Systematics of Q^2 for HAPPEX

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In this report we discuss the measurements of Q^2 for the HAPPEX runs in 1998 and 1999 (Table 1). We assumed a 2 MeV average energy loss to the center of the target. We have weighted the Q^2 by ADC pulse heights according to $Q^2 = (\Sigma Q_i^2 A_i)/(\Sigma A_i)$ where A_i are ADC amplitudes in bin i and Q_i^2 is the corresponding measurement. This weighting shifted Q^2 by -0.38 ± 0.05 %. Some typical Q^2 distributions are shown in figure 1. In this report we discuss the systematic errors due to VDC calibration and efficiency, spectrometer surveys, angle reconstruction, momentum reconstruction, beam energy, beam position and size, and trigger bias.

TABLE 1. **Q^2 for 1998 and 1999 HAPPEX Runs**

	1998 Run	1999 Run (part I)	1999 Run (part II)
Incident Energy $E - (\frac{dE}{dx} \times \bar{x})$ (GeV)	3.345	3.353	3.316
E-arm Angle (degrees)	12.528	12.527	12.527
H-arm Angle (degrees)	12.558	12.562	12.562
E-arm Q^2 $(\text{GeV})^2$	0.473 ± 0.006	0.477 ± 0.006	0.466 ± 0.006
H-arm Q^2 $(\text{GeV})^2$	0.475 ± 0.006	0.477 ± 0.006	0.466 ± 0.006

I INGREDIENTS IN Q^2

The four-momentum transfer squared is $Q^2 = 2EE'(1 - \cos(\theta))$ where E is the incident energy, E' is the final momentum of the relativistic electron and θ is the scattering angle. For the elastic peak one may eliminate one of the three variables, which provides a consistency check.

One ingredient we need is the beam energy. The two energy measuring apparatus, ARC and e-P, are supposedly accurate to 1 MeV. We have assumed that beam energy was correctly measured in the 1999 run, and then found that an -8 MeV or -0.2% adjustment was needed for the 1998 run to be consistent.

A second ingredient is the scattered momentum. It has been fairly widely assumed within HAPPEX that the momentum scale of the spectrometer would contribute negligible error. But we have found that it was necessary to adjust the momentum scale by a few tenths of a percent in order to satisfy the missing mass constraint for elastic scattering. The magnet constants we obtained agree within 0.1% with the values recently obtained by Nilanga Liyanage.

Finally, one needs to know the scattering angle. There are two ingredients here: 1) Surveys are done to measure the angle of the optic axis relative to the incident beam direction; and 2) The spectrometer reconstruction code, ESPACE, reconstructs the horizontal and vertical angles at the target, where these angles are relative to the optic axis. In order to calibrate the ESPACE angle reconstruction, a sieve slit run is done, as explained below, and this requires additional survey information.

II SYSTEMATIC ERRORS

A Time Calibration and VDC Efficiency

The effect of drift time calibration was studied, see figure 2. The $T0$ offset was varied by several nanoseconds to see what effect this had on the χ^2 of the track fit, the efficiency of tracking, and shifts in the Q^2. As seen in the figure, it is easy to adjust $T0$ within a few nsec, and the systematic error is negligible. We did not study the effects of varying the drift velocity or non-linear corrections near the sense wires. A lack of sensitivity to calibration is plausible because the track angle depends mainly on the average wire position in the top and bottom VDC chambers.

The efficiency for ESPACE to make a trigger into a well-fit track in our Q^2 range is 98.5%. The rest of the events are mostly mis-reconstructions due to a variety of small problems such as chamber problems (one sees a zigzag of hits that cannot make a line) and confusion from noise and multiple tracks. The background level from showering in the spectrometer is about 0.2% – there is a separate report on that. These problems do not bias Q^2.

TABLE 2. Survey Data from 1999 HAPPEX Run

Date	E-arm Angle	E-arm mispointing	H-arm Angle	H-arm mispointing
	degrees	mm upstream	degrees	mm downstream
April 5–7	12.527	1.11	12.561	2.83
April 28	12.527	0.95	12.564	3.28
June 1–2	12.526	1.26	12.563	3.23
July 20–21	12.527	0.92	12.560	2.64

B Spectrometer Surveys

Shown in table 2 are the surveys from 1999 (only). This displays the stability and reproducibility. The systematic error comes from three sources: 1) About 0.5 mrad from the apparent variation in the mispointing which impacts on the angle calibration (sieve slit run). 2) Less than 0.3 mrad from the apparent variation in the surveyed central angle. 3) There have been observed ≈ 0.5 mm variations in the position of the sieve slit hole relative to the optic axis over a 3 year period (see the information at www.jlab.org/Hall-A/news/minutes/collimator-distance.html). Whether all these observed variations are due to movement or to lack of reproducibility of the survey is unknown.

Adding the three contributions listed above in quadrature, the total spectrometer survey error is 0.8 mrad. This will have to be added to the other quantities that enter into the angle (see next subsection).

C Angle Reconstruction

The angle reconstruction is checked with a sieve slit run. The least ambiguous check is to use a thin solid target, like our carbon target, together with the sieve slit whose holes define the angles. Figure 3 shows the geometry of the setup. We assume that when the EPICS variables for the X and Y location of the beam say that X = Y = 0, that this is accurate to 0.5 mm. Further, the beam position varies on this scale. (Note: one should look at these variables during HAPPEX analysis and take care if the position is more than 1 mm off.) The corresponding angle error is 0.5 mrad. For the sieve calibration we also need to know the carbon target Z position. We assume it is at Z = 0 to within 2 mm.

The angle error from spectrometer surveys mentioned in the previous section, 0.8 mrad, has to be added in quadrature to the angle errors arising from beam position error (0.5 mrad) and target position error (0.2 mrad). Thus the total angle error is ≈ 1 mrad which corresponds to a 1% error in Q^2 because

$dQ^2/Q^2 \approx 2\,d\theta/\theta$ and $\theta \approx 219$ mrad. Note that the angle error is the main source of error for Q^2.

Figure 4 shows a sieve slit run from 1999. The scribe marks show the expected location of the central holes based on the survey data. The reconstructed angles from ESPACE were adjusted by ≈ 1 mrad for both E-arm and H-arm in 1999. This adjustment depended slightly (± 0.3 mrad) depending on which database was tried. For 1998, no adjustment was needed for E-arm, and a ≈ 1 mrad adjustment was needed for H-arm. The plot shows the data *after* adjustment. No adjustment was made for vertical angle, but it has a very small effect. Several sieve slit runs were performed during the 1999 run. We have checked 4 of them, and they were consistent within 0.1 mrad. This gives some indication of the good stability of the apparatus. Other indications of stability are from the Z target distribution, both for the LH2 target and for the empty (dummy) target, and from the stability of Q^2 shape and average, which is shown later.

