DVCS Amplitude and Generalized Parton distributions

in Position Space

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- Light-front wave functions and DVCS
- Simple example : electron at one loop
- DVCS amplitude in σ space
- Parton distributions in impact parameter space
- Simulated bound state calculations

ECT Trento; June 11-15, 2007

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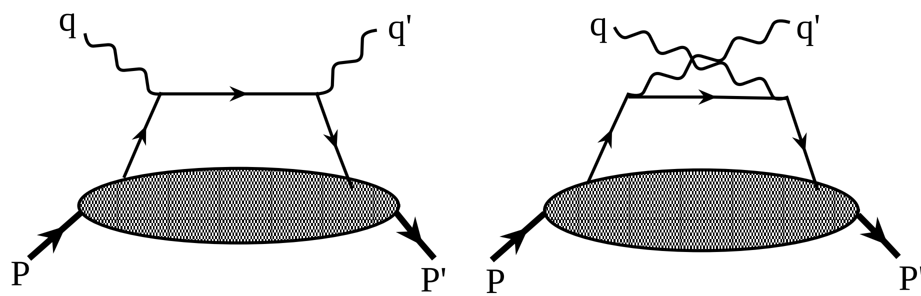
Deeply Virtual Compton Scattering (DVCS) Amplitude

Deeply virtual Compton scattering :



Incident photon highly virtual, final photon real; momentum transfer $\Delta = P - P'$, $t = \Delta^2$

Momenta of initial and final proton :

$$P = \left(P^+ \, , \, \vec{0}_\perp \, , \, \frac{M^2}{P^+} \right) \, , P' = \left((1-\zeta) P^+ \, , \, -\vec{\Delta}_\perp \, , \, \frac{M^2 + \vec{\Delta}_\perp^2}{(1-\zeta) P^+} \right)$$

$$t \; = \; 2 P \cdot \Delta \; = \; - \frac{\zeta^2 M^2 + \vec{\Delta}_\perp^2}{1 - \zeta}, \qquad \frac{Q^2}{2 P \cdot q} = \zeta$$

ζ : skewness variable

For DVCS, $-q^2 = Q^2$ is large compared to the masses and $\mid t \mid$

Choose a frame where the incident space-like photon carries $q^+ = 0$

DVCS (contd.)

DVCS amplitude

$$M^{IJ}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = \epsilon^I_\mu \, \epsilon^{*J}_\nu \, M^{\mu\nu}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = -e_q^2 \, \frac{1}{2\bar{P}^+} \int_{\zeta-1}^{1} \mathrm{d}x$$

$$\times \quad \left\{ t^{IJ}(x,\zeta) \, \bar{U}(P') \left[H(x,\zeta,t) \, \gamma^+ + E(x,\zeta,t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \right] U(P) \right\},$$

where $\bar{P} = \frac{1}{2}(P' + P)$,
x is the fraction of the proton momentum carried by the active quark
For circularly polarized initial and final photons

$$t^{\uparrow\uparrow}(x,\zeta) = \quad t^{\downarrow\downarrow}(x,\zeta) = \frac{1}{x - i\epsilon} + \frac{1}{x - \zeta + i\epsilon}$$

Contributions from longitudinal pol. photons are suppressed

$$F_{\lambda,\lambda'} = \int \frac{dy^-}{8\pi} \, e^{ixP^+y^-/2} \, \langle P', \lambda' | \bar{\psi}(0) \, \gamma^+ \, \psi(y) \, | P, \lambda \rangle \Big|_{y^+=0, y_\perp=0}$$

$$= \quad \frac{1}{2\bar{P}^+} \, \bar{U}(P', \lambda') \left[H(x,\zeta,t) \, \gamma^+ + E(x,\zeta,t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \right] U(P, \lambda),$$

Overlap Representation

• The target state is expanded in terms of multiparticle light-front wave functions in Fock space; choose light-front gauge

DVCS amplitude is given in terms of overlaps of the light-front wave functions

Diehl, Feldman, Jacob, Kroll (2001);
Brodsky, Diehl, Huang (2001)

• Diagonal parton number conserving $n \to n$ overlap in the kinematical regime $\zeta < x < 1$ and $\zeta - 1 < x < 0$

Off-diagonal $n+1 \to n-1$ overlap for $0 < x < \zeta$ where the parton number is decreased by two.

• Consider a dressed electron state instead of a proton

State is expanded in Fock space : $\mid e^- \gamma \rangle$ and $\mid e^- e^- e^+ \rangle$ contribute to $O(\alpha)$

• Generalized form of QED : mass M to the external electrons, m to the internal electron lines λ to the internal photon lines $\to$ composite fermion state with mass M : a fermion and a vector ‘diquark" constituents

Brodsky, Drell (1980)

• Two and three particle LFWFs are systematically evaluated in perturbation theory : $2 \to 2$ and $3 \to 1$ contributions

In the kinematical region $\zeta < x < 1$ one has $2 \to 2$ contribution

$$F^{22}_{++} = \frac{\sqrt{1-\zeta}}{1-\frac{\zeta}{2}} H_{(2\to 2)}(x,\zeta,t) - \frac{\zeta^2}{4(1-\frac{\zeta}{2})\sqrt{1-\zeta}} E_{(2\to 2)}(x,\zeta,t)$$

$$= \int \frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3} \Big[\psi^{\uparrow *}_{+\frac{1}{2}+1}(x',\vec{k}'_\perp)\psi^{\uparrow}_{+\frac{1}{2}+1}(x,\vec{k}_\perp) + \psi^{\uparrow *}_{+\frac{1}{2}-1}(x',\vec{k}'_\perp)\psi^{\uparrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp) + \psi^{\uparrow\ *}_{-\frac{1}{2}+1}(x',\vec{k}'_\perp)\psi^{\uparrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp)\Big],$$

$$F^{22}_{+-} = \frac{1}{\sqrt{1-\zeta}} \frac{(\Delta^1 - i\Delta^2)}{2M} E_{(2\to 2)}(x,\zeta,t)$$

$$= \int \frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3} \Big[\psi^{\uparrow *}_{+\frac{1}{2}-1}(x',\vec{k}'_\perp)\psi^{\downarrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp) + \psi^{\uparrow *}_{-\frac{1}{2}+1}(x',\vec{k}'_\perp)\psi^{\downarrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp)\Big],$$

where

$$x' = \frac{x-\zeta}{1-\zeta}, \quad \vec{k}'_\perp = \vec{k}_\perp - \frac{1-x}{1-\zeta}\,\vec{\Delta}_\perp .$$

DVCS in QED at one loop (contd.)

