Proton Magnetic Form Factor from Existing Elastic e-p Cross Section Data

Longwu Ou, Barak Schmookler, Shalev Gilad (MIT)

Eric Christy (Hampton University)

Cynthia Keppel, Bogdan Wojtsekhowski (JLab)

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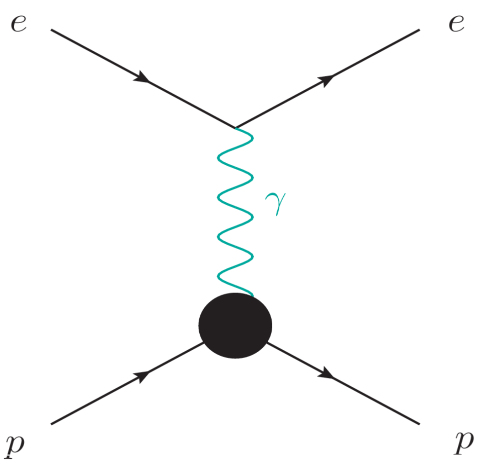
Nucleon Electromagnetic Form Factors

- Fundamental quantities defined in the context of one-photon exchange
  - Related to the spatial distributions of charge and magnetization in the nucleon in non-relativistic limit
  - Precision data for ground state QCD studies
  - Input to nuclear structure and parity violation experiments
  - New information on basic hadron structure as first moments of GPD
- Experimental measurements have been conducted for several decades → unresolved puzzles still exist

Measurement of Proton Form Factor



Rosenbluth separation method:

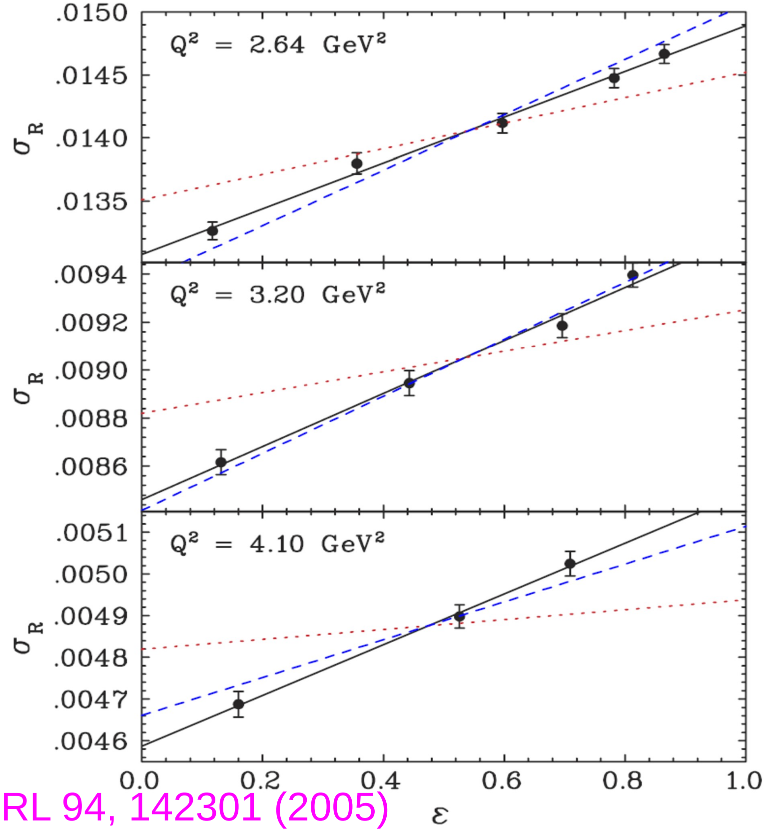
Electron-proton elastic cross section in one-photon exchange approximation:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{\epsilon (G_E^p)^2 + \tau (G_M^p)^2}{\epsilon (1+\tau)}$$

$$\sigma_R \equiv \epsilon (1+\tau) \frac{d\sigma}{d\Omega} / \left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \epsilon (G_E^p)^2 + \tau (G_M^p)^2$$

$$\tau = Q^2/4M_p^2 \qquad \epsilon = [1+2(1+\tau)\tan^2(\theta/2)]^{-1}$$

- Measure angular dependence of *ep* elastic cross section at fixed Q^2
- Extract electric and magnetic form factor from linear dependence of reduced cross section on ε