D Using Missing Mass to Adjust Momentum

As we will show in the next section, the missing mass is a very sensitive constraint on the kinematic variables. The missing mass squared $dm^2 = 2m_p(E - E') - Q^2$ should be zero. Since 1998 we were given at least three databases which we were told had good kinematic reconstruction; they were called “db vcs new10”, “db opt tc4”, and “db cebaf 3.0”. According to Nilanga the latter is the best, but since all three databases are the result of a lot of effort it seemed reasonable to compare them and get a feel for the instability. All three databases do about the same quality of angle reconstruction, requiring similar corrections of about 1 mrad as explained in previous section. Depending on which database is tried, the missing mass squared was typically off by $\pm$ 0.02 GeV^2. To understand the significance of this shift, we mention three possible reasons for a shift of 0.02 GeV^2: 1) The beam energy could be wrong by 10 MeV; 2) The momentum could be wrong by 10 MeV; or 3) The Q^2 could be wrong by 4%. This was alarming. Note that an angle systematic of ≈ 1 mrad cannot explain a 4% shift in Q^2.

One can see from figure 5 that nearly all the shifts in missing mass were due to the momentum scale which were different between the databases. We will assume that for the 1999 run, the beam energy was measured with sufficient accuracy and we will adjust the momentum scale using the database “db cebaf 3.0”. For this database we needed to decrease the E-arm momentum by 0.6% and increase the H-arm momentum by 0.2%. Figure 6 shows the shifts in missing mass squared as a percentage of Q^2 before and after the momentum scaling. This did a good job for the 1999 run in the six measurements used. However, for the two 1998 measurements, the missing mass was still not zero

TABLE 3. Sensitivity $\Delta D_i/Q_0^2$ expressed as a percentage

difference (index i)	sensitivity to E ($\Delta E = 10$ MeV)	sensitivity to E' ($\Delta E' = 10$ MeV)	sensitivity to θ ($\Delta\theta = 1$ mrad)
1	+0.28 %	-0.33 %	-0.24 %
2	-0.30 %	+0.36 %	+0.17 %
3	+3.6 %	-4.2 %	+0.94 %

after momentum adjustments. Since the same database was used, the same momentum scale must be used. We have already adjusted the 1998 data for angle. The only quantity left is beam energy. We therefore assume that the 1998 beam energy was 8 MeV lower than was reportedly measured. The results are shown in the lower plot in figure 6. The scale of the residuals dm^2/Q^2 is 0.5% FWHM. In figure 7 are some representative missing mass plots from the 1999 and 1998 run after the corrections. The peaks are fairly well centered at zero.

To check the mathematical consistency of our assumptions we have applied this same procedure to all three databases. Just to remind you, the assumptions are: 1) The 1999 energy is correct; 2) The sieve slit run is used to correct the angle reconstruction (this is done separately for each database); and 3) We have to scale the momentum to make the missing mass zero. If this is the correct procedure, then each of the three databases should give the same result for Q^2. The comparison is done on the same sample of data using the same header file. The result, averaging over three databases and two spectrometers, is that there was a residual discrepancy in Q^2 of ±0.4% which we presume is due to instability in the reconstruction matrix elements. We will assign a 0.4% error due to spectrometer matrix elements in the summary table 6.

III COMPARISON OF 4 METHODS

For elastic scattering we only need 2 variables to determine Q^2, for example energy and angle. Thus there are four ways of measuring Q^2:

$$(0) \qquad Q_0^2 = 2EE'(1-\cos(\theta)) \qquad \text{(uses all variables)}$$

$$(1) \qquad Q_1^2 = 2E^2 f_r (1-\cos(\theta)) \qquad \text{(independent of } E'\text{)}$$

$$(2) \qquad Q_2^2 = 2E'^2 f'_r (1-\cos(\theta)) \qquad \text{(independent of E)}$$

$$(3) \qquad Q_3^2 = 2m_p(E - E') \qquad \text{(independent of } \theta)$$

Here f_r and f_r' are recoil factors defined as $f_r = 1/(1 + (E/m)(1 - \cos(\theta)))$ and $f_r' = 1/(1 - (E'/m)(1 - \cos(\theta)))$.

To compare these methods it is only meaningful to consider the *differences* between them, because the differences will show an elastic peak plus a radiative tail. The shift from zero in the peaks in these differences have certain sensitivities to the systematics in E, E', and θ. Let us define the differences as $D_i = Q_i^2 - Q_0^2$. The sensitivities of D_i with respect to E, E', and θ are shown in table 3 where we have computed $\Delta D_i = (\partial D_i/\partial E)(\Delta E)$ and similarly for E' and θ, and we have expressed these quantities as a percentage of Q^2 and assumed $\Delta E = \Delta E' = 10$ MeV and $\Delta\theta = 1$ mrad to set the scale. It is clear that the differences which involve the factor $1 - cos(\theta)$ are about 10 times less sensitive to the kinematics as D_3. Note that D_3 is simply the missing mass squared. Therefore the missing mass is the most sensitive constraint at forward angle.

IV BEAM SIZE AND POSITION

During the run we varied the beam size (raster spot size) and beam position to measure the effect on Q^2. When the spot size was decreased from 5 mm square to 2 mm square, the average Q^2 changed by -0.05 ± 0.18 %. The changes observed with position are shown in table 4 in an X-Y grid of measured and expected results. The expected deviation was estimated roughly to be 1.5% per 2 mm if we use the central angle and the formula $dQ^2/Q^2 \approx 2\, d\theta/\theta$. The changes we saw were smaller than this formula by a factor of 2 or 3. This is probably because the angles we sample must be weighted by the cross section, and the naive formula is wrong. The E-arm and H-arm spectrometers had oppositely signed changes, and the sign of the changes correlated well to the sign of the X direction.

Tests were performed after the experiment, when HAPPEX detector was removed, to see if the rastered spot size, shape, frequency, or modulation method (breathing versus TV pattern) affected Q^2. No effects were seen. The Q^2 distribution with the standard trigger setup looked like it did during HAPPEX.

V TRIGGER BIAS

During the 1999 run we sometimes ran the HAPPEX trigger with scintillator planes S1 and S2 HV turned on. This was a mistake because signals from S1 caused a non-uniform inefficiency which distorts the momentum distribution. Other types of triggers are clean, however, see table 5.

TABLE 4. Percent (%) Deviation in Q^2 with Position

	shown is:	E-arm (H-arm)	errors statistical
	X = −2.6 mm	X = 0	X = +2 mm
Y = +2.5 mm			
measured		$+0.02 \pm 0.36$ ($+0.02 \pm 0.38$)	
estimated		≤ 0.1 %	
Y = 0			
measured	$+0.72 \pm 0.25$ (-0.47 ± 0.25)	0 (by definition)	-0.77 ± 0.23 ($+0.64 \pm 0.24$)
estimated	+2.0 % (−2.0%)	0 (by definition)	−1.5 % (+1.5%)
Y = −0.3 mm			
measured		-0.21 ± 0.26 ($+0.13 \pm 0.26$)	
estimated		≤ 0.1 %	

TABLE 5. Triggers Used to Measure Q^2

Symbol	Description	Bias
H-S(off)	HAPPEX Trigger, Scint S1 and S2 off (cleanest measurement of Q^2)	none
Standard S-Ray Trigger	HAPPEX Detector Removed (e.g. used for raster tests)	none
S2	Require S2. Leave S1 either on or off (used for alignments)	none
H-S1(off)-S2(on)	HAPPEX Trigger, S1 off, S2 on (used for 1998 Q^2 measure)	small
H-S1(on)-S2(on)	HAPPEX Trigger, S1 and S2 on (used during 1999 part I)	severe

The bias in HAPPEX triggers from leaving scintillators turned on are due to incorrect timing at the MLU. The MLU (memory lookup unit) is a logic device that decides whether to make a trigger based on a programmed response to logic levels at its inputs. The HAPPEX trigger requires the signal from the HAPPEX detector. All signals, including scintillators, send strobes to the MLU. At the instant the MLU receives a strobe, it looks at its inputs and makes its decision. The HAPPEX detector is 65 nsec slower than the scintillators. Even though the strobes from scintillators are delayed several nsec, they are too early for the HAPPEX trigger. Therefore, any signal on the scintillator will kill this trigger.

