Absolute Calibration Of the Central Momentum

of the Hadron HRS

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Abstract

It is shown that the central momentum of the Hadron High Resolution Spectrometer in Hall A of the Jefferson Laboratory is given by the expression:

$$P_0 = \rho_0 B_0 c\ Kk'$$

where:

- P_0 is the central momentum in units of eV/c
- ρ_0 is the design bend radius of the spectrometer ($\rho_0 = 8.4$ m)
- B_0 is the central magnetic field in the spectrometer in Tesla
- c is the speed of light in ms^{-1}
- K is a correction factor for fringing fields in the dipole (K = 1.00708)
- k' is a correction factor for a known misalignment of the Q_3 quadrupole. (k'= 0.999833)

The uncertainty on the absolute value of P_0 is estimated to be $\pm 2x10^{-3}$ of the value of P_0. An expression is also given for the determination of B_0 based on the field read in the "low field" NMR probe.

Simple Calculation of P_0:

By design the HRS spectrometers bend the central trajectory through a 45º angle with a bend radius (ρ_0) of 8.40 m. The central momentum, assuming the effective length of the magnet is exactly as designed, is simply given by:

$$P_0 = \rho_0 B_0 c \qquad \textbf{Equation 1}$$

In what follows I will try to evaluate how wrong this simple picture can be and how well we can measure B_0, the ultimate goal being a simple expression to determine P_0 from the measured field in the dipole.

Magnet Effective Length

Corrections:

The original design effective length (L) of the dipole is given by $L = \frac{\pi}{4} \cdot \rho_0$. However, due to an error in the magnetostatic design that wasn't caught until too late, the effective length of the dipole is in fact 5 cm longer than what is given by the simple expression above. This extra length was confirmed in the electron arm at the ~2 mm level during mapping in April 1995. In figure 1 the solid circles show the location of the exit effective boundary of the dipole as measured in April 1995 in a coordinate system where the center of the entrance effective field boundary is at z = 3 mm and y = 0 mm. The solid triangle shows where in that coordinate system the center of the EFB would be if the magnet's length was given by $L = \frac{\pi}{4} \cdot \rho_0$.

