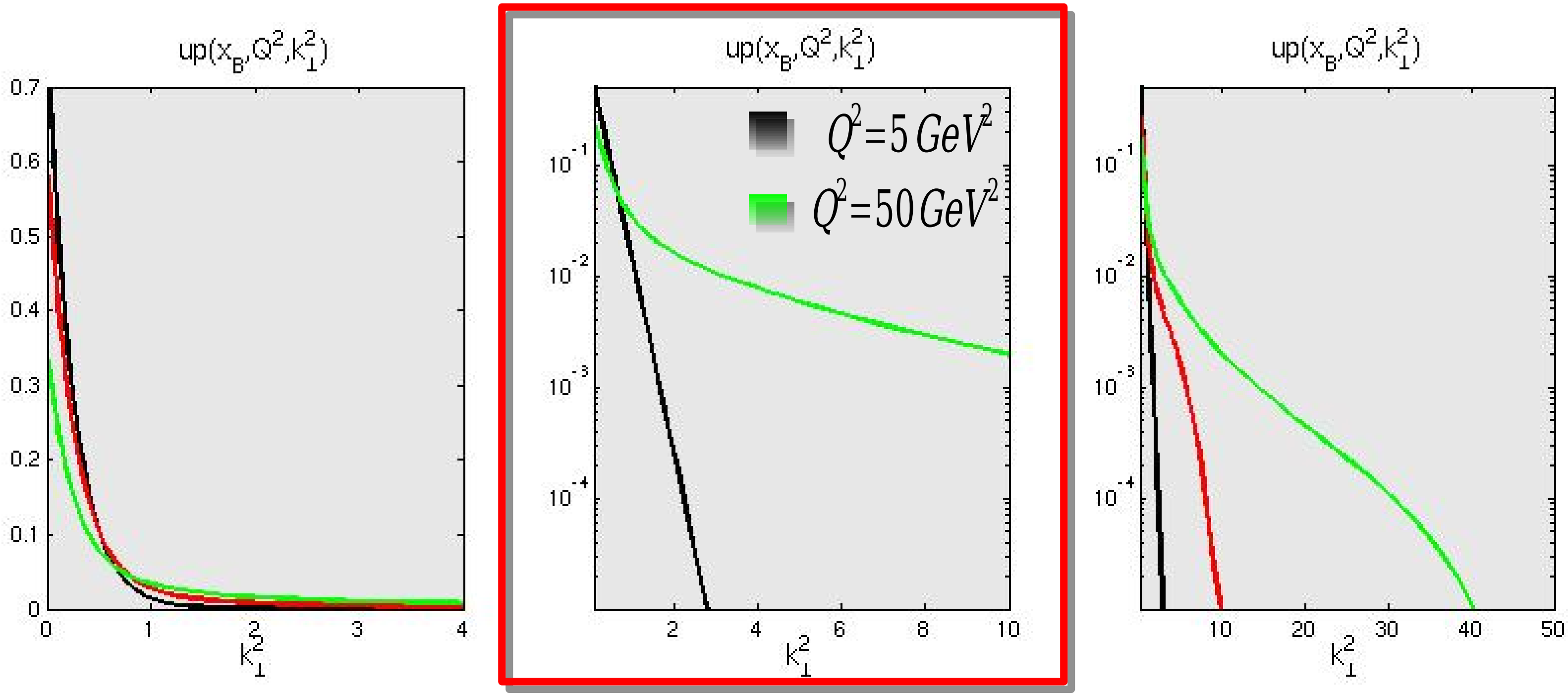
Simulation Setup : $x_B = 0.1, 5 < Q^2 < 50\, Gev^2, k^2_{\perp, max} = 50\, GeV^2$



- smooth broadening of the TMD ditributions;
- the broadening is faster at small Q^2 due to the higher value of $\alpha_s(Q^2)$ (as for standard PDF)

General remark: Pre-asymptotics effect.
Experiment at similar, low, scale may give different answer

Simulation Setup : $x_B=0.1, 5<Q^2<50\, Gev^2, k^2_{\perp, max}=50\, GeV^2$



- gaussian form not preserved under evolution;
- the perturbative tail behaves as $\approx \frac{a}{(k_\perp^2)^b+c}, b=0.130$

(coefficients, fitted to simulation, are given by pQCD and depend on x_B and Q^2 and ansatz)