In the kinematical region $0 < x < \zeta$ contribution comes from $3 \to 1$ overlap :

$$
\begin{aligned}
F^{31}_{++} &= \frac{\sqrt{1-\zeta}}{1-\frac{\zeta}{2}}\, H_{(3\to1)}(x,\zeta,t) - \frac{\zeta^2}{4(1-\frac{\zeta}{2})\sqrt{1-\zeta}} E_{(3\to1)}(x,\zeta,t) \\
&= \sqrt{1-\zeta}\int \frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3}\Big[\psi^{\uparrow}_{+\frac{1}{2}\;+\frac{1}{2}\;-\frac{1}{2}}(x,1-\zeta,\zeta-x,\ \vec{k}_\perp,-\vec{\Delta}_\perp,\vec{\Delta}_\perp-\vec{k}_\perp) \\
&\qquad + \psi^{\uparrow}_{-\frac{1}{2}\;+\frac{1}{2}\;+\frac{1}{2}}(x,1-\zeta,\zeta-x,\vec{k}_\perp,-\vec{\Delta}_\perp,\vec{\Delta}_\perp-\vec{k}_\perp)\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{31}_{+-} &= \frac{1}{\sqrt{1-\zeta}}\,\frac{(\Delta^1 - i\Delta^2)}{2M}\, E_{(3\to1)}(x,\zeta,t) \\
&= \sqrt{1-\zeta}\int \frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3}\Big[\psi^{\downarrow}_{+\frac{1}{2}\;+\frac{1}{2}\;-\frac{1}{2}}(x,1-\zeta,\zeta-x,\ \vec{k}_\perp,-\vec{\Delta}_\perp,\vec{\Delta}_\perp-\vec{k}_\perp) \\
&\qquad + \psi^{\downarrow}_{-\frac{1}{2}\;+\frac{1}{2}\;+\frac{1}{2}}(x,1-\zeta,\zeta-x,\vec{k}_\perp,-\vec{\Delta}_\perp,\vec{\Delta}_\perp-\vec{k}_\perp)\Big]
\end{aligned}
$$

third region $\zeta - 1 < x < 0$ does not contribute : corresponds to the emission and reabsorption of a positron from the physical electron

Light-front Wave Functions

Dressed electron state : 2 particle LFWF

$$\left\{ \begin{array}{l} \psi^{\uparrow}_{+\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}\,\frac{-k^1+ik^2}{x(1-x)}\,\varphi\,, \\ \psi^{\uparrow}_{+\frac{1}{2}\,-1}(x,\vec{k}_\perp) = -\sqrt{2}\,\frac{k^1+ik^2}{1-x}\,\varphi\,, \\ \psi^{\uparrow}_{-\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}\,(M-\frac{m}{x})\,\varphi\,, \\ \psi^{\uparrow}_{-\frac{1}{2}\,-1}(x,\vec{k}_\perp) = 0\,, \end{array} \right\}.$$

$$\varphi(x,\vec{k}_\perp) = \frac{e}{\sqrt{1-x}}\;\frac{1}{M^2-\frac{\vec{k}_\perp^2+m^2}{x}-\frac{\vec{k}_\perp^2+\lambda^2}{1-x}}\,.$$

3 particle LFWF is also known

One particle wave fn : renormalization of the state ; contributes in the $3-1$ overlaps.

Real and Imaginary Parts of the Amplitude

- Real and imaginary parts of the DVCS amplitude are calculated separately using the principal value prescription for the propagator and using the explicit form of the two-and three-particle LFWFs

$$\int_0^1 dx \frac{1}{x-\zeta+i\epsilon} F(x,\zeta) = P\int_0^1 dx \frac{1}{x-\zeta} F(x,\zeta) - i\pi F(\zeta,\zeta)$$

Here P denotes the principal value defined as

$$P\int_0^1 dx \frac{1}{x-\zeta} F(x,\zeta) = \lim_{\epsilon\to 0}\left[\int_0^{\zeta-\epsilon} \frac{1}{x-\zeta} F(x,\zeta) + \int_{\zeta+\epsilon}^1 \frac{1}{x-\zeta} F(x,\zeta)\right]$$

where

$$\begin{aligned} F(x,\zeta) &= F_{ij}^{31}(x,\zeta,\Delta_\perp), \text{ for } 0 < x < \zeta \\ &= F_{ij}^{22}(x,\zeta,\Delta_\perp), \text{ for } \zeta < x < 1 \end{aligned}$$

$ij = ++$ for helicity non-flip and $ij = +-$ for helicity flip amplitudes

Parton Distributions in Impact Parameter Space

Impact parameter dependent parton distributions are defined from the GPDs by taking a FT in $\Delta_\perp$ when $\zeta = 0$

$$q(x, b_\perp) = \frac{1}{(2\pi)^2} \int d^2\Delta_\perp e^{-ib_\perp \cdot \Delta_\perp} H(x, t),$$

$$e(x, b_\perp) = \frac{1}{(2\pi)^2} \int d^2\Delta_\perp e^{-ib_\perp \cdot \Delta_\perp} E(x, t),$$

$b_\perp$ is the impact parameter conjugate to $\Delta_\perp$

Soper(1977); Burkardt (2000).

- Give simultaneous information on the distribution of quarks as a function of x and the transverse distance $b_\perp$ of the parton from the center of the proton in the transverse plane

- Obey certain positivity constraints, can interprete them as probability densities

- Transversely polarized target : distribution of partons in the impact space no longer axially symmetric; deformation is described by $e(x, b_\perp) \rightarrow$ model calculation shows that it is connected with Sivers Effect

Burkardt (2002); Meissner, Metz, Goeke (2007)

Fourier Transform of DVCS Amplitude in ζ

- In order to obtain the DVCS amplitude in y^- space, we take a Fourier transform in ζ as,