The standard S-Ray trigger and the S2 trigger have no such inefficiency because their strobes are properly timed. (See table 5 for definitions of trigger types.) Strobes from S1 are delayed about 25 nsec from S2 strobe and do not harm the S2 trigger. For the H-S1(off)-S2(on) trigger used in 1998, one could imagine a small distortion if the probability for a track to cause a hit in S2 was dependent on the momentum. This is evidentally a very small effect because the momentum distribution, as well as Q^2 distribution, in 1998 look identical in the H-S1(off)-S2(on) and S2 triggers. By the same token, the H-S(off) and the S2 triggers look identical. If there is differential absorption along the apparatus, its effect would be opposite for the H-S1(off)-S2(on) and S2 triggers: Suppose that if one goes from momentum p1 to p2 the probability to reach S2 increased; then one would have a higher efficiency in the S2 trigger and a correspondingly lower efficiency in the H-S1(off)-S2(on) trigger. Preliminary Geant simulations done by Eugene Chudakov have so far confirmed that the probability to reach S2 is reasonably flat.

The H-S1(on)-S2(on) does have a severe bias in momentum. This is partly because S1 had paddles 5 and 6 removed, and is partly a geometrical effect that paddle 1 is much farther away than 4. As a consequence, a large elastic peak appears on paddle 4 and produces a hole in the momentum at this point. We can reproduce the observed distortions in the H-S1(on)-S2(on) momentum distribution with the following analysis (see fig 8). First we use either the S2 trigger or the H-S(off) trigger to measure the true momentum distribution (fig 8a). Using the S2 trigger, we observe the momentum distribution for events with S1 missing. This would be the momentum distribution if we had used the HAPPEX trigger and left only S1 on (and not S2); the logic is that if S1 is missing it does not cause an inefficiency in the HAPPEX trigger. Thus we know the shape of momentum imposed by S1 strobes. To obtain the observed spectra for H-S1(on)-S2(on) we have to add the real and the S1-imposed momentum spectra with a weighting that takes into account the probability that a hit occurs in S1 but not S2. This probabilty can be deduced from two pieces of information: Using the S2 trigger we find the probabilty that there is also a hit in S1 is 86%. This measures a correlated component of the rate. Using the scalers which measure raw strobe rates we find that the S1 rate is 1.9 times higher than S2. Using these factors, we get a predicted momentum spectra in fig 8b which agrees fairly well with the observed spectra in 8c. The showers after HAPPEX detector have a lot of soft photons, as evidenced by the low amplitude in the scintillators and the large uncorrelated component of the rate.

The H-S1(on)-S2(on) triggers were not used to compute the average Q^2. Instead we rely on the S2 triggers for most of the first half of 1999. In the second half of 1999 both the S2 trigger and the H-S(off) can be used to measure Q^2.

VI STABILITY IN TIME

Figure 9 shows the meaurements of Q^2 over time in our 1999 run. We also show the time evolution of the missing mass expressed as a percentage of Q^2. Only the statistical errors are shown. We have not displayed the Q^2 measurements that were done with the incorrect H-S1(on)-S2(on) trigger, but we have shown the missing mass for all runs because the trigger bias problem does not affect the missing mass peak. The deviations from zero missing mass are indicative of residual systematic errors and drifts which affect Q^2 by of order 0.5%.

VII SUMMARY OF ERRORS

In table 6 we summarize the errors which add in quadrature to 1.2% or ± 0.006 GeV^2 for each spectrometer. The largest error is from the scattering angle. The beam energy error might seem too large since we have heard it quoted as 1 MeV. However, we had to adjust the 1998 energy by 8 MeV, and since there has been some recent doubt cast on the 1 MeV accuracy, based on comparisons throughout the history of e-P and arc measurements, it seems like a good idea to leave this error 10 MeV for now. The matrix element error, estimated by comparing different databases available, could possibly be eliminated by optimizing our own database, but it would be a big effort.

TABLE 6. Summary of Errors in Q^2

Error Source	Error (in source units)	Percent Error in Q^2
Timing Calibration	$\leq$ 5 nsec	$\leq$ 0.1%
Affecting the Angle :		
Beam Position	0.5 mm	0.5%
Survey of Spectr. Angle	0.3 mrad	0.3%
Survey of Mispointing	0.5 mm	0.5%
Survey of Collimator	0.5 mm	0.5%
Target Z position	2 mm	0.3%
Momentum Scale	3 MeV	0.1 %
Beam Energy	10 MeV	0.3 %
Matrix Elements		0.4 %
Drifts in Time		0.5 %
Total Systematic Error		1.2 %
Statistical Error		$\leq$0.1%
TOTAL ERROR		1.2%

