

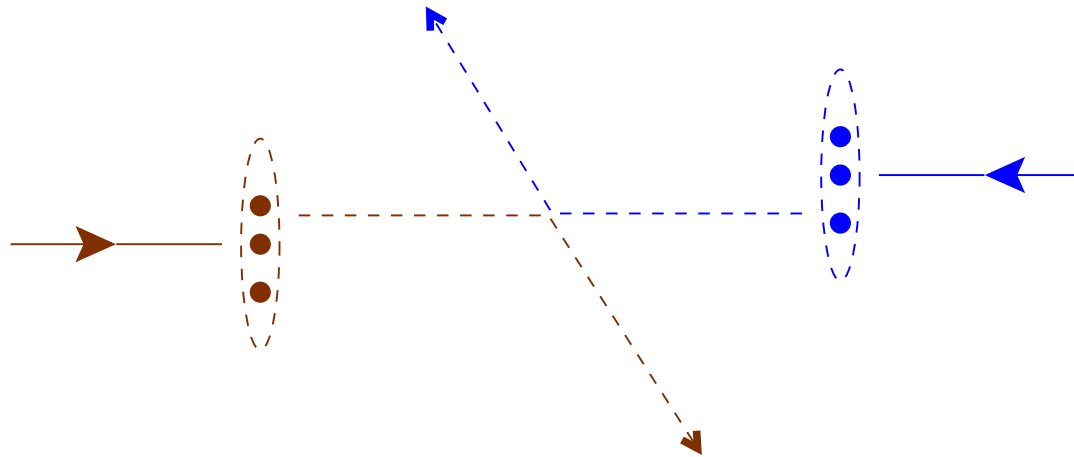
# Unintegrated Parton Densities

John Collins (Penn State)

# Summary

- Factorization; unintegrated pdfs: When & why?
- Eikonal propagation of fast partons
  - SSA etc
  - Evolution
- Factorization breaking

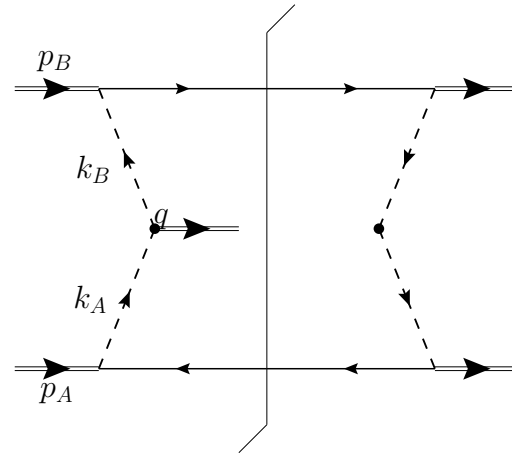
# Basic idea of factorization



- Time dilated and Lorentz contracted beam particles
- Short-distance hard scattering
- Pdf  $f_{i/p}(x)$  as probability density
  - Depends on longitudinal momentum fraction
  - Parton  $k_T$  and  $k^2$  integrated over (neglected in hard scattering)
  - Fragmentation fn. similarly.
- Factorization formula for e.g.,  $H_1 + H_2 \rightarrow H_3 + H_4 + X$ , high  $p_T$
- Predictions: from universality of pdfs, perturbative calculations of hard scattering

# Unintegrated parton densities

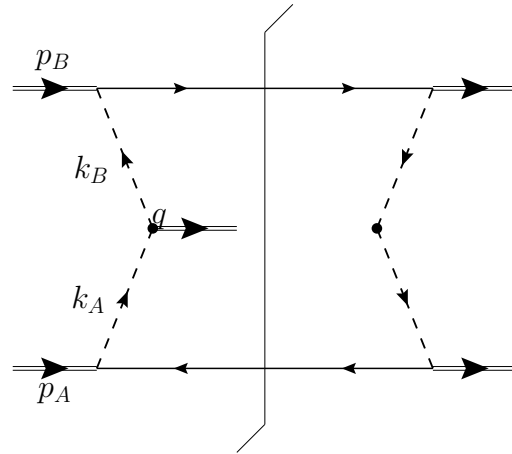
- E.g., Drell-Yan cross section with measured  $q_T$  (in model):
- In model:



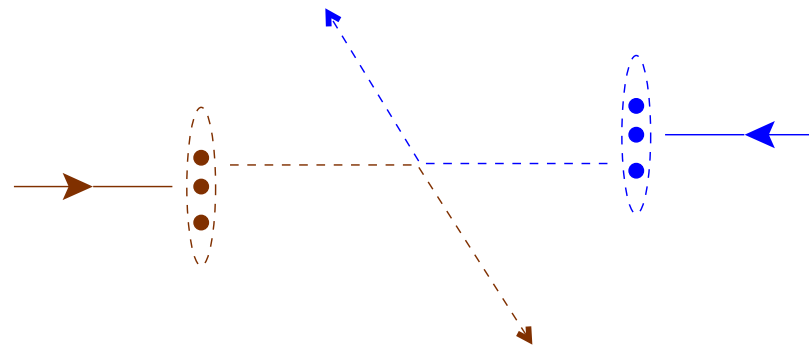
- Can *not* neglect parton  $k_T$  in hard scattering
- $\frac{d\sigma}{d^4q} \propto \int d^2k_T P_{\bar{q}}(x_1, k_T) P_q(x_2, q_t - k_T) \hat{\sigma}_{q\bar{q}} \dots$

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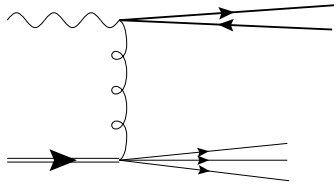


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- Similarly for out-of-plane  $p_T$  in  $H_1 + H_2 \rightarrow H_3 + H_4 + X$ :



# Where need for unintegrated pdfs etc shows up:

E.g.:  $\gamma$ -prod. of  $c\bar{c}$ :

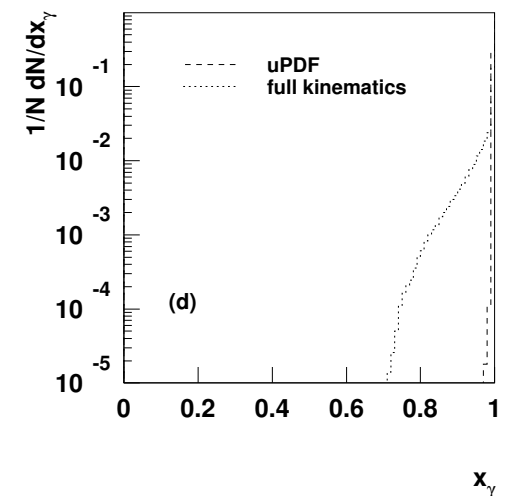
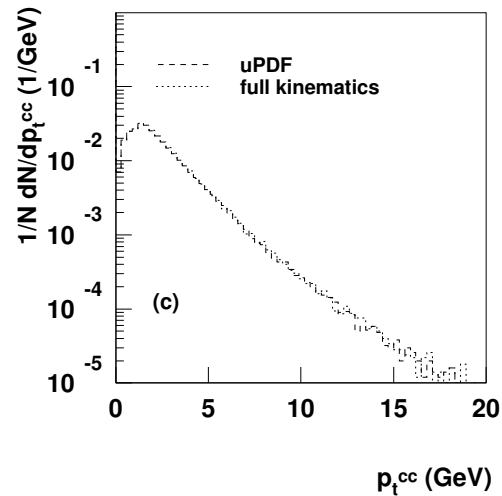
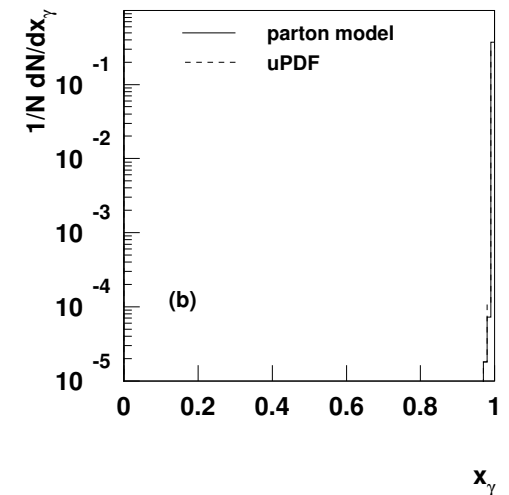
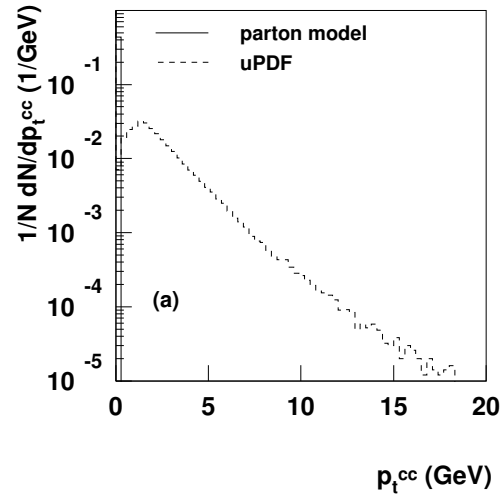