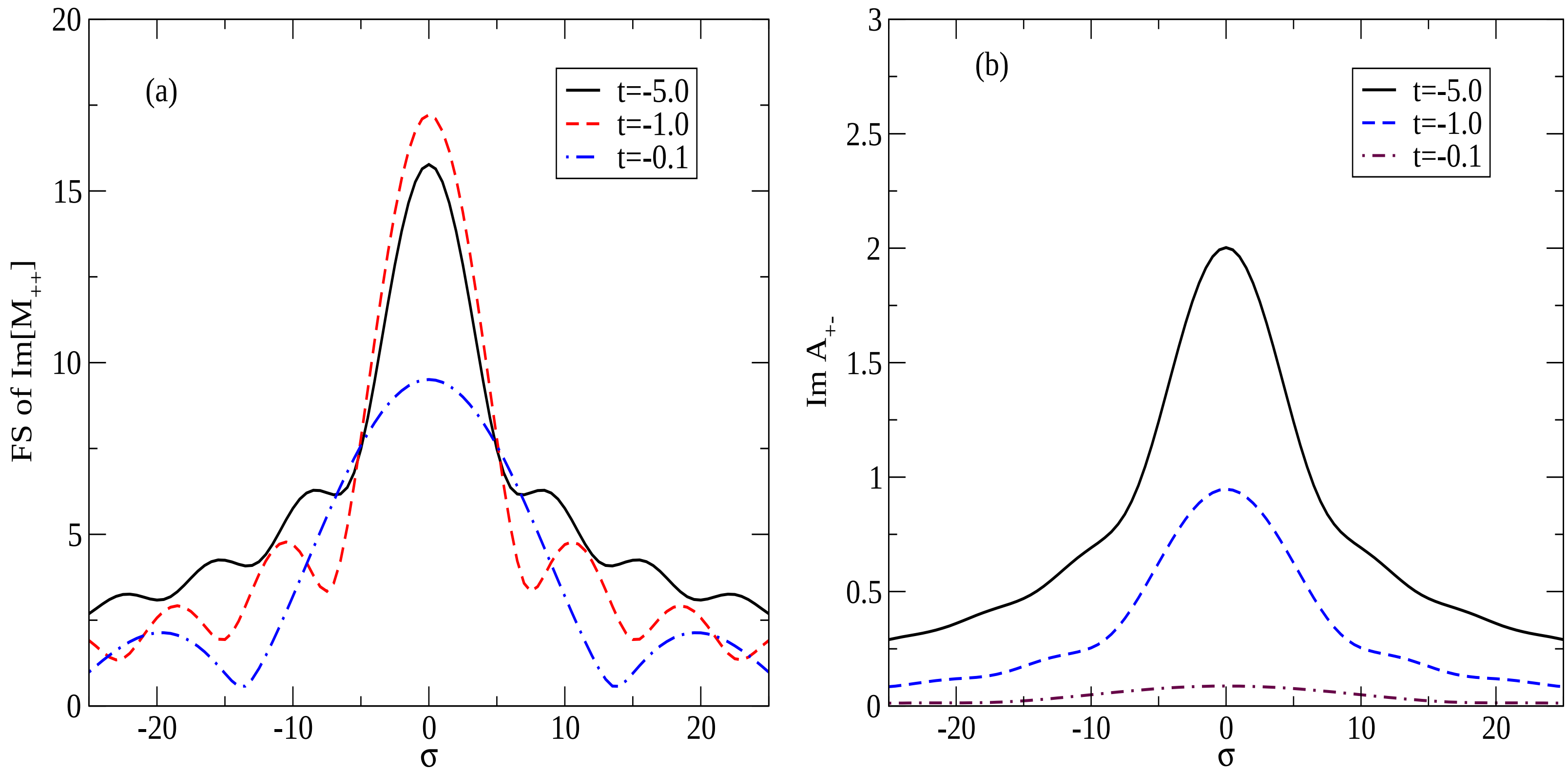
$$A_{++}(\sigma,\Delta^\perp) = \frac{1}{2\pi}\int_{\varepsilon_2}^{1-\varepsilon_2} d\zeta\, e^{i\sigma\zeta}\, M_{++}(\zeta,\Delta^\perp),$$
$$A_{+-}(\sigma,\Delta^\perp) = \frac{1}{2\pi}\int_{\varepsilon_2}^{1-\varepsilon_2} d\zeta\, e^{i\sigma\zeta}\, M_{+-}(\zeta,\Delta^\perp),$$

where $\sigma = \frac{1}{2}P^+y^-$ is the (boost invariant) longitudinal distance on the light cone

- Fourier transforms have been performed by numerically calculating the Fourier sine and cosine transforms and then calculating the resultant by squaring them, adding and taking the square root

- Helicity non-flip part of the amplitude depends on the scale Λ

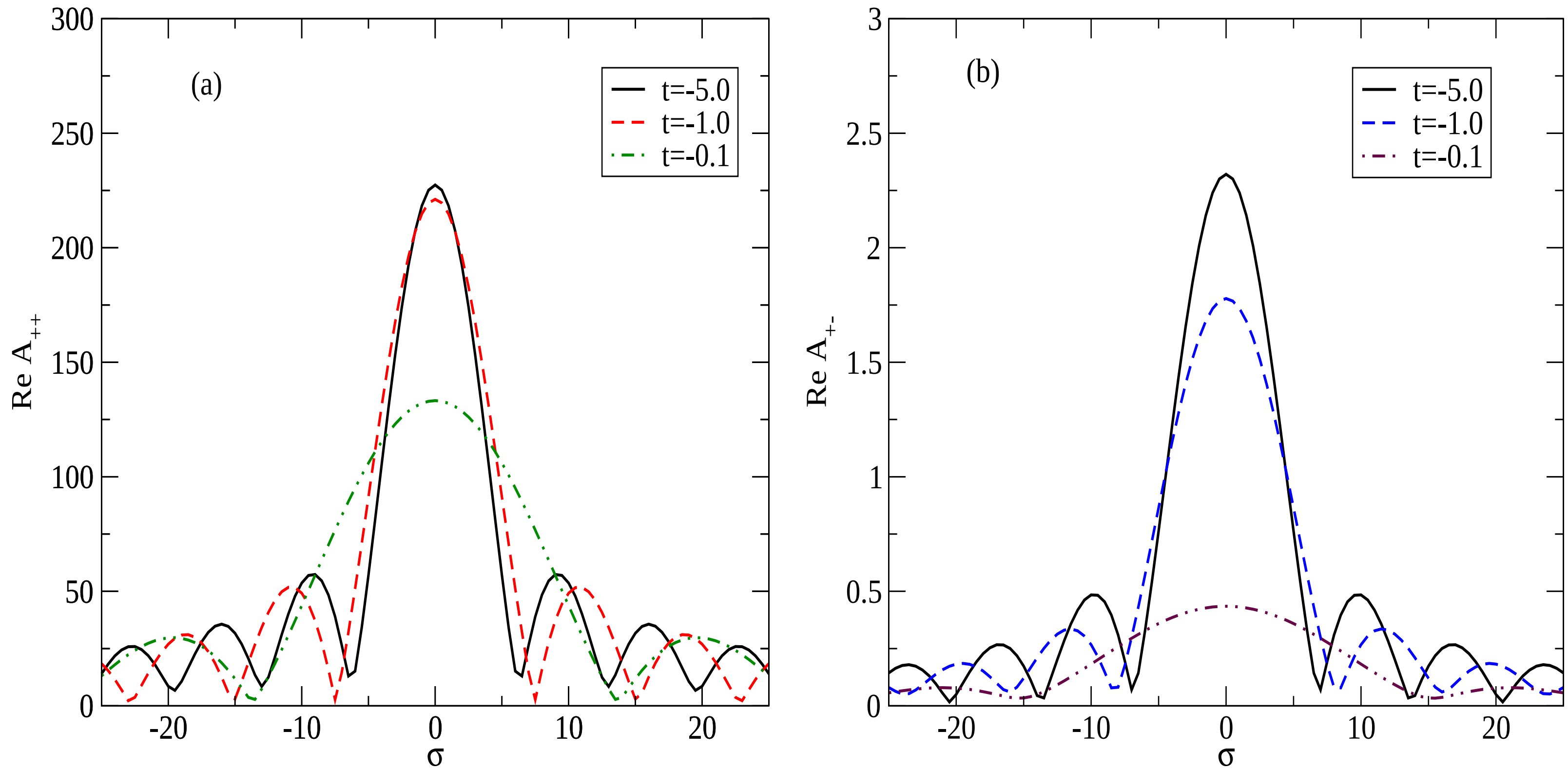
Imaginary Part of the DVCS Amplitude in σ Space



- (a) When the electron helicity is not flipped, (b) helicity is flipped
- $M = 0.51$ MeV, $m = 0.5$ MeV, $\lambda = 0.02$ MeV, t is in MeV^2

Brodsky,Chakrabarti,Harindranath,AM,Vary (2006).

