catch the light

technical reprint R/P064

photomultipliers - space charge effects & transit time spread

electron tubes

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introduction

For the high current case there are two effects to be considered:

- saturation, and
- fatigue

Saturation may be regarded as an AC or pulse phenomenon whereas fatigue may be regarded as a DC or steady state phenomenon. Under AC or pulsed conditions, saturation usually limits the photomultplier operation before fatigue occurs, whereas under steady state conditions the reverse occurs[12].

This paper deals only with saturation, but in order to understand the conditions under which it is likely to occur and its overall effects, this will involve a discussion of transit time spread, the dynode chain circuit and the output circuit.

We shall confine ourselves to a qualitative approach since we are more concerned with the practical use of photomultipliers.

It will be assumed that adequate steps have been taken to ensure that the photomultplier is operating under stable conditions. In particular, it is assumed that the interdynode voltage is held constant during the high current pulse. This may be accompished by:

(a) high dynode chain current
(b) decoupling capacitors across the dynode chain
(c) use of zener diodes in dynode chain
(d) other special techniques[3]

1 saturation

1.1 cause of saturation

The Child-Langmuir law shows that for plane parellel electrodes, placed a distance, d, apart and with a potential difference, V, between them, the current, I, under conditions of space charge limitation, or saturation is given by:

$$I = \frac{16\pi\varepsilon_o}{9}\left(\frac{2e}{m}\right)^{1/2}\frac{V^{3/2}}{d^2} \qquad \ldots(1)$$

$$I = \frac{2.34V^{3/2}}{d^2}\mu A cm^{-2} \qquad \ldots(2)$$

For $d = 1\ cm,\ V = 100\ V$

$I = 2.34\ mA\ cm^{-2}$

The above was initially derived with thermionic valves in mind, but may be applied to the interdynode stages of a photomultiplier giving saturation curves. (**figure 1**).

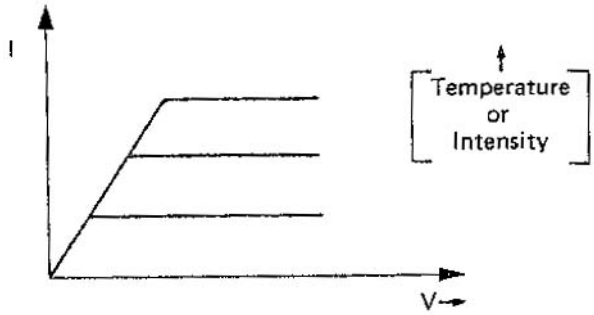

figure 1

Equation (1) assumes that:
(a) the current is limited by saturation only.
(b) infinite electrode area.
(c) initial electron velocity is zero.

The Child-Langmuir law itself does not explain why a limiting current is reached. With space charge limiting, at a point distance, x, from one electrode,

$$\frac{dV}{dx} = \frac{4}{3}\left(\frac{9I}{16\delta\in_o}\right)^{2/3}\left(\frac{m}{2e}\right)^{1/3}x^{1/3} \qquad \ldots(3)$$

i.e. $$\frac{dV}{dx} \propto x^{1/3} \qquad \ldots(4)$$

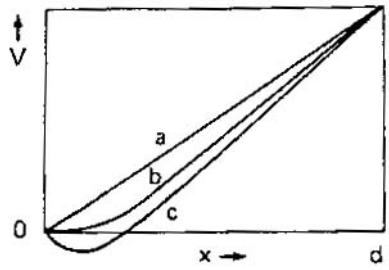

figure 2 variation of voltage between parallel electrodes:
(a) space charge saturation absent.
(b) space charge saturation (theoretical).
(c) space charge saturation (experimental).

Thus, at $x = o$, the accelerating field is zero, **figure 2**, curve b, and, subject to c, above, further electron current is impossible. (N.B. equation (4)) predicts the sharp limiting action shown in **figure 1**.
In practice the initial electron velocity is not zero. Because of the initial velocities of the electrons the current increases slightly beyond the value predicted by Child's law and the space charge field at the emitting (cathode) surface slighty exceeds the applied field. This results in a negative field in the immediate vicinity of the cathode, **figure 2**, curve c. Furthermore, the assumption of infinite electrode area is invalid. The overall effect of these imperfections is to give saturation characteristics which do not exhibit the sharp cut-off, **figure 3**.