Qattan et al.PRL 94, 142301 (2005)

Measurement of Proton Form Factor

Polarization transfer method:

The ratio of electric to magnetic form factors are determined by transverse (P_t) and longitudinal (P_l) polarization components of the recoil proton

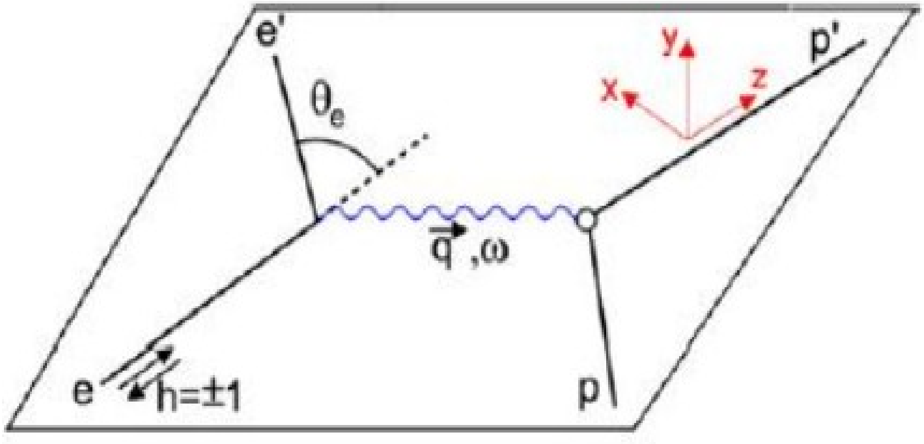
$$I_O P_l = \frac{(E+E')}{M_p}\sqrt{\tau(1+\tau)}\,(G_M^p)^2\tan^2\left(\frac{\theta_e}{2}\right)$$

$$I_O P_t = -2\sqrt{\tau(1+\tau)}\,G_E^p G_M^p \tan\left(\frac{\theta_e}{2}\right)$$

$$I_O = (G_E^p)^2 + \frac{\tau}{\epsilon}(G_M^p)^2$$

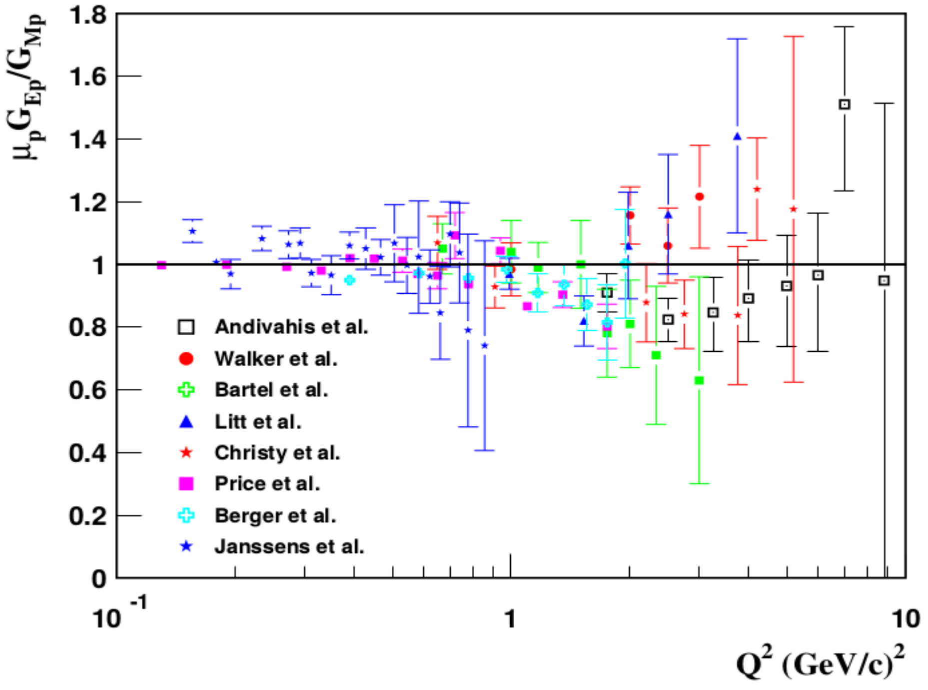
$$\frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l}\frac{(E+E')}{2M_p}\tan\left(\frac{\theta}{2}\right)$$

