# *Spin and TMD azimuthal asymmetries in SIDIS*

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TRENTO, ITALY
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Castello di Trento ("Trint"), watercolour, 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495) C. British Museum, London.

**Transverse momentum, spin, and position distributions of partons in hadrons**

Jefferson Lab Trento, June 11-15, 2007

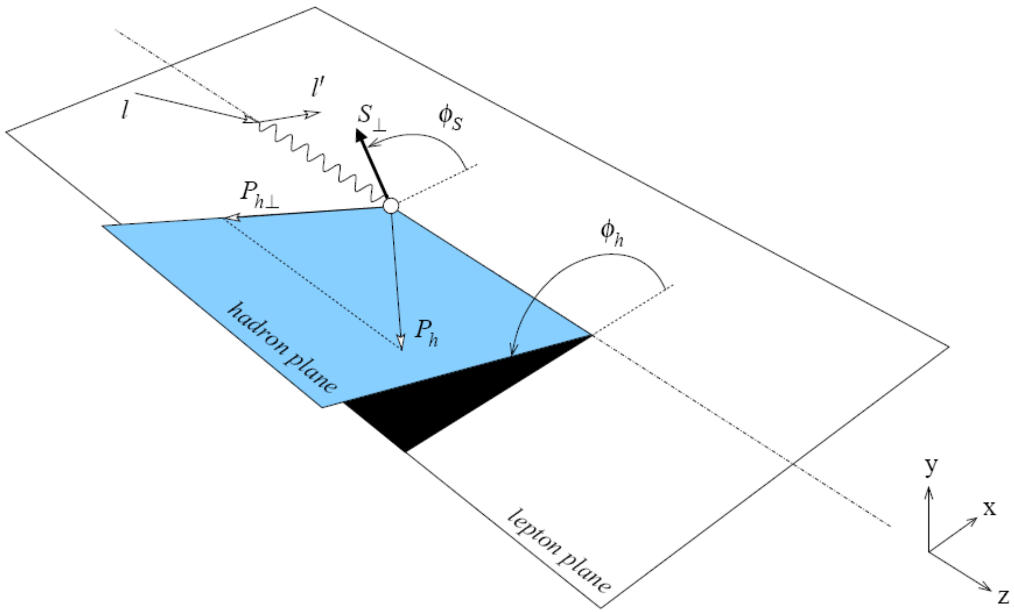
- Polarized SIDIS cross-section
  - Azimuthal asymmetries
  - Parton model for CFR
    - Quark-Diquark model for DFs
  - TFR
- Conclusions

# General expression of polarized SIDIS cross-section (1)

One photon exchange approximation: $\mathcal{M} \propto J_\mu^{lept} \frac{1}{q^2} J^\mu_{hadr}$

A.K. *NP B441 (1995)*

$$l_{\mu\nu} W^{\mu\nu} = L^{ab} \left( H_{ab}^{(0)} + S^\rho e^i_\rho H_{abi}^{(S)} \right)$$




$$\mathcal{L}^{00} = \epsilon,$$

$$\mathcal{L}^{01} = \sqrt{\frac{\epsilon(1+\epsilon)}{2}} \cos\phi_h^l + i\lambda\sqrt{\frac{\epsilon(1-\epsilon)}{2}} \sin\phi_h^l,$$

$$\mathcal{L}^{02} = -\sqrt{\frac{\epsilon(1+\epsilon)}{2}} \sin\phi_h^l + i\lambda\sqrt{\frac{\epsilon(1-\epsilon)}{2}} \cos\phi_h^l,$$

$$\mathcal{L}^{11} = \tfrac{1}{2} + \tfrac{1}{2}\epsilon\cos 2\phi_h^l,$$

$$\mathcal{L}^{12} = -\frac{\epsilon}{2}\sin 2\phi_e^h + i\frac{\lambda}{2}\sqrt{1-\epsilon^2},$$

$$\mathcal{L}^{22} = \tfrac{1}{2} - \tfrac{1}{2}\epsilon\cos 2\phi_h^l,$$

Using current conservation + parity conservation + hermiticity one can show that

18 independent Structure Functions describe one particle SIDIS.

Moreover, the dependences on azimuthal angle of produced hadron and of the target nucleon polarization were calculated explicitly and factorized

# General expression of polarized SIDIS cross-section (2)

Bacchetta, Diehl, Goeke, Metz, Mulders and Schlegel JHEP 0702:093,2007

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+P_{beam}\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

$$+P_L\Big[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\Big]$$

$$+P_L P_{beam}\Big[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\Big]$$

$$+|\boldsymbol{P}_T|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{P}_T|P_{beam}\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\},$$

$$\varepsilon=\frac{1-y-\frac{1}{4}y^2\gamma^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}y^2\gamma^2}$$

$$\gamma=2x_B M_p/Q$$

This is a general expression which is also valid for exclusive reactions and for entire phase space of SIDIS (TFR, CFR)

## Azimuthal modulations:

2 polarization independent

1 single beam polarization dependent

2 single target longitudinal polarization dependent

1 double beam + target longitudinal polarization dependent

5 single target transverse polarization dependent

3 double beam + target transverse polarization dependent

# Measured Structure Functions and Asymmetries

$F_{UU,T}$ - Yes

$F_{UU,L}$ - No

$F_{UU}^{\cos\varphi_h}$ - Yes

$F_{UU}^{\cos 2\varphi_h}$ - Yes

$A_{LU}^{\sin\varphi_h}$ - Yes, HERMES & JLab

$A_{UL}^{\sin\varphi_h}$ - Yes, HERMES & JLab

$A_{UL}^{\sin 2\varphi_h}$ - Yes, HERMES & JLab

$A_{LL}$ - Yes

$A_{LL}^{\cos\varphi_h}$ - Yes, JLab preliminary

$A_{UT,T}^{\sin(\varphi_h-\varphi_s)}$ - Yes, HERMES & COMPASS

$A_{UT,L}^{\sin(\varphi_h-\varphi_s)}$ - No

$A_{UT}^{\sin(\varphi_h+\varphi_s)}$ - Yes, HERMES & COMPASS

$A_{UT}^{\sin(3\varphi_h-\varphi_s)}$ - Yes, COMPASS preliminary

$A_{LT}^{\cos(\varphi_h-\varphi_s)}$ - Yes, COMPASS preliminary

$A_{LT}^{\cos\varphi_s}$ - Yes, COMPASS preliminary

$A_{UT}^{\sin\varphi_s}$ - Yes, COMPASS preliminary

$A_{LT}^{\cos(2\varphi_h-\varphi_s)}$ - Yes, COMPASS preliminary

$A_{UT}^{\sin(2\varphi_h-\varphi_s)}$ - Yes, COMPASS preliminary

# Parton model for SIDIS in CFR

$$d\sigma^{l+N\to l'+h+X} \propto DF \otimes d\sigma^{l+q\to l'+q'} \otimes FF$$

At twist-two

$$\mathcal{P}_N^q(x,\mathbf{k}_T) = f_1^q(x,k_T^2) + f_{1T}^{\perp q}(x,k_T^2)\frac{[\mathbf{k}_T \times \hat{\mathbf{P}}_N]\cdot S_T^N}{M},$$

$$f_1^q(x,k_T^2)s_L^q(x,\mathbf{k}_T) = g_{1L}^q(x,k_T^2)\lambda_N + g_{1T}^{\perp q}(x,k_T^2)\frac{\mathbf{k}_T\cdot\mathbf{S}_T^N}{M},$$

$$f_1^q(x,k_T^2)\mathbf{s}_T^q(x,\mathbf{k}_T) = h_{1T}^q(x,k_T^2)\mathbf{S}_T^N + [h_{1L}^{\perp q}(x,k_T^2)\lambda_N + h_{1T}^{\perp q}(x,k_T^2)\frac{\mathbf{k}_T\cdot\mathbf{S}_T^N}{M}]\frac{\mathbf{k}_T}{M} + h_1^{\perp q}(x,k_T^2)\frac{[\mathbf{k}_T\times\hat{\mathbf{P}}_N]}{M}$$

Often used: $h_1^q(x,k_T^2) = h_{1T}^q(x,k_T^2) + \frac{k_T^2}{2M^2}h_{1T}^{\perp q}(x,k_T^2)$

$$\mathcal{P}_{q\uparrow}^h(z,\mathbf{P}_{Tq}^h) = D_q^h(z,P_{Tq}^h) + H_{1q}^{\perp h}(z,P_{Tq}^h)\frac{[\mathbf{P}_{Tq}^h\times\hat{\mathbf{k}}']\cdot\mathbf{s}_T'}{M} = D_q^h(z,P_{Tq}^h) + s_T'\frac{P_{Tq}^h}{M}H_{1q}^{\perp h}(z,P_{Tq}^h)\sin(\phi_{Collins})$$

A. Bacchetta, M. Boglione, A. Henneman and P. J. Mulders, Phys. Rev. Lett. **85**, 712 (2000)

$$\frac{\mathrm{k}_T^2}{M^2}\left(g_{1T}^q(x,k_T^2)\right)^2 + \frac{\mathrm{k}_T^2}{M^2}\left(f_1^{q\perp}(x,k_T^2)\right)^2 \le \left(f_1^q(x,k_T^2)\right)^2 - \left(g_{1L}^q(x,k_T^2)\right)^2$$

# Twist-two contributions

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \;
& \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{ \boxed{F_{UU,T}} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
& + \varepsilon\,\boxed{\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}} + P_{beam}\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
& + P_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\boxed{\sin\phi_h\,F_{UL}^{\sin\phi_h}} + \varepsilon\,\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
& + P_L P_{beam}\left[\sqrt{1-\varepsilon^2}\,\boxed{F_{LL}} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
& + |\boldsymbol{P}_T|\Big[\boxed{\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\right.} + \left.\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad + \varepsilon\,\boxed{\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}} + \varepsilon\,\boxed{\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
& + |\boldsymbol{P}_T|P_{beam}\Big[\sqrt{1-\varepsilon^2}\,\boxed{\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\},
\end{aligned}
$$