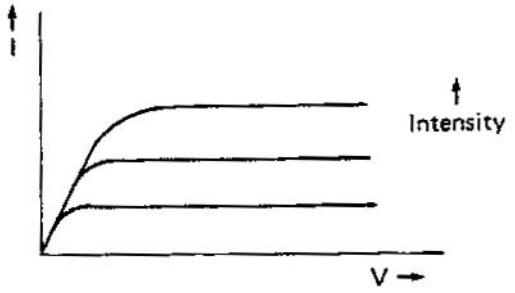

figure 3

The same arguments may be applied to saturation in photomultipiers although the assumption of zero or low initial electron velocity is certainly invalid[15].

Experimental data[4] shows reasonable agreement with **figure 3**.

In discussing saturation in photomultpliers it has become customary to take in terms of cathode (K) to first dynode (d1) saturation, and interdynode saturation. The reason for this division is that additional problems, which give rise to saturation, occur in the K – d1 region. The major problem is concerned with semitransparent cathodes which have ahigh surface resistance – typically $\approx 10^{10}$ Ωcm^{-2}. Hence for relatively high cathode currents (0.02 $\mu A\ cm^{-2}$) a considerable voltage drop across the K – d1 region occurs which can give rise to a saturation (4,5) effect. Furthermore, in unfocused photomultipliers the weaker electric field in the peripheral region of the cathode can accentuate saturation.

1.2 effects of saturation

The effects of saturation are:

1.2.1 **non-linearity** That non-linearity between light input and anode current output arises is obvious from the above discussion. In practical terms the non-linearity is described rather loosely, e.g. "for a given dynode chain operating at a given voltage the departure from linearity at an anode current of 8 mA is 10% and at 20 mA is 50%". What occurs outside these specified conditions is something one has to hazard an intelligent guess at using Child's law.

1.2.2 **distortion of pulse shape** This occurs due to saturation increasing the transit time spread of the current pulse. This is dealt with later.

1.3 reduction and elimination of saturation

Saturation may be reduced by:

1.3.1 **suitable choice of photomultiplier** Manufacturers produce photomultpliers specifically designed to handle high currents. It should be pointed out that focused photomultpliers have better saturation characteristics than venetian blind or box and grid types. Also side window tubes are to be preferred since the effective cathode surface resistance is less than that of the semitransparent end-on photomultplier.

1.3.2 **non-linear dynode chain** This is really concerned with interdynode saturation. The photomultiplier is operated with the last few dynode stages operated at progressively higher voltages (6). One method of achieving this is to use a "non-linear" dynode chain as shown in **figure 4** below. Alternatively zener diodes may be employed.

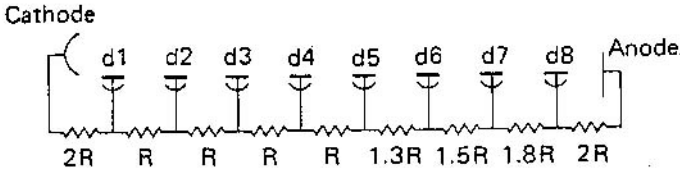

figure 4

1.3.3 **special techniques** A variety of special techniques have been tried. Some of these are shown in **figure 5**. The first two methods, 5(a) and 5(b), employ a similar form of dynode configuration. The photomultipier output signal is taken from the central region of the dynode chain and, more importantly, in 5(b), the secondary electrons emitted by the output dynode are extracted (and undergo further amplification) by subsequent dynode stages. That these methods give increased linearity at high dynode currents suggests that in standard circuits, (e.g. **figure 4**) saturation in the final stages of the photomultplier is as much a consequence of low velocity secondary electrons emitted from the anode as "conventional" saturation described by the Child-Langmuir relationship. This view is further endorsed by the fact that the circuit of **figure 5(b)**, which employes a further two stages of amplfication

beyond the output dynode, gives a higher output current (50 mA) that that of 5(a) (≈6 mA).