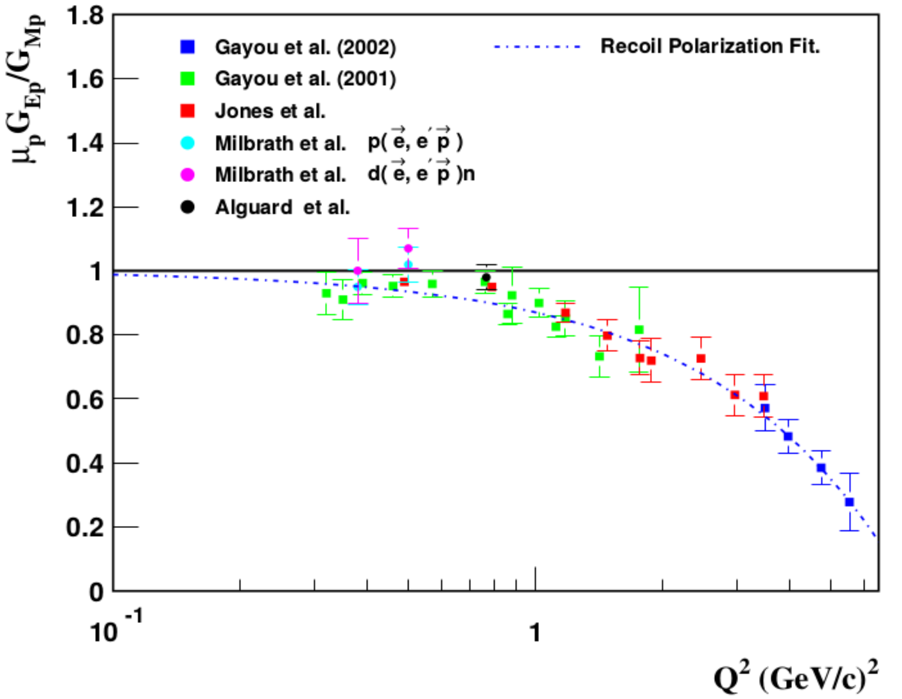
Longitudinally polarized electrons scatters elastically from unpolarized proton target



Proton Form Factor "Crisis"

- Two methods yield significantly different results for form factor ratio in the region $Q^2 \geqslant 1.0 (GeV/c)^2$