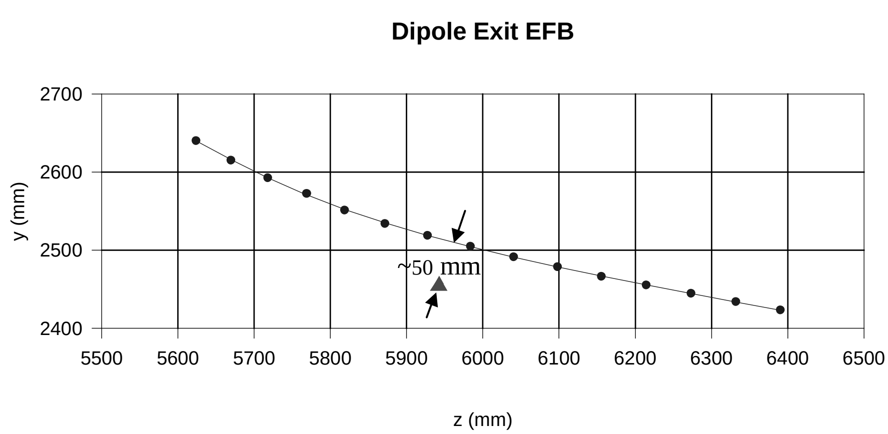

Figure 1

The effect of this on the central momentum is small. A back of the envelope estimate simply assuming B_0L changes to B'_0L' gives a correction factor of $\frac{L'}{L} = 1.0076$. This is in fact a slight overestimate. The actual trajectory that we call the "central trajectory" (**i.e.** the trajectory with x_0, y_0, θ_0, & $\phi_0 = 0$ that crosses x = 0 at the first VDC) doesn't move along a perfect arc. The extra length along with the effect of the fringe fields of the dipole and the fact that this trajectory doesn't hit Q3 exactly on axis, even when Q_3 is perfectly aligned, all add small corrections to the 1.0076 number. Raytracing studies using RAYTRACE[1] and SNAKE[2] independently determine correction factors of 1.00705 and 1.00708, respectively, for this effect. In these tests the fringe fields in the dipole and the magnetic field in Q_3 were turned off so that the dipole magnetic field had the unrealistic character of being a step function as one crossed the EFB. The magnetic field in the dipole, B_0, was then set such that for a given momentum the axial ray crossed the first VDC plane at x = 0. In SNAKE this was found to correspond exactly to what one would expect from equation 1 above. RAYTRACE gave a slightly different value attributable to an incorrect value for the speed of light as stored in RAYTRACE. Then the fringe fields and Q_3 were turned on. The dipole fringe fields add 2.5 cm to the length of the magnet at each end and give a reasonable approximation to the observed fall off of the field as measured in the mapping. Then B_0 was adjusted to again bring the axial ray to x = 0 at the first VDC plane. The ratio of B_0 needed with no fringe fields and Q_3 turned off divided by the value with them turned on is the correction factor given above. Since the SNAKE analysis was a little more straight forward, c is correct in SNAKE, I'm inclined to prefer the SNAKE value (given the final estimated errors it hardly makes a difference). It should be noted that the effect of turning Q_3 on and off is roughly 1/10 that of turning the dipole fringe fields on and off. Also, in this analysis Q_3 was assumed to be perfectly aligned.
With this correction equation 1 above becomes:

$$P_0 = \rho_0 B_0 c\, K \quad \textbf{Equation 2}$$

Where K = 1.00708

Quadrupole Misalignment

To first order a quadrupole in the spectrometer that is misaligned appears to all trajectories as the same quadrupole with a dipole component equal to the field gradient in the magnet times the displacement superimposed on it. For displacements along the dispersive plane, vertical in HRS, this extra dipole serves to add a small additional bend to all trajectories. The effect of such displacements is tabulated in the CDR[3] . For known misalignments this effect can be included in a further correction factor in equation 2 above. A review of the data[4] shows that Q_3 is in fact shifted -1 mm relative to the optic axis. Taking this into account equation 2 becomes:

$$P_0 = \rho_0 B_0 c\, Kk' \quad \textbf{Equation 3}$$

Where k'=0.999833.

Errors:

There is of course some uncertainty in the actual realized effective length of the dipole magnet. As mentioned above the effective length of the Electron Dipole was measured. Not only is there an inherent uncertainty in these measurements but also these measurements were not made on the Hadron Dipole. One must also consider the possibility that the effective length of the magnet can change with field strength. A second mapping of the Electron Dipole in June 1996 focused on the entrance and first half of the magnet. Q_3's presence prevented measurement of the exit of the magnet. Those measurements showed that the entrance effective field boundary (EFB) was stable at the few mm level and did not move around more than a few mm with excitation. Figures 2 and 3 show the deviation of the mapped entrance effective field boundary from the design location (taking the extra 5 cm into account) at 600 and 900 A. Note that what matters for the present purposes is the location of the EFB for the central trajectory, **i.e.** at x = 0.