A third method, shown in **figure 5(c)**, is primarily intended to improve the time response of nanosecond photomultpliers. However, it is found that in addition the circuit arrangement shown also inreases the amplification and saturation current of the photomultplier. The anode of the photomultiplier is enclosed in a coaxial shield with a shield grid window which permits electrons to enter the anode region. The secondary electrons which are emitted from the anode (which is at a potential of –500V w.r.t. the shield) are collected by the shield and form the photomultiplier output signal.

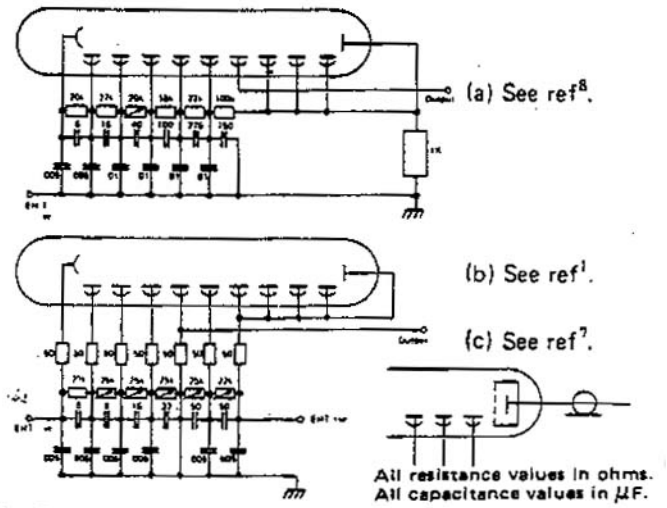

figure 5

1.4 uses of saturation

In the majority of cases saturation creates limits to photomultplier performance. However, it has been employed to some use in providing a photomultplier with “logarithmic” output charactertistic and in pulse shape discrimination (9).

2 transit time and transit time spread

The output (anode current) of a photomultiplier when a short pulse (ideally a delta function) of light is incident upon the cathode has the form shown in **figure 6**.

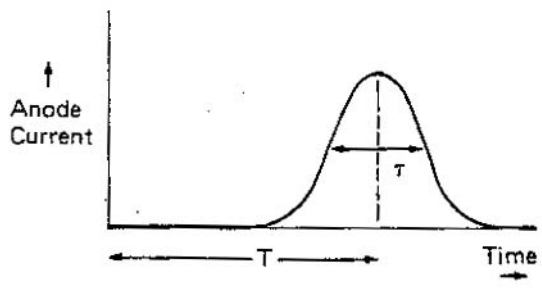

figure 6

The transit time, T, is defined as the time taken to collect half the total number of electrons at the output (10) (Gaussian statistics assumed) and the transit time spread, τ, as the full width at half height. Typically, $T - 50\ ns, = \tau\ \ 20\ ns$ for “standard” photomultipliers.

From a practical point of view the transit time, which represents the time for the current pulse to pass from cathode to anode, merely introduces a delay within the experimental system whereas the transit time spread, representing statistical fluctuations in T, introduces limitations in the form of resolving time and pulse shape distortion.

Consider two infinite parallel plane electrodes placed a distance, d, apart with a potential difference of V between them.