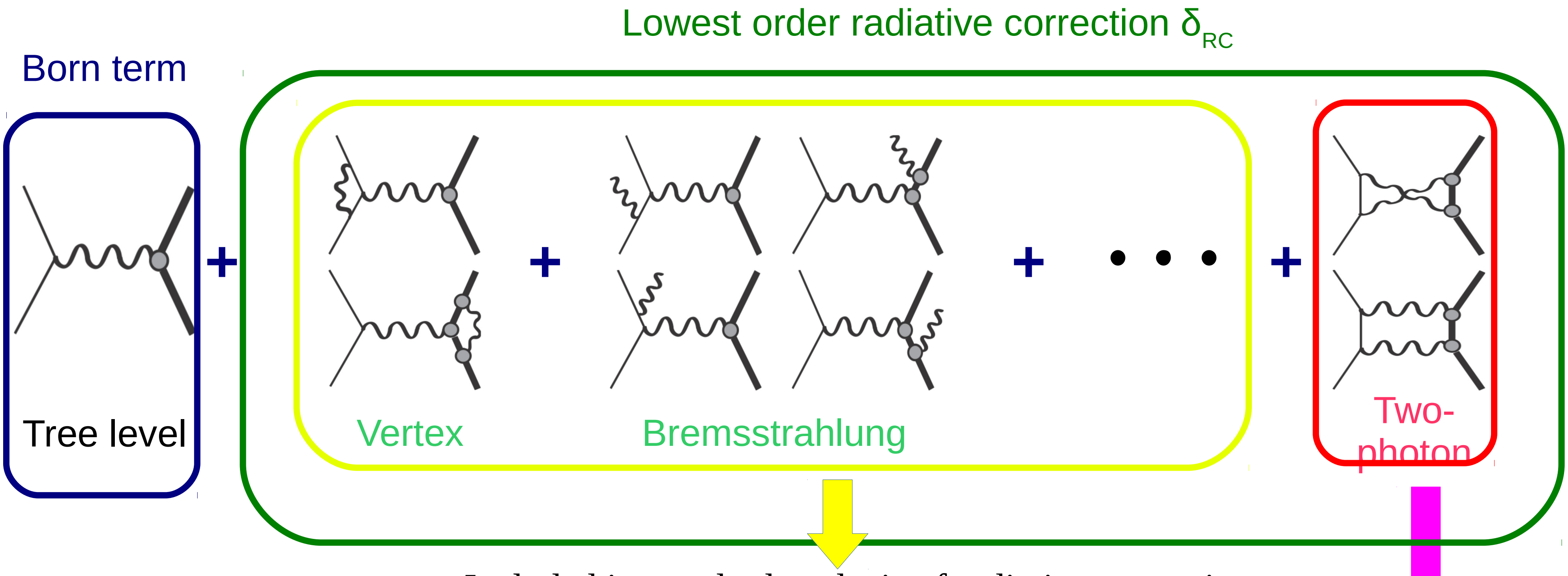




- Likely explanation: Two-photon exchange (TPE) effects

Two-Photon Exchange

Radiative corrections for ep scattering: $$\left(\frac{d\sigma}{d\Omega}\right)_{obs} = \left(\frac{d\sigma}{d\Omega}\right)_{Born} (1+\delta_{RC})$$



Included in standard analysis of radiative correction (Mo&Tsai, Rev. Mod. Phys. 41, 205(1969))

Full theoretical calculation is complicated ⇐ *Two-hard-photon exchange is not included in Mo&Tsai's calculation* ⇐ *ε-dependence is modified*

