JLAB-TN-00-024

Calibration of Beam Energy and Spectrometer Central Angles using Hall A HRS in ^{1}H(e,e′p)

Paul E. Ulmer and Hassan Ibrahim

Department of Physics
Old Dominion University
Norfolk, VA 23529-0116

Nilanga Liyanage

Jefferson Laboratory
Newport News, VA 23606

October 12, 2000

Abstract: During the period April 1–2, 2000, a set of five elastic ^{1}H(e,e′p) measurements spanning a large angular range of both Hall A spectrometers was performed. All five data sets were acquired at the same nominal beam energy of 3085.8 ± 0.6 MeV as determined by measurement using the Hall A arc (this is the value after applying the recently discovered $+10^{-3}$ correction for proper inclusion of the dipole fringe fields). The goal of these measurements was to determine absolute angular offsets for each spectrometer as well as the beam energy. The large angular ranges chosen gave rise to substantial variation of the energy-angle derivatives, maximizing sensitivity to electron angle at one extreme and to proton angle at the other extreme.

Contents

1 The Method

For elastic scattering the beam energy can be determined from knowledge of the scattering angle and the angle of the recoiling nucleus relative to the beam. For a hydrogen target we have for the beam energy:

$$e = m_p(-\cot\frac{\theta_e}{2}\cot\theta_p - 1) + E_s + E_r \tag{1}$$

where θ_e is the electron scattering angle, θ_p is the angle of the recoiling proton with respect to the beam and m_p is the proton mass. E_s is a correction for energy straggling of the incident electron due to the target and entrance walls and E_r is the energy loss due to internal bremsstrahlung for "pre-radiation".

The energy straggling cannot be corrected perfectly since it is a statistical process. Instead the average energy loss calculated using the material up to the measured vertex is substituted for E_s. In order to minimize the influence of internal bremsstrahlung, a tight cut is placed on the missing energy ($\epsilon_m = \omega - T_p$, where ω is the electron energy transfer and T_p is the proton kinetic energy) which selects only low energy radiated photons. Future refinements would be to use knowledge of the emission angle of the recoiling photon to select only events which "post"-radiated (this assumes peaking approximation) since photons radiated after the elastic scattering vertex are of no relevance, or to approximately correct for the photon energy since it can be determined from the elastic kinematics. In any case, the sensitivity to the missing energy cut was investigated and the results were found to be almost completely insensitive to this cut (from $\delta\epsilon_m$ from 5 MeV to 100 MeV).

The accuracy of the extracted energy depends on the accuracy in the angles and the error derivatives:

$$\frac{\partial e}{\partial\theta_e} = \frac{m_p}{2}(1+\cot^2\frac{\theta_e}{2})\cot\theta_p \tag{2}$$

$$\frac{\partial e}{\partial\theta_p} = m_p\cot\frac{\theta_e}{2}(1+\cot^2\theta_p). \tag{3}$$

The angles must be corrected for mispointing of the spectrometers (see below). Each scattering event within the coincidence acceptance gives an angle pair (θ_e, θ_p) and ideally every such pair should lead to the same value of the energy, e. In practice, effects due to multiple scattering, detector resolution, traceback errors and residual effects of energy straggling and internal bremsstrahlung produce a broad distribution of energies.

Each of the two angles is determined from the in-plane and out-of-plane Transport [1] angles at the target as well as the beam angles:

$$\cos\theta_e = (\cos\theta_0^e - \tan(\phi_{tg}^e - \phi_{tg}^b)\sin\theta_0^e)/\eta_e \tag{4}$$

$$\cos\theta_p = (\cos\theta_0^p - \tan(\phi_{tg}^p - \phi_{tg}^b)\sin\theta_0^p)/\eta_p \tag{5}$$

where θ_0^e and θ_0^p are the central angles of the electron and proton spectrometers respectively, ϕ_{tg}^b, ϕ_{tg}^e and ϕ_{tg}^p are the beam, scattered electron and proton in-plane angles at the target, θ_{tg}^b, θ_{tg}^e and θ_{tg}^p are the beam, scattered electron and proton out-of-plane angles at the target, and

$$\eta_e = \sqrt{1+\tan^2(\theta_{tg}^e - \theta_{tg}^b) + \tan^2(\phi_{tg}^e - \phi_{tg}^b)} \tag{6}$$

$$\eta_p = \sqrt{1 + \tan^2(\theta^p_{tg} - \theta^b_{tg}) + \tan^2(\phi^p_{tg} - \phi^b_{tg})}. \quad (7)$$

Here, a small correction given by the products of squared tangents for the in-plane and out-of-plane angles has been ignored.

2 The Fitting Procedure

The goal of these measurements is to determine the absolute angle offsets of the spectrometers as well as the beam energy. This is possible since the entire kinematic set was performed at a single beam energy. However, for this to work, the angle offsets must be constant for each kinematics. Such a constant offset could be due, for example, to errors in the survey of the sieve slits which are used to determine the optics of each spectrometer. All angles are measured relative to the spectrometer central axis and this axis, in turn, is measured relative to the location of the sieve center hole. Clearly, random errors in the angles will introduce a scatter about the fit values.