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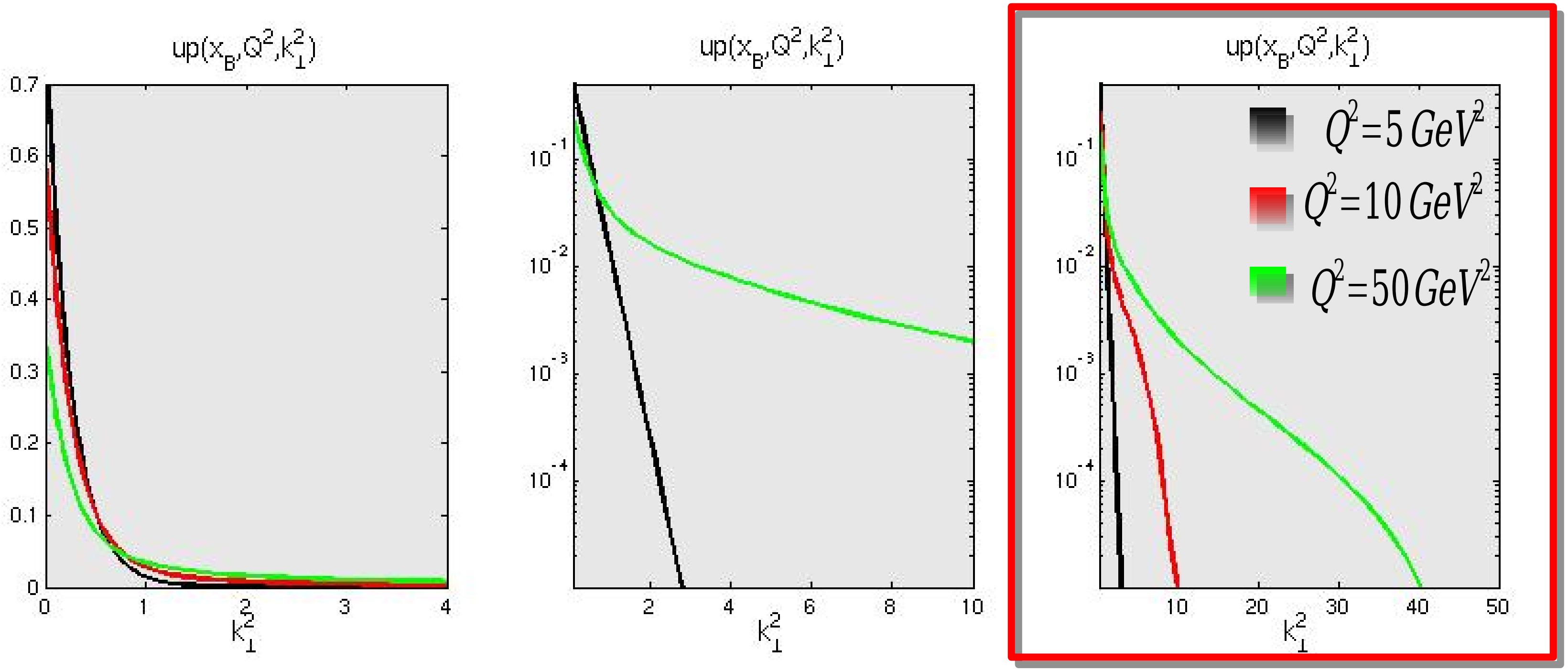