Two-Hard-Photon Exchange

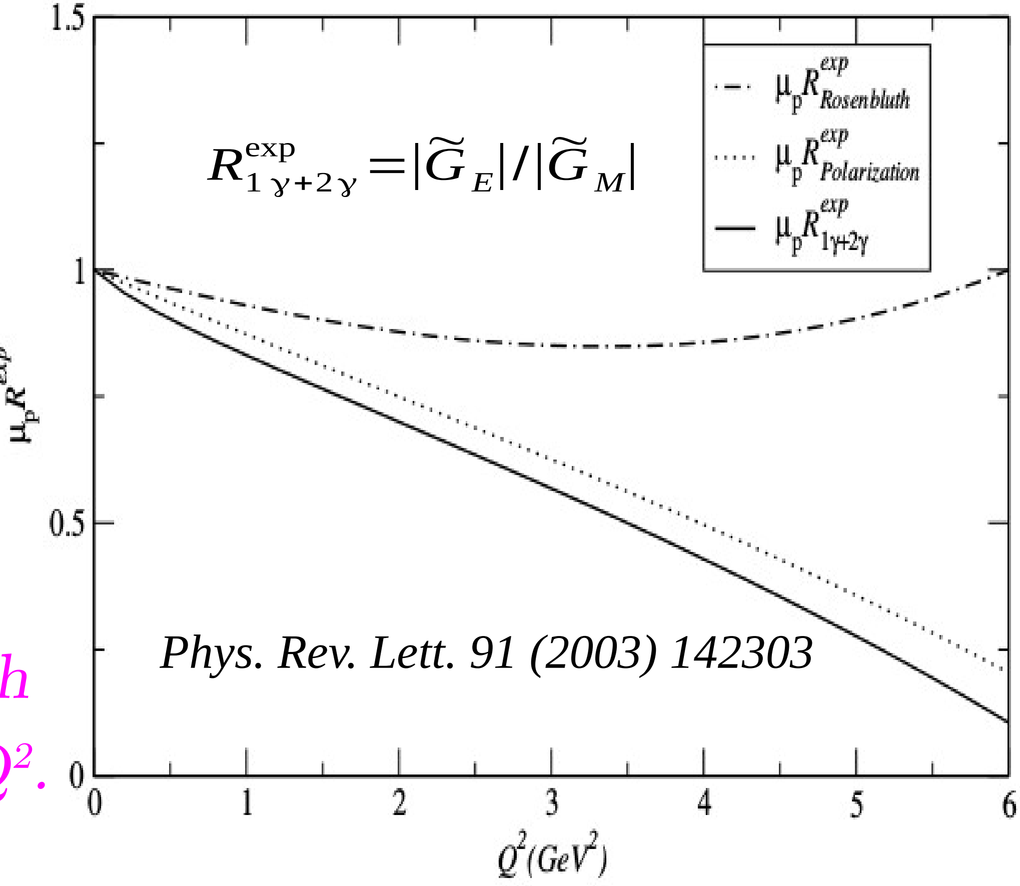
TPE effects affect the ratio of form factors obtained from Rosenbulth separation method and polarization transfer technique in different ways (*Guichon & Vanderhaeghen, Phys. Rev. Lett. 91 (2003) 142303*)

$$Y_{2\gamma}(\epsilon,Q^2)=\Re\left(\frac{\nu\tilde{F}_3}{M^2|\tilde{G}_M|}\right)\sim\alpha\simeq 1/137 \qquad \nu=M^2\sqrt{(1+\epsilon)/(1-\epsilon)}\sqrt{\tau(1+\tau)}$$

$$R^{exp}_{polarization}=\frac{|\tilde{G}_E|}{|\tilde{G}_M|}+\left(1-\frac{2\epsilon}{1+\epsilon}\frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right)Y_{2\gamma}$$

$$\left(R^{exp}_{Rosenbluth}\right)^2=\frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2}+2\left(\boxed{\tau}+\frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right)Y_{2\gamma}$$

TPE correction impact Rosenbluth ratios more significantly at high Q^2. Polarization result is less affected



Phys. Rev. Lett. 91 (2003) 142303

Cross Section with TPE Correction

- Experimental observations fail to find non-linearity in the ε-dependence of reduced cross section
- At ε=1, TPE corrections to the reduced cross section should vanish from theory (JHEP04(2013)029)

TPE effect on reduced cross section can be expressed as a linear function proportional to (1-ε):

$$(\sigma_R)_{\exp} = (\sigma_R)_{Born} + (\sigma_R)_{2\gamma}$$

$$(\sigma_R)_{\exp} = \underbrace{\boxed{\tau (G_M^p)^2 \left(1 + \frac{\epsilon}{\tau} R^2\right)}}_{\text{Born term}} + \underbrace{\boxed{a(Q^2)(1-\epsilon)\tau (G_M^p)^2}}_{\text{TPE correction}}$$

$$\boxed{\sigma_{fit} = \sigma_{Mott} \frac{\tau}{\epsilon(1+\tau)} (G_M^p)^2 \left[1 + \frac{\epsilon}{\tau} R^2 + a(Q^2)(1-\epsilon)\right] \qquad R = G_E / G_M}$$

