

Effective use of JLab Hall A HRS acceptance with R-functions

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December 5, 2001

Abstract

A method for placing complicated variable cuts on data is presented. It is used to find an optimal cut on HRS acceptance, which selects maximum region of “flat” acceptance. The method is based on utilizing R-function formalism, that allows to use equations of boundaries of a geometrical object to construct functions that are equal to 0 on the boundary of the object, have different signs inside and outside the object and in absolute value approximate distance functions to the boundary of the object.

1 Introduction

Let’s consider a charged particle entering a Hall A spectrometer that is set at a central momentum value. In a simple approximation trajectory of the particle is determined only by deflection of the particle in the spectrometer magnetic field. Then for every initial line of flight and momentum of the particle at the spectrometer entrance there is a definite trajectory of the particle, which results in either particle passing through spectrometer to its focal plane, or getting absorbed by internal spectrometer apertures. It is clear that in this approximation particles having exactly the same line of flight, momentum and charge at the spectrometer entrance will all pass through spectrometer to its focal plane or will all get absorbed by apertures, and thus spectrometer “acceptance function” defined in the space of initial particle lines of flight and momenta assumes values of either 0 or 1.

Although this approximation neglects many probabilistic effects such as multiple scattering, absorption in spectrometer entrance and exit windows, rescattering, fluctuations of the magnetic field, and also scintillator, tracking, trigger and other efficiencies, it does take into account geometry of the (average) magnetic field in the spectrometer and geometry of spectrometer apertures. One can say that this approximation attempts to separate geometrical aspects of spectrometer acceptance.

In the above approximation the region where acceptance function is equal to 1 (“flat” or “good” acceptance region) can be described as a region in the 5-dimensional space of

“target variables” [1] $x_{tg}, y_{tg}, \phi_{tg}, \theta_{tg}, \delta_{tg}$ (variables $x_{tg}, y_{tg}, \phi_{tg}, \theta_{tg}$, uniquely specify initial line of flight of particle in 3D space and δ_{tg} specifies particle momentum).

For purposes of cross-section extraction software cuts are usually placed on the target variables to select a region inside the “good acceptance” region. Placing the cuts so that they define a large region but still lying within the “good acceptance” region allows to include more particles in the cross-section analysis.

It turns out that if the software cuts are defined in terms of R-functions, then it is possible to easily “squeeze” and “unsqueeze” the cut region (i.e. uniformly enlarge or decrease the region of the cut across all its boundaries) and also, by comparing to a simulation, to see at what point of “unsqueezing” the cut region starts to get out of “good acceptance” region. With a wisely defined initial cut, this allows to find an optimal cut.

This method of finding optimal cut on HRS acceptance was applied to E89044 ${}^3\text{He}(e,e'p)$ data and is described below along with basic information on R-functions.

2 R-functions

An R-function [2, 3, 4, 5] is a real-valued function whose sign is completely determined by signs of its arguments.

For example, function $\min(x, y)$ is an R-function, because it is possible to predict its sign just from knowing signs of its arguments x and y . Function $\min(x, y) - 1$ is not an R-function because it is not possible to predict its sign from signs of its arguments. There are infinitely many R-functions acting on any number of arguments, examples are [4]: $xyz, x + y + \sqrt{xy + x^2 + y^2}, xy + z + |z - yx|$.

It is easy to see that R-functions act on signs of arguments in the same manner as Boolean functions (functions of the Boolean algebra of logic) act on logical variables (considering positive argument values as “true” or “1” and negative as “false” or “0”). In fact, for every R-function $f(x_1, \dots, x_n)$ there exists a Boolean function $F(X_1, \dots, X_n)$ such that

$$S(f(x_1, \dots, x_n)) = F(S(x_1), \dots, S(x_n))$$

for all x_1, \dots, x_n , where $S(x)$ is a “step” function:

$$S(x) = \begin{cases} 0, \text{ or “false”} & \text{if } x < 0 \\ 1, \text{ or “true”} & \text{if } x > 0. \end{cases}$$

Such Boolean function $F(X_1, \dots, X_n)$ is said to be a companion to R-function $f(x_1, \dots, x_n)$.