- $F_{UU,T}$ → $f_1^q \otimes D_q^h$
- $\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$ → $h_1^{\perp q} \otimes H_{1q}^{\perp h}$
- $\sin\phi_h\,F_{UL}^{\sin\phi_h}$ → $h_{1L}^{\perp q} \otimes H_{1q}^{\perp h}$
- $F_{LL}$ → $g_{1L}^q \otimes D_q^h$
- $\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)}$ → $f_{1T}^{\perp q} \otimes D_q^h$
- $\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$ → $h_1^q \otimes H_{1q}^{\perp h}$
- $\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$ → $h_{1T}^{\perp q} \otimes H_{1q}^{\perp h}$
- $\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$ → $g_{1T}^{\perp q} \otimes D_q^h$

# Orbital momentum and $g_{1L}$

Model by Brodsky, Hwang, Ma & Schmidt, NPB 593 (2001) 311

Spin decomposition of the $J^z = +\frac{1}{2}$ electron

| Configuration | Fermion spin $s_f^z$ | Boson spin $s_b^z$ | Orbital ang. mom. $l^z$ |
|---|---|---|---|
| $\left\|+\frac{1}{2}\right\rangle \to \left\|+\frac{1}{2}+1\right\rangle$ | $+\frac{1}{2}$ | $+1$ | $-1$ |
| $\left\|+\frac{1}{2}\right\rangle \to \left\|-\frac{1}{2}+1\right\rangle$ | $-\frac{1}{2}$ | $+1$ | $0$ |
| $\left\|+\frac{1}{2}\right\rangle \to \left\|+\frac{1}{2}-1\right\rangle$ | $+\frac{1}{2}$ | $-1$ | $+1$ |

$$g_{1L}(x,k_T^2)_{\text{spin-1 diquark}} \propto \left[\frac{k_T^2}{x^2(1-x)^2} + \frac{k_T^2}{(1-x)^2} - \left(M - \frac{m}{x}\right)^2\right]|\varphi|^2$$

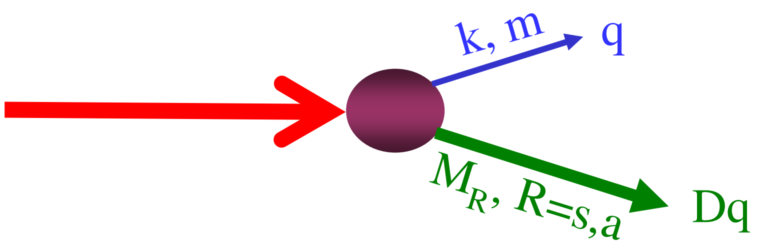
Spin decomposition of the $J^z = +\frac{1}{2}$ fermion in Yukawa theory

| Configuration | Fermion spin $s_f^z$ | Boson spin $s_b^z$ | Orbital ang. mom. $l^z$ |
|---|---|---|---|
| $\left\|+\frac{1}{2}\right\rangle \to \left\|+\frac{1}{2}\right\rangle$ | $+\frac{1}{2}$ | $0$ | $0$ |
| $\left\|+\frac{1}{2}\right\rangle \to \left\|-\frac{1}{2}\right\rangle$ | $-\frac{1}{2}$ | $0$ | $+1$ |

$$g_{1L}(x,k_T^2)_{\text{spin-0 diquark}} \propto \left[\left(M + \frac{m}{x}\right)^2 - \frac{k_T^2}{x^2}\right]|\varphi|^2$$

# Quark-Diquark model

R. Jakob, P. Mulders & Rodrigues NP A626, 937 (1997)




Choose exponential form-factor:

$$N\frac{k^2-m^2}{\left|k^2-\Lambda^2\right|^{\alpha}} \Rightarrow N'(k^2-m^2)\exp\left(\frac{k^2}{2\Lambda^2}\right)$$

$$f_1(x,k_T^2)=q_+(x,k_T^2)+q_-(x,k_T^2)=f_0(x)\left[(xM+m)^2+k_T^2\right]\exp\left(-\frac{k_T^2}{(1-x)\Lambda^2}\right)$$

$$g_{1L}(x,k_T^2)=q_+(x,k_T^2)-q_-(x,k_T^2)=a_R f_0(x)\left[(xM+m)^2-k_T^2\right]\exp\left(-\frac{k_T^2}{(1-x)\Lambda^2}\right)$$

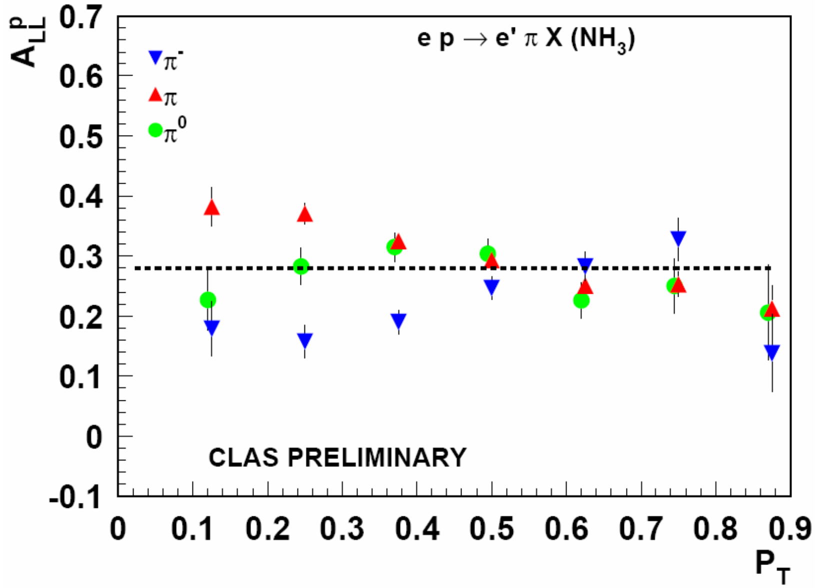
$$f_0(x)=\frac{N^2}{(1-x)}\exp\left(-\frac{x(1-x)M^2-xM_R^2}{(1-x)\Lambda^2}\right)$$

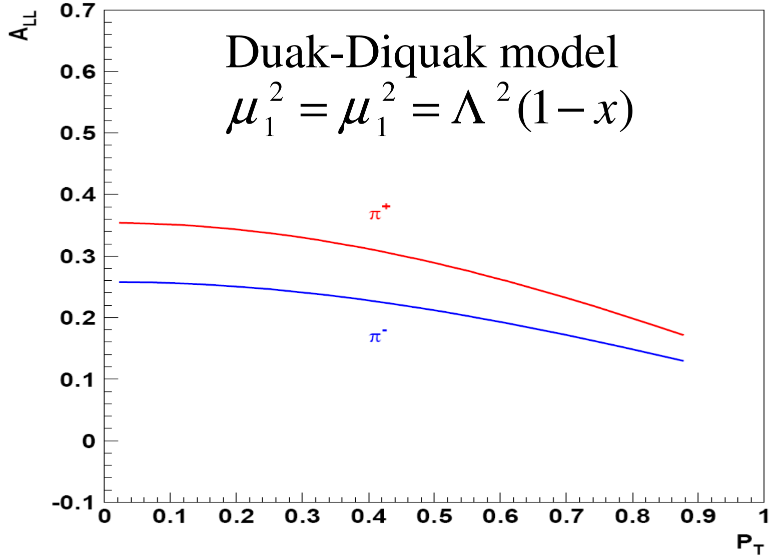
$$q_+(x,k_T^2)=\frac{1}{2}f_0(x)(xM+m)^2\exp\left(-\frac{k_T^2}{(1-x)\Lambda^2}\right)$$

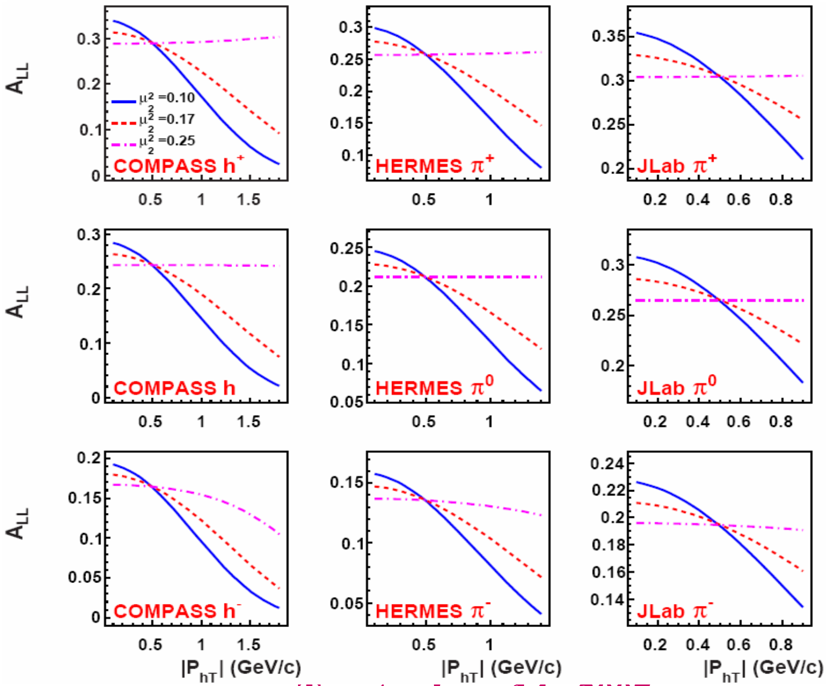
$$q_-(x,k_T^2)=\frac{1}{2}f_0(x)k_T^2\exp\left(-\frac{k_T^2}{(1-x)\Lambda^2}\right)$$

No x-$k_T$ factorization!