Differential cross section:

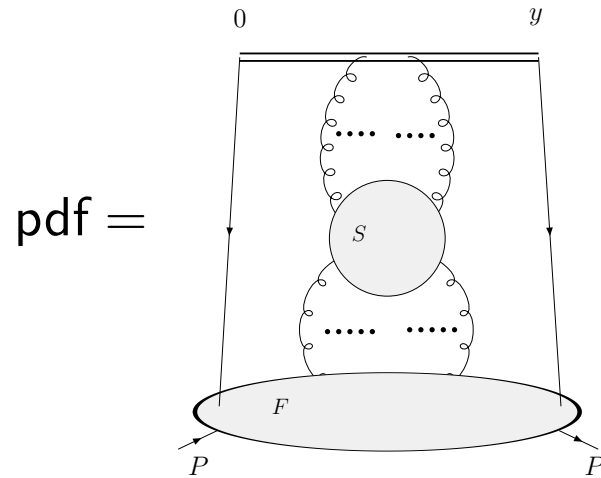
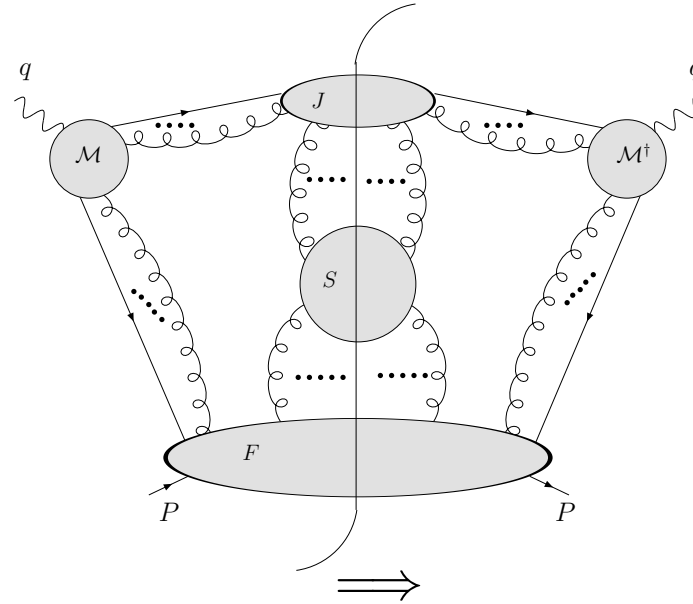
$p_T \implies$  unint. pdf

$x_\gamma \implies$  parton correl. fn.