Real Part of the DVCS Amplitude in σ



- (a) When the electron helicity is not flipped, (b) helicity is flipped
- $M = 0.51$ MeV, $m = 0.5$ MeV, $\lambda = 0.02$ MeV, t is in MeV^2
- As $\mid t \mid$ increases the first minima move in, positions of minima independent of target helicity

Brodsky,Chakrabarti,Harindranath,AM,Vary (2006).

Simulated model for a hadron

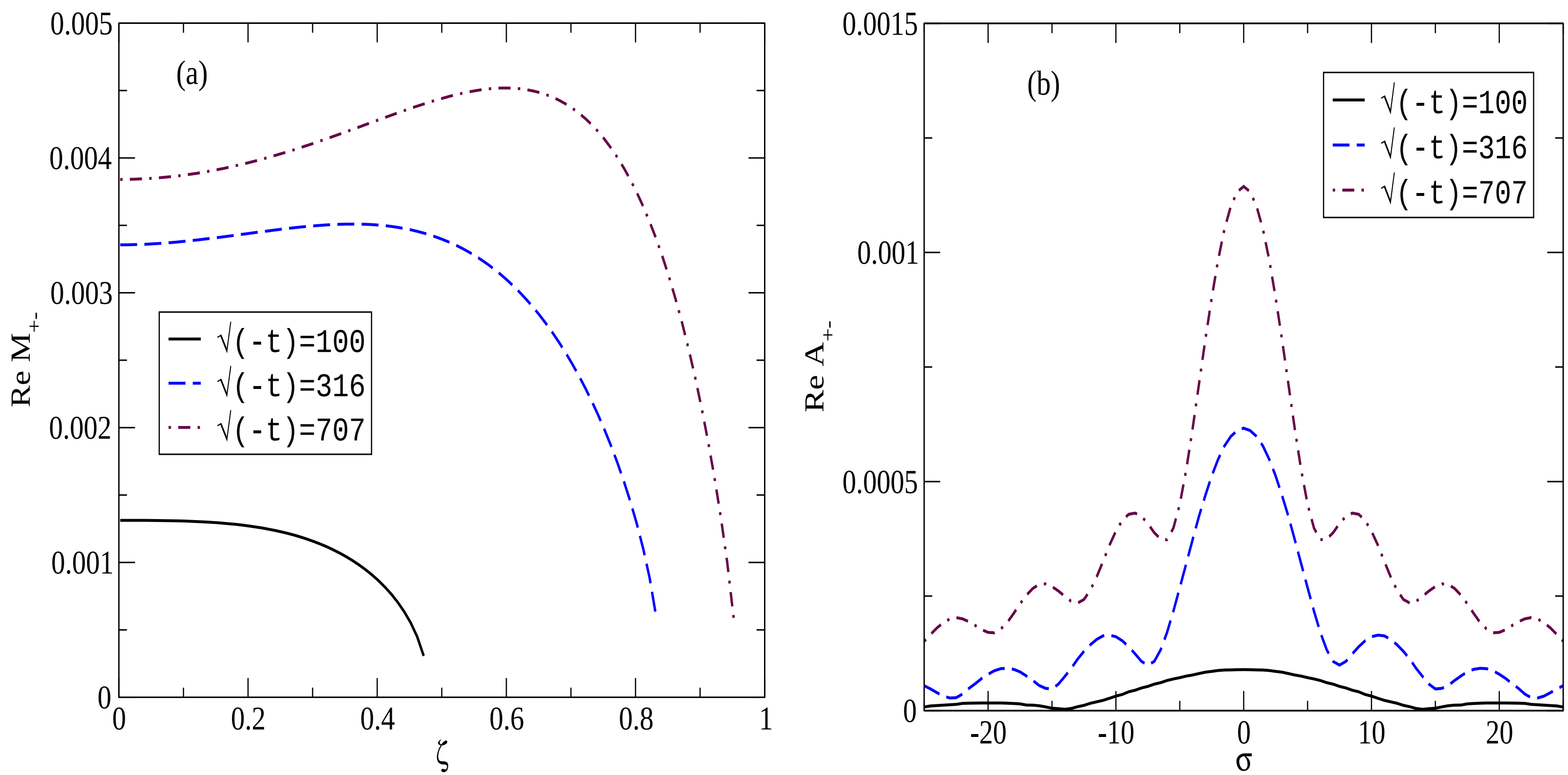
- Wave function of a dressed electron depends on bound state mass square M^2 (denominator) : differentiation wrt M^2 increases the fall-off of the LFWFs near the end points as well as improves the $k_\perp$ behaviour; this simulates the wave functions of a meson-like hadron (model 1)

- Differentiation wrt m^2 and λ^2 simulates the fall-off at short distances which matches the fall-off wavefunction of a baryon : form factor $F_1(Q^2)$ computed from the Drell-Yan-West formula will fall-off like $\frac{1}{Q^4}$; analog of a two-parton quark plus spin-one diquark model of a baryon (model 2)

- Convolutions of these wave functions gives the DVCS amplitude as well as the GPDs for the simulated hadron model