The fit is performed by first linearizing the energy-angle formula:

$$e = e_i + \left(\frac{\partial e}{\partial \theta^e_0}\right)_i \delta\theta^e_0 + \left(\frac{\partial e}{\partial \theta^p_0}\right)_i \delta\theta^p_0 \quad (8)$$

where e_i is the energy extracted from a given angle pair (labeled by the index i) satisfying the missing energy cut and corrected for energy loss but before correction of absolute angular offsets. The electron and proton spectrometer angle offsets, $\delta\theta^e_0$ and $\delta\theta^p_0$, are assumed constant (and thus do not depend on the index i). Due to out-of-plane angular acceptance, the derivatives must be expanded (the proton formulas are identical in form to the electron ones, so only the latter are given here; also, the index i has been dropped for simplicity):

$$\frac{\partial e}{\partial \theta^e_0} = \left(\frac{\partial e}{\partial \theta_e}\right)\left(\frac{\partial \theta_e}{\partial \theta^e_0}\right) \quad (9)$$

where $\partial e/\partial\theta_e$ was given above and

$$\frac{\partial \theta_e}{\partial \theta^e_0} = \frac{\sin\theta^e_0 + \tan\phi^e_{tg}\cos\theta^e_0}{\eta_e \sin\theta_e}. \quad (10)$$

A χ^2 minimization with respect to e (the "actual" beam energy), $\delta\theta^e_0$ and $\delta\theta^p_0$ is then performed:

$$\chi^2 = \sum_i \left[e - e_i - \left(\frac{\partial e}{\partial \theta^e_0}\right)_i \delta\theta^e_0 - \left(\frac{\partial e}{\partial \theta^p_0}\right)_i \delta\theta^p_0\right]^2 / \sigma_i^2 \quad (11)$$

where σ_i is the uncertainty in the energy for the ith measurement. The uncertainties are given by:

$$\sigma_i^2 = \left(\frac{\partial e}{\partial \theta^e_0}\right)^2_i (\delta\theta^e_0)^2 + \left(\frac{\partial e}{\partial \theta^e_0}\right)^2_i (\delta\theta^e_0)^2 \quad (12)$$

assuming that the entire error comes from uncorrelated errors in the electron and proton spectrometer central angles. (The *ep* program, described below, can be run with or

without weighting according to these uncertainties.) Setting derivatives of χ^2 with respect to each fit quantity to zero results in a set of three linear equations. Inversion of the resulting 3×3 matrix then yields the fit quantities. Use of this analytic method, as opposed to various grid search techniques for example, is justified since there are no sharp structures in the energy-angle equation and since the offsets are expected to be small.

3 Dealing with Resolution Effects

As mentioned above, the extracted energies for a given kinematic setting form a broad distribution due to resolution effects. Points far from the peak of this distribution have a large influence on χ^2 and their inclusion can heavily bias the fit. Using the entire distribution leads, in fact, to absurd values of the fit energy. Therefore a study of the extracted energy and offsets was made where the fit region for each kinematics was varied from several sigma about the mean to a small fraction of one sigma. The conclusion was that one must include significantly less than $\pm 1\sigma$ in the fitting interval. However, this can lead to a relatively large statistical error in the fit. To circumvent this a different procedure was used.

The entire distribution (counts *vs.* extracted energy) was used to establish the mean energy for a given kinematics (again, corrected for mean energy loss and cut on missing energy). This energy plays the role of e_i above, where now i runs over the five kinematic settings, rather than individual events. The electron angle is taken to be the spectrometer central angle. From this angle and the mean energy, one can use the energy-angle formula to compute the conjugate proton angle. The resulting electron-proton angle pair then represents the entire data set for a given setting. The resulting five angle pairs (one for each of the five settings) are used in the χ^2 minimization procedure. As a consistency check, this method was compared with using all data in a tight interval about the mean. There were no significant differences in the extracted fit quantities.

4 Monte Carlo Test of Validity

To test the procedure in the presence of resolution effects which can conceivably bias the fit, the Monte Carlo program MCEEP [2] was used to generate pseudo-data for each of the five settings. MCEEP was run with energy straggling, multiple scattering and internal bremsstrahlung. Effects in the target walls, target material, spectrometer exit window and VDCs were included. An additional gaussian contribution to the beam, scattered electron and proton angles was needed to obtain an extracted energy distribution of comparable width to that of the data. The sigmas of these added gaussians were all arbitrarily taken to be equal (for the three particles, but not the same for all five settings). After adding artificial, constant offsets to the electron and proton angles, the resulting pseudo-data were fit with exactly the same procedure as for the real data. Results are given below.