# $A_{LL}$ @ JLab




M.Anselmino, A.Efremov, A.K & B.Parsamyan

PRD 74, 074015 (2006)

$$f_1^q(x,k_T^2) = \frac{1}{\pi\mu_1^2}\exp(-\frac{k_T^2}{\mu_1^2})f_1^q(x)$$

$$g_{1L}^q(x,k_T^2) = \frac{1}{\pi\mu_2^2}\exp(-\frac{k_T^2}{\mu_2^2})g_{1L}^q(x)$$

$\mu_1^2 = 0.25$; $\mu_2^2 = 0.25$; $\mu_2^2 = 0.17$; $\mu_2^2 = 0.10$; $(\text{GeV/c})^2$

The case $\mu_1^2 = 0.25$; $\mu_2^2 = 0.17$ $(\text{GeV/c})^2$ is very similar to Quark-Diquark results

# Interpretation of target transverse spin asymmetries

Twist-2:

Sivers

Collins

$$A_{UT}^{\sin(\varphi_h-\varphi_s)} \propto f_{1T}^{\perp q} \otimes D_{1q}^{h}$$

$$A_{UT}^{\sin(\varphi_h+\varphi_s)} \propto h_1^{q} \otimes H_{1q}^{\perp h}$$

$$A_{LT}^{\cos(\varphi_h-\varphi_s)} \propto g_{1T}^{q} \otimes D_{1q}^{h}$$

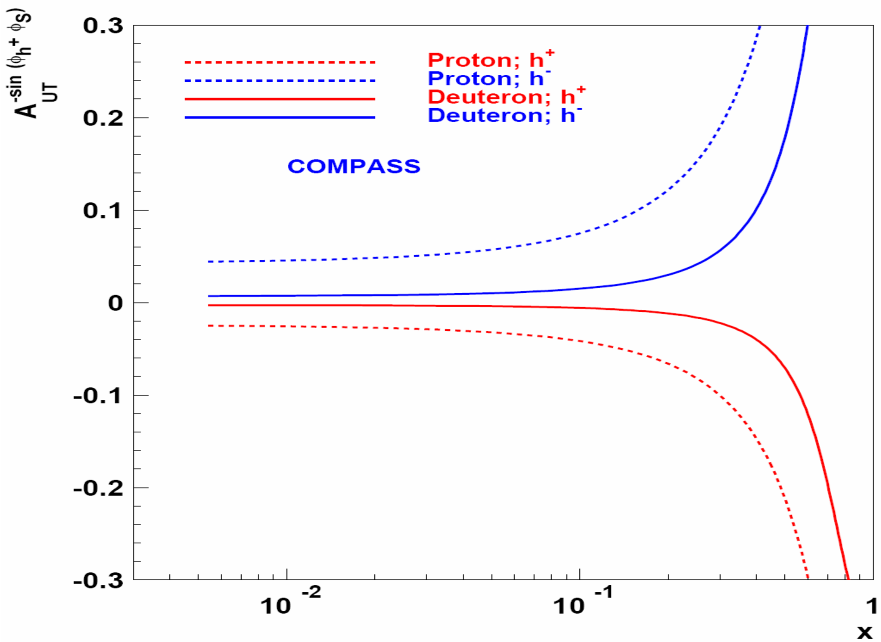
$$A_{UT}^{\sin(3\varphi_h-\varphi_s)} \propto h_{1T}^{\perp q} \otimes H_{1q}^{\perp h}$$

# $A_{UT}^{-\sin(\varphi_h-\varphi_s)}$ Collins asymmetry @ COMPASS

$$A_{UT}^{\sin(\varphi_h+\varphi_s)} \propto h_1^q \otimes H_{1q}^{\perp h}$$

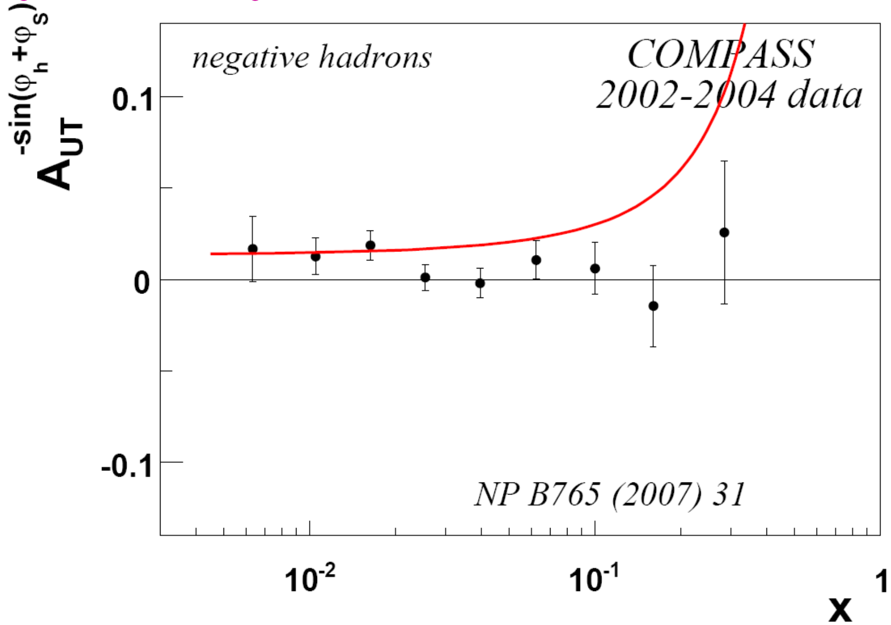
$$h_1^q(x,k_T^2) = a_R f_0(x)(xM+m)^2 \exp(-\frac{k_T^2}{(1-x)\Lambda^2})$$

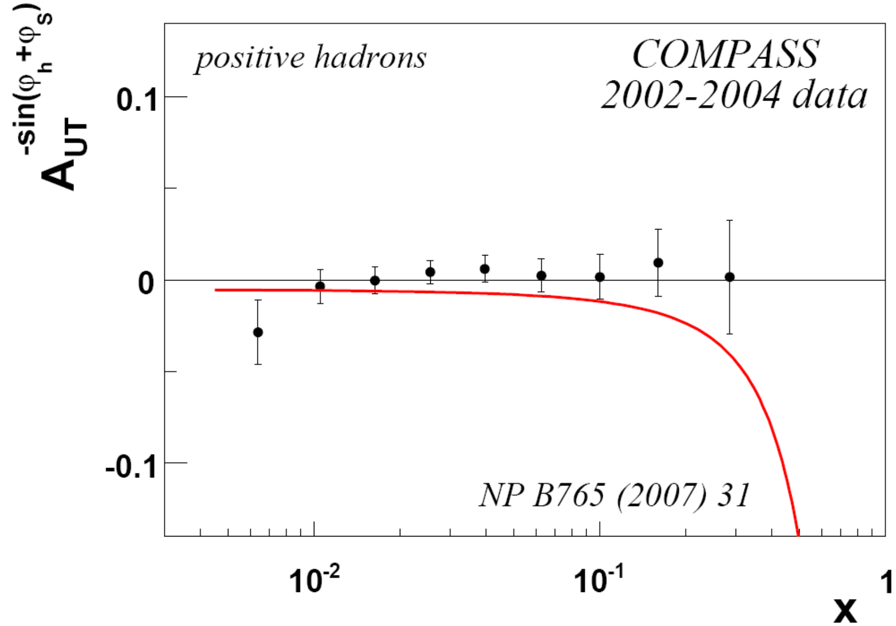
$$f_0(x) = \frac{N^2}{(1-x)} \exp\left(-\frac{x(1-x)M^2 - xM_R^2}{(1-x)\Lambda^2}\right)$$