- 2 particle wave function is normalized to 1

Helicity-flip DVCS Amplitude in the Simulated Model

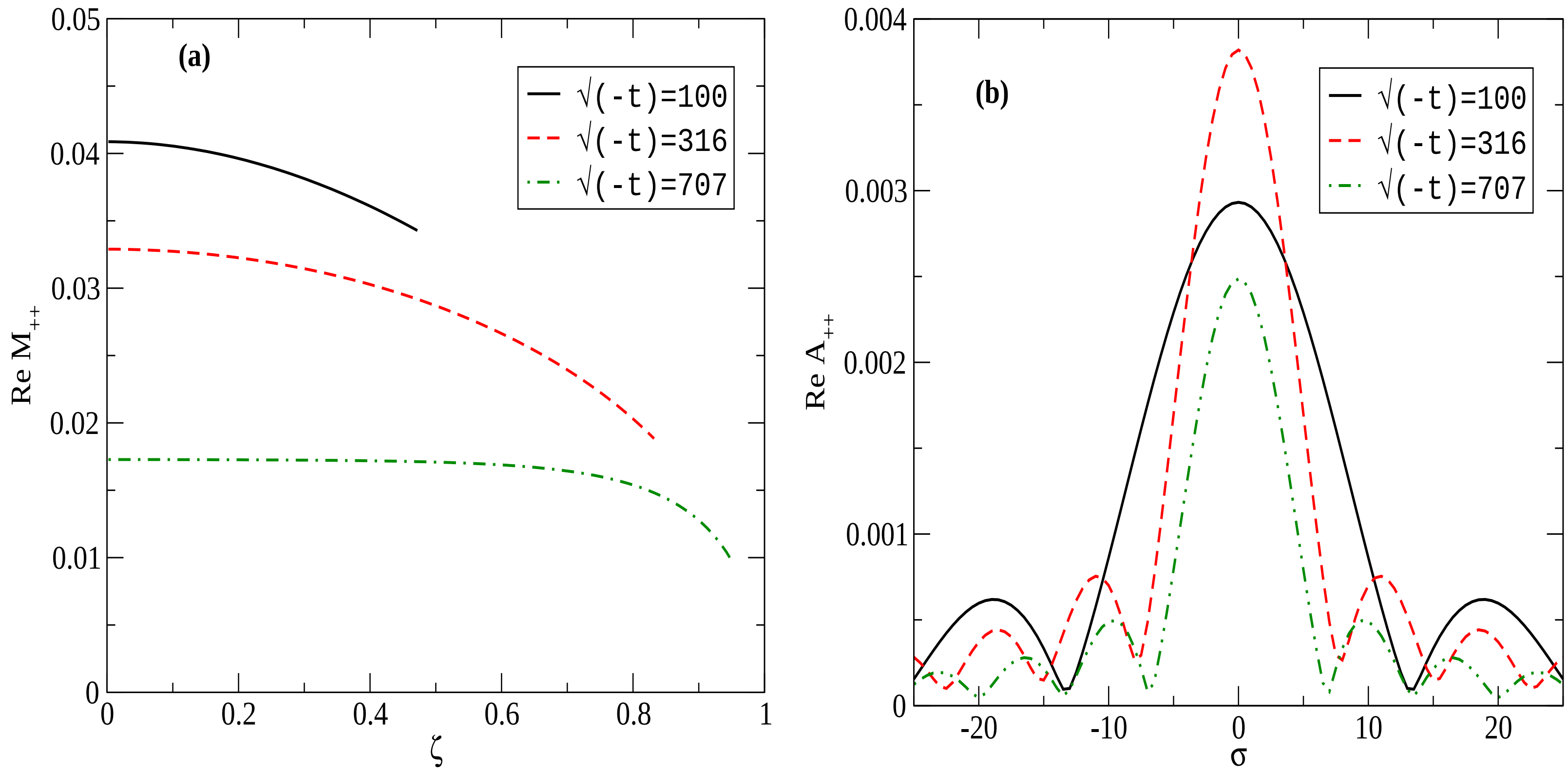


- Mass parameters $M = 150, m = \lambda = 300$ MeV
- Differentiation wrt M^2 brings an extra factor of $x - \zeta$ in the numerator : imaginary part of the DVCS amplitude vanishes in this model

- Note : Differentiation of the single particle wave function gives zero : so the $3 \to 1$ overlap vanishes in this model

Brodsky,Chakrabarti,Harindranath,AM,Vary (2006).

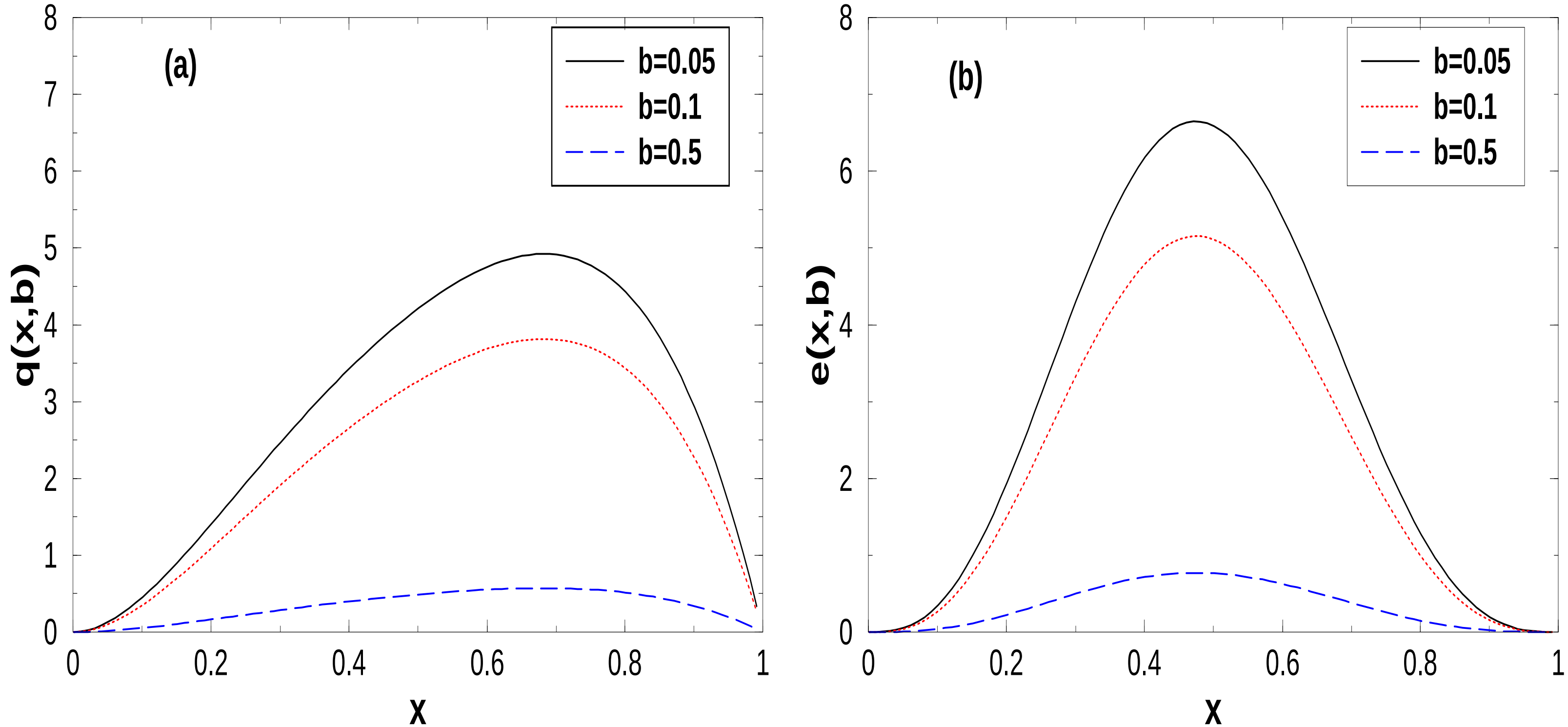
Helicity Non-flip DVCS Amplitude in the Simulated Model



- Mass parameters $M = 150, m = \lambda = 300$ MeV
- Helicity non-flip amplitude decreases as ζ increases for fixed $\mid t \mid$
- Diffraction pattern in σ

Brodsky,Chakrabarti,Harindranath,AM,Vary (2006).