It turns out [2] that every Boolean function is a companion to infinitely many R-functions. For example, Boolean “and” (\wedge) function is a companion to R-functions $\min(x, y), x + y - \sqrt{x^2 + y^2}, (x + y)^3 - |x - y|^3$ and many others; Boolean “or” (\vee) function is a companion to $\max(x, y), x + y + \sqrt{x^2 + y^2}, (x + y)^3 + |x - y|^3$ and many others; Boolean “not” (\neg) function is a companion to $-x, -x^3, e^{-x} - e^x$ and many others.

Just as Boolean functions, R-functions are closed under composition. This means that combination of several R-functions is another R-function. Any Boolean function

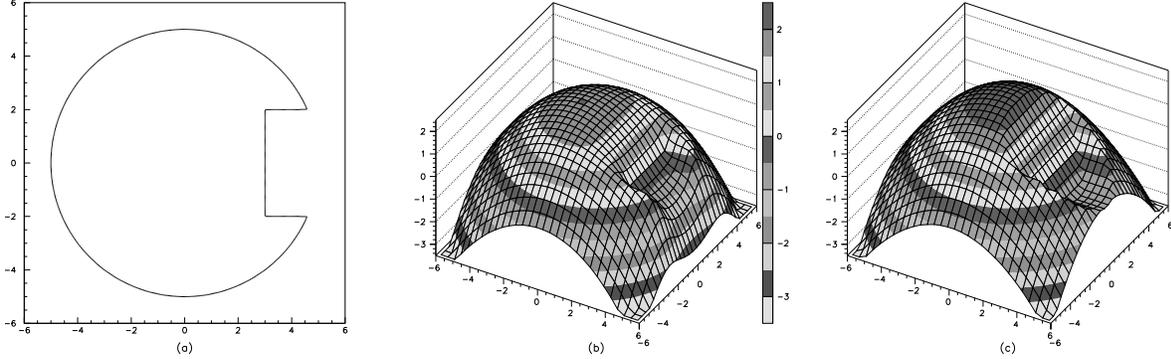


Figure 1: (a) Geometric domain and (b), (c) approximations to its distance function.

F can be constructed from combination of logical negation, conjunction and disjunction (\neg, \vee, \wedge) acting on its arguments, and an R-function f corresponding to this Boolean function F (i.e. an R-function f such that Boolean function F is its companion) can be constructed by formal substitution of Boolean \neg, \vee, \wedge by their corresponding R-functions and by substituting logical arguments by real arguments.

These properties allow to create functions describing complex geometrical objects by writing equations of boundaries of an object, converting them to “greater or equal to 0” inequalities describing regions of space bounded by the boundaries, writing Boolean expression combining these regions of space into the geometrical object and then formally substituting Boolean functions by corresponding R-functions. The resulting function is equal to 0 on the boundary of the geometrical object, greater then 0 inside the object and less then 0 outside the object. Depending on choice of R-functions various differential properties of the resulting function can be obtained. Moreover, the absolute value of the resulting function can be made approximately equal to distance to the nearest boundary of the geometrical object.

For example [5], boundaries of geometrical domain in Figure 1(a) can be described by equations:

$$\omega_1 \equiv \frac{1}{10}(25 - x^2 - y^2) = 0, \quad \omega_2 \equiv x - 3 = 0, \quad \omega_3 \equiv \frac{1}{4}(4 - y^2) = 0.$$

Regions of space defined by inequalities $\omega_1 \geq 0$, $\omega_2 \geq 0$ and $\omega_3 \geq 0$ can be combined as $\Omega_1 \wedge (\Omega_2 \wedge \Omega_3)$ to form the region inside the domain. Formally substituting Boolean function \wedge by corresponding R-function $x + y - \sqrt{x^2 + y^2}$, Boolean negation \neg by R-function $-x$ and Boolean arguments $\Omega_1, \Omega_2, \Omega_3$ by $\omega_1, \omega_2, \omega_3$, one obtains a function (Figure 1(b))

$$f(x, y) = \omega_1 + (-\omega_2 + \omega_3 - \sqrt{\omega_2^2 + \omega_3^2}) - \sqrt{\omega_1^2 + (-\omega_2 + \omega_3 - \sqrt{\omega_2^2 + \omega_3^2})^2},$$

where $\omega_1, \omega_2, \omega_3$ are defined above. This function is analytic everywhere except the corner points, is equal to 0 on the boundary of the domain in Figure 1(a), is positive

inside the domain and negative outside. It is also normalized on all regular points of the boundary in the sense, that

$$\frac{\partial f}{\partial \nu|_{\partial\Omega}} = 1,$$

where ν is the inner normal to the boundary $\partial\Omega$. This means that function $f(x, y)$ approximates distance function in the vicinity of non-corner points of the boundary.