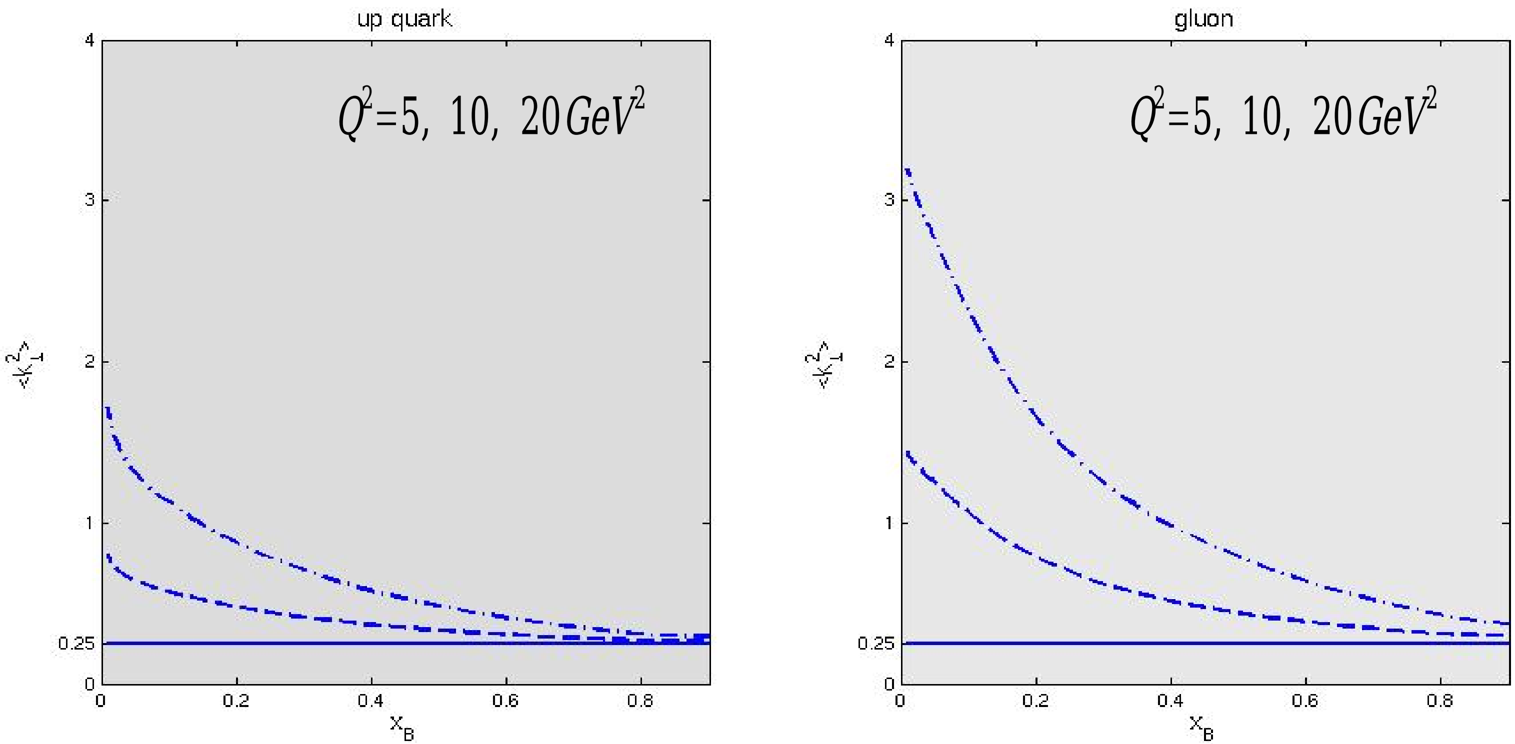
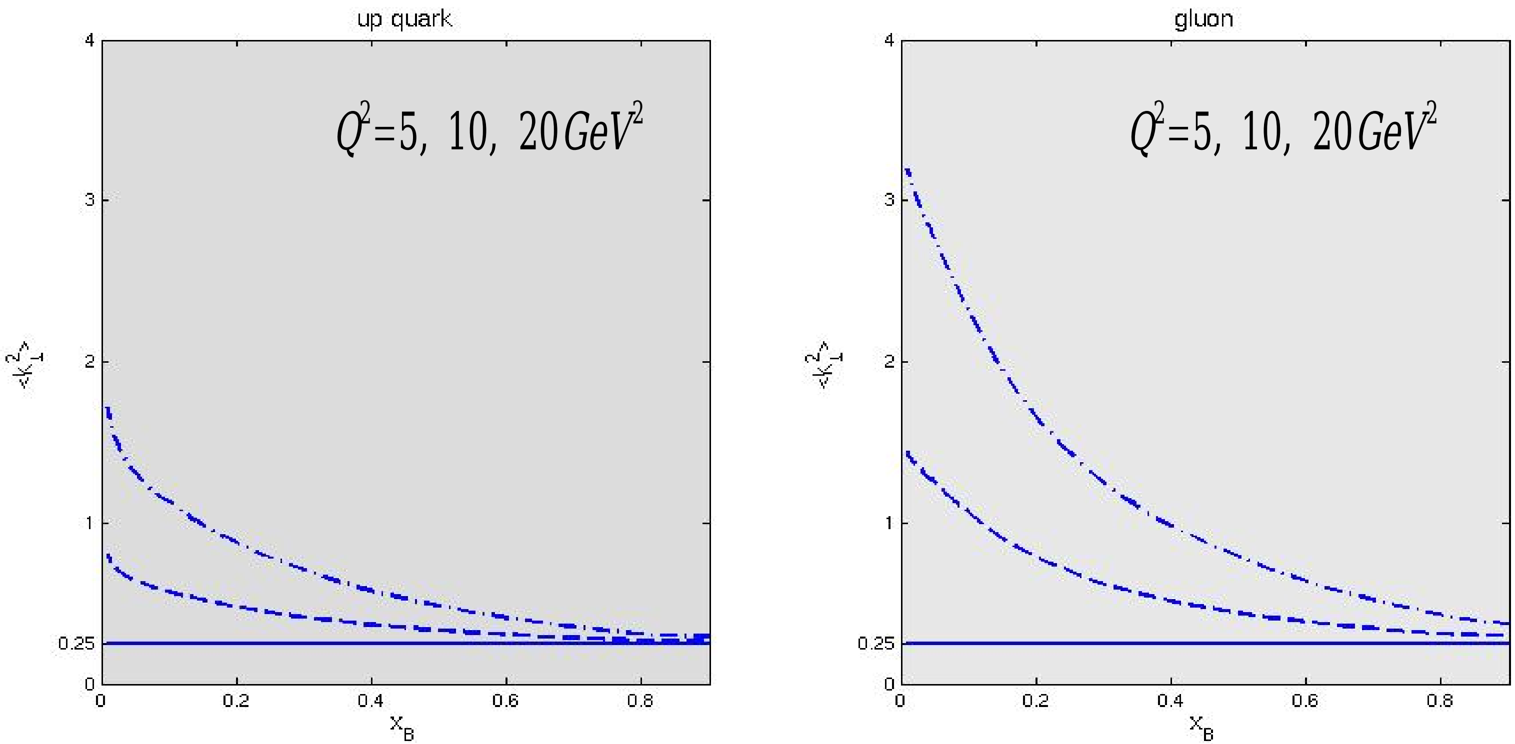