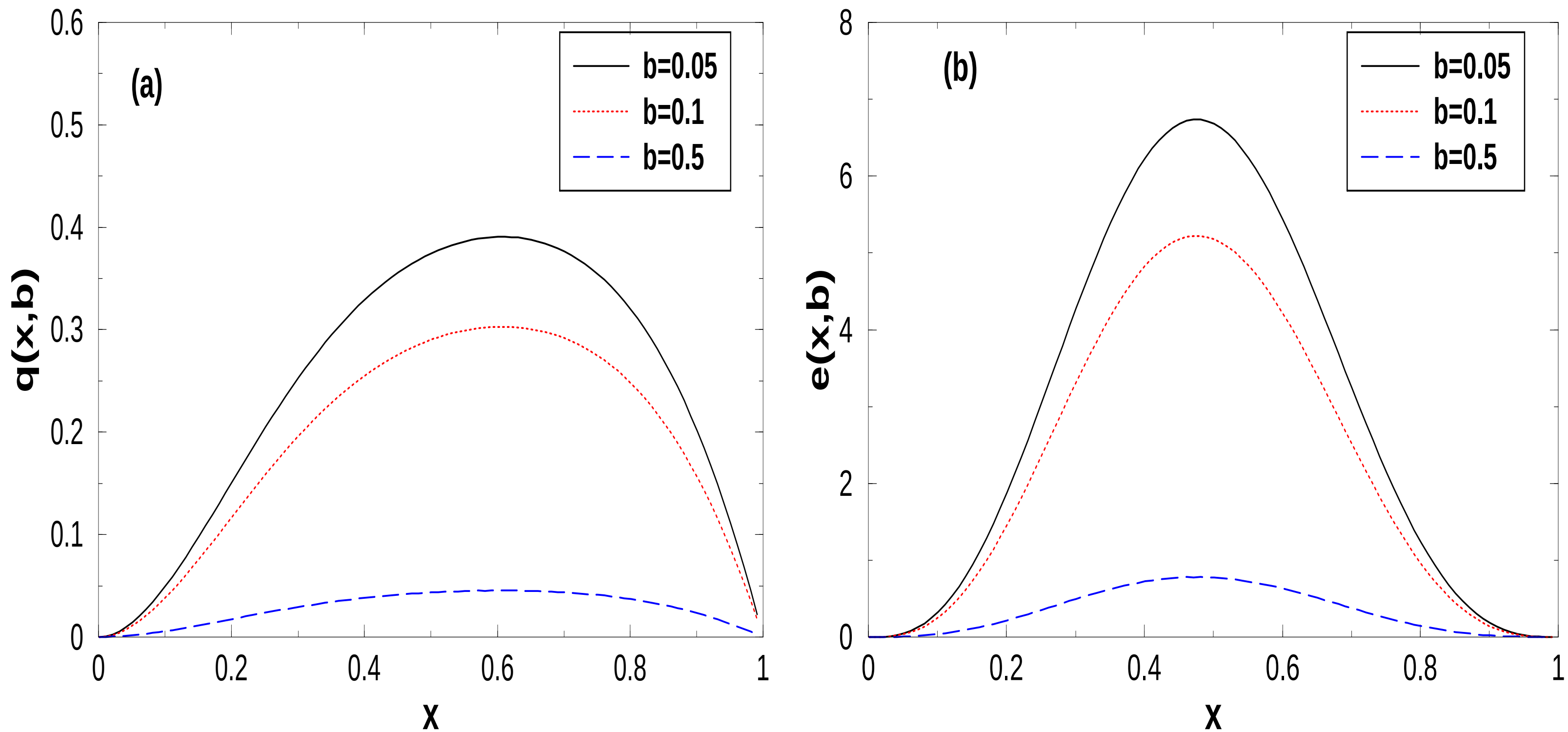
IPDPDFs in the Simulated Model (1)



- Mass parameters $M = 150, m = \lambda = 300$ MeV; $b_\perp$ is in MeV^{-1}.
- (a) Helicity non-flp; (b) helicity flip
- Skewness $\zeta = 0$ here; $2-2$ overlap

H. Dahiya, AM, S. Ray (2007).

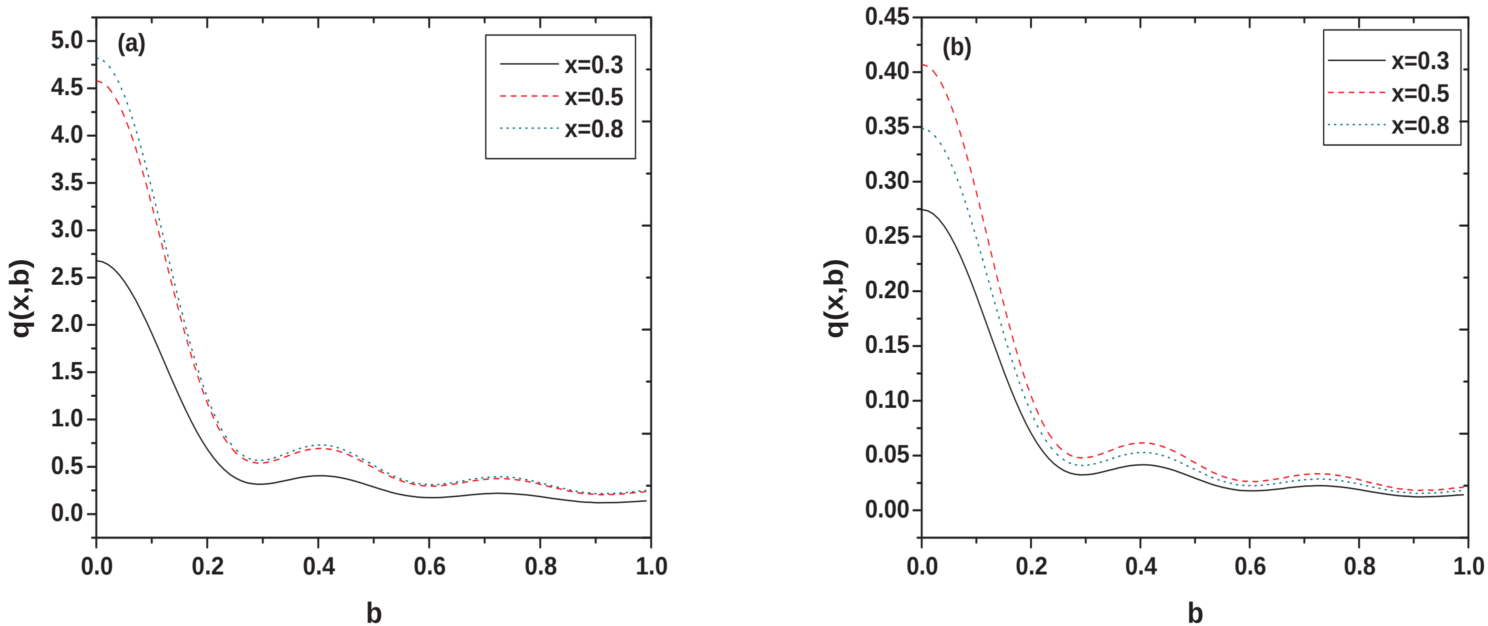
IPDPDFs in the Simulated Model (2)



- Mass parameters $M = 150, m = \lambda = 300$ MeV; $b_\perp$ is in MeV^{-1}.
- (a) Helicity non-flp; (b) helicity flip
- Skewness $\zeta = 0$ here

H. Dahiya, AM, S. Ray (2007).

IPDPDFs in the Simulated Model



- Mass parameters $M = 150, m = \lambda = 300$ MeV; $b_\perp$ is in MeV^{-1}.
- (a) model (1); (b) model (2); helicity non-flip
- Skewness $\zeta = 0$, x fixed

H. Dahiya, AM, S. Ray (2007).

Light Front Wave Function in Holographic QCD

Normalized holographic QCD LFWF for the meson ($q\bar{q}$) from AdS/CFT

$$\Psi_{L,k}(x, b_\perp) = B_{L,k}\sqrt{x(1-x)}J_L(\xi\beta_{L,k}\Lambda_{QCD})$$

Brodsky,Teramond (2006)

$B_{L,k} = \Lambda_{QCD}\Big[(-1)^L \pi J_{1+L}(\beta_{L,k})J_{1-L}(\beta_{L,k})\Big]^{-1/2}$,
$\xi = \sqrt{x(1-x}|b_\perp|$,
$\beta_{L,k}$ is the k-th zero of Bessel function J_L.