In general, function f is said to be normalized up to m -th order when the following conditions are satisfied:

$$f|_{\partial\Omega} = 0; \quad \frac{\partial f}{\partial \nu|_{\partial\Omega}} = 1; \quad \frac{\partial^k f}{\partial \nu^k|_{\partial\Omega}} = 1; \quad k = 2, 3, \dots, m$$

with the definitions above. There are several techniques for constructing normalized implicit functions, including a general method for recursively increasing order of normalization of any given function [2]. A simpler approach is based on observation that many R-functions tend to preserve normalization properties of their arguments. In the example above function ω_1 , ω_2 , ω_3 are normalized (at least to the first order), and R-functions used tend to preserve normalization.

If in the example above one used R-function $\min(x, y)$ for Boolean \wedge , one would obtain a function (Figure 1(c)):

$$f(x, y) = \min(\omega_1, (-\min(\omega_2, \omega_3))),$$

which is easier to compute and behaves more like distance function but does not have as good differential properties as the function in Figure 1(b).

This example shows that if equations of boundaries of a multidimensional geometrical object are known, R-functions allow to easily construct a few functions that are equal to 0 on the boundary of the object, have different signs inside and outside the object and in absolute value approximate distance to the boundary of the object. For more information on the theory of R-functions and its applications see [2, 3, 4, 5].

3 Initial acceptance cut

The 5-dimensional HRS acceptance region is a complicated region that is not easy to visualize. During E89044 x_{tg} was constrained to interval (-4.5mm, 4.5mm) by beam position, spectrometer angles and target geometry. This provided a natural tight x_{tg} cut and x_{tg} could be excluded from acceptance cuts. With very constrained x_{tg} spectrometer acceptance is a 4-dimensional region of variables $y_{tg}, \phi_{tg}, \theta_{tg}, \delta_{tg}$, main features of which can be seen at $(\theta_{tg}, \delta_{tg}), (\phi_{tg}, \delta_{tg}), (\phi_{tg}, y_{tg}), (\theta_{tg}, \phi_{tg})$ plots of singles data that covers as much acceptance as possible (Figure 2, proton singles in right (hadron) spectrometer set at 48.3 degrees and central momentum 1480 MeV/c, quasielastic knockout of protons from ^3He gas in 10cm “tuna can” target, beam energy 4807 MeV, open collimator).

The acceptance region exhibits a number of features that can be explained from physical considerations:

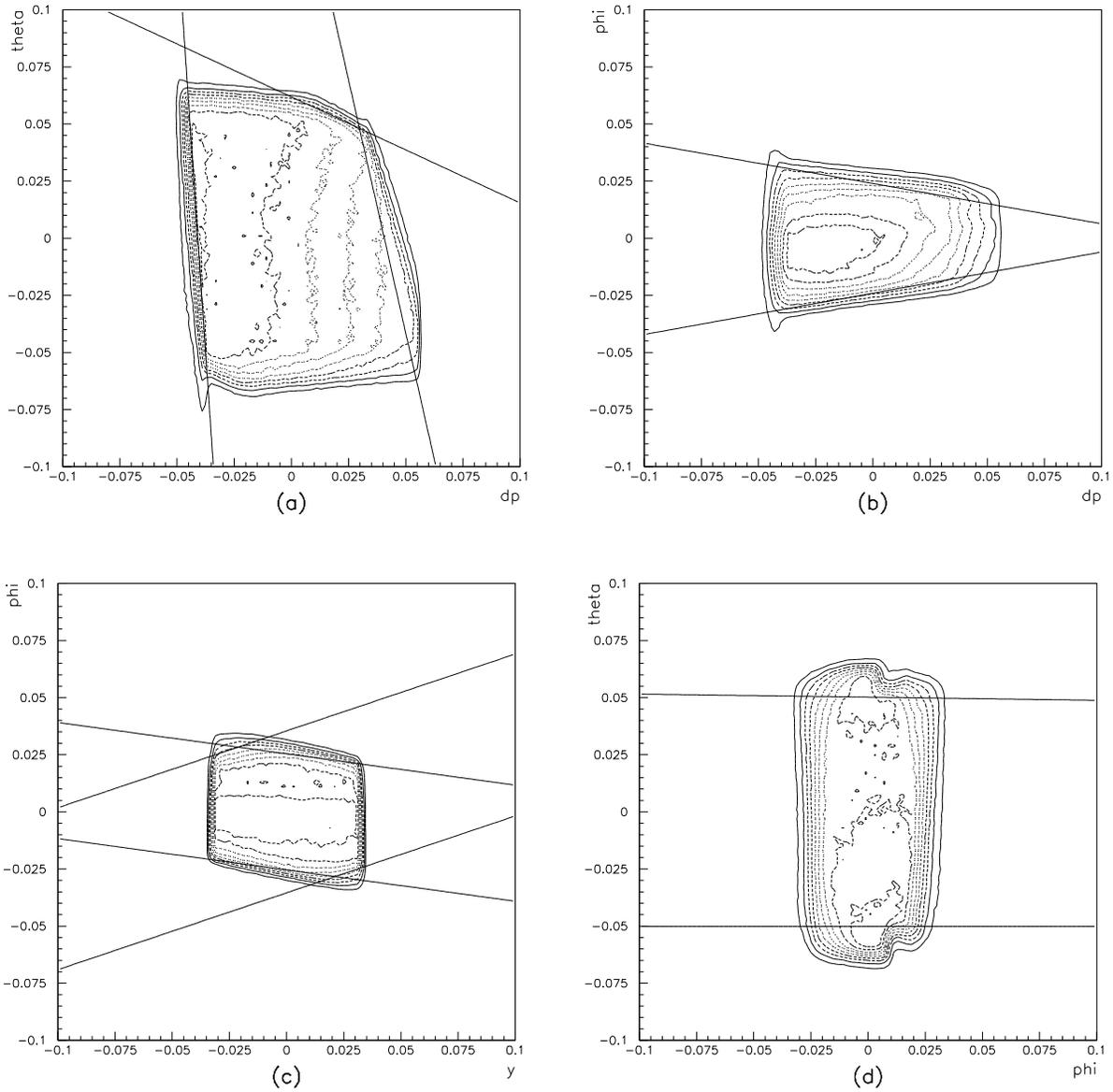


Figure 2: Proton singles distributions in target variables (contour plots) and initial acceptance cut (solid lines).

- dependence of accepted δ_{tg} on θ_{tg} (Figure 2(a)) is caused by the fact, that trajectories of higher momentum particles have lower curvature in the dipole, and in order for them to “fit” into spectrometer they have to have lower θ_{tg} (negative θ_{tg} corresponds to upward flying particles in lab coordinate system); for similar reasons lower momentum particles (with higher curvature in the dipole) have to have higher θ_{tg} to reach the focal plane;
- dipole magnet has trapezoidal cross-section and higher momentum particles tend to fly closer to its shorter base (higher magnetic field) side, and this explains why accepted range of ϕ_{tg} decreases with increasing δ_{tg} (Figure 2(b));
- increasing (decreasing) y_{tg} requires decreasing (increasing) ϕ_{tg} in order for the particle to get into spectrometer entrance window (Figure 2(c)).

11 linear cuts shown as solid lines in Figure 2 were placed on $(\theta_{tg}, \delta_{tg})$, (ϕ_{tg}, δ_{tg}) , (ϕ_{tg}, y_{tg}) , (θ_{tg}, ϕ_{tg}) pairs of variables to restrict events to region that seems to be a “good acceptance” region. Acceptances of the 2 spectrometers appear to be slightly different, but for simplification cuts were chosen so that they restrict events to “good acceptance” region in both spectrometers at the same time (left (electron) arm data is not shown). Combination of the 11 cuts on each of the 2 spectrometers defines a region in 8-dimensional space of variables $y_{tge}, \phi_{tge}, \theta_{tge}, \delta_{tge}, y_{tgh}, \phi_{tgh}, \theta_{tgh}, \delta_{tgh}$. This region is finite and is bounded by 22 hyperplanes and is the initial acceptance cut. Of course it is difficult to judge shape of a 4-dimensional region by its 2-dimensional projections and a better initial cut could be placed. But as will be shown later even this simplified approach produces good results.