(Plots: JCC & Jung,  
arXiv:hep-ph/0508280)

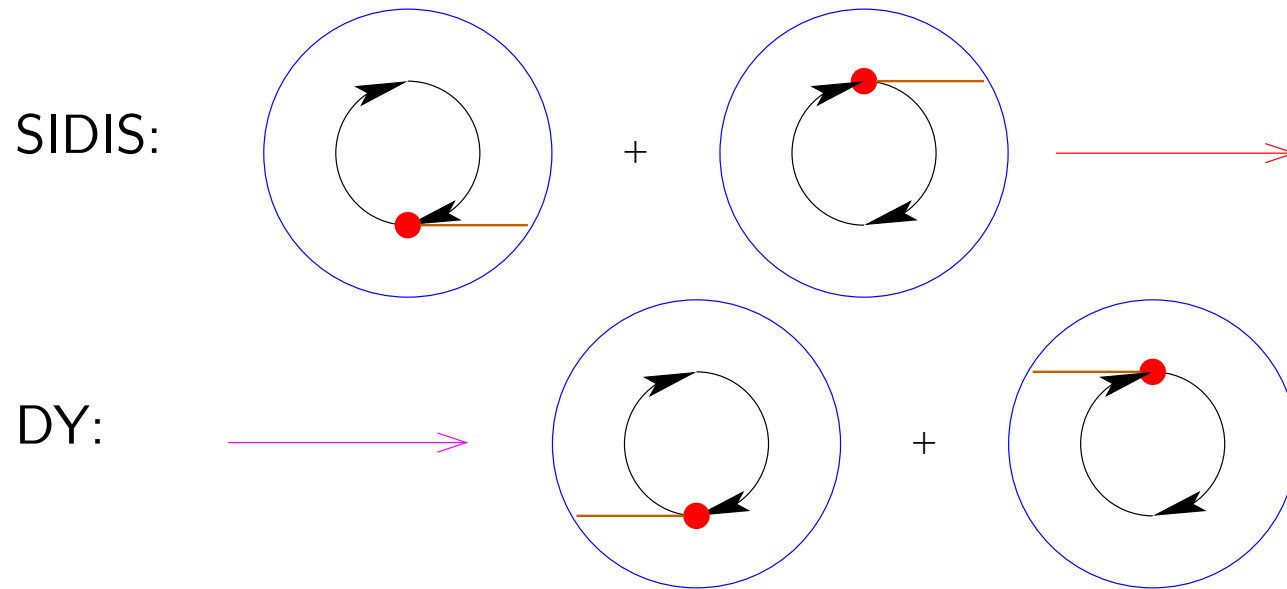


# Propagation of quark; Wilson lines in definition of pdf



$$= \text{FT of } \langle p, s | \bar{\psi}(y) W(y, \infty)^\dagger W(0, \infty) \psi(0) | p, s \rangle$$

# DIS v. DY; Sivers function; modified universality



Future- v. past-pointing propagation

$$P(x, \mathbf{k}_T) = P_{\text{unpol}}(x, |k_T|) + P_{\text{Sivers}}(x, |k_T|) \sin(\phi - \phi_S)$$