~2 times lager than best fit

from Anselmino *et al.* analysis



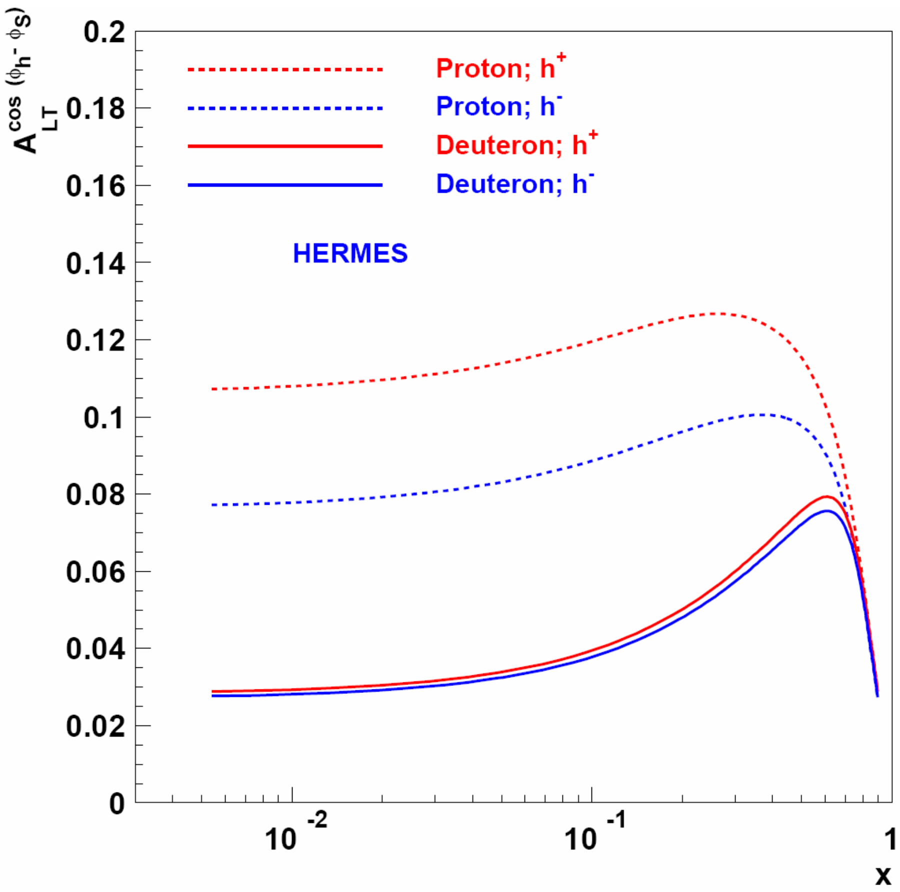



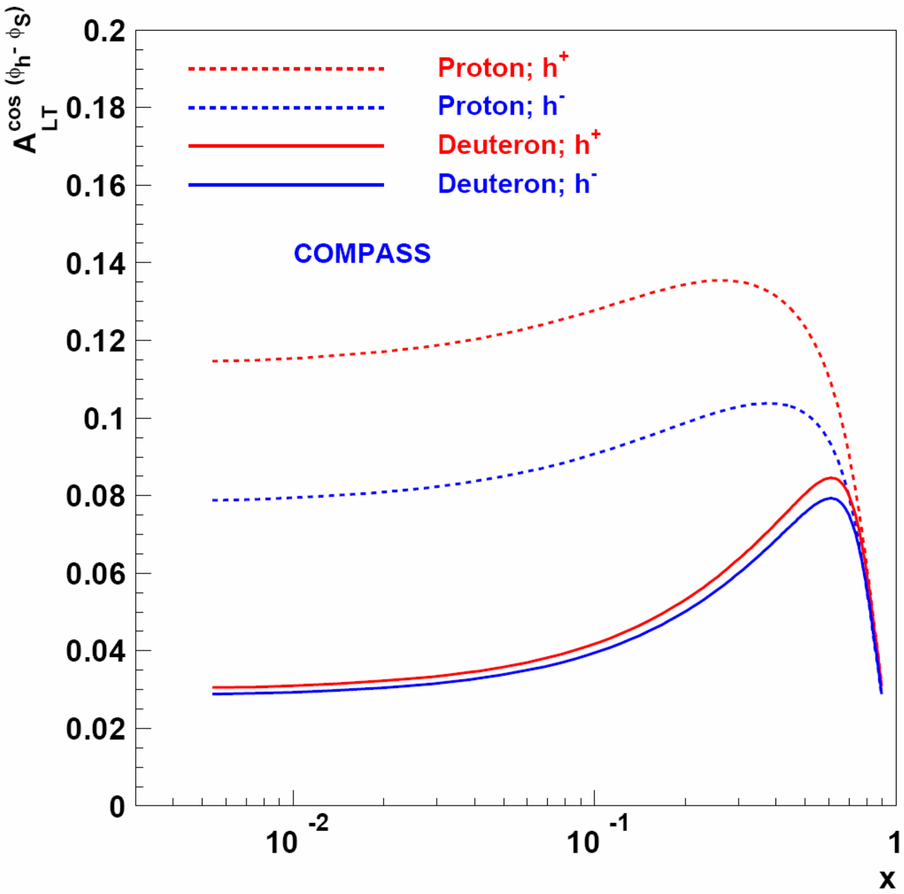
# $A_{LT}^{\cos(\varphi_h-\varphi_s)}$ in quark-diquark model

$$A_{LT}^{\cos(\varphi_h-\varphi_s)} \propto g_{1T}^q \otimes D_{1q}^h$$

$$g_{1T}^q(x,k_T^2) = 2a_R f_0(x) M(xM+m)\exp\left(-\frac{k_T^2}{(1-x)\Lambda^2}\right)$$

$$f_0(x) = \frac{N^2}{(1-x)}\exp\left(-\frac{x(1-x)M^2 - xM_R^2}{(1-x)\Lambda^2}\right)$$



# $A_{LT}^{\cos(\varphi_h - \varphi_s)}$ @ COMPASS

First estimations by

A.K. & P.Mulders,

PRD 54, 1229 (1996)

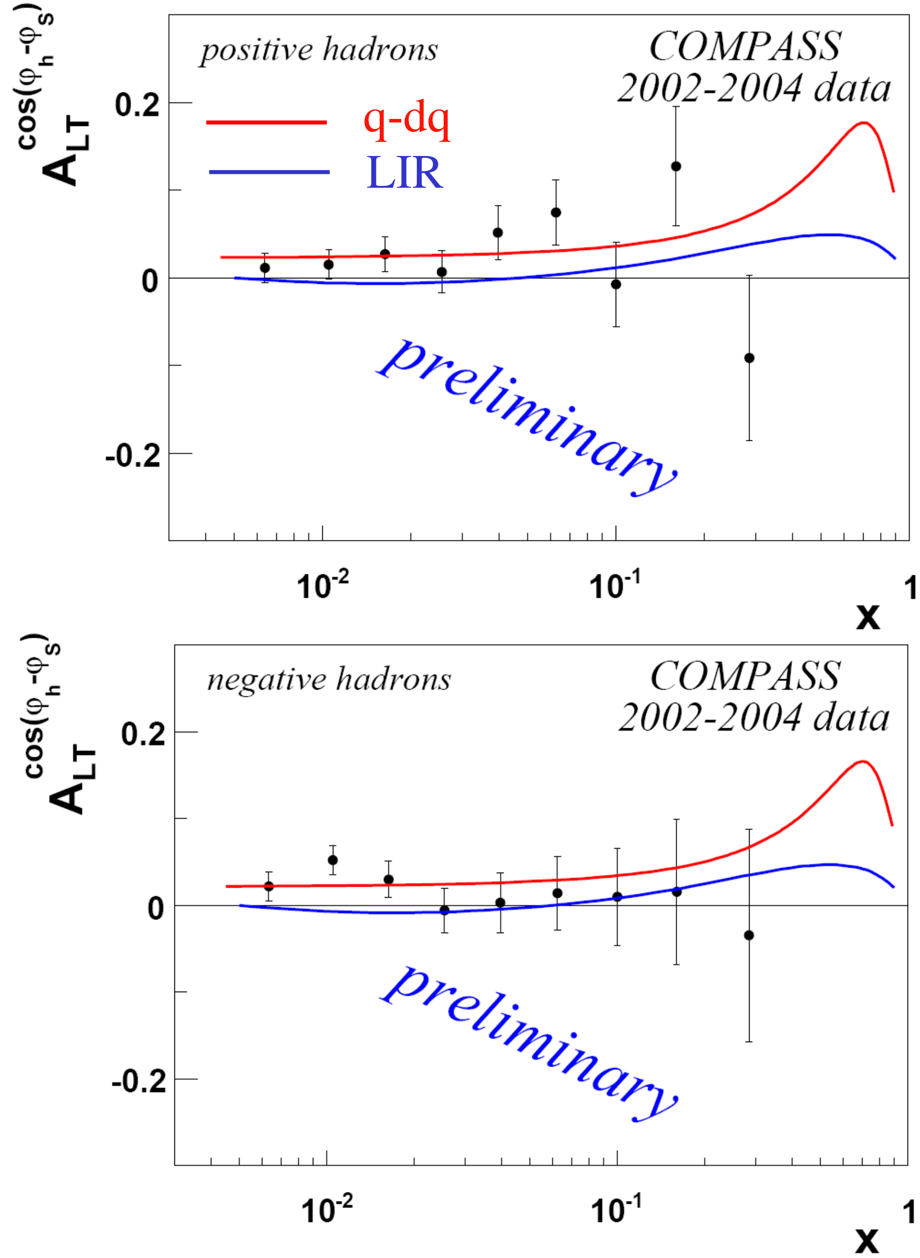
**Lorentz Invariance Relations:**

$$g_2^q(x) = \frac{d}{dx} g_{1T}^{q(1)}(x)$$

$$g_{1T}^{q(1)}(x, k_T^2) \approx x \int_x^1 dy \frac{g_1^q(y)}{y}$$

A.K., Parsamyan & Prokudin,

**PRD73:114017,2006**



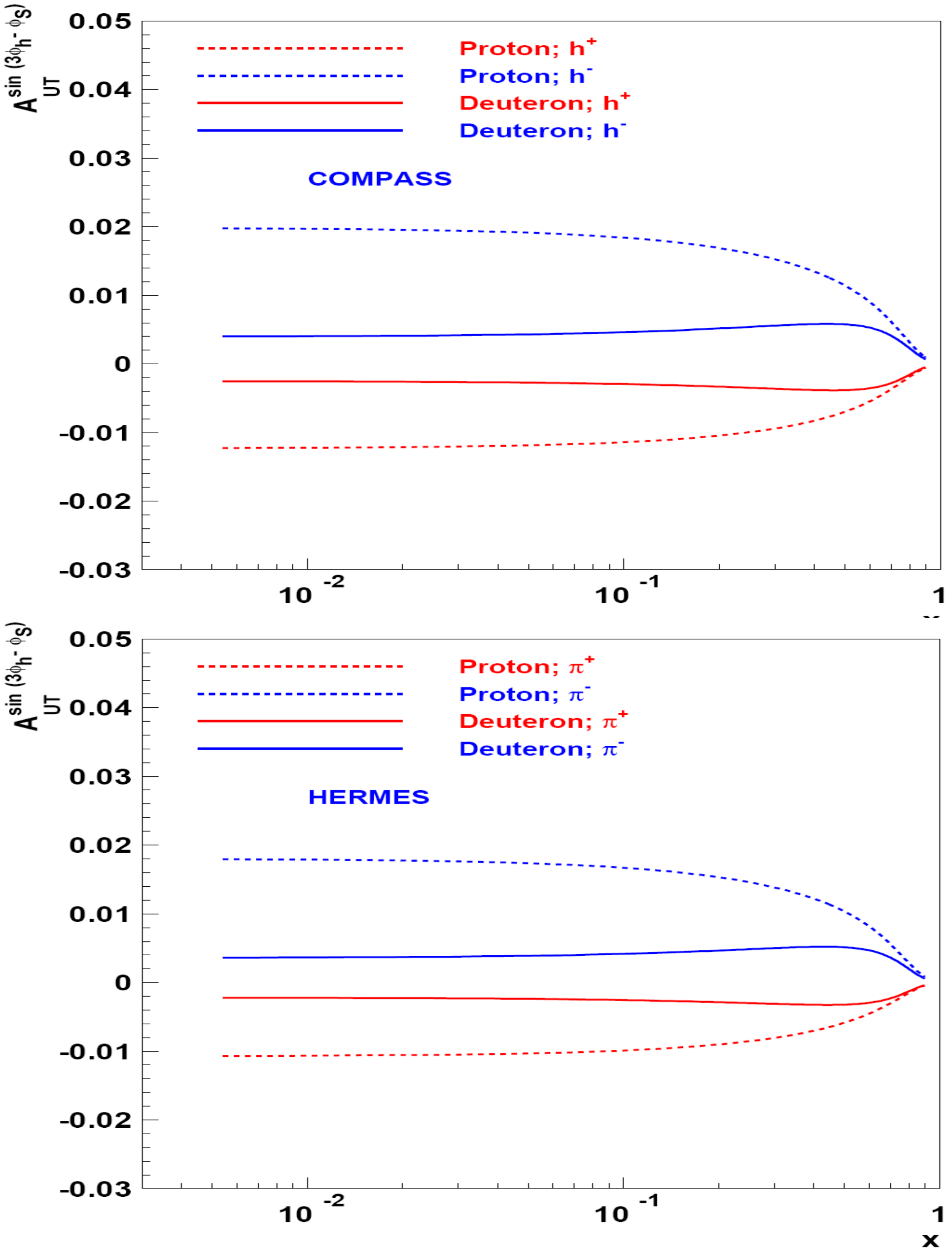
# $A_{UT}^{\sin(3\varphi_h-\varphi_s)}$ in quark-diquark model

$$A_{UT}^{\sin(3\varphi_h-\varphi_s)} \propto h_{1T}^{\perp q} \otimes H_{1q}^{\perp h}$$