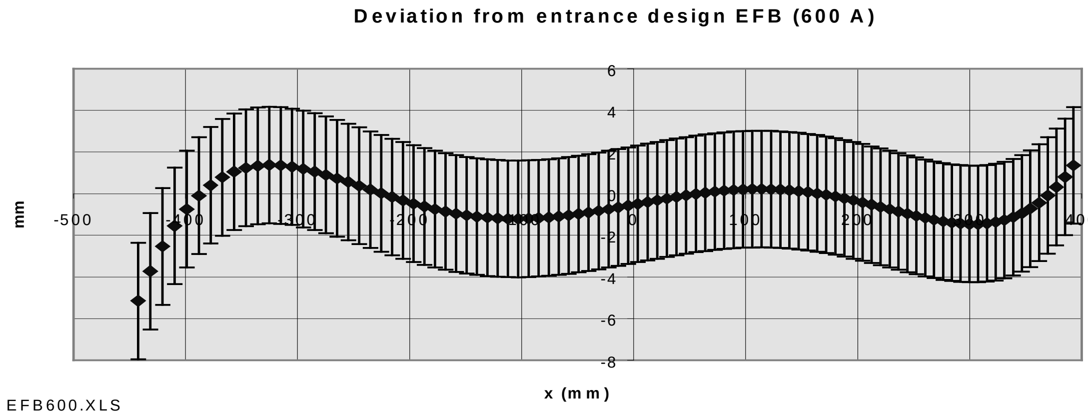

Figure 2

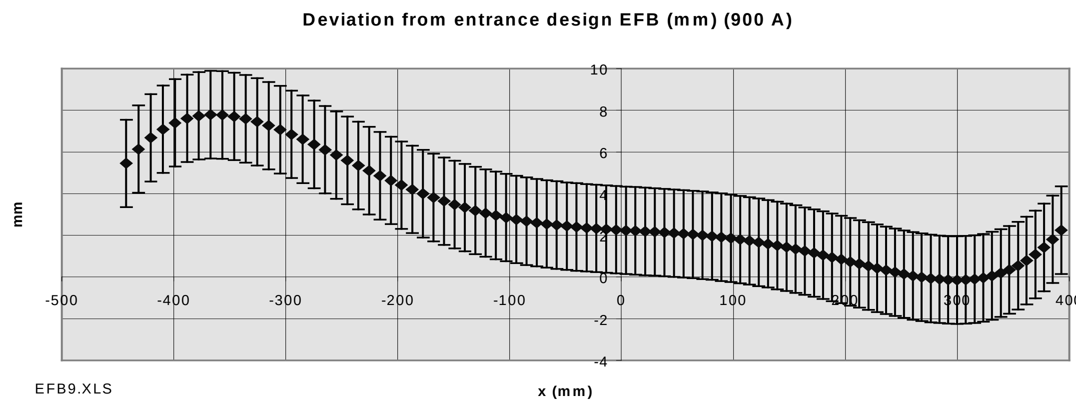

Figure 3

Here we make two leaps of faith:

1) that the exit EFB was as well behaved as the entrance
2) that the Hadron Dipole field boundaries behave the same as those of the electron dipole.

The justification of the first leap is that the one measured exit EFB and its associated entrance EFB agree very well with the design. The justification of the second leap is the fact that the empirically determined optics of the two spectrometers have been found to be almost identical. In the following table (Table 1) the empirically determined first and second order coefficients of the reconstruction tensor[5] for the Hadron and Electron spectrometers is compared. First order terms are listed in italics. In the third and fourth column calculated values for the terms used to determine momentum are given. The calculation involved setting up a model for the spectrometer in the Raytracing program SNAKE using the measured effective lengths and field gradients for the Quadrupoles[6] corresponding to the current settings used in our empirical setup for each spectrometer and a field index n = -1.26 in the dipole along with the design values for the effective field boundaries and measured positions of the quadrupoles relative to the dipoles.

	Electron	Hadron	Hadron Calculated	Electron Calculated
$\mathbf{D_{1000}}$	*0.0853*	*0.0837*	*0.0840*	*0.0833*
$\mathbf{D_{2000}}$	0.011	0.011	0.011	0.0110
$\mathbf{D_{0100}}$	*-0.040*	*-0.037*	*-0.042*	*-0.025*
$\mathbf{D_{1100}}$	0.259	0.285	0.251	0.246
$\mathbf{D_{0200}}$	-1.373	-1.81	-1.93	-1.73
$\mathbf{D_{0002}}$	0.183	-0.04	0.13	-0.080