5 Pointing Correction to Angles

As the front end of the spectrometer is not firmly fixed to the pivot the pointing can vary from measurement to measurement. The system which normally measures this point-

ing was not operating correctly during these measurements, so the determination was made by scattering from a thin (50 mg/cm^2) carbon foil target instead. A measurement of the transverse position (y_t) of the interaction point as seen by the spectrometer, in combination with a survey of the position of the target along the beamline (z_t) and the horizontal beam position (x_b) from the BPMs nearest the target suffices. See Figure 1 for the geometry. All positions are defined relative to a coordinate system with z-axis along the nominal unrastered beam, y-axis pointing vertically upward and x-axis pointing left (right-handed system). The origin is defined by the intersection of the nominal unrastered beam with the nominal spectrometer optical axis. A bit of trigonometry gives the spectrometer object point as

$$x_o = -\delta \cos\theta_a \tag{13}$$

$$z_o = +\delta \sin\theta_a \tag{14}$$

where θ_a is the actual spectrometer angle and

$$\delta = y_t + z_t \sin\theta_a - x_b \cos\theta_a . \tag{15}$$

The actual angle can be computed from the nominal one (which assumes no mispointing) as

$$\theta_a = \theta_n + \delta/L \tag{16}$$

where L (=9.9 m) is the distance from the pivot to the point along the spectrometer where the nominal angle determination is made (*i.e.* the vernier). Here it is assumed that the angle is determined by this fixed point at distance L and the pointing offset and that the spectrometer is at a constant radial distance from the pivot.

6 Kinematics

The kinematics are given in Table 1 (see the table caption for details). The set of kinematics involves a large range of electron and proton angles. The forward electron angles maximize sensitivity to the electron arm offset whereas the backward electron angles maximize sensitivity to the proton arm offset. The kinematic derivatives are given in Table 2.

7 Results

As a test of the method, the results of the extracted energies for each kinematic setting are shown for the MCEEP pseudo-data in Figure 2. The program was run with artificial, fixed offsets for the electron and proton angles to test whether the fitting procedure can deduce these offsets. The figure shows the results before (circles) and after (stars) incorporation of the angular offsets determined from the fit. The final energies clearly show less scatter and are much closer to the input value. The input value of energy was 3.0888 GeV. Before application of the angular offsets, the average value obtained was 3.0664 ± 0.0060 GeV whereas the final value (which corrects for the offsets) was 3.0869 ± 0.00089 GeV. (The uncertainty quoted here is the standard deviation.) The input values of the offsets were