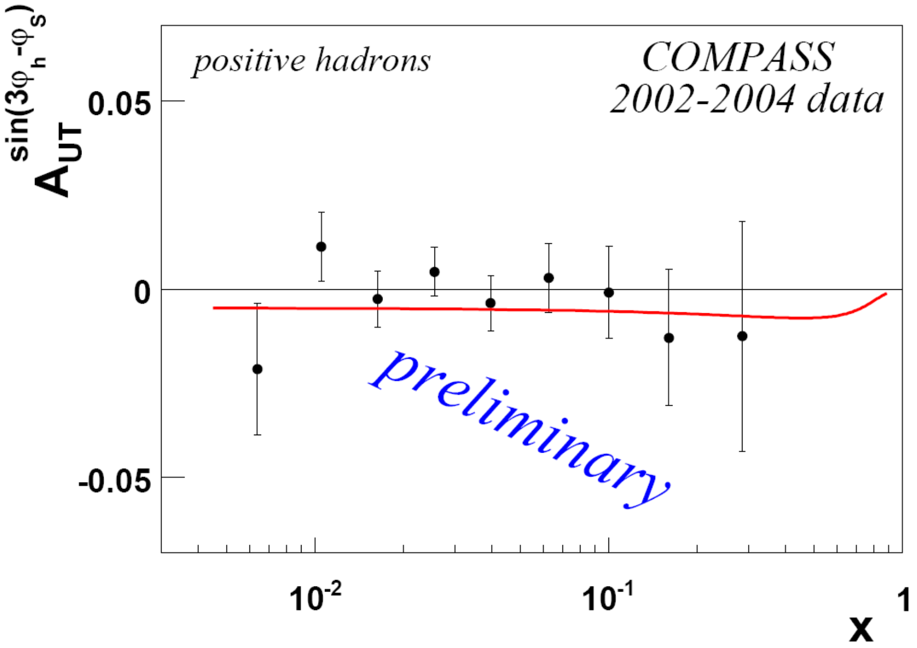
$$h_{1T}^{\perp q}(x,k_T^2) = -2a_R f_0(x) M^2 \exp\left(-\frac{k_T^2}{(1-x)\Lambda^2}\right)$$

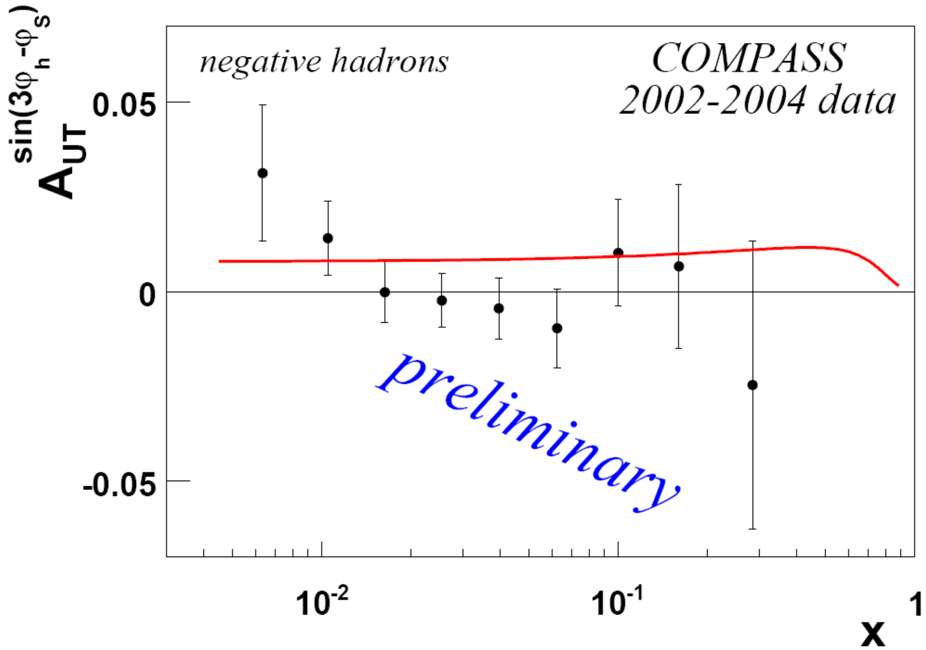
$$f_0(x) = \frac{N^2}{(1-x)} \exp\left(-\frac{x(1-x)M^2 - xM_R^2}{(1-x)\Lambda^2}\right)$$

$H_{1q}^{\perp h}$ from Anselmino *et al.* global analysis



# $A_{UT}^{\sin(3\varphi_h-\varphi_s)}$ @ COMPASS



# "Twist-three" Structure Functions

- Conan Doyle, *A Scandal in Bohemia*
  - "I have no data yet. It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts."
- There is no reasons do not extract "twist-three" structure functions!

# Subleading twist (from paper (2))

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\tilde{D}^\perp}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\tilde{H}}{z}\Big)\Big]$$

$$F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f_1 D_1\Big]$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0,$$

$$\begin{aligned}
F_{UT}^{\sin\phi_S} &= \frac{2M}{Q}\,\mathcal{C}\Big\{\Big(xf_T D_1 - \frac{M_h}{M}\,h_1\frac{\tilde{H}}{z}\Big) \\
&\quad - \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xh_T H_1^\perp + \frac{M_h}{M}g_{1T}\frac{\tilde{G}^\perp}{z}\Big) - \Big(xh_T^\perp H_1^\perp - \frac{M_h}{M}f_{1T}^\perp\frac{\tilde{D}^\perp}{z}\Big)\Big]\Big\},\\
F_{UT}^{\sin(2\phi_h-\phi_S)} &= \frac{2M}{Q}\,\mathcal{C}\Big\{\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\Big(xf_T^\perp D_1 - \frac{M_h}{M}\,h_{1T}^\perp\frac{\tilde{H}}{z}\Big) \\
&\quad - \frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xh_T H_1^\perp + \frac{M_h}{M}g_{1T}\frac{\tilde{G}^\perp}{z}\Big) \\
&\quad + \Big(xh_T^\perp H_1^\perp - \frac{M_h}{M}f_{1T}^\perp\frac{\tilde{D}^\perp}{z}\Big)\Big]\Big\},\\
F_{LT}^{\cos\phi_S} &= \frac{2M}{Q}\,\mathcal{C}\Big\{-\Big(xg_T D_1 + \frac{M_h}{M}\,h_1\frac{\tilde{E}}{z}\Big) \\
&\quad + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xe_T H_1^\perp - \frac{M_h}{M}g_{1T}\frac{\tilde{D}^\perp}{z}\Big) + \Big(xe_T^\perp H_1^\perp + \frac{M_h}{M}f_{1T}^\perp\frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\},\\
F_{LT}^{\cos(2\phi_h-\phi_S)} &= \frac{2M}{Q}\,\mathcal{C}\Big\{-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\Big(xg_T^\perp D_1 + \frac{M_h}{M}\,h_{1T}^\perp\frac{\tilde{E}}{z}\Big) \\
&\quad + \frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xe_T H_1^\perp - \frac{M_h}{M}g_{1T}\frac{\tilde{D}^\perp}{z}\Big) \\
&\quad - \Big(xe_T^\perp H_1^\perp + \frac{M_h}{M}f_{1T}^\perp\frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\}.
\end{aligned}$$

$$\begin{aligned}
xe &= x\tilde{e} + \frac{m}{M}f_1,\\
xf^\perp &= x\tilde{f}^\perp + f_1,\\
xg'_T &= x\tilde{g}'_T + \frac{m}{M}h_{1T},\\
xg_T^\perp &= x\tilde{g}_T^\perp + g_{1T} + \frac{m}{M}h_{1T}^\perp,\\
xg_T &= x\tilde{g}_T - \frac{p_T^2}{2M^2}g_{1T} + \frac{m}{M}h_1,\\
xg_L^\perp &= x\tilde{g}_L^\perp + g_{1L} + \frac{m}{M}h_{1L}^\perp,\\
xh_L &= x\tilde{h}_L + \frac{p_T^2}{M^2}h_{1L}^\perp + \frac{m}{M}g_{1L},\\
xh_T &= x\tilde{h}_T - h_1 + \frac{p_T^2}{2M^2}h_{1T}^\perp + \frac{m}{M}g_{1T},\\
xh_T^\perp &= x\tilde{h}_T^\perp + h_1 + \frac{p_T^2}{2M^2}h_{1T}^\perp.\\
xe_L &= x\tilde{e}_L,\\
xe_T &= x\tilde{e}_T,\\
xe_T^\perp &= x\tilde{e}_T^\perp + \frac{m}{M}f_{1T}^\perp,\\
xf'_T &= x\tilde{f}'_T + \frac{p_T^2}{M^2}f_{1T}^\perp,\\
xf_T^\perp &= x\tilde{f}_T^\perp + f_{1T}^\perp,\\
xf_T &= x\tilde{f}_T + \frac{p_T^2}{2M^2}f_{1T}^\perp,\\
xf_L^\perp &= x\tilde{f}_L^\perp,\\
xg^\perp &= x\tilde{g}^\perp + \frac{m}{M}h_1^\perp,\\
xh &= x\tilde{h} + \frac{p_T^2}{M^2}h_1^\perp.
\end{aligned}$$