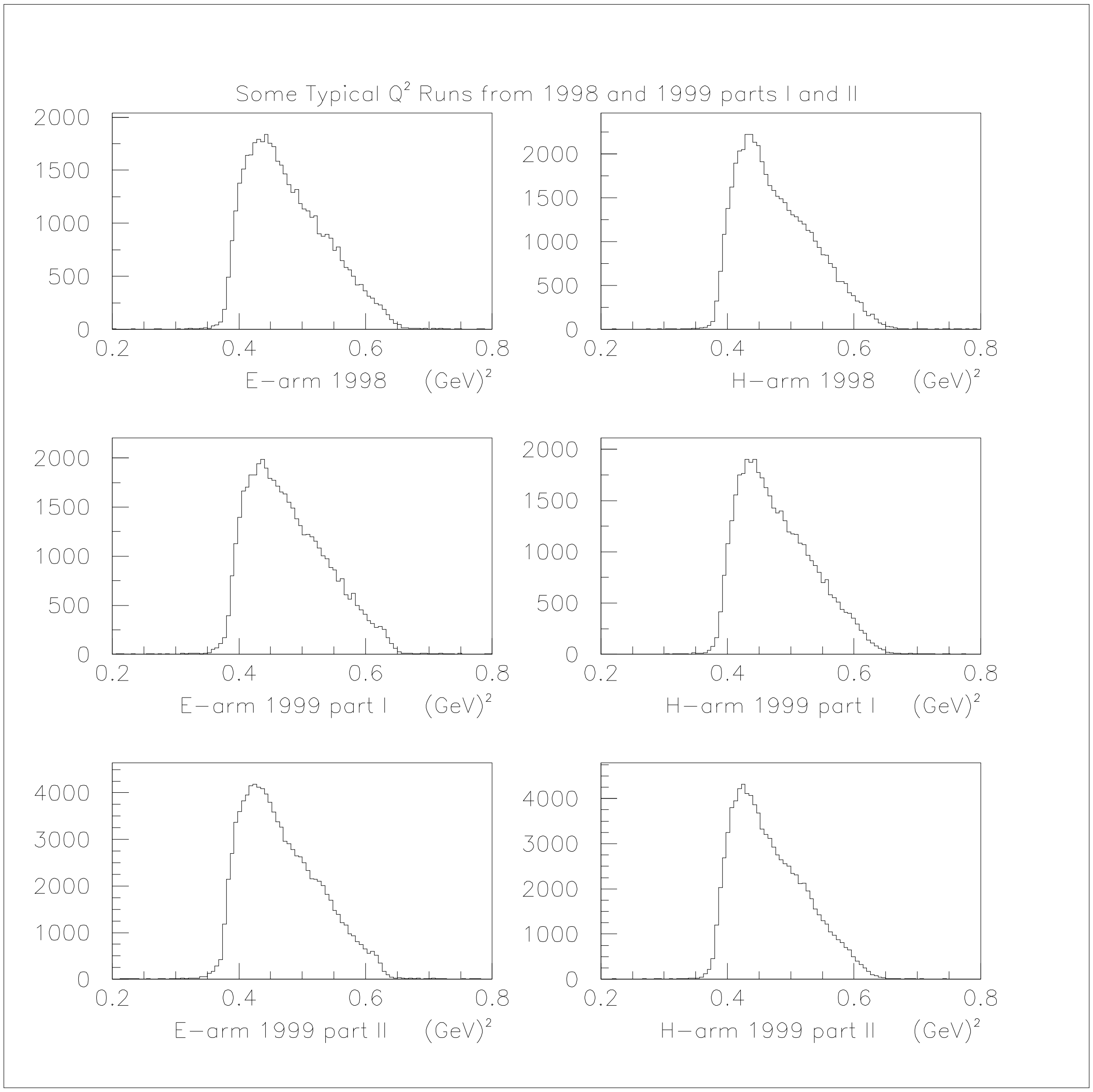

FIGURE 1. Some typical Q^2 measurements from 1998 (top), 1999 part I (middle) and 1999 part II (lower). Note: These distributions are not ADC weighted. For the 1999 part I we used a scintillator trigger run with the requirement that the HAPPEX detector had a hit. For the other plots the HAPPEX trigger was used.

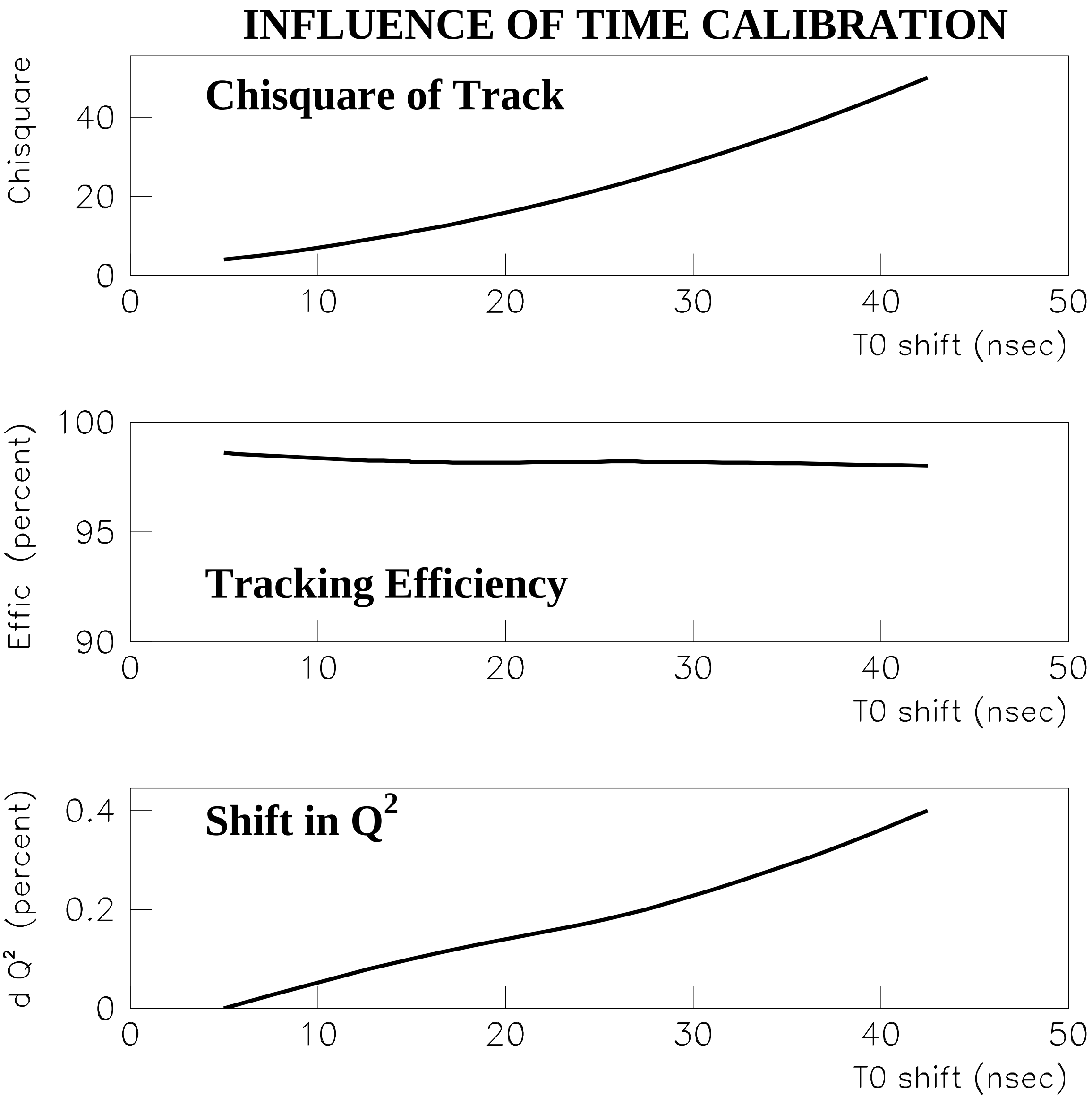

FIGURE 2. The influence of time calibration on the error in Q^2 is negligible.