▸ with this setup, the very-high $k^2_\perp$ region is not still populated;



- a x-independent initial condition leads to $< k_\perp^2 (x_B) >$
- this effect is expected: TMD DGLAP mixes longitudinal and transverse degree of freedom:

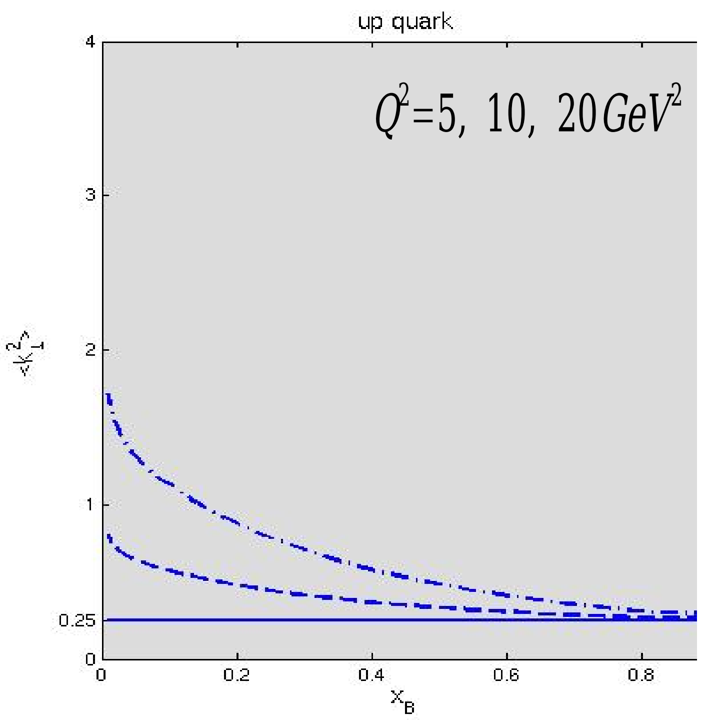
$$Q^2 \frac{\partial F_a^b(Q^2, x_B, \mathbf{k_T})}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} \int \frac{d^2 \mathbf{q_T}}{\pi} \delta[(1-u)Q^2 - \mathbf{q_T^2}] P_a^c(u) F_c^b\left(Q^2, \frac{x_B}{u}, \frac{\mathbf{k_T} - \mathbf{q_T}}{u}\right)$$

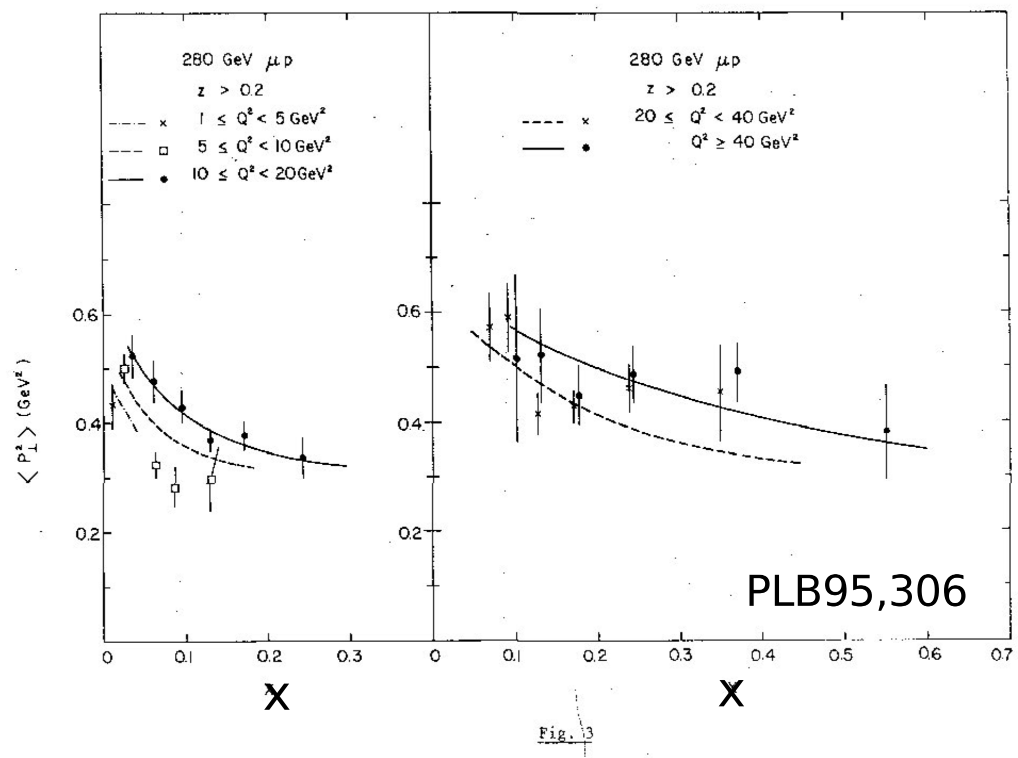


- the evolution suggests $< k_\perp^2(x_B) > = < k_{\perp,0}^2 > x_B^{\gamma}, \quad \gamma < 0$
- the factorization scale at which we make use of the factorized ansatz is arbitrary