In E89044 analysis there is 1 more implicit acceptance cut: cut to remove contributions from target walls (normally $|reactz| < 0.045$) that effectively limits y_{tg} range.

4 R-cuts

Applying 22 cuts on 8 variables might seem as a tedious job, but R-functions allow to reduce it to 1 cut (“R-cut”¹) on the value of the function $f(y_{tge}, \phi_{tge}, \theta_{tge}, \delta_{tge}, y_{tgh}, \phi_{tgh}, \theta_{tgh}, \delta_{tgh})$ describing the whole 8-dimensional region bounded by the cuts. Such function can be easily constructed (analogously to the examples in Section 2) in a number of different ways from left hand sides of “equal to zero” equations of cut lines. If equations of cut lines are normalized (with coefficient $\frac{1}{\sqrt{a^2+b^2}}$ for equation $ax + by + c = 0$) and R-functions used in construction tend to preserve normalization (e.g. functions $\min(x, y)$, $x + y - \sqrt{x^2 + y^2}$), then modulus of the constructed function will approximate distance from a point to the nearest boundary of the region. Then cutting on a positive f_0 value of the function will “squeeze” the region of the cut approximately uniformly across all its boundaries by f_0 , cutting on negative values $-f_0$ will “unsqueeze” the cut region by f_0 .

PAW definition of function f_1 that describes the 8-dimensional region of “initial cut” on variables $y_{tge}, \phi_{tge}, \theta_{tge}, \delta_{tge}, y_{tgh}, \phi_{tgh}, \theta_{tgh}, \delta_{tgh}$ is given in Appendix A. f_1 was constructed from left hand sides of normalized equations of cut lines in Figure 2 by using

¹This term seems to have been introduced by Zhengwei Chai.

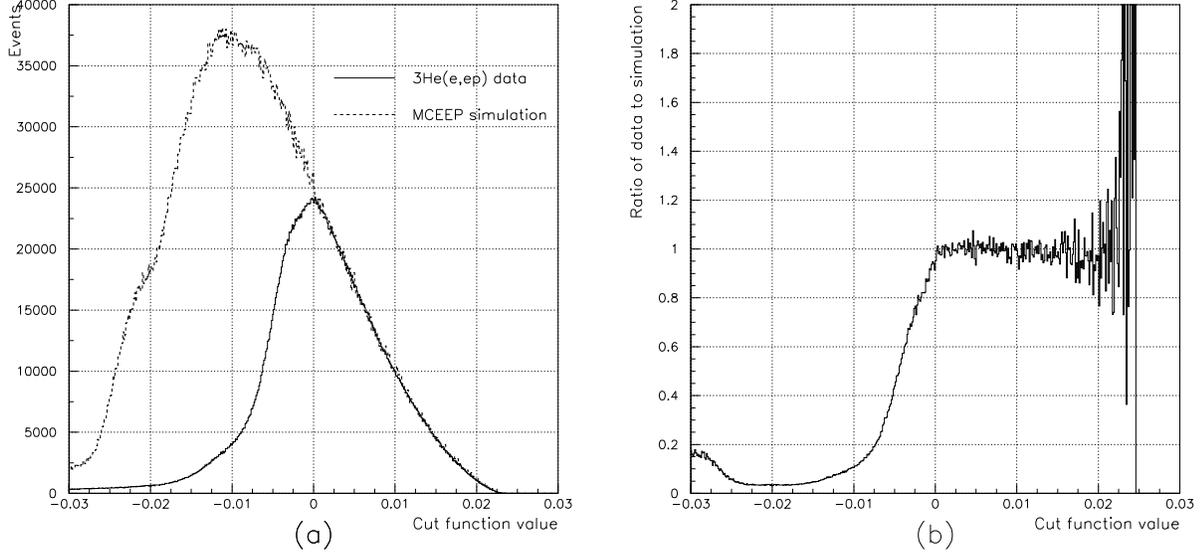


Figure 3: (a) Cut function f_1 distributions for data and simulation and (b) ratio of the 2 distributions (beam energy 4807 MeV).