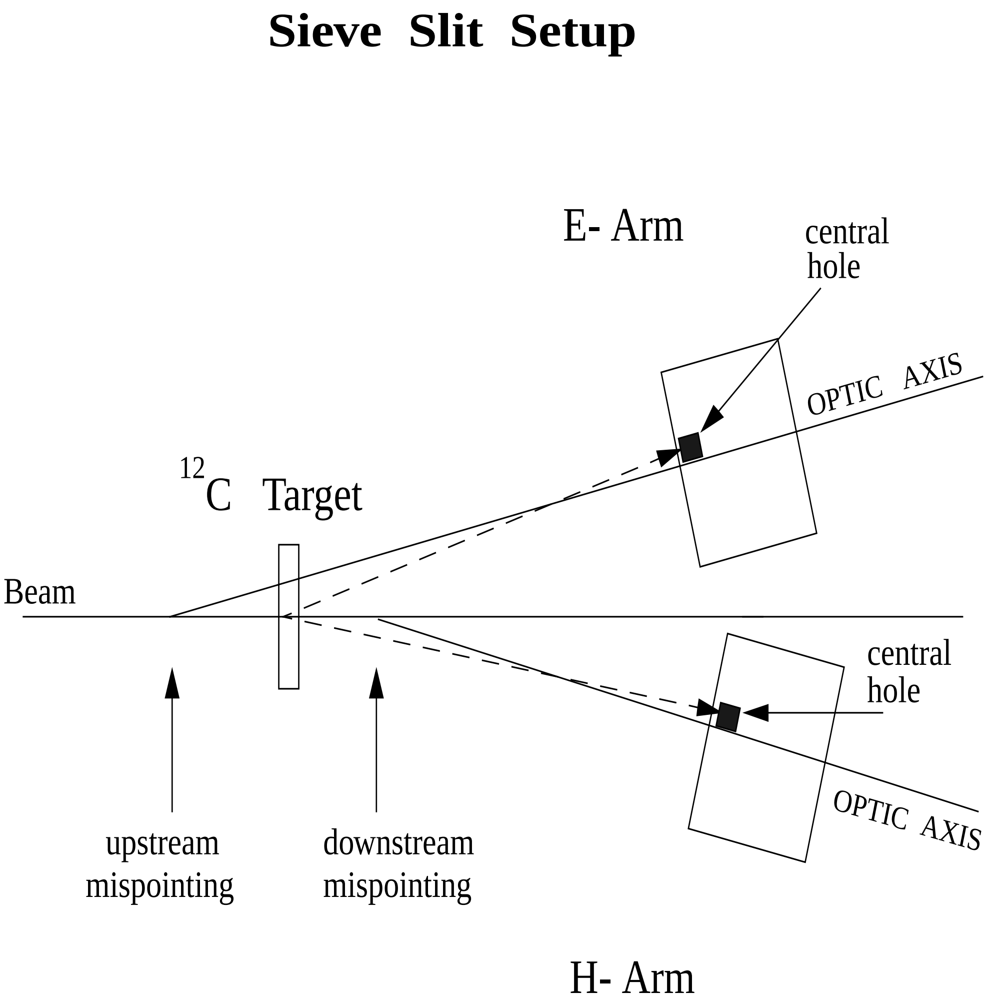

FIGURE 3. Sieve Slit Setup for calibrating the angle reconstruction. Surveys give the angle of the optic axis relative to the beam direction and the “mispointing error” which is the distance along the beam axis from the center of the target to where the optic axis intersects the beam (see table 2). Surveys also give the deviation of the central hole of the sieve slit from the optic axis (0.84 and 0.73 mm for E-arm and H-arm respectively). The signs are as drawn.

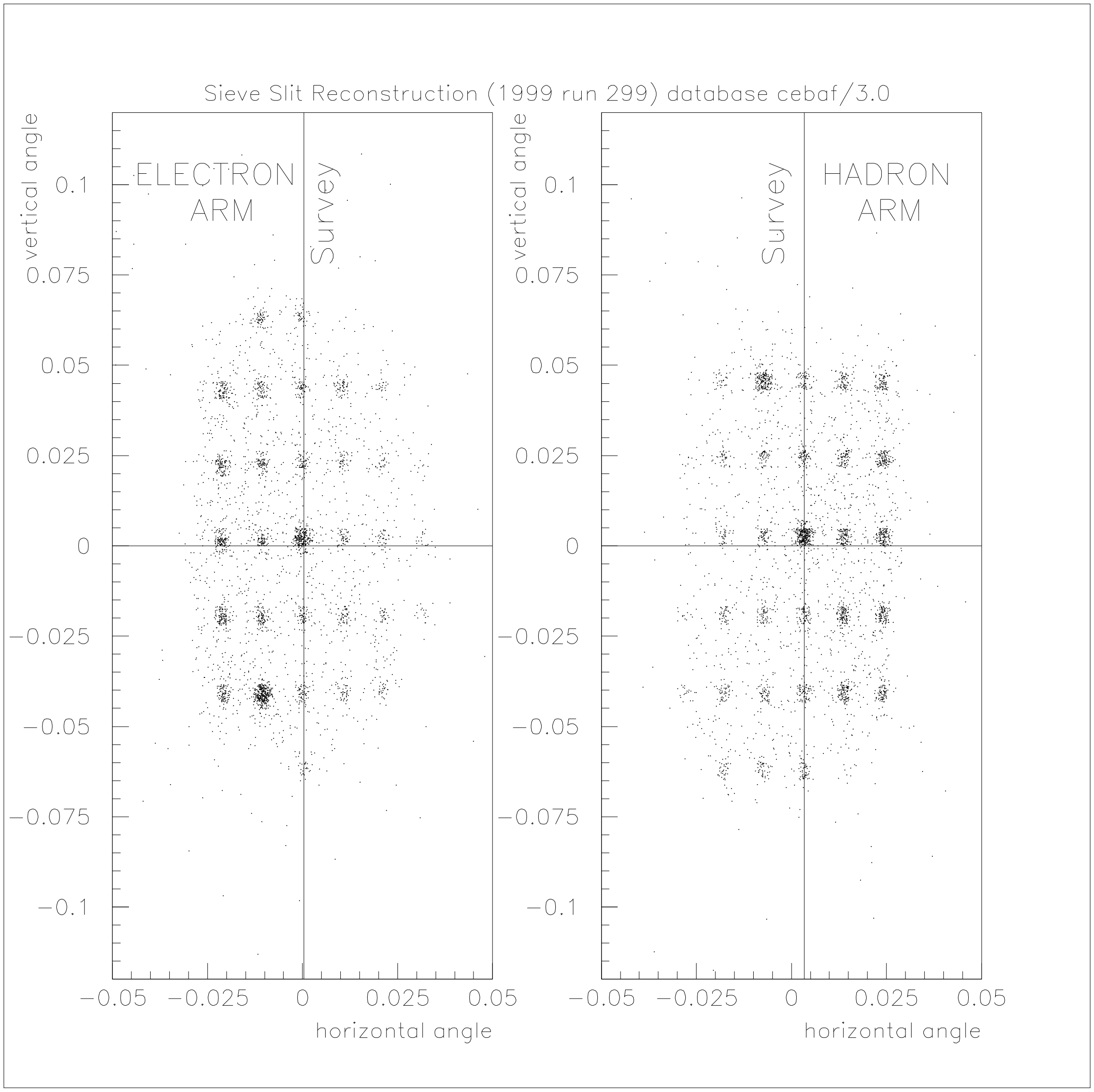

FIGURE 4. Sieve Slit construction for a 1999 run using the "cebaf 3.0" database. The scribe marks indicate the location of the central hole according to the survey. The data was corrected in horizontal angle (only) by about 1 mrad. The result after correction is shown.