▼

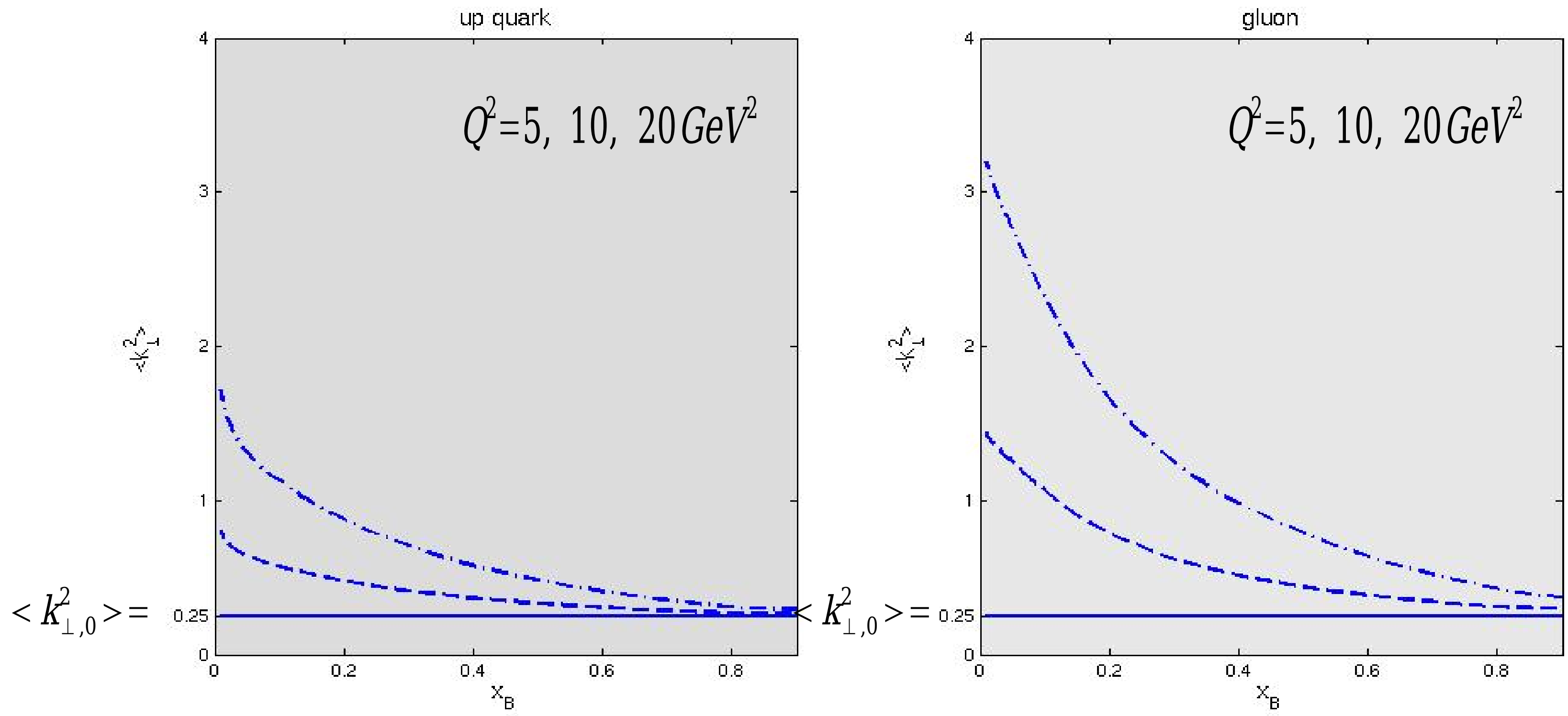
strong indications for x-dependent initial condition





▸ the evolution suggests $<k_\perp^2(x_B)> = <k_{\perp,0}^2> x_B^\gamma, \quad \gamma < 0$

Suggested also experimentally by EMC'91 data.



☛ quark channel at large x: soft gluons do not generate transverse momentum, allow an accurate extraction of $< k^2_{\perp,0} >$

First, however, they must be resummed to all orders......

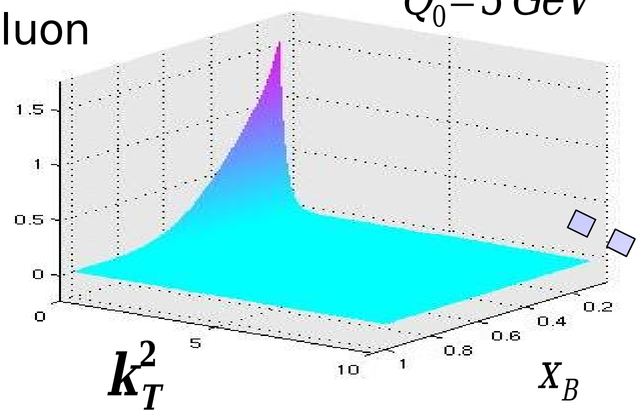
$\alpha_s(Q^2) \rightarrow \alpha_s(k^2_\perp(1-x_B))$ (see for example Catani&Trentadue)

N.B. No PDF set which include soft gluon resummation...