R-function $\min(x, y)$ for its companion Boolean \wedge . f_1 is equal to 0 on the boundary of the 8-dimensional “initial cut” region, increases toward the center of the region and decreases outward of the center of the region.

In Figure 3(a) the value of this function is plotted for $^3\text{He}(e,e'p)$ coincidence data (left (electron) arm at 16.4 degrees and 3966 MeV/c, right (hadron) arm at 48.3 degrees and 1480 MeV/c, beam energy 4807 MeV, open collimators) as a solid line, the value of this function for MCEEP simulation with no spectrometer apertures is plotted as dashed line ($^3\text{He}(e,e'p)$ cross-section model used in simulation was factorization of CC1 by spectral function fitted to the data).

The value of the function for a coincidence event characterizes how close is the event to the center of the combined left/right spectrometer acceptance defined by cuts in Figure 2. Events with positive values of the function lie inside the region of the initial 8-dimensional acceptance cut, events with negative values of the function lie outside of the acceptance cut. From Figure 3(a) one can see that in region closer to the center of acceptance data and simulation agree, and as one departs from the center at some value of f_1 yield from data starts to get cut by spectrometer apertures. From Figure 3(b) (ratio of the 2 curves in Figure 3(a)) one can find that this value of f_1 is equal to 0.001, i.e. a cut to restrict events to a true “good acceptance” region should be $f_1 > 0.001$.

Function f_1 is defined in terms of target variables that are “tied” to spectrometers. This suggests that changing angle and momentum of spectrometers should not change the form of the acceptance cut. To check this idea in Figure 4 function f_1 is plotted for different $^3\text{He}(e,e'p)$ data and corresponding MCEEP simulation (left (electron) arm

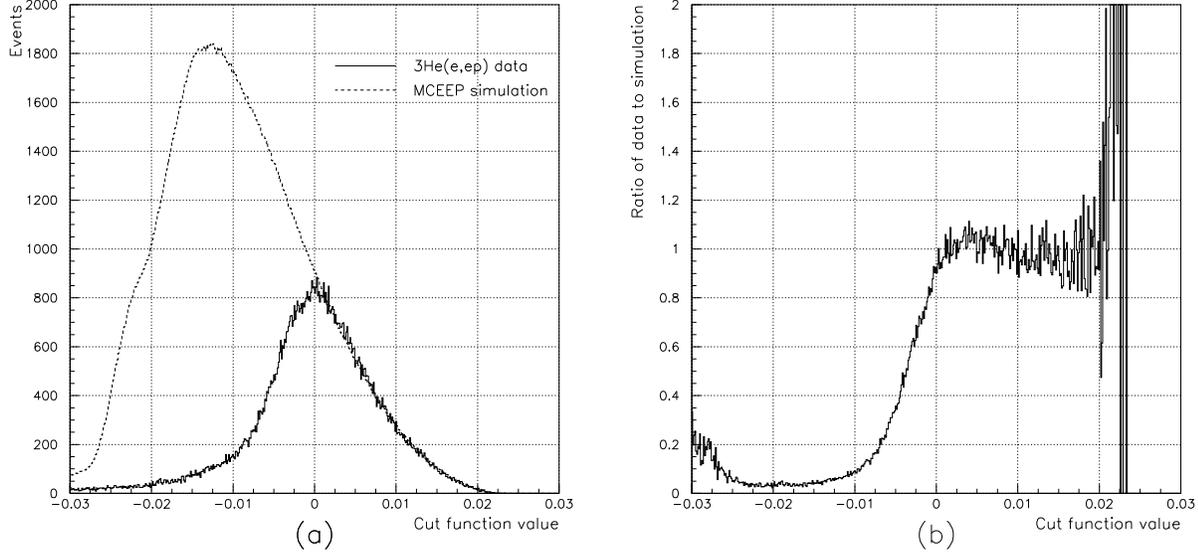


Figure 4: (a) Cut function f_1 distributions for data and simulation and (b) ratio of the 2 distributions (beam energy 1255 MeV).