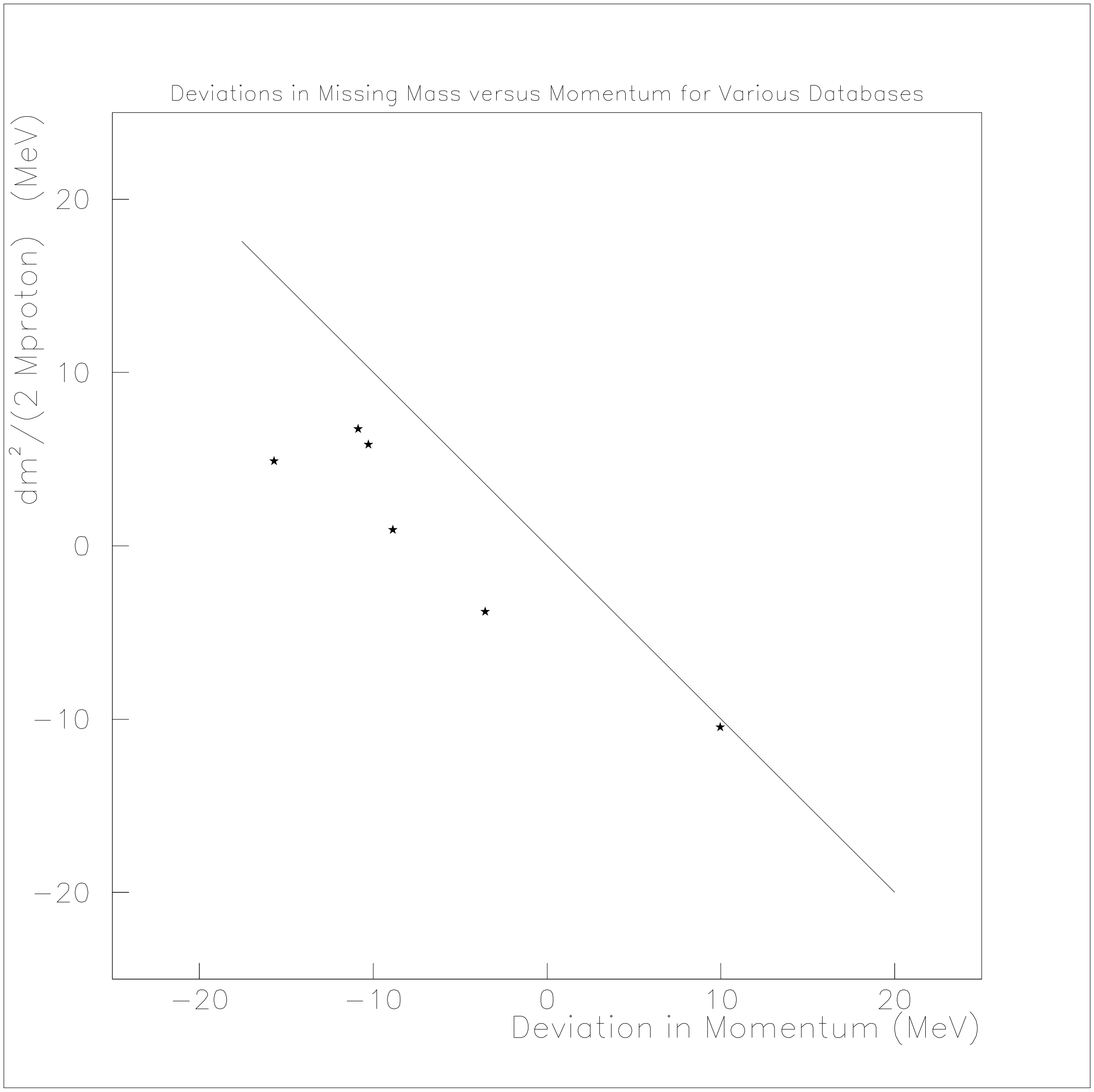

FIGURE 5. Shifts in missing mass square from zero (Y axis) versus the shifts in momentum scale in the databases which we have tried. Three databases and two spectrometers are shown (total of 6 points). Comparing these databases indicates an apparent variation in the momentum scale of the HRS as the main cause for the shifts.