D_{0020}	0.55	0.59	0.12	0.30
D_{0011}	0.33	0.50	-0.56	0.43
T_{0100}	*-2.380*	*-2.208*		
T_{1100}	0.490	0.59		
T_{0200}	-8.129	0.65		
T_{0002}	0.276	1.12		
T_{0011}	1.117	-0.12		
T_{0020}	0.123	-0.37		
P_{0001}	*-0.694*	*-0.620*		
P_{1001}	-0.083	-0.13		
P_{0010}	*-0.293*	*-0.282*		
P_{1010}	0.342	0.38		
P_{0110}	3.622	2.60		
P_{0101}	6.177	6.00		
Y_{0001}	*0.666*	*0.724*		
Y_{1001}	-1.279	-1.24		
Y_{0010}	*-1.172*	*-1.304*		
Y_{1010}	-0.736	-0.73		
Y_{0110}	-12.006	-14.39		
Y_{0101}	-0.559	-2.68		

Table 1

The reader should note that this table only summarizes the first and second order terms. In fact terms up to fourth order have been determined by fitting a polynomial to limited data. Sometimes small variations in the first order terms can be offset by variations in the third order terms etc. This is difficult to see by inspection as the number and complexity of terms increases as one goes to higher orders. Nevertheless, the lower order terms are very similar for both spectrometers. The calculated terms also agree rather well with the empirically determined ones. Here discrepancies can easily be attributed to uncertainties in the actual momentum being analyzed as well as uncertainties in the absolute values of the quadrupole field settings. The D_{0100} term is especially sensitive in this regard. In fact a variation of $\pm 2\times10^{-3}$ in only the central momentum[‡] of the spectrometer as used in the calculation will cause this term to vary by ±0.006.
So finally, the error assigned to the knowledge of the effective length of the dipole is ± 5 mm which corresponds to a relative error of $\pm 7.6\times10^{-4}$.

Determination of B_0

The NMR probes[*] used to measure the magnetic field are extremely accurate and by themselves do not contribute significantly to the error in the determination of B_0. There are however several possible errors associated with this particular setup. First of all, as with all NMR probes while the field reading is very good it is difficult to know their exact location very well. The probes sit in Aluminum racks that have been carefully assembled and mounted to the rails intended to guide the mapper through the magnet. The positions of the rails are known to better than a mm. However, owing to the finite size of the sensitive element in the probe and the assembly tolerances in putting together the racks holding the probes the estimated uncertainty in the radial position of the probes is ± 2 mm. Considering the field gradient in the dipole this alone would result in a relative uncertainty of $\pm 3\cdot10^{-4}$ (This uncertainty is implicitly included in

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[‡] The value of the central momentum used in the calculation presented here was 837.6 MeV/c along with the spectrometer settings used to put the elastic peak from ^{12}C at 16° with a "nominal" beam energy of 845 MeV at x = 0 in the first VDC. The reason for this choice will become clear later in this document.
[*] METROLAB PT 4025 with 1060-3,4,5 probes.

the following discussion of variations in ratios of NMR probe readings but stands as a lower limit of what is possible).

Furthermore, in order to get the NMR probes to work in the field gradient of the dipole each probe is enclosed in a gradient compensating coil package which provides a magnetic field gradient opposite to the magnet's gradient in the limited region around the probe. Ideally these coils should be positioned such that the zero crossing point of the field generated by the coil is centered on the sensitive element of the probe. In this way they would have no effect on the accuracy of the reading obtained from the probe. Naturally, perfect positioning is nearly impossible. Some compensating coil current dependence of the probe reading has been observed but it is very small. The relative difference between field readings when the NMR signal is optimized, compensating current adjusted to maximize the signal, and when the compensating coil current is noticeably off the optimum value is typically $<5\times10^{-5}$. In practice the compensating coil current for the Hadron Dipole NMR probes is set by the field regulation software to the known optimum value for the field setting desired so the 5×10^{-5} above is larger than the true error coming from the compensating coil placement. This effect is small compared to the other effects considered here.