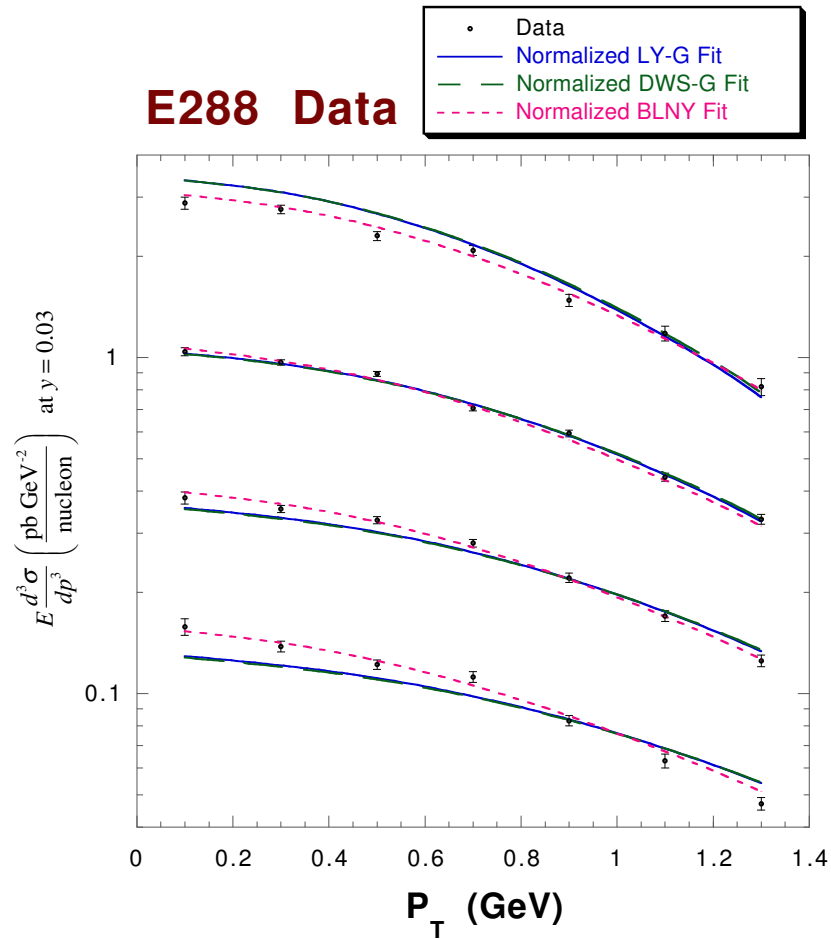
Time-reversal

⇒ Unpolarized density: same numerical value

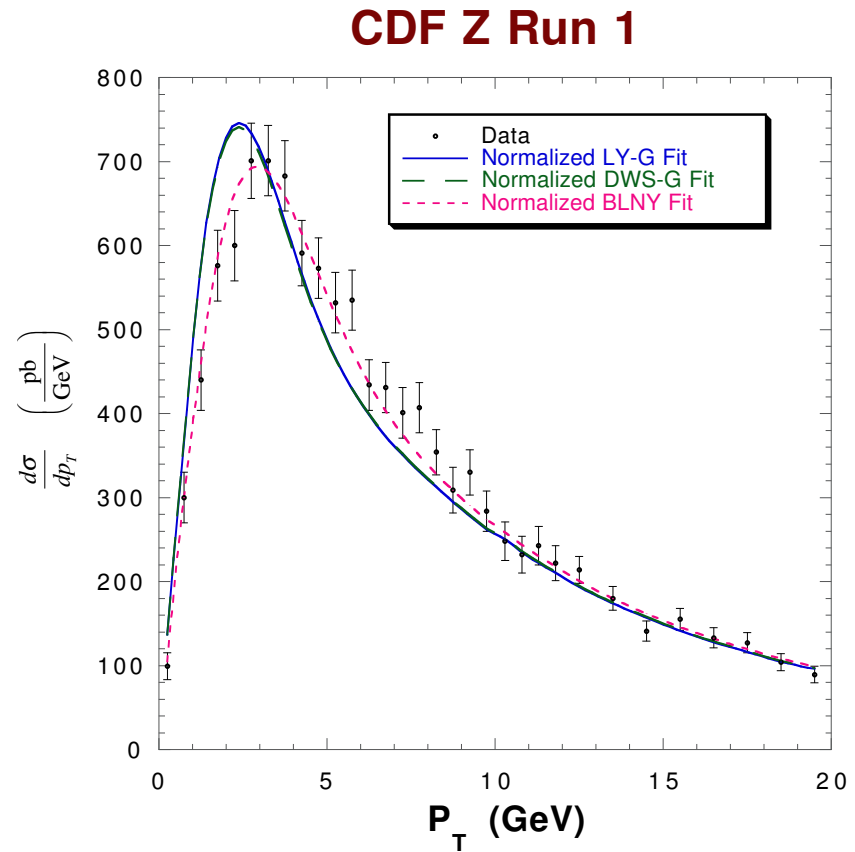
⇒ SSA: opposite



# But CSS evolution



$$\sqrt{s} = 27.4 \text{ GeV}$$



$$\sqrt{s} \sim 900 \text{ GeV}$$

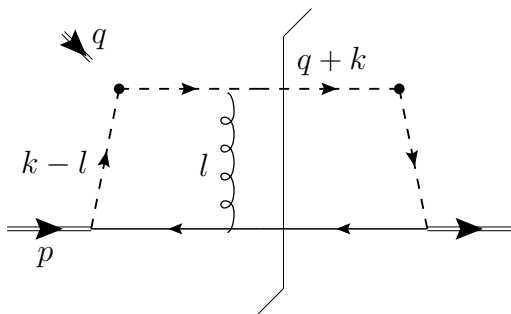
[Landry et al., Phys. Rev. D67, 073016 (2003)]

# Hadron production: Counterexample to factorization I

(JCC & Qiu, arXiv:0705.2141)