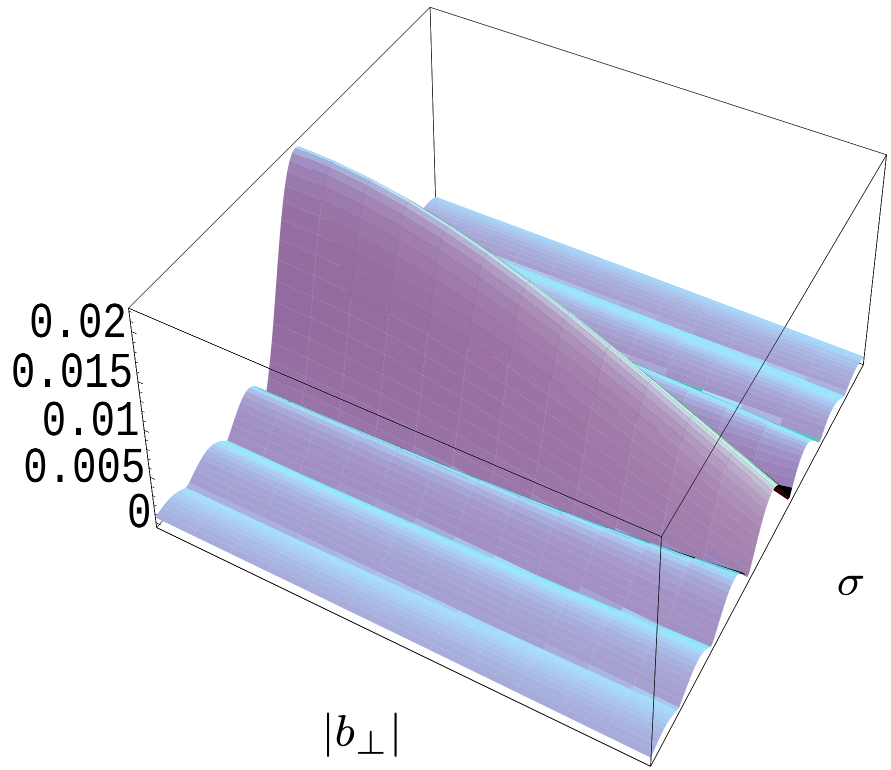
For ground state $L = 0, k = 1$ and we have

$$\phi(x, b_\perp) = \Psi_{0,1}(x, b_\perp) = \Lambda_{QCD}\sqrt{x(1-x)}\frac{J_0(\xi\beta_{0,1}\Lambda_{QCD})}{\sqrt{\pi}J_1(\beta_{0,1})}$$

Corresponding momentum space LFWF

$$\psi(x, \kappa_\perp) = \sqrt{4\pi^2}\int d^2b_{1\perp}\, e^{-ib_{1\perp}\cdot\kappa_\perp}\, \phi(x, b_{1\perp})\ .$$

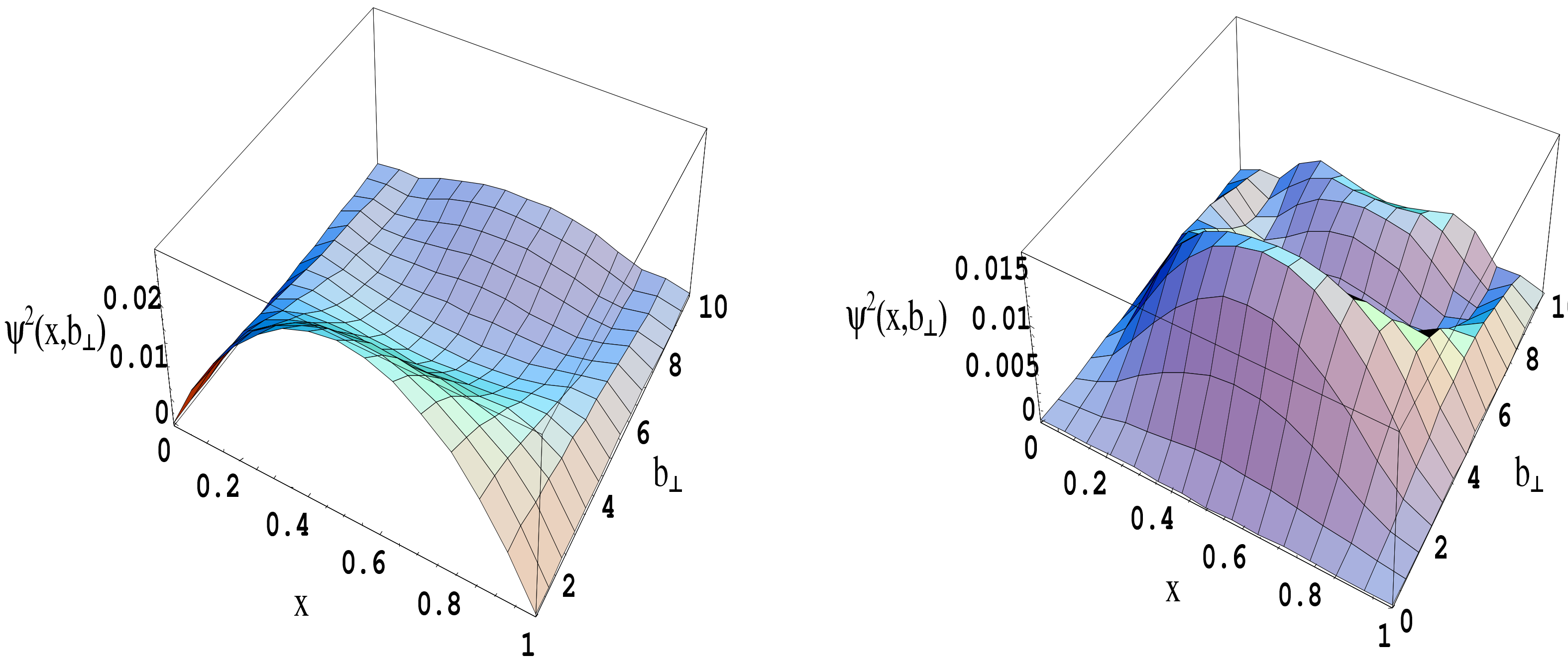
Light Front Wave Function in Holographic QCD



Ground state ($L = 0, k = 1$) of two parton holographic light front wave function in 3D space. We have taken $\Lambda_{QCD} = 0.32$ GeV. $|b_\perp|$ runs from 0.001 to 6.0 GeV^{-1} and σ from -25 to 25.