If an electron is emitted with a velocity u_o then its potential at a point x is given by:

$$V(x) = V_o + \frac{x}{d}V \quad where\ V_o = \frac{mu_o^{\,2}}{2e}$$

its velocity at x, $U(x)$, is:

$$U(x) = \sqrt{\frac{2e}{m}\left(V_o + \frac{x}{d}V\right)} \qquad \ldots(5)$$

and the transit, T,

$$T = \int_o^d \frac{dx}{U(x)} = \int_o^d \frac{dx}{\sqrt{\frac{2e}{m}\left(V_o + \frac{x}{d}V\right)}} \qquad \ldots(6)$$

$$T = \frac{2d}{V}\sqrt{\frac{m}{2e}}\left[\sqrt{V_o + V} - \sqrt{V_o}\right] \qquad \ldots(7)$$

In practice $V >> V_o$

$$\therefore T = \frac{d}{V}\sqrt{\frac{2m}{e}}\left[\sqrt{V} - \sqrt{V_o}\right] \qquad \ldots(8)$$

$$T = d\sqrt{\frac{2m}{eV}}\left[1 - \sqrt{\frac{v_o}{V}}\right] \qquad \ldots(9)$$

Despite all the implied assumptions in deriving (9), good agreement with experiment is observed[11,12,13].

Typically for one stage, $d = 1\ cm,\ V = 100\ volts.\ V_o \gg 2.0\ eV\ and\ T = 3.0\ ns$.

2.1 causes of transit time speed

There are four basic causes:

2.1.1 **spread in emission time of the secondary electrons** For all practical purposes this contri-

bution to transit time spread may be ignored since the time of emission $\approx 10^{-11}$ seconds (10,14).

2.1.2 **spread in emission velocities of the secondary electrons.** From (9) we have:

$$\frac{\Delta T}{T} = \sqrt{\frac{\Delta V_o}{V_o}} \qquad \ldots(10)$$

If ΔV_o represents the full width at half height of emission distribution then $\Delta T \approx \tau$.

Typically, $T = 3.0\ ns,\ \Delta V_o = 0.75\ eV$ (15)m giving $\tau = 1.8\ ns$ per stage for N stages, $\tau_{10} \approx 5.8\ ns$.

At the cathode an additional spread in electron velocities occurs due to the variation of emission velocity with wavelength $(V_o \propto \lambda^{-½})$. Equation **(6)** assumes that the field between the electron emitter and collector is constant. In the real case this is untrue, particularly for the K – d_1 region, whch will also contribute to τ.

2.1.3 **spread in electron path length** This problem is very complex since it depends not only upon the point of electron emission but also upon the angle of emission. The greatest contributions to τ arise in the K – d_1 region, **figure 7**.

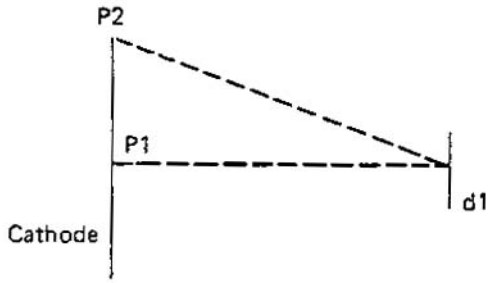

figure 7

For a potential difference of 100 volts between the cathode and d_1, the time spread in electrons emitted along the trajectory as shown is approximately 3 ns (5" photomultiplier geometry). However, this does not take into account thte time difference between electrons emitted normally at P1 and tangentially at P2. Furthermore, the weaker electric field in the region of P2 will cause the electron to execute a longer patch before reaching d1.

2.1.4 **space charge effects** In the presence of space charge (5) becomes

$$U(x) = \sqrt{\frac{2e}{m}\left[V_o + \left(\frac{x}{d}\right)^{4/3} V\right]} \qquad \ldots(11)$$

yielding

$$T = d\ \sqrt{\frac{9m}{eV}}\left[1 - 0.85\left(\frac{V_o}{V}\right)^{0.25}\right] \qquad \ldots(12)$$

and $$\frac{\Delta T}{T} = 0.85\left(\frac{\Delta V_o}{V_o}\right)^{0.25} \qquad \ldots(13)$$

Using $T = 3\ ns,\ \Delta V_o = 0.75\ eV,\ V_o = 2.0\ eV$ gives $\tau = 5.5\ ns$ per stage.

Thus with space charge the transit time spread is increased by a factor of about 3.5.

2.2 **effects of transit time spread** The overall effect of transit time spread must be determined for each particular case. One case will be considered here as an example – that of the simple scintillation counter. The output for a scintillation counter may be considered as the sum of a series of exponential decays (16).