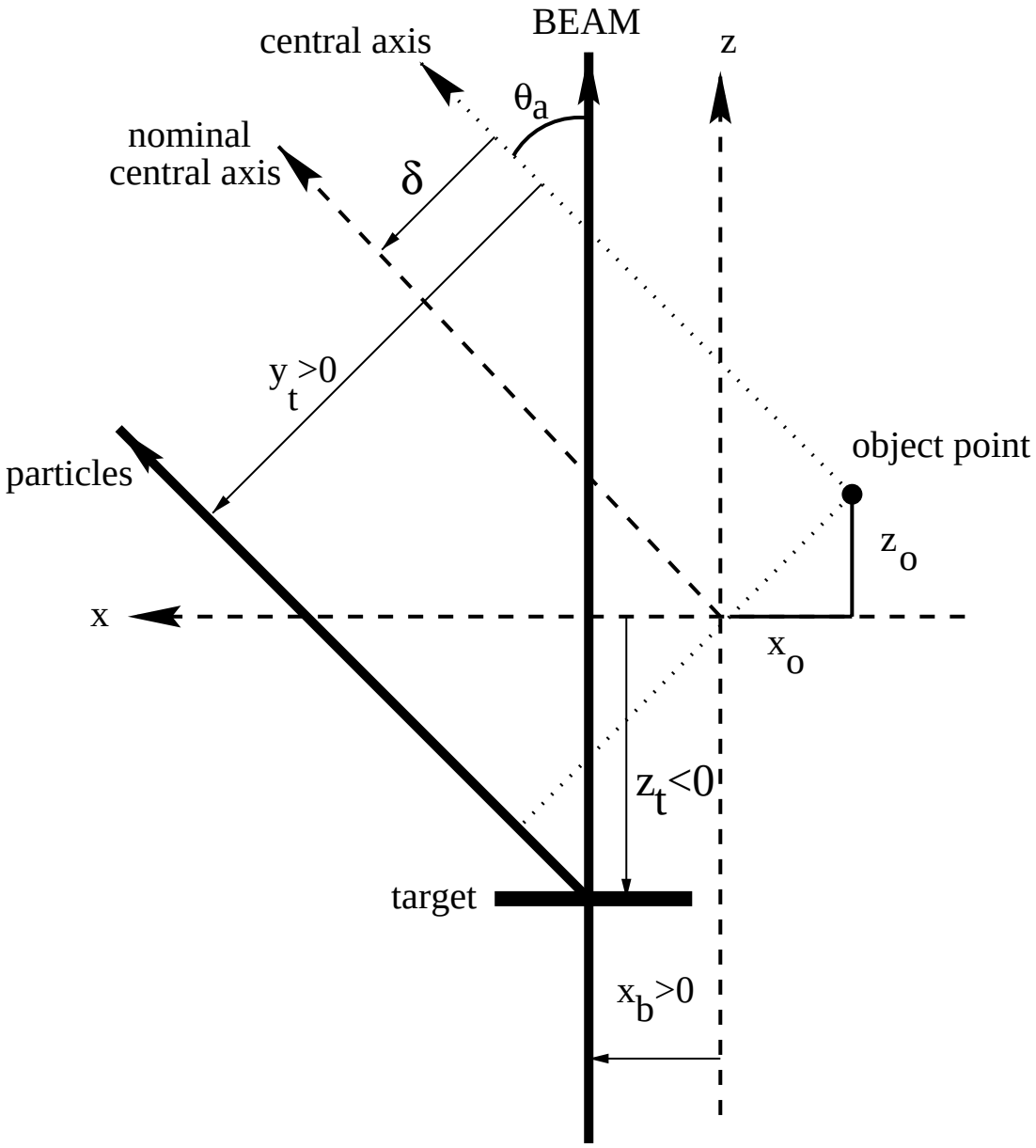

Figure 1: Geometry for determining the pointing offset of a spectrometer.

Run #	θ_n^e deg	θ_a^e deg	p_e GeV/c	θ_n^p deg	θ_a^p deg	p_p GeV/c
3132	12.9744	12.9969	2.841	−63.9619	−63.9687	0.7116
3135	19.9623	19.9844	2.582	−52.8125	−52.8187	1.109
3141	32.9616	32.9834	2.019	−38.1156	−38.1202	1.783
3143	49.9587	49.9809	1.420	−26.4202	−26.4224	2.440
3144	80.9628	80.9783	0.8172	−15.2196	−15.2213	3.078

Table 1: Kinematics. The nominal beam energy was 3.0858 GeV. Here, θ_n^e and θ_n^p are the "nominal" electron and proton central angles respectively (*i.e.* before pointing correction) and θ_a^e and θ_a^p are the "actual" electron and proton central angles respectively (*i.e.* after pointing correction).

Run #	$de/d\theta_e$ MeV/mr	$de/d\theta_p$ MeV/mr
3132	−17.86	10.19
3135	−11.77	8.37
3141	−7.39	8.29
3143	−5.25	10.07
3144	−4.08	15.87

Table 2: Derivatives of the beam energy with respect to the electron and proton angles.

Run #	Energy GeV	Deviation from Arc $\times 10^{-4}$
3132	3.1000	45.9
3135	3.0976	38.2
3141	3.0950	29.7
3143	3.0923	21.1
3144	3.0950	30.0

Table 3: Mean energies and deviations from arc value **before** application of the absolute angle offsets. The average energy is 3.0960 ± 0.0029 GeV. (The uncertainty quoted here is the standard deviation.)

-1.43 mr and 0.75 mr for the electron and proton arms respectively compared to the fit values of -1.31 mr and 0.80 mr. Note that without the additional resolution contribution, these results lie even closer to the input values.

The extracted energies before any offset correction are shown for each kinematic setting in Figure 3 for the real data. The gaussian fits shown were restricted to data within $\pm 1.5\sigma$ of the mean values. The mean energies were then combined with the central electron angles to obtain the conjugate proton angles in the manner described above.

The mean energies and deviations from the arc value before application of the fit angular offsets are shown in Table 3. The results after putting in the offsets are shown in Table 4 and Figure 4. These values result from a weighted fit, where the spectrometer central angle uncertainties were taken to be 0.10 mr for both arms. The resulting χ^2 per degree of freedom was 1.0 compared to the value of 4.1 before adjustment of the angles due to the fit offsets. The angular offsets obtained were 0.59 ± 0.17 mr and -0.26 ± 0.27 mr for the electron arm and hadron arm respectively. The uncertainties quoted here also assume random errors in the angles of 0.10 mr. These offsets must be added to the corresponding spectrometer central angles to obtain the correct absolute angles. The fit value of the energy was 3.0878 ± 0.0035 GeV ($+6.5 \pm 11.3 \times 10^{-4}$ compared to the arc value). (The energy uncertainty assumes the 0.10 mr per point angular uncertainties as well – it is NOT a standard deviation here.) The results for both the 3- and 2-parameter fits are shown in Table 5.

8 Conclusions

Although it seems unlikely that an energy measurement significantly better than 10^{-3} could be obtained from this technique (using the Hall A HRS spectrometers), calibration of the absolute spectrometer central angles at the 0.1 mr level seems possible. However, this level of accuracy would require an independent measurement of the beam energy at the few $\times 10^{-4}$ level. In order to assess the reproducibility of the results, more data are needed. It would be desirable to have a larger number of angle pairs (at least a dozen