- BDGMMSch:
  - “Whether and how the tree-level factorization used in the present paper extends to subleading level in 1/Q is presently not known.”
- Simple approach – Twist-two +Cahn kinematical corrections

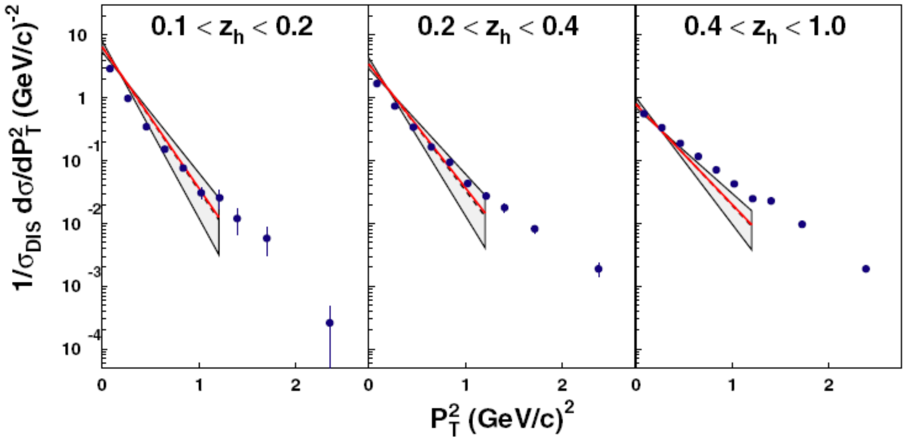
$$\frac{d\sigma^{lq\to lq}}{dy\,d\varphi_q} \propto \frac{\hat{s}^2+\hat{u}^2+\lambda\lambda^q(\hat{s}^2-\hat{u}^2)}{\hat{t}^2}$$

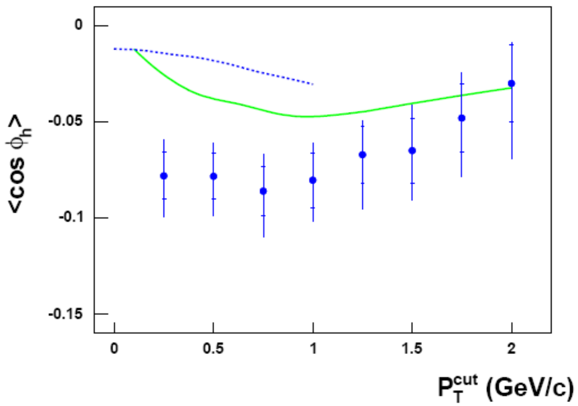
$$\begin{aligned}
\hat{s} &\approx 2MEx\left[1-2\sqrt{1-y}\frac{|\mathbf{k}_T|}{Q}\cos\varphi_q\right],\\
\hat{t} &= -Q^2 = -2MExy,\\
\hat{u} &\approx -2MEx(1-y)\left[1-\frac{2|\mathbf{k}_T|}{Q\sqrt{1-y}}\cos\varphi_q\right]
\end{aligned}$$

# Higher twist example 1: unpolarized SIDIS

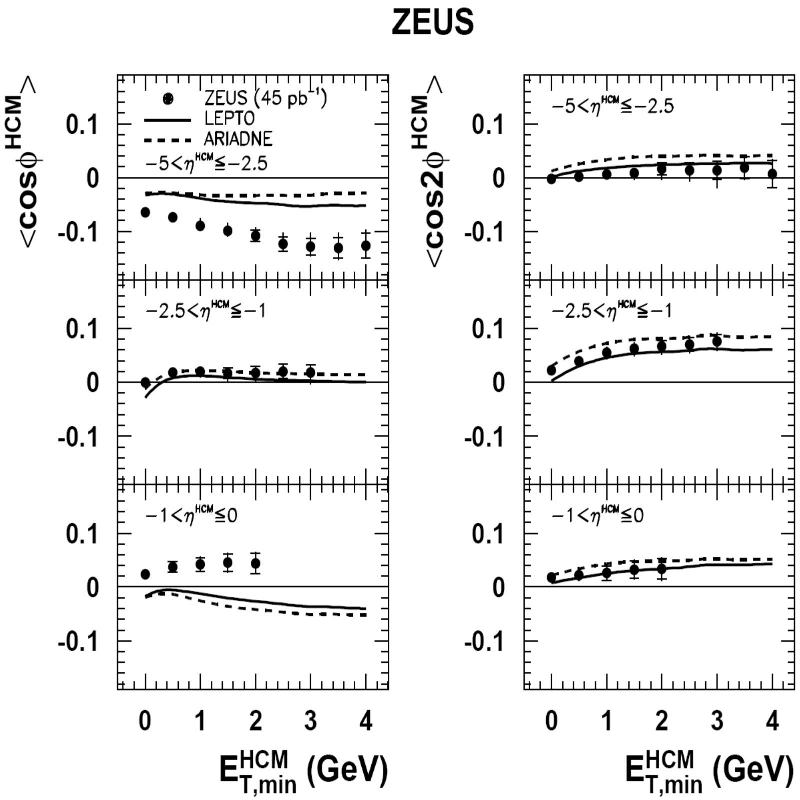
Cahn effect (tw-2 DFs and FFs + kinem. corr. )contributions

$$\left[1+(1-y)^2+4(1-y)\frac{\mu_0^2}{Q^2\mu_h^2}\left(\mu_D^2+\frac{P_{hT}{}^2 z^2\mu_0^2}{\mu_h^2}\right)-4\sqrt{1-y}(2-y)\frac{\left|\vec{P}_{hT}\right| z\mu_0^2}{Q\mu_h^2}\cos\varphi_h+4(1-y)\frac{P_{hT}{}^2 z^2\mu_0^4}{Q^2\mu_h^4}\cos 2\varphi_h\right]\frac{1}{\pi\mu_h^2}e^{-\frac{P_{hT}{}^2}{\mu_h^2}}\sum_q e_q^2 f_q(x)D_q^h(z)$$



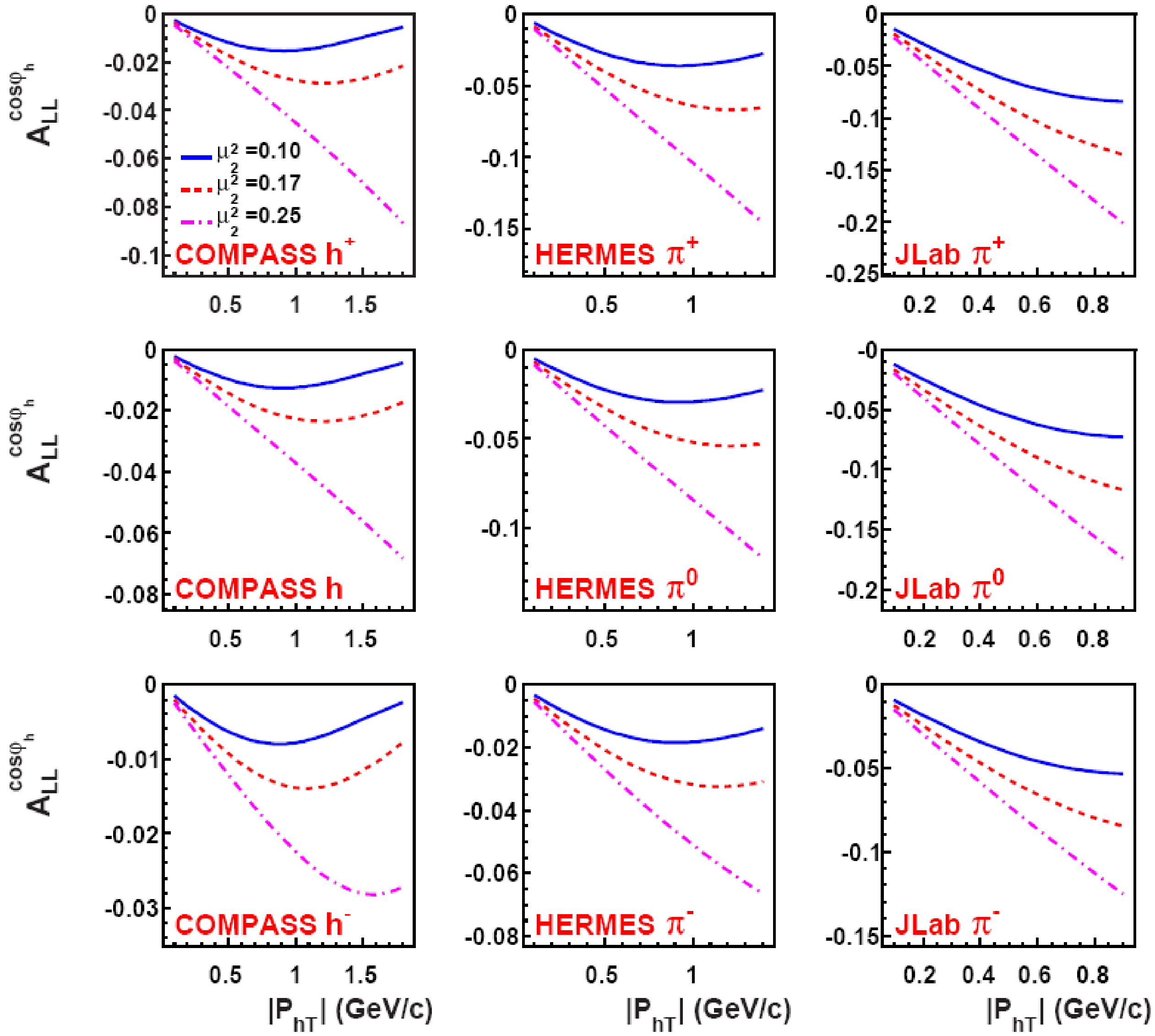

ZEUS, $Q^2 > 180\ (\mathrm{GeV}/c)^2$




η<0 -- CFR

# Higher twist example 2: Cahn effect in $A_{LL}$

M. Anselmino, A. Efremov
A.K. & B. Parsamyan,
PRD 74, 074015 (2006)



(The HERMES Collaboration)