✇ Numerical solution of TMD space-like evolution equations

➥ $g(x_B, Q_0^2, \boldsymbol{K}_\perp) = g(x_B, Q_0^2) \dfrac{e^{-\boldsymbol{k}_\perp^2 / \langle \boldsymbol{k}_{\perp,0}^2 \rangle}}{\pi \langle \boldsymbol{k}_{\perp,0}^2 \rangle}$

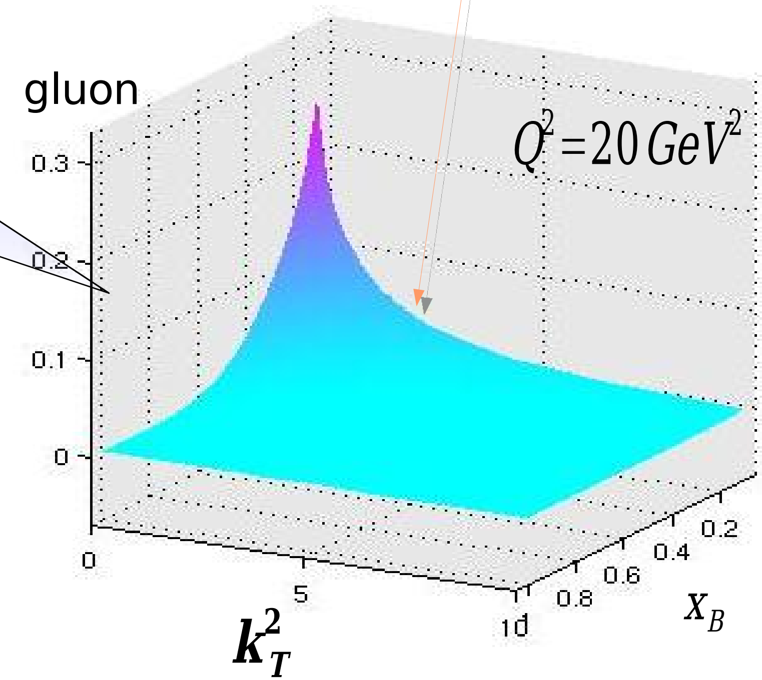
➥ $\int d^2 \boldsymbol{k}_T F(x_B, Q^2, \boldsymbol{k}_T) = F(x_B, Q^2), \quad \forall Q^2$



☛ The spread is enhanced by small-x gluon dynamics:

$$P_{gg}(x \to 0) \sim x^{-1}$$

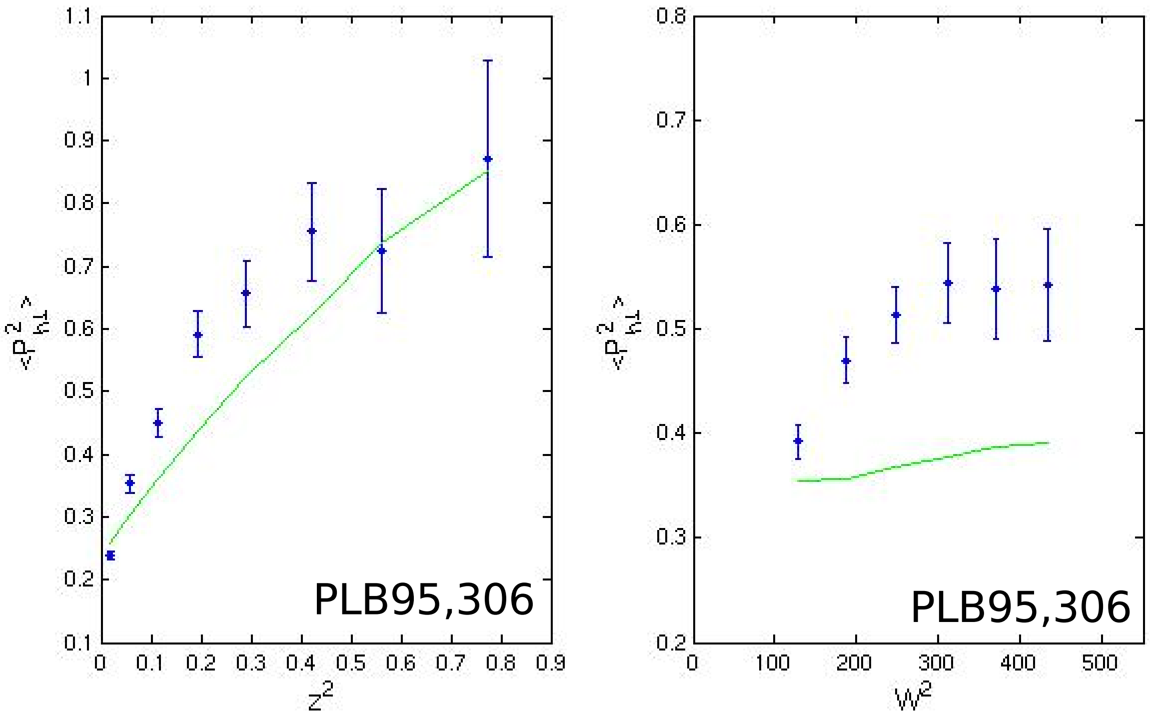
δQ^2



Need a physically motivated initial condition for the gluon at intermediate x. Probably not a gaussian there!!