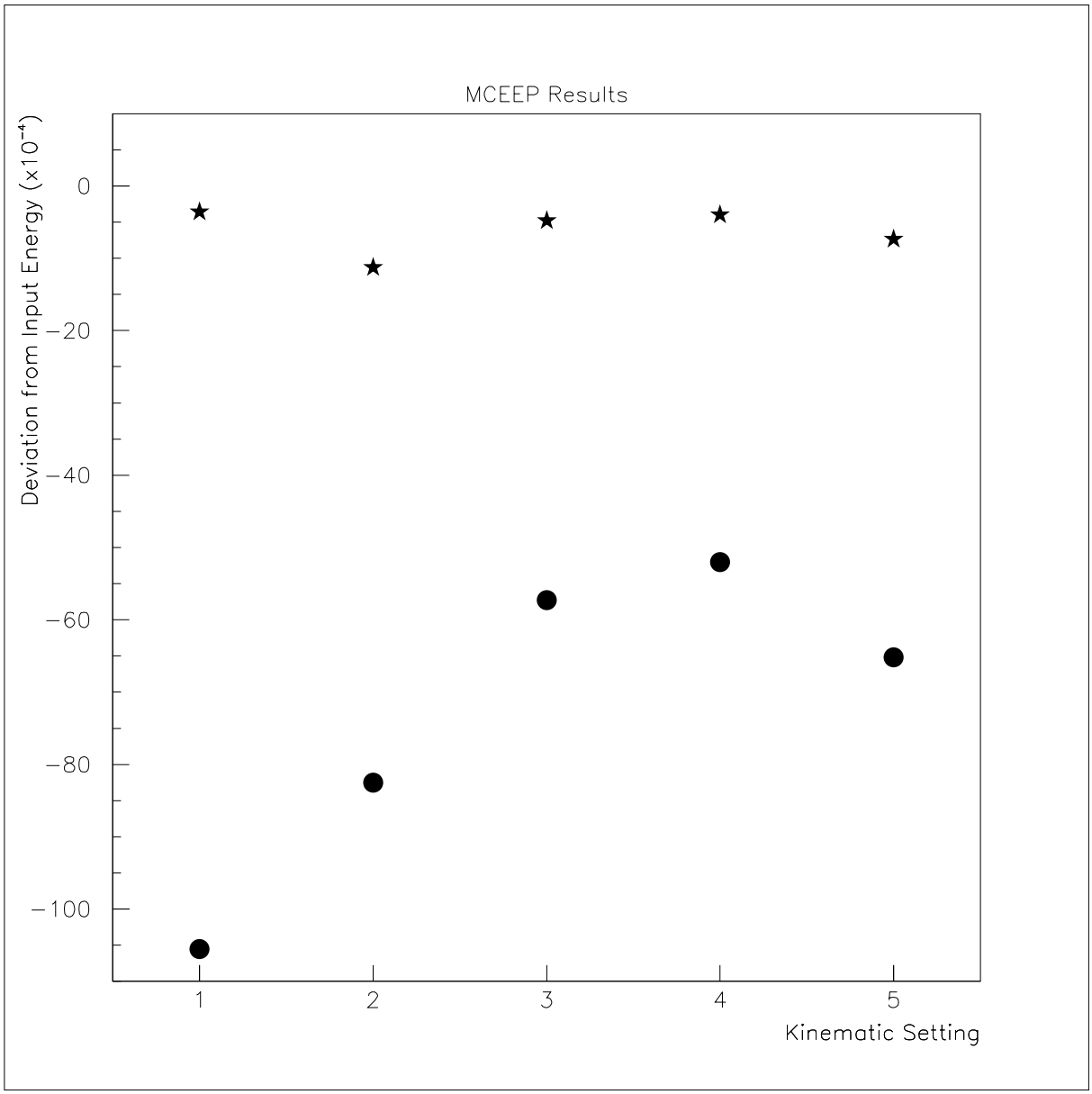

Figure 2: Energy deviations from input energy before (circles) and after (stars) incorporation of the absolute angular offsets for the MCEEP pseudo-data. See the text for details.