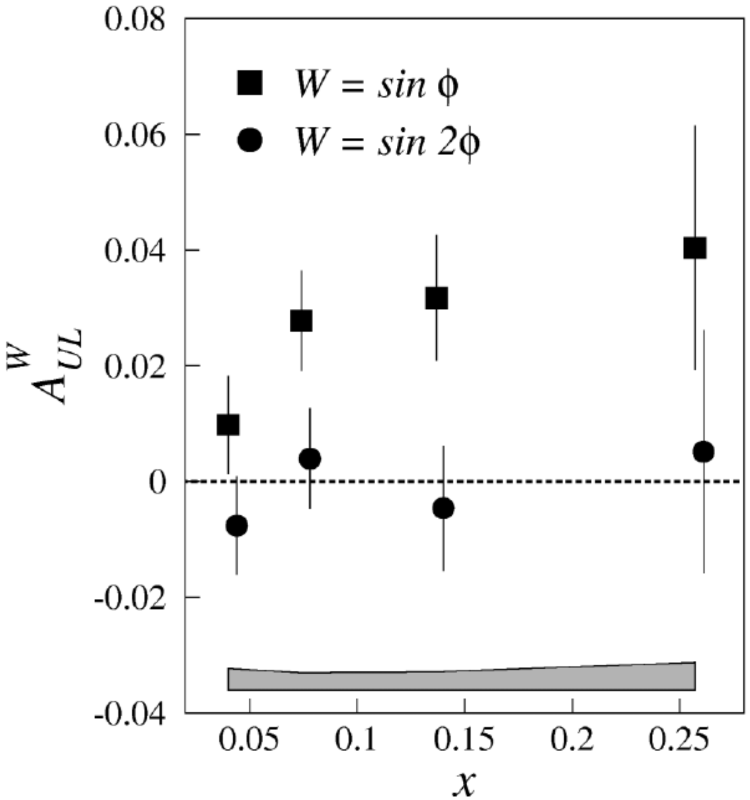



FIG. 2. Target-spin analyzing powers for $\pi^+$: $A_{UL}^{\sin\phi}$ (squares) and $A_{UL}^{\sin 2\phi}$ (circles) as a function of Bjorken $x$. Error bars show the statistical uncertainty and the band represents the systematic uncertainties for $A_{UL}^{\sin\phi}$. As shown in Table II, $\langle Q^2 \rangle$ varies with $x$.

Revisit interpretation

# $A_{LU}$ @ HERMES & CLASS

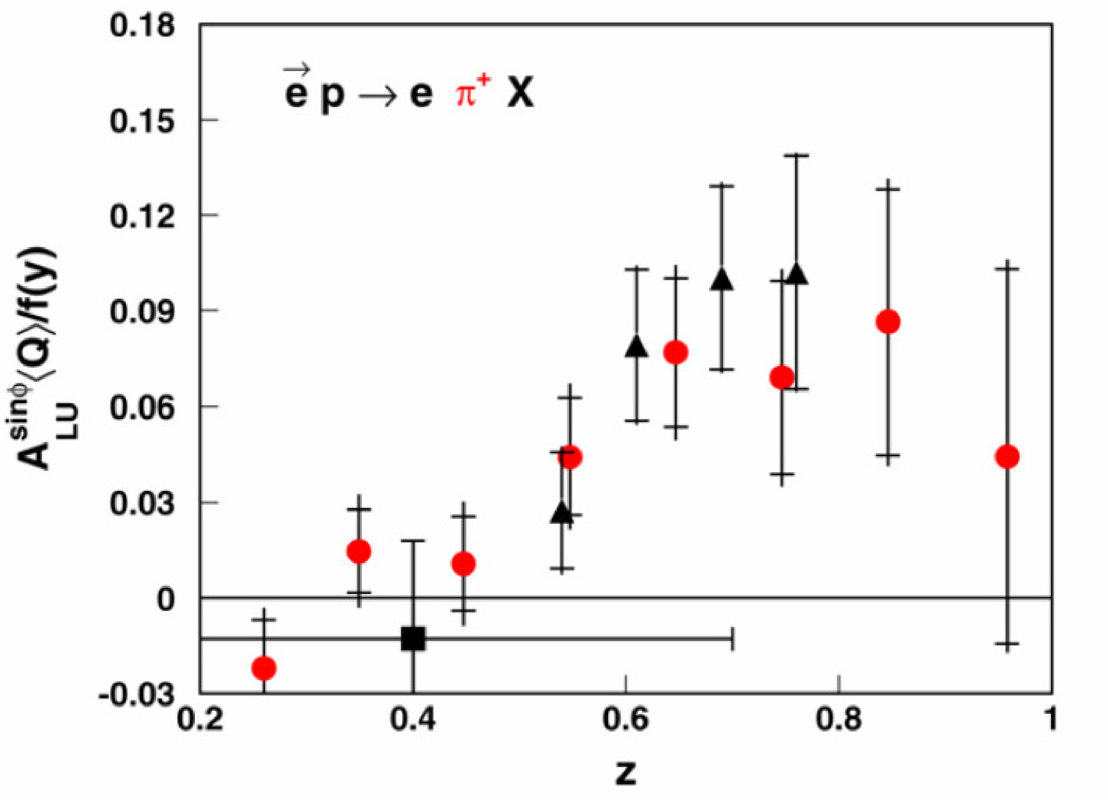



Fig. 6. Comparison of the kinematically rescaled asymmetry amplitudes $A_{LU}^{\sin\phi} \cdot \langle Q\rangle / f(\langle y\rangle)$ for $\pi^+$ between the HERMES (circles) and CLAS (triangles) measurements. The full square represents a previous HERMES measurement [14], averaged over the indicated large $z$ range $(0.2 < z < 0.7)$. The outer error bars represent the quadratic sum of the systematic uncertainty and the statistical uncertainty (inner error bars).

# Interpretation of target transverse spin asymmetries

Twist-2 + $k_T/Q$ kinematical corrections:

$$A_{LT}^{\cos(\varphi_s)} \propto \frac{M}{Q} g_{1T}^{q} \otimes D_{1q}^{h}$$

$$A_{LT}^{\cos(2\varphi_h-\varphi_s)} \propto \frac{M}{Q} g_{1T}^{q} \otimes D_{1q}^{h}$$

$$A_{UT}^{\sin(\varphi_s)} \propto \frac{M}{Q} \left( h_1^q \otimes H_{1q}^{\perp h} + f_{1T}^{\perp q} \otimes D_{1q}^{h} \right)$$

$$A_{UT}^{\sin(2\varphi_h-\varphi_s)} \propto \frac{M}{Q} \left( h_{1T}^{\perp q} \otimes H_{1q}^{\perp h} + f_{1T}^{\perp q} \otimes D_{1q}^{h} \right)$$

# Higher twist example 2: predictions for cos($\varphi_s$) asymmetry

**Chiral-odd fragmentation functions in single pion inclusive electroproduction**

Leonard P. Gamberg[a], Dae Sung Hwang[b], Karo A. Oganessyan[c,d]

Physics Letters B 584 (2004) 276–284

Spectator model

$$A_{LT}^{\cos(\varphi_s)}$$

$$\sigma_{UU} = \langle 1 \rangle_{UU} = \frac{[1+(1-y)^2]}{y} f_1(x) D_1(z),$$

$$\sigma_{LT} = \langle 1 \rangle_{LT} = \lambda_e |S_T| \sqrt{1-y}\, \frac{4}{Q} \cos\phi_S \left[ M x g_T(x) D_1(z) + M_h h_1(x) \frac{E(z)}{z} \right]$$

Cahn correction

for $g_{1T}$ contribution

$$A_{LT} \equiv \frac{\int d^2 P_{h\perp} (\sigma^{\leftarrow}(\phi_S) + \sigma^{\rightarrow}(\pi + \phi_S) - \sigma^{\leftarrow}(\phi_S) - \sigma^{\rightarrow}(\pi + \phi_S))}{\int d^2 P_{h\perp} (\sigma^{\leftarrow}(\phi_S) + \sigma^{\rightarrow}(\pi + \phi_S) + \sigma^{\leftarrow}(\phi_S) + \sigma^{\rightarrow}(\pi + \phi_S))} = \frac{\sigma_{LT}}{\sigma_{UU}}$$