Brodsky,Chakrabarti,Harindranath,AM,Vary (2006).

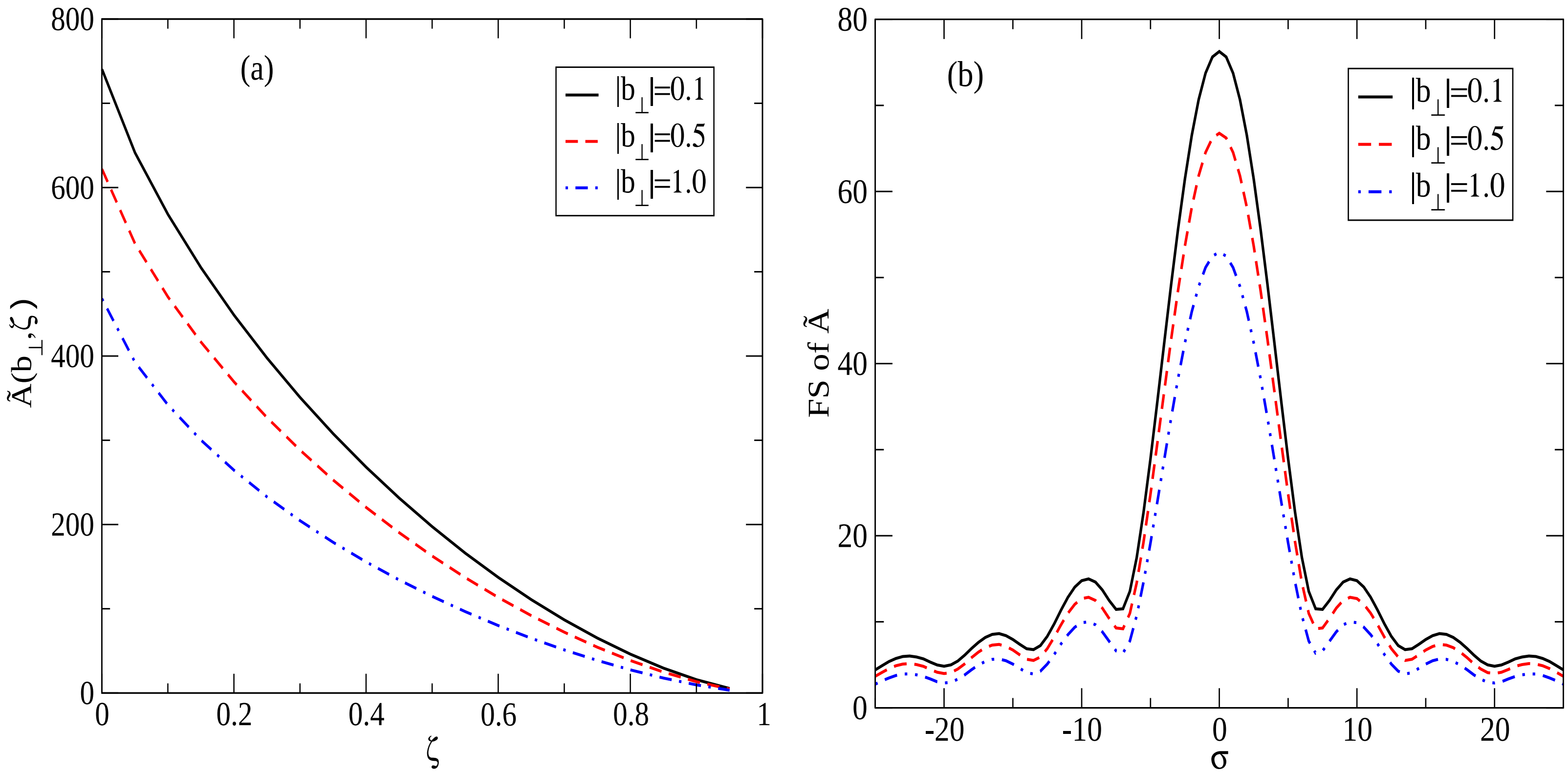
IPDPDFs in Holographic QCD model



(a) ground state $L = 0, k = 1$; (b) first exited state $L = 1, k = 1$
$\Lambda_{QCD} = 0.32$ GeV and $b_\perp$ is given in GeV^{-1}.

H. Dahiya, AM, S. Ray (2007).

DVCS amplitude in Holographic QCD model



The DVCS amplitude vs. ζ and Fourier spectrum of the DVCS amplitude in the σ space using the light front wave function for meson obtained from holographic QCD.

$\Lambda_{QCD} = 0.32$ GeV. Plots are in unit of e_q^2. $b^\perp$ is given in units of GeV^{-1}.

Brodsky,Chakrabarti,Harindranath,AM,Vary (2006).

Optics Analog for σ dependence

- Analogy with optics :

(i) Finite range of ζ integration act as a slit of finite width and provides a necessary condition for the occurrence of diffraction pattern in the Fourier transform of the DVCS amplitude.
(ii) In analogy with optical diffraction, where the positions of the first minima are inversely proportional to the slit width, here we expect their positions to be inversely proportional to ζ_{max}. Since ζ_{max} increases with -t, the position of the first minimum moves to a smaller value of σ
(iii) For fixed $-t$, higher minima appear at positions which are integral multiples of the lowest minimum : in analogy with diffraction in optics.

- Fourier transforming the amplitude in ζ at fixed $\Delta_\perp$ and then Fourier transforming $\Delta_\perp$ to impact space $b_\perp$ will give the analog of a three-dimensional scattering center.

- Scattering photons in DVCS provides the complete Lorentz-invariant light front coordinate space structure of a hadron.

Summary and Conclusions

•Fourier transform of the Deeply Virtual Compton Scattering (DVCS) amplitude with respect to the skewness variable ζ provides a unique way to visualize the light-front wavefunctions (LFWFs) of the target state in the boost-invariant longitudinal coordinate space variable ($\sigma = \frac{P^+ y^-}{2}$).

• As a specific example, we consider a fermion state at one loop in QED. We then simulate the wavefunction for a hadron by differentiating the above LFWFs with respect to M^2 and study the corresponding DVCS amplitudes in σ space.

•Results are analogous to the diffractive scattering of a wave in optics in which the dependence of the amplitude on σ measures the physical size of the scattering center of a one-dimensional system.

• If one combines this longitudinal transform with the Fourier transform of the DVCS amplitude with respect to the transverse momentum transfer $\Delta^\perp$, one can obtain a complete three-dimensional description of hadron optics at fixed light-front time $\tau = t + z/c$.