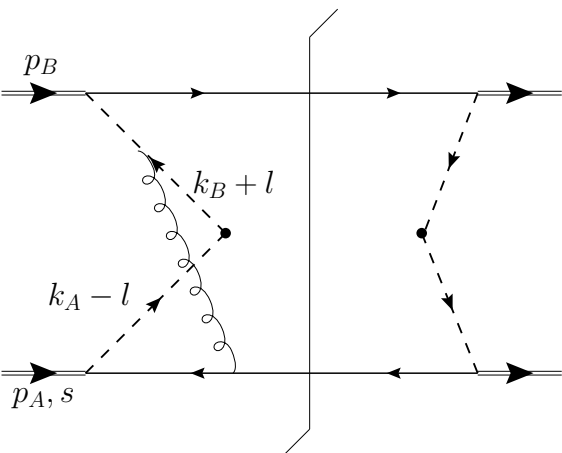
- Spectator model, with 1-extra-gluon exchange
- SSA starts with imaginary part (from on-shell intermediate state)
- In processes with factorization

– SIDIS:



$$\implies \frac{g \times 2q^-}{(q+k-l)^2 - m^2 + i\epsilon} \implies g \times (-i\pi)$$

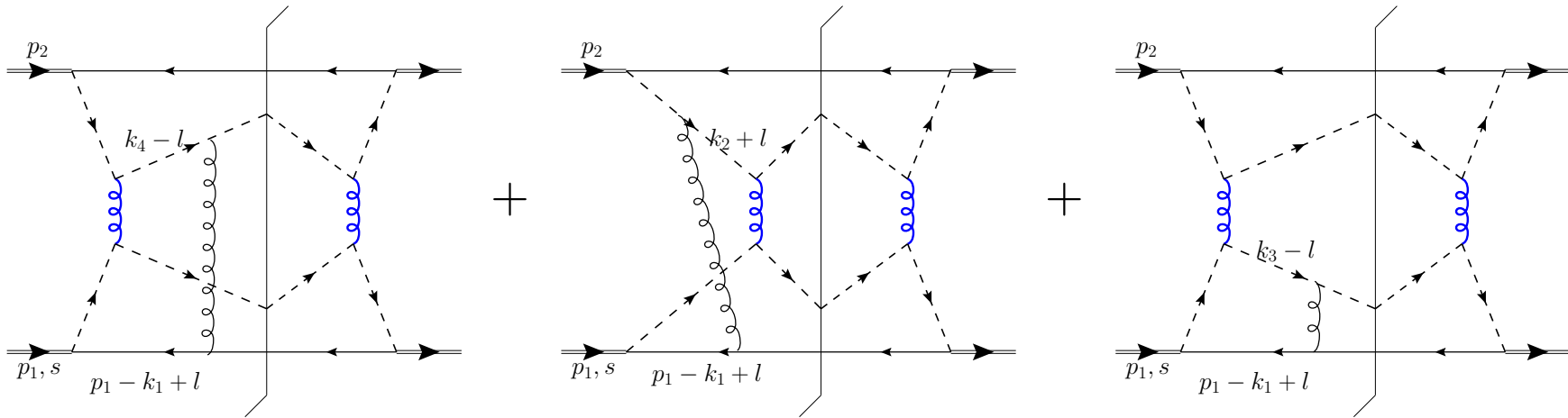
– DY:



$$\implies \frac{-g \times 2k_3^-}{(k_3+l)^2 - m^2 + i\epsilon} \implies -g \times (-i\pi)$$