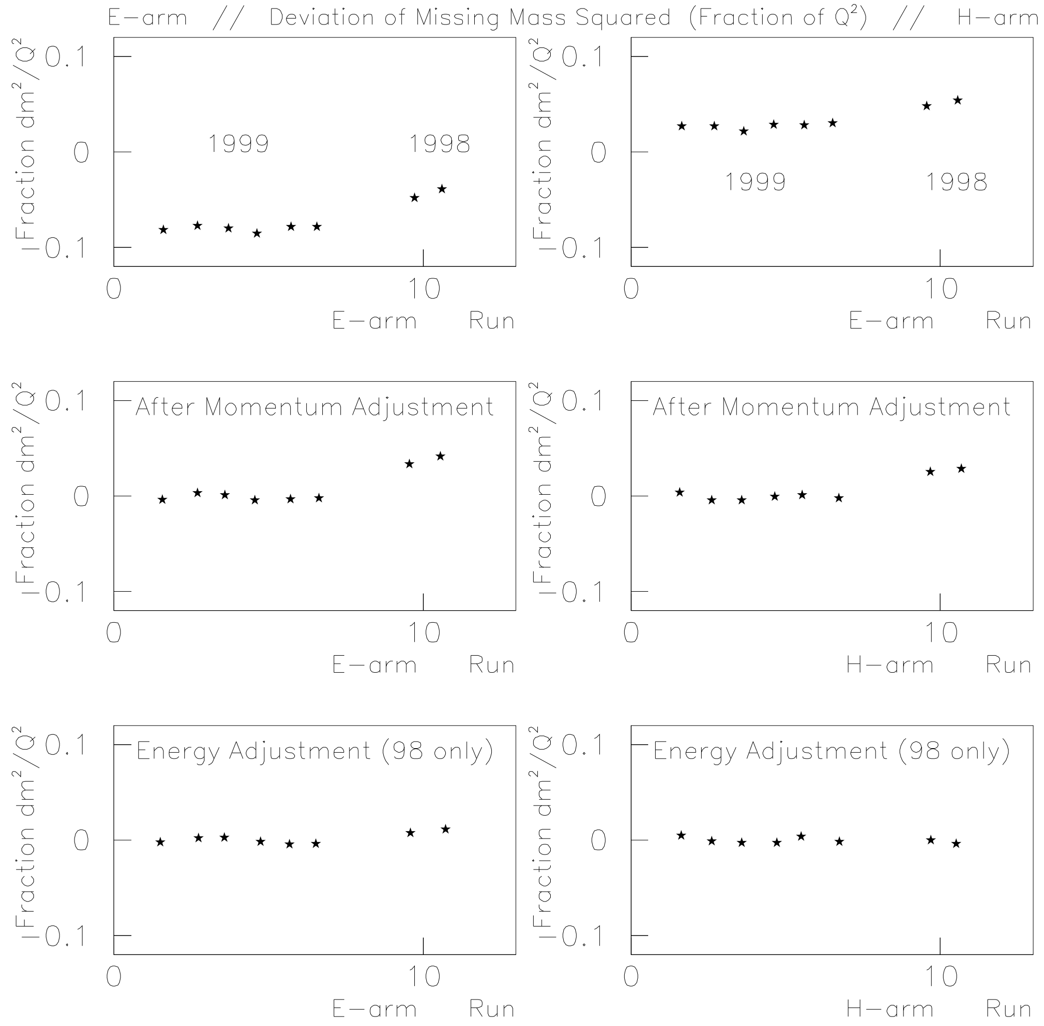

FIGURE 6. Shifts in missing mass squared (as a percentage of Q^2) for various runs. The E-arm is in the left plots; the H-arm on the right. The top two plots are with no correction to the momentum scale (but the angles were corrected). In each plot the first 3 points are runs from 1999 part I, the next 3 points are runs from 1999 part II, and the last 3 points are for 1998. The middle group of 2 plots are from after corrections to the momentum scale (of order 0.5%). Here the 1998 missing mass is still nonzero. The bottom group of two plots is after adjusting 1998 beam energy by −8 MeV (−0.2%).

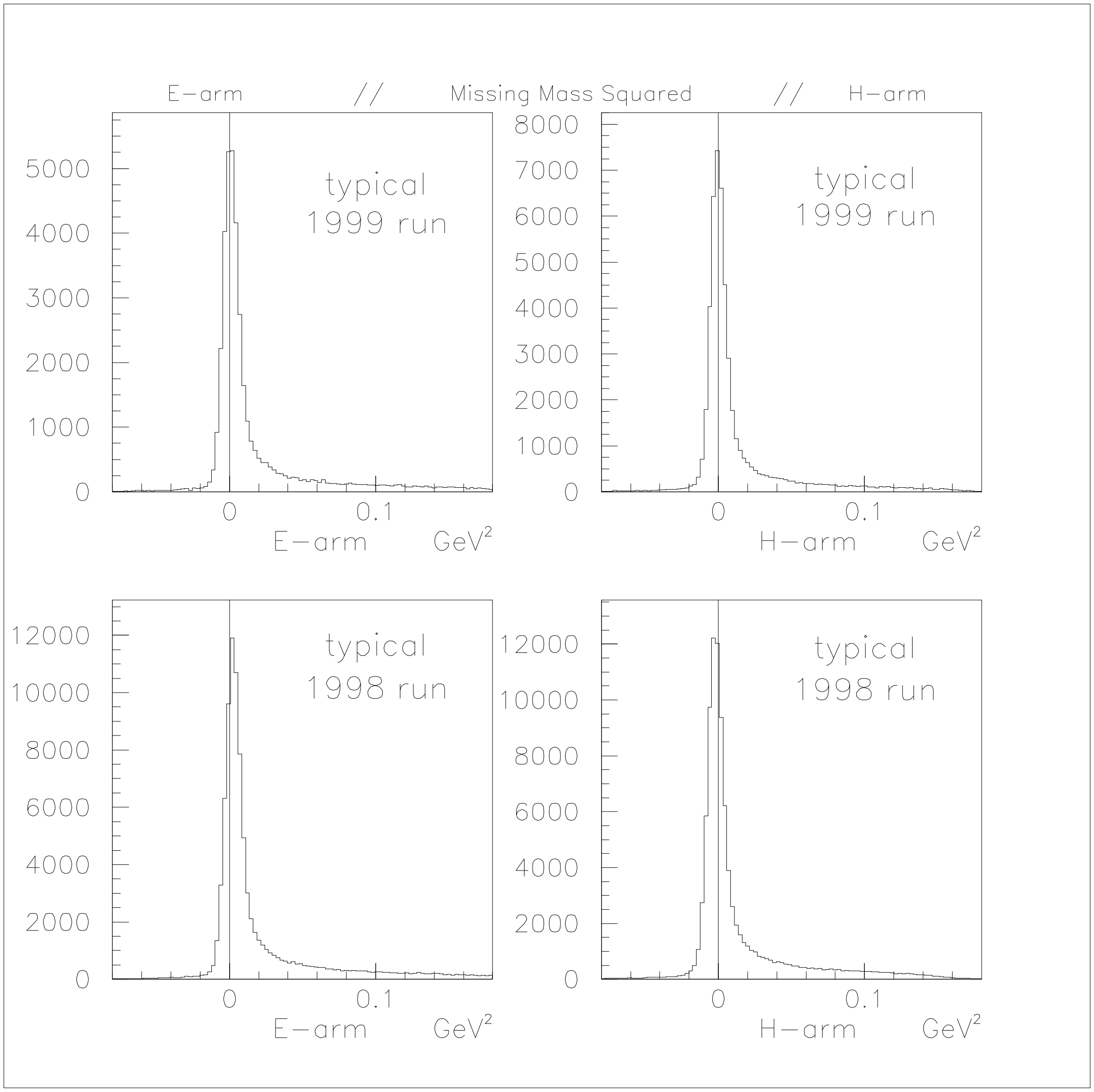

FIGURE 7. Missing Mass Peaks for a typical run from 1998 and 1999 for E-arm and H-arm after adjustments of a few tenths of a percent to the momentum scale (see text). For 1998, a −8 MeV shift (−0.2%) in the beam energy was also necessary. For 1999, we assumed the beam energy was measured perfectly.