Our Parametrization: $\frac{G_M^p(Q^2)}{\mu_p} = \frac{1 + b_0 \tau}{1 + b_1 \tau + b_2 \tau^2 + b_3 \tau^3}$ *J. J. Kelly, Phys. Rev. C 70, 068202 (2004)*

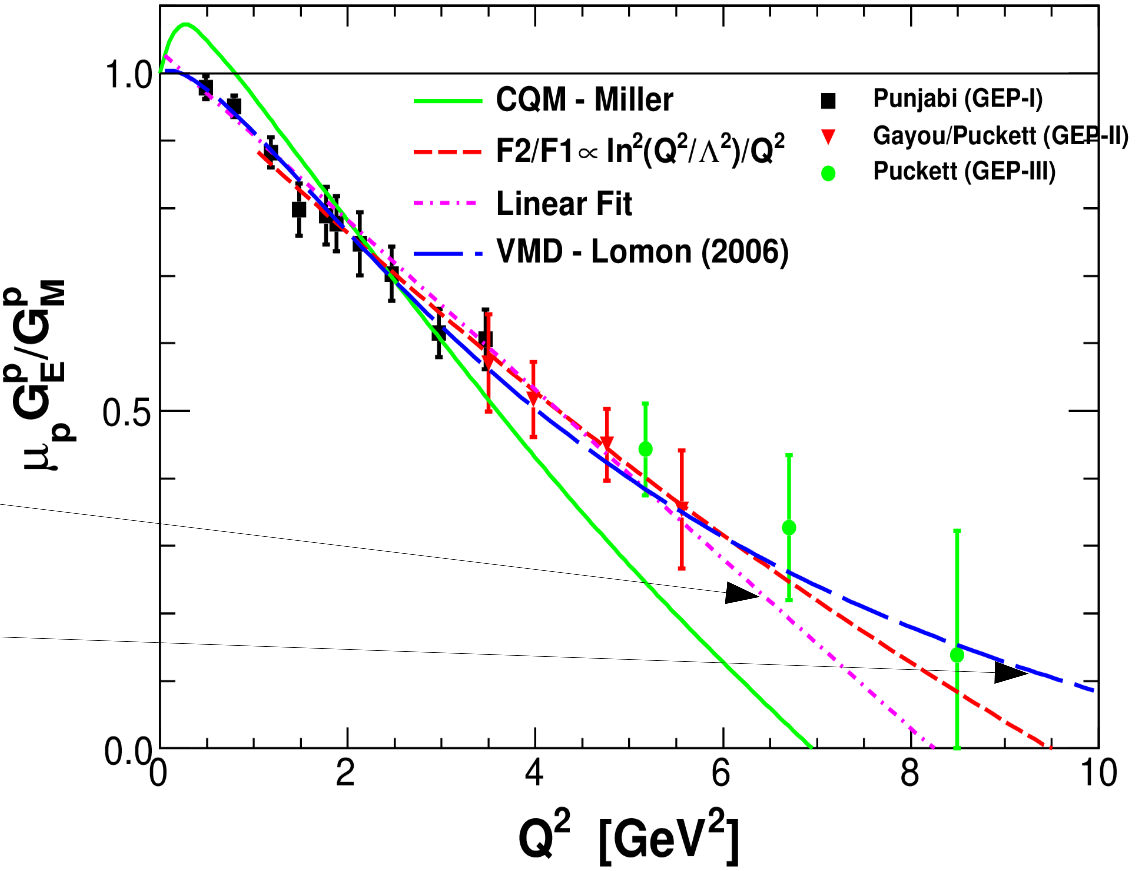
$$a(Q^2) = p_0 + p_1 Q^2 + p_2 Q^4 + p_3 Q^6 \qquad \tau = \frac{Q^2}{4 M_p^2}$$