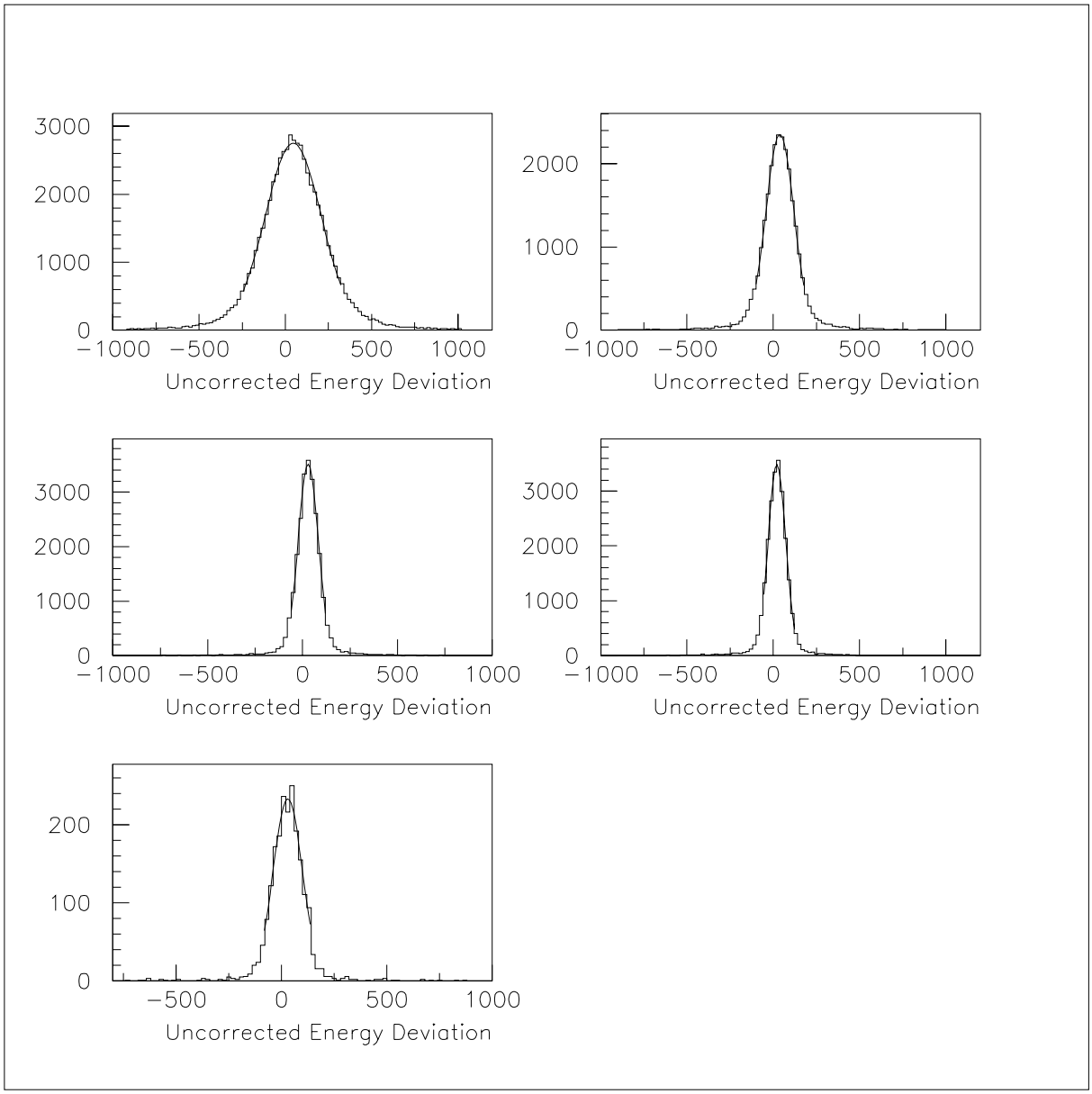

Figure 3: Distribution of extracted energy for each kinematic setting along with a gaussian fit over a $\pm 1.5\sigma$ interval. The energy plotted on each horizontal axis is given by $10^{-4} \times (e_i - e_{arc})/e_{arc}$ where e_{arc} is the value obtained from the Hall A arc measurement (3085.8 ± 0.6 MeV). The kinematic settings are ordered reading first across from left to right and then down.

Run #	Energy GeV	Deviation from Arc $\times 10^{-4}$
3132	3.0868	3.2
3135	3.0885	8.7
3141	3.0884	8.6
3143	3.0866	2.6
3144	3.0885	8.7

Table 4: Mean energies and deviations from arc value **after** application of the absolute angle offsets. The average energy is 3.0878 ± 0.0035 GeV. The angular offsets obtained were 0.59 ± 0.17 mr and -0.26 ± 0.27 mr for the electron arm and hadron arm respectively. These offsets must be added to the corresponding spectrometer central angles to obtain the correct absolute angles. The uncertainties quoted here assume random errors in the angles of 0.10 mr (this value results in a χ^2 of 1.0). (The energy uncertainty assumes this as well – it is NOT a standard deviation here.)

Parameter	Units	Energy Fit	Energy Fixed
e	GeV	3.0878 ± 0.0035	3.0858
$\frac{e}{e_{arc}} - 1$	10^{-4}	$+6.5 \pm 11.3$	0
$\delta\theta_e$	mr	$+0.59 \pm 0.17$	$+0.65 \pm 0.12$
$\delta\theta_p$	mr	-0.26 ± 0.27	-0.40 ± 0.11

Table 5: Summary of fit results for 3-parameter and 2-parameter fits.