at 118.7 degrees and 417 MeV/c, right (hadron) arm at 14.2 degrees and 1480 MeV/c, beam energy 1255 MeV, open collimators). From this figure one can see that the optimal acceptance cut should be $f_1 > 0.0014$, i.e. a little tighter than in the case above, which can be explained by a slightly different left spectrometer optics and optical databases used at the new momentum setting. One can also observe that ratio of data to simulation (Figure 4(b)) deviates from 1 by as much as 5%. This might suggest that optical databases used for these spectrometer settings might need reoptimization and that cross-sections extracted with these databases might vary by $\sim 5\%$ depending on acceptance cut used.

To be on the safe side one can choose a cut $f_1 > 0.002$ for both cases. This acceptance cut selects 52% of all coincidence events for the case in Figure 3 and 44% events for the case in Figure 4. Usual “square” acceptance cuts (e.g. $|\phi_{tg}| < 0.02$, $|\theta_{tg}| < 0.04$, $-0.035 < \delta_{tg} < 0.03$, y_{tg} is limited by $|\text{reactz}| < 0.045$) include 28% and 22% events for these 2 cases respectively, that is cut $f_1 > 0.002$ is almost 2 times more effective than “square” acceptance cuts. Also one can see that using R-cuts allows to judge how well simulation agrees with data in different regions of acceptance by plotting value of the cut function for data and simulation.

As was mentioned above, one can use different ways to combine left hand sides of equations of linear cuts in Figure 2 to construct function f describing the acceptance. If one desires to obtain a function that changes C times faster than distance function in the vicinity of a boundary, one can simply multiply normalized expression for the boundary in the function definition by C .

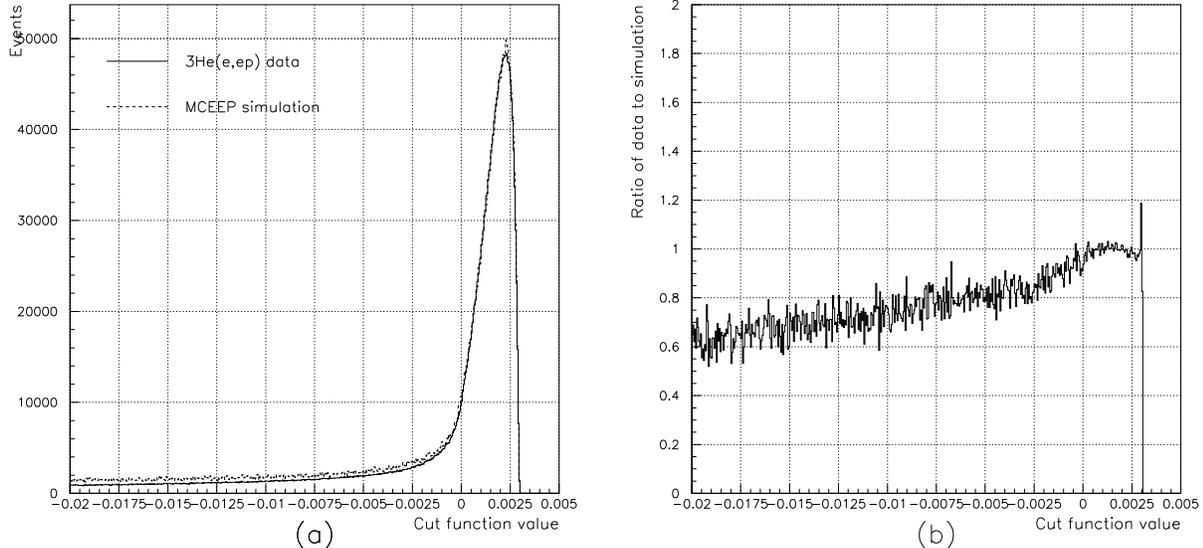


Figure 5: (a) Cut function f_2 distributions for data and simulation and (b) ratio of the 2 distributions (beam energy 4807 MeV).

If one used R-function $x + y - \sqrt{x^2 + y^2}$ instead of $\min(x, y)$ in the definition of f_1 , one would obtain a function f_2 , which is also equal to 0 on the boundary of the “initial cut” region, increases toward the center of the region and decreases in the opposite direction. It is plotted for data and simulation in Figure 5 (data and simulation are the same as in Figure 3). One can see that this function is not as convenient for acceptance cuts as f_1 .