Since the dipole is a conical magnet with a field index, the field doesn't vary linearly across the gap but rather as follows:

$$B = \frac{B_0}{1 + n\frac{\Delta\rho}{\rho_0}} \qquad \textbf{Equation 4}$$

where B_0 is the central field (the quantity of interest); n is the field index; and $\Delta\rho$ is the radial displacement from the central layout radius (ρ_0). The field index has been measured to be $n=-1.257\pm0.003$ at 600 A. Figure 4 shows data extracted from a map taken at 600 A and used to determine n.

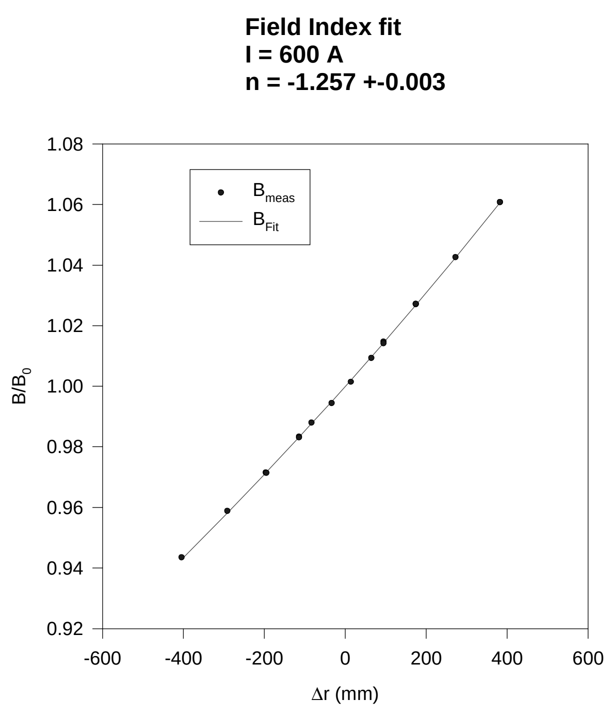

Figure 4

In the dipole two NMR probes are placed at slightly more than 0.4 m from the centerline of the magnet, one on the high field side and the other on the low field side. This second NMR probe (B_{low}) is normally used to set and monitor the magnetic field of the Hadron dipole during experiments. The ratio of the low NMR field reading to the high NMR field reading has been measured to vary from 0.883 to 0.882 in going from 0.2 to 0.8 Tesla. The ratio of the 2 probe readings can be used along with the measured value of the field index to determine the locations of the probes. Assuming n = -1.257 and ρ_0 = 8.4 m, this corresponds to values of between $\Delta\rho=\pm0.415$ m and ±0.418 m, where the low NMR has a negative $\Delta\rho$ and the high NMR has a positive $\Delta\rho$. (This range of values of $\Delta\rho$ suggest a better knowledge of the probe locations than discussed above.†) To get B_0 from the measured value of B_{low} use the following expression:

$$B_0 = B_{low}(1+n\frac{\Delta\rho}{\rho_0}) \qquad \textbf{Equation 5}$$

For n=-1.257, ρ_0=8.4 m, and $\Delta\rho$=-0.415 m this becomes:

$$B_0 = B_{low}\times 1.06210 \qquad \textbf{Equation 6}$$

Changing to $\Delta\rho$=-0.418 m gives:

$$B_0 = B_{low}\times 1.06255 \qquad \textbf{Equation 7}$$

Using n = -1.254 the multiplicative factors in equations 6 and 7 become 1.06195 and 1.6240, respectively. If one goes to the other extreme and sets n = -1.26 they become 1.06225 and 1.06270. The difference between the maximum and minimum of the above values divided by their average is 7×10^{-4} which can be taken as the uncertainty in the determination of B_0 based on the field measurements (**i.e.** $\pm3.5\times10^{-4}$). In summary the value of B_0 to be used in equation 3 above should be determined by equation 5 above using n = -1.257 and $\Delta\rho$ =- 0.4165 m. **i.e.:**

$$\mathrm{B_0} = \mathrm{B_{low}}\times 1.06233 \qquad \textbf{Equation 8}$$

The resulting relative uncertainty is $\pm3.5\times10^{-4}$.