Description of the Ratio G_E/G_M

- Ratio of electric to magnetic form factors is measured by polarization transfer experiments at Q^2 up to 8.5 GeV^2
- A model describing the evolution of R is needed to extend the analysis to high Q^2 region
  - Linear fit of R below 8.3 GeV^2 and assume R=0 above that
  - Lomon's VMD fit (arXiv:nucl-th/0609020v2 *(2006)*)

$$\sigma_{fit}=\sigma_{Mott}\frac{\tau}{\epsilon(1+\tau)}\left(G_M^p\right)^2\left[1+\frac{\epsilon}{\tau}R^2+a(Q^2)(1-\epsilon)\right]$$

$$R=G_E/G_M$$



Fitting Procedure

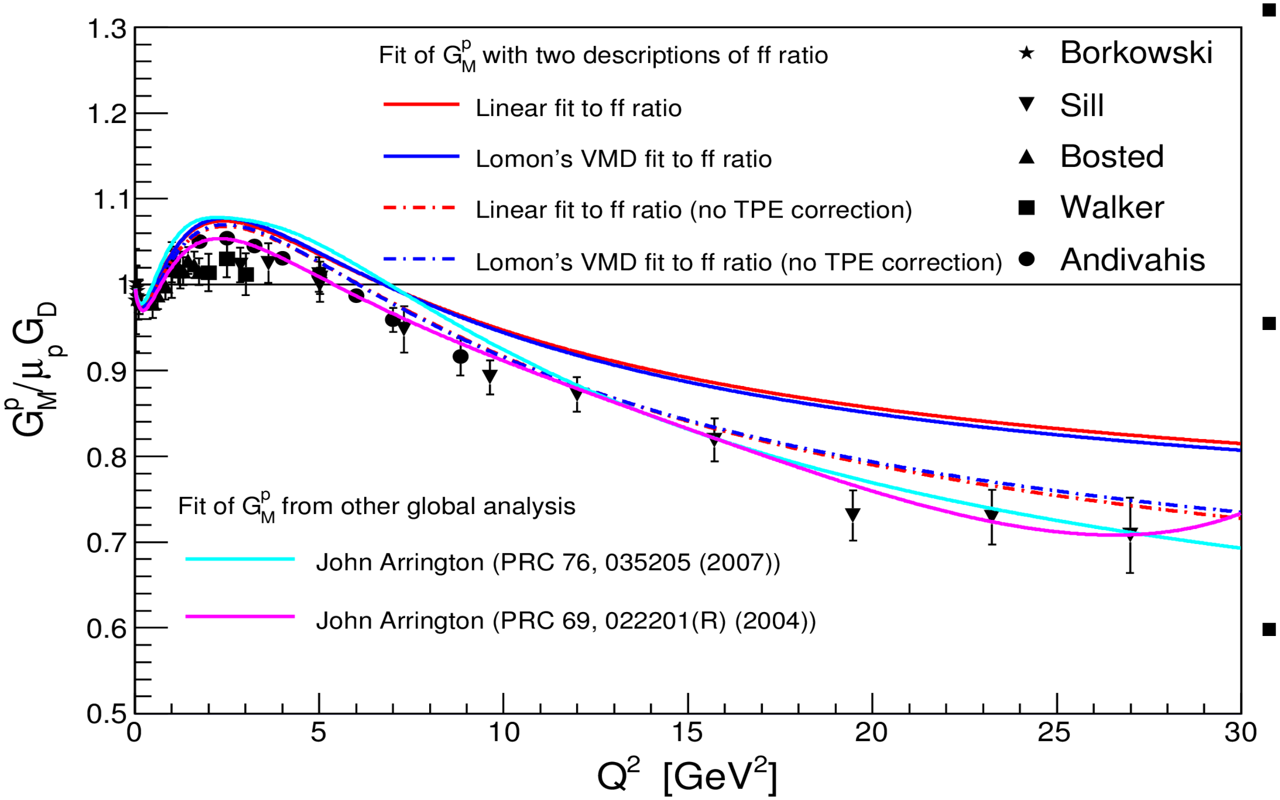
Parameters to be fit:

- Parameters describing the form factor G_M
- Parameters characterizing the TPE corrections
- Normalization factor η for each dataset

These parameters are determined by minimizing the following χ^2:

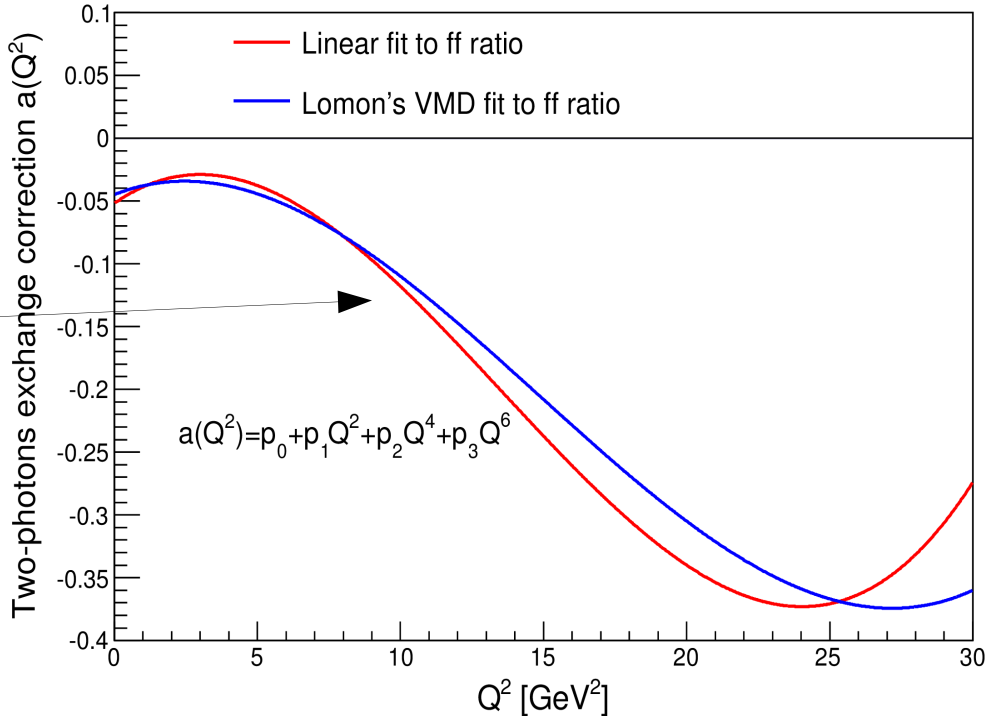
$$\chi^2 = \sum_{j=1}^{N_{expt}} \sum_{i=1}^{N_{data\ point}} \left[\frac{(\sigma_i - \sigma_{fit}/\eta_j)^2}{(d\sigma_i)^2} + \frac{(\eta_j - 1)^2}{(d\eta_j)^2} \right]$$

Normalization constant for each experiment



- Total χ^2 with TPE correction: 487 (linear fit to ff ratio) and 491 (Lomon's VMD fit to ff ratio) for 435 degrees of freedom
- Total χ^2 without TPE correction: 562 (linear fit to ff ratio) and 566 (Lomon's VMD fit to ff ratio) for 439 degrees of freedom
- The largest deviation of fitted normalization factors to 1 is less than 1.5σ

$a(Q^2)$ from the global fit using two descriptions of ff ratio



Summary

Summary:

- A phenomenological ansatz is proposed to describe the TPE effect in the calculation of *ep* elastic cross section
- The proton magnetic form factor G_M is extracted over a wide Q^2 range and found to change by about 15% at high Q^2 when comparing with the Rosenbluth separation results
- The TPE amplitude $a(Q^2)$ is fitted to a polynomial for Q^2 up to 30 GeV^2 (analysis of error band will be performed soon)