# Hadron production: Counterexample to factorization II

- Simplified model with abelian gluons, different quark charges,
- Maximally simple calculation from:



$$\implies \text{SSA from } \frac{g_2 \times 2k_4^-}{(k_4 - l)^2 - m^2 + i\epsilon} + \frac{g_2 \times 2k_2^-}{(k_2 - l)^2 - m^2 + i\epsilon} + \frac{g_1 \times 2k_3^-}{(k_3 - l)^2 - m^2 + i\epsilon}$$

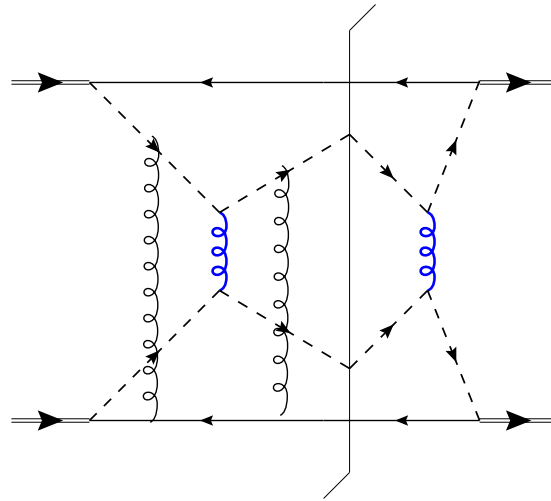
- Real part eikonalizes at leading power
- Imaginary part from  $(2g_2 + g_1) \times (-i\pi)$

$\implies$  Non-universality of pdfs or non-factorization

# Non-factorization

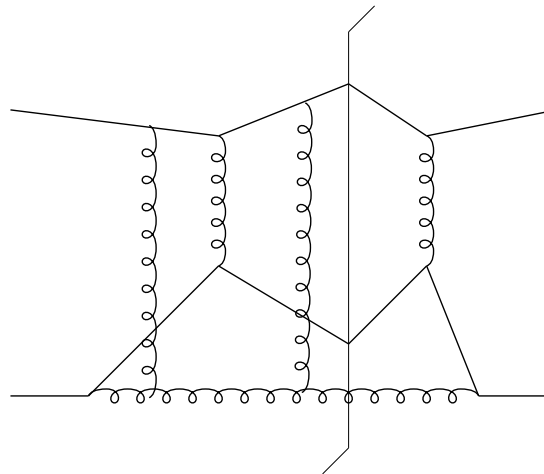
- Correction of parton model by eikonal propagation of fast partons through glue
  - Factorization only by absorbing this in Wilson lines in definitions of pdfs, etc.
  - SSA with TMD especially sensitive
- Counter-example in model theory:
  - Spectator-related phenomenon: Pdf effect
  - Dependence on color of other parton: Cannot use Wilson line
- Kills proof of factorization with TMD in hadroproduction of hadrons
- Also for unpolarized cross section, resummation, etc, etc.

- Minimum factorization breaking graphs in unpolarized cross section:



etc. (Check!)

- Collinear factorization (without TMD sensitivity): **Must check carefully!**.  
(Derivations in literature quite inexplicit at the difficult points.)
- Ordinary unpolarized quark-gluon calculations only affected at NNLO:



etc

# Conclusions

- TMD distributions natural and normal from parton model
- Needed in DY etc, where kinematics wrong at LO without parton  $k_T$
- Give greater sensitivity to beyond-naive-parton-model effects
- Especially SSA
- Factorization breaking: Big opportunity
- SIDIS v. DY comparison vital