DIS current fragmentation: $(h^+ + h^-)$ lepto-production

$$H_2(x_B, z_h, \boldsymbol{P}_{hT}, Q^2) = \sum_{i=q,\bar{q}} e_i^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^{(2)}(z_h \boldsymbol{k}_T + \boldsymbol{p}_T - \boldsymbol{P}_{hT}) F_P^i(x_B, Q^2, \boldsymbol{k}_T) D_i^h(z_h, Q^2, \boldsymbol{p}_T)$$



$$W^2 = Q^2(1 - x_B)/x_B$$

$$Q^2 > 5\, GeV^2$$

$$< \boldsymbol{k}^2_{T,0} >_{q,g} = 0.25\, GeV^2$$

$$< \boldsymbol{p}^2_{T,0} >_{q,g} = 0.20\, GeV^2$$

X. Ji, J. Ma, F. Yuan *Phys. Rev.* **D71**, 034005 (2005)

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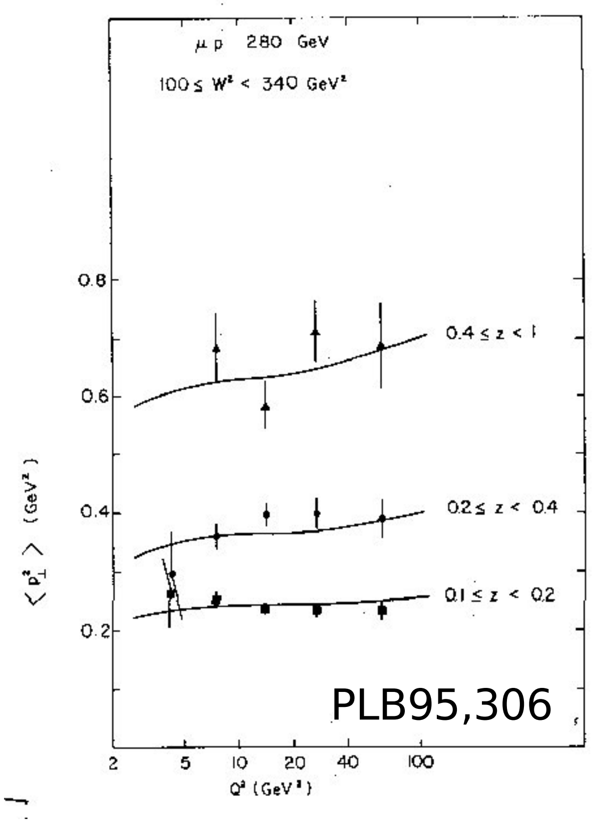
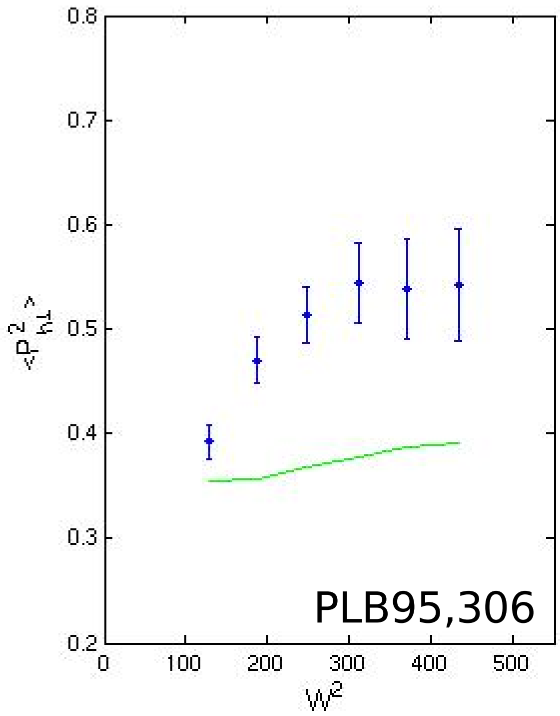