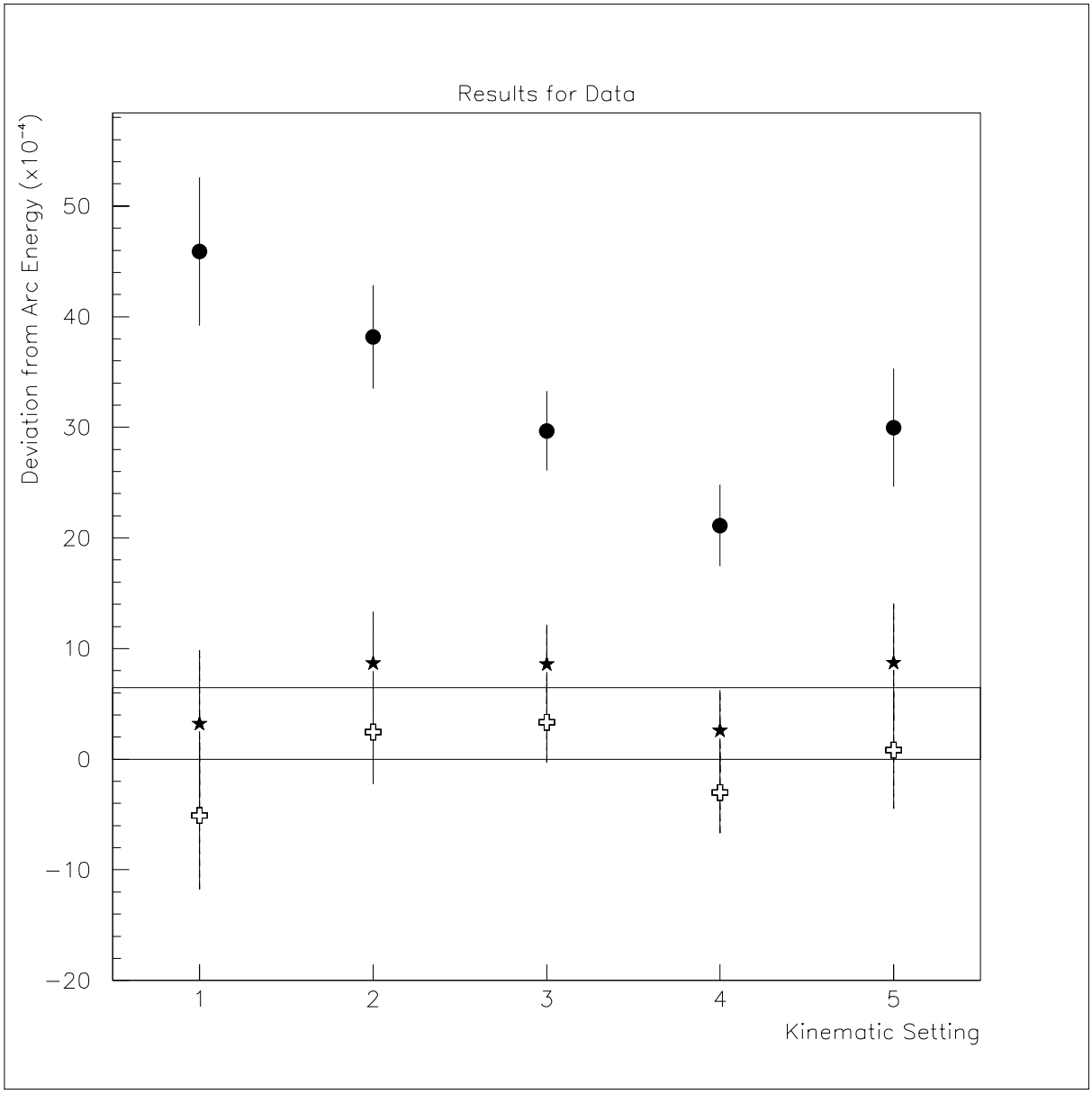

Figure 4: Energy deviations from arc before (circles) and after (stars) incorporation of the absolute angular offsets. The uncertainties shown assume uncorrelated errors in the spectrometer central angles of 0.10 mr for each spectrometer. The crosses show the results if the energy is constrained to the arc value and only the angular offsets are fit (the angular offsets in this case are 0.65 ± 0.12 mr and -0.40 ± 0.11 mr for the electron and hadron arm respectively, compared to 0.59 ± 0.17 mr and -0.26 ± 0.27 mr for the 3-parameter fit). The horizontal lines indicate the mean energies for the respective fits (2- or 3-parameter).

or so) to assess the quality of the final fit. In addition, an angle scan at a different (but constant) beam energy may also allow one to assess the systematic error on the fit angular offsets.

9 Using the Programs

The following two programs may be obtained from one of the authors (P.E. Ulmer) to aid in the analysis of similar calibration data sets.

9.1 Program: "offsets"

This program determines the spectrometer object point and, more relevant to the energy calibrations, the offset corrected spectrometer central angles. The program is standard Fortran and should compile on most platforms. It requires modification of the input file, "offsets.inp". Examination of the existing file should suffice to explain the input and format. The first line is the foil position along the z-axis (*i.e.* the beam axis) in meters (downstream is positive). Following this is a table of numbers. Each row should contain: an identifier (character*4), the nominal spectrometer central angle (deg), the LVDT value (cm), the position of the centroid of the y_tgt coordinate (cm) and the horizontal position of the beam (cm) with beam left positive. The LVDT value is removed since it is currently assumed that this number is not meaningful. The program produces an output file: "offsets.out". For each input file row, the offset corrected central angle is given (theta_a) along with the x and z coordinates of the spectrometer object point (these axes are the nominal laboratory ones - see ESPACE [3]).