Quadrupole Misalignment

Except for Q_3, the quadrupoles in the Hadron arm were installed such that there are no known misalignments, but there is an uncertainty of ± 1 mm in the actual location of the magnets. Due to its difficult location that uncertainty for Q_3 is ± 2 mm. These potential misalignments each contribute to the uncertainty in the central momentum of the spectrometer. The couplings between these misalignments and changes in momentum are tabulated in the CDR. Exact values are reported in Table 2 below..

Vertical Positioning

This is a simple uncertainty resulting from a less than perfect knowledge of the overall vertical position of the spectrometer relative to the beam. In optics terms a nonzero value of x_0, the vertical displacement of the beam relative to the spectrometer, is indistinguishable from a change in momentum. A shift in x at the target translates directly into a shift in x at the focal plane and a shift in x at the focal plane is recognized as a change in momentum. That shift in momentum is given by:

$$\Delta\delta = \frac{\langle x|x_0\rangle x_0}{\langle x|\delta\rangle} \quad \textbf{Equation 9}$$

Where $\delta = \frac{\Delta P}{P}$ as usual, and $\langle x|x_0\rangle$ & $\langle x|\delta\rangle$ are the standard first order matrix elements representing the magnification and dispersion respectively in the dispersive (vertical for HRS) plane. Assuming ± 0.5 mm for the combined total uncertainty in the beam vertical position and the central axis of the spectrometer leads to an uncertainty of $\pm 9\cdot10^{-5}$ in the absolute value of the central momentum.

† What is actually varying here is the product n$\Delta\rho$. Most likely n is varying since the probes shouldn't move with excitation. The variation is discussed in terms of a variation in $\Delta\rho$ for conceptual reasons.

VDC Positioning

An uncertainty in the position of the VDC translates into an uncertainty in the momentum by dividing that translation uncertainty by the dispersion. Survey data[7] indicate that the VDC's are positioned correctly to $\pm$ 1 mm. This leads to a $\pm 6\cdot10^{-5}$ uncertainty in the central momentum.

Sieve Vertical Alignment

This actually only contributes a very small amount but is included here for completeness. If we had a perfect focus, $\langle x|\theta\rangle = 0$, the effect would be zero to first order. This is because a vertical displacement of the sieve causes an error in the assumption of which trajectory corresponds to $\theta_0 = 0$. However, with $\langle x|\theta\rangle = 0$, the trajectory falsely labeled as having $\theta_0 = 0$ arrives at the same x on the focal plane as the true $\theta_0 = 0$ trajectory. In fact our empirically determined tune has a slight overfocus such that at the focal plane $\langle x|\theta\rangle = -0.2$ ("natural units"). This leads to small changes in x at the focal plane with variation of θ_0. Still the effect is very small. Assuming $\pm$ 2 mm uncertainty in the vertical alignment of the sieve to the spectrometer axis results in only $\pm 3\cdot10^{-5}$ uncertainty in the central momentum.

Summary of Errors

Errors	+-		Relative Error on P0
Dipole effective length	5.0	mm	7.6E-4
Central Field			3.5E-4
Q1 vertical alignment	1.0	mm	2.1E-4
Q2 vertical alignment	1.0	mm	1.9E-4
Q3 vertical alignment	2.0	mm	3.3E-4
Vertical Positioning (X0)	0.5	mm	9.3E-5
VDC Positioning	1.0	mm	5.9E-5
Sieve vertical alignment	2.0	mm	3.0E-5
		Sum	2.0E-3

Table 2

Comparison with Estimated Beam Energy

When electrons scatter elastically from a nucleus the momentum of the scattered electron is given by the expression:

$$P_0 = \frac{P_{beam}}{1 + \frac{2P_{beam}}{M}\sin^2\frac{\theta}{2}} \qquad \textbf{Equation 10}$$

Where P_0 is the final momentum as seen by the spectrometer, P_{beam} is the beam momentum, M is the mass of the scattering nucleus, and θ is the scattering angle. Using equation 10 the momentum of an 845 MeV/c electron scattered elastically through 16º from a ^{12}C nucleus is expected to have a final momentum, P_0 = 842.53 MeV/c.