For constructing other functions describing region of “initial cut” one could try to use $x + y - \sqrt{x^2 + y^2}$ and $\min(x, y)$ together in the definition of the same function (since there are 22 logical \wedge s needed to produce the function), or use different R-functions whose companion is \wedge .

5 Conclusion

R-functions provide a good practical way for placing complicated variable cuts. They can be successfully applied to defining “good acceptance” region of Hall A spectrometers.

Acknowledgement

Author would like to thank all members of MIT Nuclear Interactions Group: W. Bertozzi, Z. Chai, S. Gilad, D. Higinbotham, P. Monaghan, S. Sirca, A. Skabelin, R. Suleiman, Y. Xiao, B. Zhang, X. Zheng, J. Zhou, Z. Zhou for extraordinary daily help and X. Jiang,

M. Jones, M. Kuss, J. LeRose, N. Liyanage, R. Michaels, P. Ulmer, B. Wojtsekhowski and many others for invaluable help in understanding Hall A.

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A Definition of f_1 as a PAW function

```
REAL FUNCTION cut4()
include ?
REAL C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,PROD
REAL D1,D2,D3,D4,D5
REAL E1,E2
REAL F1,F2
REAL cute
*
C1=-0.0137*ph_tgth+0.0502-th_tgth
C2=0.0502+th_tgth

C3=-0.136*y_tgth+0.02518-0.991*ph_tgth
C4=y_tgth*0.136+0.02518+0.991*ph_tgth

C5=-0.975*dph+0.0396-0.220*th_tgth
C6=dph+0.0409+0.068*th_tgth
C9=-0.421*dph-0.907*th_tgth+0.0561

C7=-dph*0.1776+0.984*ph_tgth+0.0237
C8=-dph*0.1738+0.0236-0.985*ph_tgth

C10=0.319* y_tgth-0.948*ph_tgth+0.0336
C11=-0.319*y_tgth+0.948*ph_tgth+0.0336

D1=PROD(C1,C2)
D2=PROD(C3,C4)
D3=PROD(C5,C6)
D3=PROD(C9,D3)
D4=PROD(C7,C8)
D5=PROD(C10,C11)

E1=PROD(D1,D2)
E2=PROD(D3,D4)
E2=PROD(E2,D5)

F1=PROD(E1,E2)
F2=cute(y_tgte,ph_tgte,th_tgte,dpe)
cut4=PROD(F1,F2)
END

REAL FUNCTION cute(y_tgth,ph_tgth,th_tgth,dph)
REAL C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,PROD
REAL D1,D2,D3,D4,D5
```

```

REAL E1,E2
REAL y_tgth,ph_tgth,th_tgth,dph
*
C1=-0.0137*ph_tgth+0.0502-th_tgth
C2=0.0502+th_tgth

C3=-0.136*y_tgth+0.02518-0.991*ph_tgth
C4=y_tgth*0.136+0.02518+0.991*ph_tgth

C5=-0.975*dph+0.0396-0.220*th_tgth
C6=dph+0.0409+0.068*th_tgth
C9=-0.421*dph-0.907*th_tgth+0.0561

C7=-dph*0.1776+0.984*ph_tgth+0.0237
C8=-dph*0.1738+0.0236-0.985*ph_tgth

C10=0.319* y_tgth-0.948*ph_tgth+0.0336
C11=-0.319*y_tgth+0.948*ph_tgth+0.0336

D1=PROD(C1,C2)
D2=PROD(C3,C4)
D3=PROD(C5,C6)
D3=PROD(C9,D3)
D4=PROD(C7,C8)
D5=PROD(C10,C11)

E1=PROD(D1,D2)
E2=PROD(D3,D4)
E2=PROD(E2,D5)

cute=PROD(E1,E2)
END

REAL FUNCTION PROD(X,Y)
REAL X,Y
c   PROD=X+Y-SQRT(X*X+Y*Y+2*0.9*X*Y)
c   PROD=X+Y-SQRT(X*X+Y*Y)
PROD=MIN(X,Y)
END

```