9.2 Program: "ep"

Once the angles corrected for mispointing are obtained, the program "ep" can be used to fit out the beam energy and absolute angular offsets of each spectrometer. The program is standard Fortran and should compile on most platforms. It requires modification of the input file, "ep.inp". The program's in-line comments contain a description of the input variables. The program can be run in two modes, described below, and can perform either a two-parameter fit for the angle offsets, or a three parameter fit including the beam energy as well. In addition, it can read either Ntuples from ESPACE (dumped from PAW [4] using "ntudump.kumac") or MCEEP Ntuples (dumped from PAW using "mcdump.kumac"). The user can also elect to perform a weighted fit or an unweighted one.

The program can be run in two modes. For the first mode (using the mean energies only), it's assumed that the mean values of the energies at each kinematics are known. One can determine them by first running the program in the other mode (see below). The mean energies should be the values based on events passing the missing energy and angle cuts and corrected for beam energy loss as well as non-zero beam angles, but NOT corrected for absolute angular offsets. These values are the so-called "uncorrected" values which the code dumps into an ascii Ntuple file for the second mode of running. The mean values obtained from a gaussian fit should be put into the input file (see below) as well as the sigma values from this fit (the sigma values are only used for the second mode). The

code then uses the electron central angle and the mean energy to determine a conjugate proton angle. The resulting angle pairs (one for each kinematics setting) are then used to perform the fit and establish the absolute angular offsets (and, optionally, the beam energy).

In the second mode (answer "N" to: Use mean energies only?) all the data within a one sigma interval about the mean is used. Each angle pair is treated as an independent measurement. Since the fitting interval affects the results, one must then either extrapolate to zero interval, or use a very small interval to begin with (if statistics allow). This should give consistent results with the first mode of running. (The first mode is the recommended one, but the second mode is useful to produce the energy Ntuples which allow determining the mean energy per setting and to verify the results.) In this mode the code reads ascii (dumped) Ntuples for each kinematic setting containing: tthe_tg, tphe_tg, tthp_tg, tphp_tg, tthb, tphb, eloss and emiss where tthe_tg and tphe_tg are the tangents of the electron out-of-plane and in-plane angles at the target, tthp_tg and tphp_tg are the tangents of the proton out-of-plane and in-plane angles at the target, tthb and tphb are the tangents of the beam in-plane and out-of-plane angles in the ESPACE hall coordinate system (notice the definition of in-plane and out-of-plane is reversed for the beam compared to the detected particles), eloss is the beam mean energy loss in MeV and emiss is the missing energy in MeV (used for the cut). For MCEEP input, the angles are read in milliradians, instead of reading the tangents of the angles. In this mode, the program will produce the energy ascii Ntuples (files ending in "uc.out" are the values before application of the offsets, whereas files ending in ".out" are the values after application; the prefix is the identifier specified in the input file - see below). The entries in this file correspond to fractional deviations from the nominal value (see the input file description below) in units of 10^{-4}.

Now the input file will be described.

- Line 1: The number of points to include in the fit per kinematic setting followed by the maximum number of points per kinematic setting contained in the Ntuple. The latter number may be set large. The first number allows, for example, using the same number of points per kinematics for the fit, so that each setting will be equally weighted.

- Line 2: The "nominal" value of the energy in GeV. For the two-parameter fit the energy is constrained to this value. Also, the Ntuple entries produced by the code (for the second mode of running) are the energies relative to this value.

- Line 3: The low and high values for the missing energy cut in GeV.

- Line 4: The low and high values of the beam energy in GeV. Values outside this range are not dumped to the Ntuples.

- Line 5: The uncertainties in the electron and proton spectrometer central angles, respectively (radians). These uncertainties are used to determine the energy uncertainty (for the "mean energy" mode only) assuming that the angle errors are uncorrelated. The resulting energy uncertainties are used to weight the fit (provided the user elects to do this). Finally, the energy uncertainties for each kinematic setting are used to determine the uncertainty in the fit energy and angular offsets.

- Line 6: The number of kinematic settings, nkin.

  For each kinematic setting, a row follows including:

- Lines 7–N: The electron angle (deg), the proton angle (deg), the mean extracted energy (GeV), the sigma of the energy (GeV), the ph_tg cut value (rad), the th_tg cut value (rad) and an identifier used for the filename prefixes (character*2). The angles should be corrected for mispointing. The data will be cut so that the phi angle lies between $\pm$ the ph_tg cut value and the theta angle lies between $\pm$ the th_tg cut value (the electron and proton angle cuts are identical). The sigma values are only used for the second mode of running.

References

[1] K.L. Brown, F. Rothacher, D.C. Carey and C.H. Iselin, TRANSPORT, *A Computer Program for Designing Charged Particle Beam Transport Systems*, SLAC-91, Rev. 2, UC-28(1/A).

[2] P.E. Ulmer, computer program MCEEP, *Monte Carlo for (e,e'p) Experiments*, CEBAF Technical Note # 91-101 (1991).

[3] ESPACE, *Event Scanning Program for Hall A Collaboration Experiments*, see http://www.jlab.org/Hall-A/espace.

[4] R. Brun, O. Couet, C. Vandoni and P. Zanarini, *PAW Users Guide*, Program Library Q121, CERN (1991).