$$W^2 = Q^2(1-x_B)/x_B$$

$$Q^2 > 5\,GeV^2$$

$$<\boldsymbol{k}_{T,0}^2>_{q,g} = 0.25\,GeV^2$$

$$<\boldsymbol{p}_{T,0}^2>_{q,g} = 0.20\,GeV^2$$

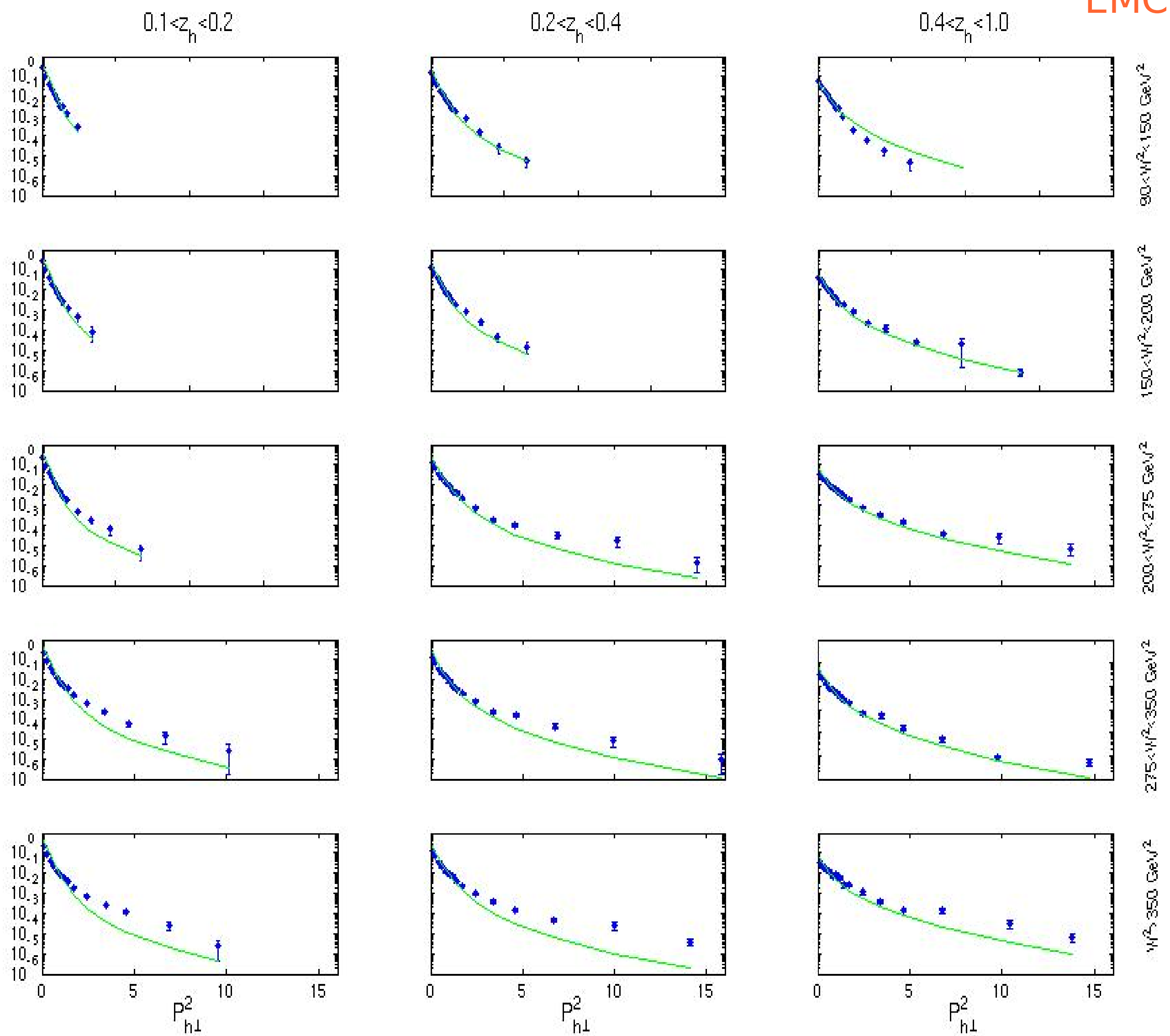
$$<P^2_{h,\perp}> \approx \log W^2 = \log Q^2 + \log 1/x_B$$

TMD DGLAP resums these log.

DIS current fragmentation: $(h^+ + h^-)$ electroproduction

EMC *Z.Phys.* **C52**, 361, (1991)

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{dP_{hT}^2}$$



Not a fit!

$$W^2 = Q^2(1 - x_B)/x_B$$

$$Q^2 > 5\,GeV^2$$

$$<\boldsymbol{k}_{T,0}^2>_{q,g} = 0.25\,GeV^2$$

$$<\boldsymbol{p}_{T,0}^2>_{q,g} = 0.20\,GeV^2$$

We reproduce scaling violations in transverse spectra!!

Conclusions:

- radiative transverse momentum can be quantitatively taken into account using TMD DGLAP evolution equations;

- encouraging agreement with EMC data;

⇓

- Strong indication of x-dependent initial condition;
perturbative inverse power-like tail at large Pt,
full spectrum reproduced;

- soft parton emission does not generate transverse momenta:
gaussian approximation valid only at large x;

- HERA data analysis ⇒ valuable information on gluon transverse spectra & DGLAP dynamics;

- Spin physics: formalism can be extended (modulo technical difficulties) to all TMD polarized distributions whose AP evolution kernels are known.