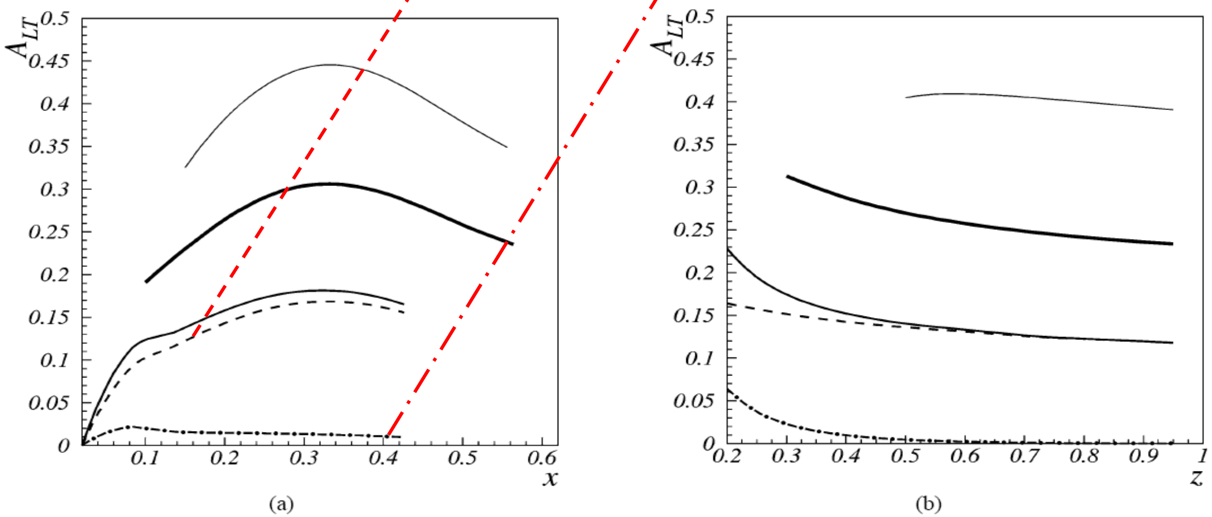
$x \in [0.3 ; 0.4]$

JLab-6: $\frac{1}{Q} \approx 0.65$ (GeV/c)$^{-1}$

JLab-12: $\frac{1}{Q} \approx 0.55$ (GeV/c)$^{-1}$

HERMES: $\frac{1}{Q} \approx 0.49$ (GeV/c)$^{-1}$

COMPASS: $\frac{1}{Q} \approx 0.24$ (GeV/c)$^{-1}$



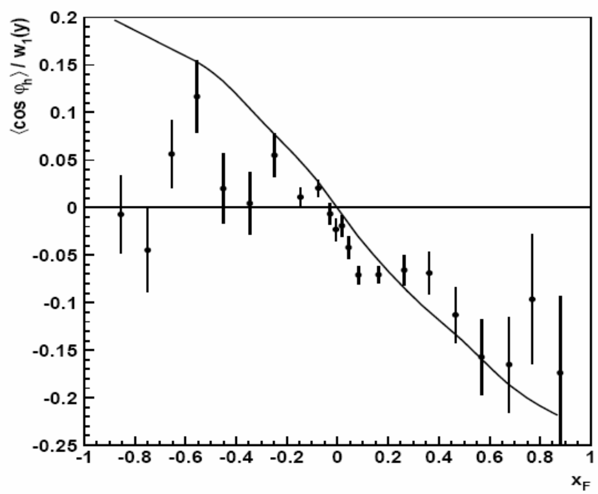

JLab 6

JLab 12

HERMES

Fig. 4. $A_{LT}$ for $\pi^+$ production as a function of $x$ and $z$ at 27.5 GeV energy. The dashed and dot-dashed curves correspond to the contributions of the two terms of Eq. (18), respectively, and the full curve is the sum of those two. The thin curve corresponds to 6 GeV and the thick curve to 12 GeV energies, respectively.

# Azimuthal asymmetries in TFR: Cahn

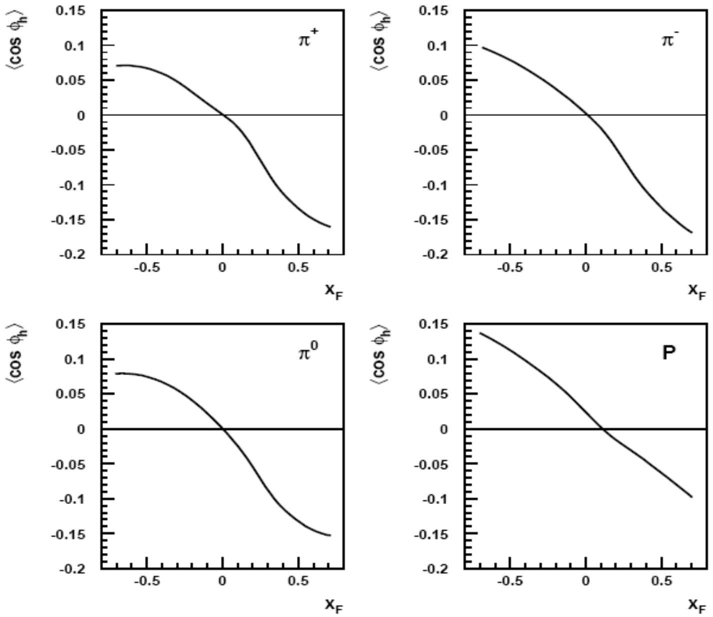



LUND String Fragmentation

Modified LEPTO

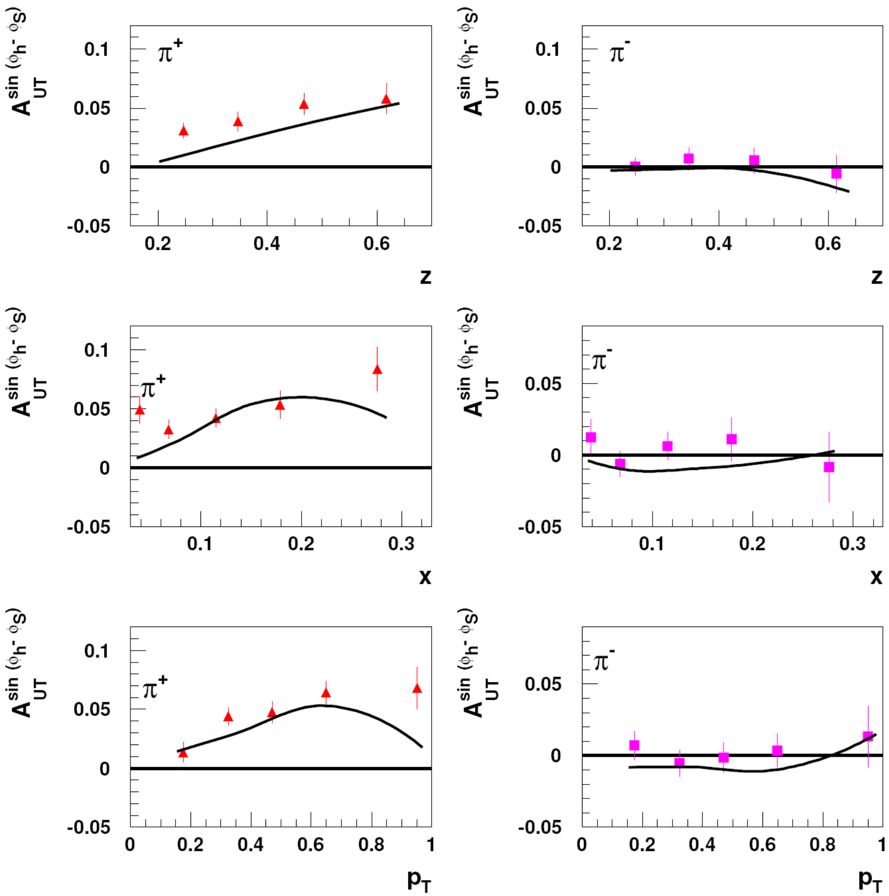
EMC Collaboration (280GeV)

$$w_1(y) = (2-y)\sqrt{1-y}/\left(1+(1-y)^2\right)$$



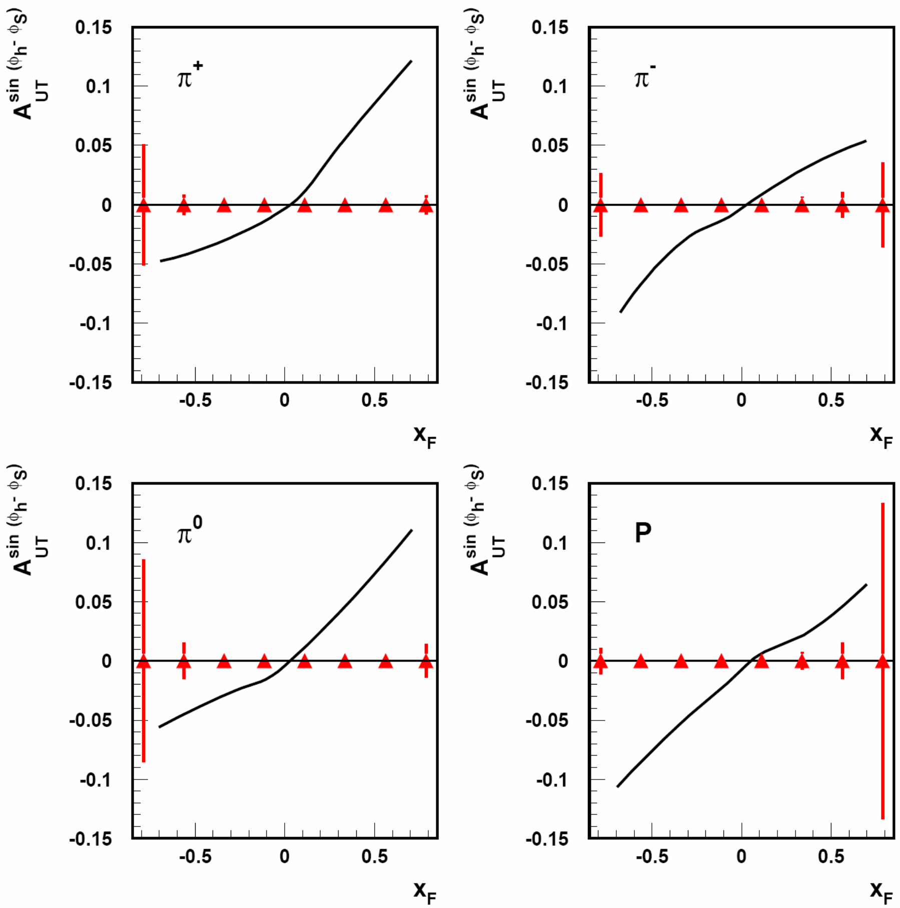

Predictions of modified LEPTO for $x_F$ dependence of $\langle \cos\phi_h \rangle$ for different hadrons produced in 12 GeV unpolarized SIDIS process

# Results: Sivers




HERMES data on $A_{UT}^{\sin(\phi_\pi - \phi_S)}$

z, $x_{Bj}$ and $P_T$ dependences




Predictions for $\mathbf{x_F}$-dependence at JLab 12 *GeV*

Red triangles with error bars – projected statistical accuracy for 1000h data taking (H.Avagyan).

# Concluding remarks

- Will be nice to have data for unpolarized x-sections and asymmetries as a function of all kinematical variables:

  $(x, z, P_T, Q^2)$ or $(x, P_T)$ , $(z, P_T)$, $(x_F, P_T)$ …
- It is important to study spin & TMD asymmetries also in $x_F<0$ region