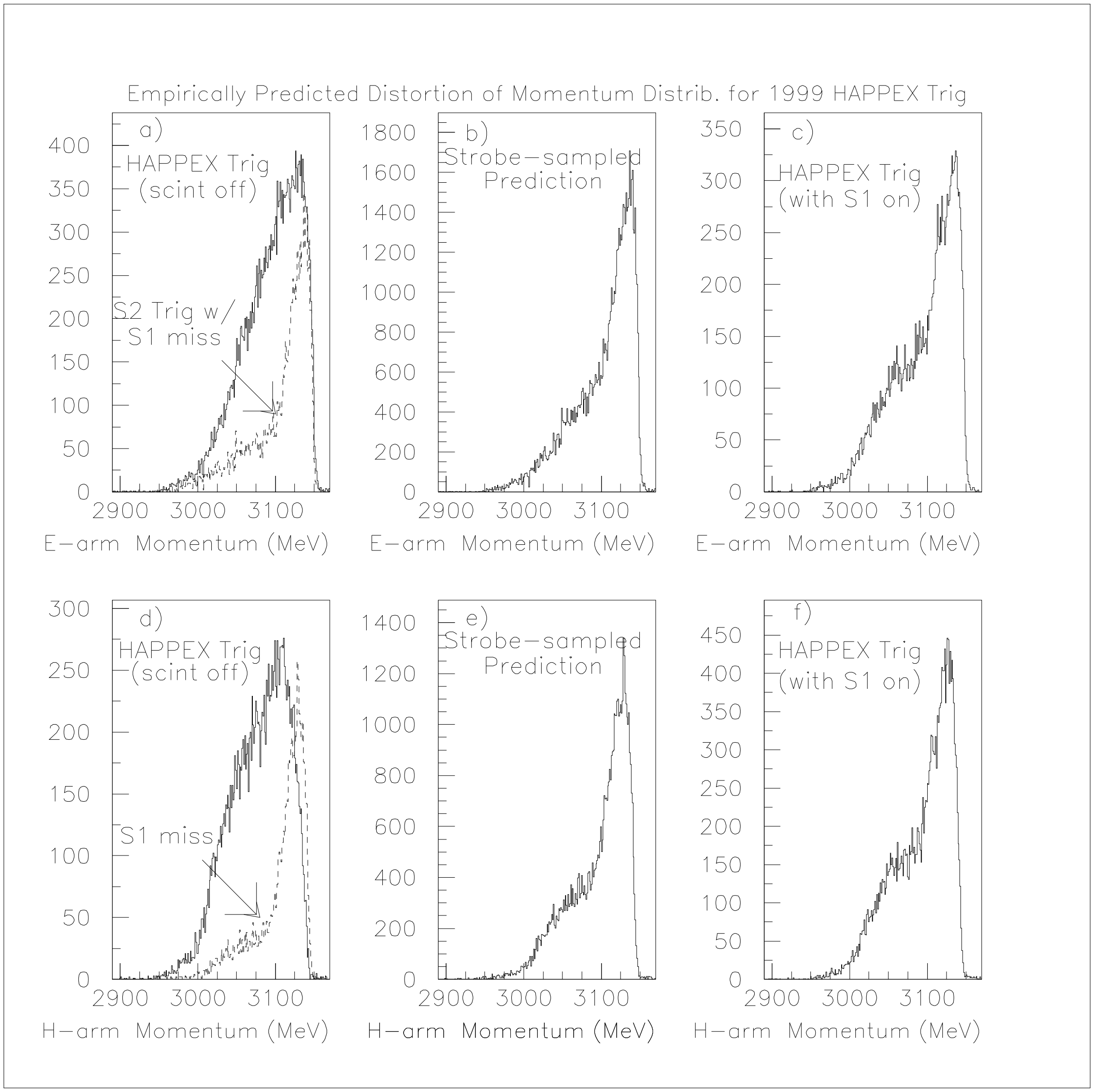

FIGURE 8. Using scintillator data to predict the distortions observed in the HAPPEX trigger when S1 is left on. Fig a) shows the momentum for HAPPEX trigger with scintillators off (our cleanest trigger). The hatched line shows the momentum for S2 trigger when S1 is missing. This would be the momentum distribution if we used the HAPPEX trigger and left S1, but not S2, turned on. The inefficiency due to S1 causes this shape. S2 also causes an inefficiency but not a distortion. Adding these two distributions together with weighting by the independent contributions of S1 yields the prediction b) to be compared to c) for HAPPEX triggers with both scintillators on. Plots d)-f) are the same for H-arm.

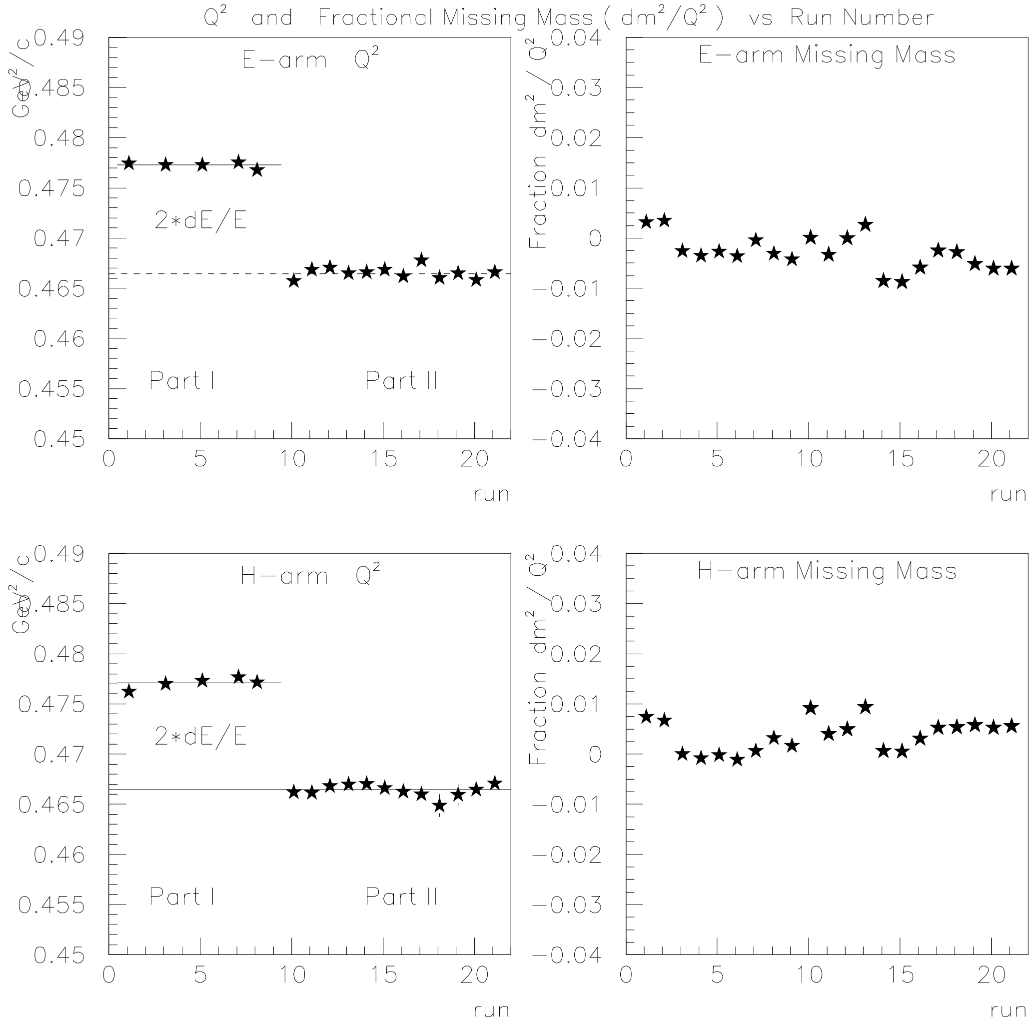

FIGURE 9. 1999 HAPPEX Q^2 measurements over time and the missing mass expressed as a fraction of Q^2. For the measurements in part I we discarded the HAPPEX trigger runs and only used the S2 trigger. For part II we could use both since we always turned scintillators off when we took HAPPEX triggers. The missing mass is plotted for all runs since it is not harmed by the trigger bias. Only statistical errors shown (typ 0.1%, the size of the point). The Q^2 changed with energy like 2 dE/E as expected.