In the late winter and spring of 1997 calibration studies were made of the Hadron spectrometer using elastic scattering of nominal 845 MeV/c electrons from a ^{12}C target at 16º as a calibrator. It was determined that the field at the low field NMR probe needed to put the elastic peak in the center of the VDC (**i.e.** at x = 0) was 0.310944 T. Using equation 8 above this corresponds to B_0 = 0.330324 T, giving P_0 = 837.59 MeV/c via equation 3 above and indicating that the beam momentum quoted as 845 MeV/c was actually 840 MeV/c $\pm$ 2 MeV/c. This discrepancy cannot be explained by a systematic error in the scattering angle. At this energy and angle the scattered momentum varies by ~300 keV/c per degree scattering angle. Effects of energy losses in the target and spectrometer windows have been ignored in this analysis because their magnitude is very small compared to the size of the observed discrepancy and the associated uncertainties.

Electron Arm

Analysis and calibration of the Electron Arm runs very much parallel to that of the Hadron Arm. The only major differences are that in the Electron Arm there is no correction factor for the misalignment of Q_3 and the variation in the observed ratio of the high and low field NMR probe readings is four times as great. So, the central momentum of the Electron arm is given by:

$$P_0 = \rho_0 B_0 c\ K \quad \textbf{Equation 11}$$

where all terms are as described above.

There is also an operational difference between the Electron and Hadron arms. In the case of the Electron arm the magnetic field in the dipole is set by the average of the high and low field probes rather than just the low field probe. This changes subtly the error analysis on the determination of B_0 and leads to the use of the following expression to determine B_0 from the measured fields:

$$B_0 = \frac{2\ B_{avg}}{\frac{1}{1+\frac{R-1}{R+1}} + \frac{1}{1-\frac{R-1}{R+1}}} \quad \textbf{Equation 12}$$

Where R is the ratio of the low field NMR reading to the high field NMR reading. Using R = 0.880 (the ratio observed during 845 MeV elastic scattering calibrations as described above for the Hadron Arm) equations 11 and 12 give a resulting value for the scattered electron momentum in the calibration runs of late winter and early spring 1997 that agrees within 50 keV/c of that found in the Hadron Arm[*] . This further supports the assumption that the dipoles on both arms are essentially identical. The total error for the Electron arm is slightly larger than that for the Hadron Arm because of the difference in the operational way the field is measured and the fact that the observed ratio of low to high field NMR's varies more in the Electron Arm than the Hadron Arm. The net effect is to roughly double the error on the determination of B_0. Thus, the total relative error for the Electron arm is $\pm 2.3 \times 10^{-3}$.

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[*] Hadron Arm and Electron Arm were calibrated against the beam simultaneously, **i.e.** both spectrometers were set at 16º and observed elastic scattering from the same ^{12}C target at the same time.

References:

[1] S. Kowalski and H.A. Enge, "RAYTRACE Users Manual", LNS/MIT, May 1986
[2] P. Vernin, RPAC (II), CEBAF, January 1987, p. 615
[3] CEBAF Conceptual Design Report, April 1990, p. A4-24.
[4] Jefferson Lab Alignment Group Data Transmittal #352, August 15, 1997.
[5] for explanation of notation see ESPACE Users Guide, E. Offermann, Chapter 6, 1997; H. Blok et al., NIM A262 (1987) 291-297; E. Offermann et al., NIM A262 (1987) 298-306.
[6] Preliminary Results of Magnetic Measurements of Electron Arm HRS Quadrupoles, QMM collaboration, July 15, 1996, and private communication, Hélène Fonvieille.
[7] Private communication, Bogdan Wojtsekhowski.