Considering a single component then the variation of intensity with time is:

$$\Phi(t) = \Phi o\ exp(-t/\tau_d)\ \alpha \qquad \ldots(14)$$

where τ is the decay time. Assuming ideal conversion at the photocathode the time distribution of electrons leaving the cathode is:

$$N(t) = \frac{N_o}{\tau_d} exp(-t/\tau_d)d \qquad \ldots(15)$$

where N_o is the total number of electrons leaving the cathode. Assuming that Gaussian statistics may be applied the shape of the current pulse at the anode for electrons emitted in the interval t to $(t+dt)$ is:

$$I = \frac{N(t)e}{\sqrt{\pi}\tau} exp - \left(\frac{t-T}{\tau}\right)^2 \qquad \ldots(16)$$

Combining (15) and (16) and replacing t in (14) by t', the current pulse at the anode is:

$$I(t) = \int_o^t \frac{N_o e}{\sqrt{\pi}\tau\tau_d} exp - \left(\frac{t-t'-T}{\tau}\right)^2 exp - \left(\frac{t'}{\tau_d}\right) dt' \ldots(17)$$

Details of the evaluation of (17) are given by Millman (10) and th results are best displayed graphically, **figure 8**.

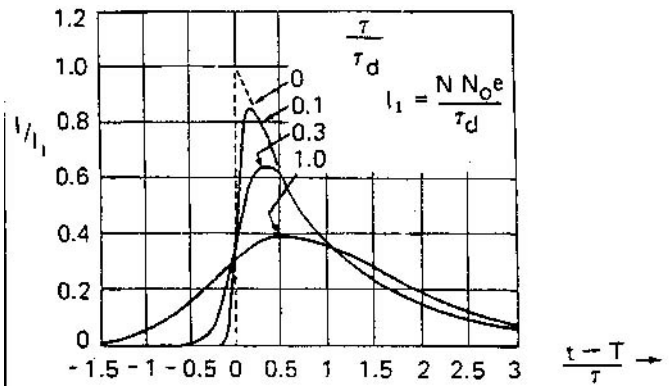

figure 8

2.3 **reduction of transit time spread**

Transit time spread my be reduced by:

2.3.1 **high inter-stage voltages**, which decrease T (equation (9)) and consequently ΔT (equation (10)). However, there are limits to the maximum inter-stage voltage which may be employed, set by breakdown and the fact that the secondary emission coefficient decreases at high field strenghts (15).

2.3.2 **focused systems** These are designed to make electron trajectories of the same (physical) length and to provide uniform electric, accelerating fields. It is found that for focused structures the electron multiplication efficiency is reduced, which suggests that not only does the focused system make path-lengths the same but at the same time secondary electrons emitted at wide angles are rejected.

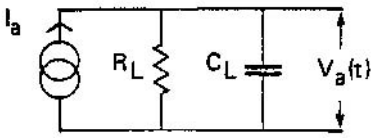

figure 9

2.3.3 **avoiding saturation**, taking care to operate the photomultiplier in its linear region.

3 output circuit

The equivalent output circuit of a photomultiplier is shown in **figure 9**.

From Kirchhoff's laws

$$I_a(t) = \frac{V_a}{R_L} + C_L \frac{dV_a}{dt} \qquad \ldots(18)$$

Assuming that $Ia(t)$ is a single exponential of the form

$$I_a(t) = I_o \exp - (t/\tau_d) \qquad \ldots(19)$$

then substituting (19) in (10) and solving for Va

$$V_a(t) = \frac{I_o \hat{o}_d R_L}{\hat{o}_d - R_L C_L}\left[(\exp-(t/\hat{o}_d) - (\exp-(t/R_L C_L)\right) \qquad \ldots(20)$$

There are two cases of (20 to be considered)

(a) $R_L C_L \gg \tau_d$

$$V_a(t) = \frac{I_o \hat{o}_d}{C_L} \exp-(t/R_L C_L) \qquad \ldots(21)$$

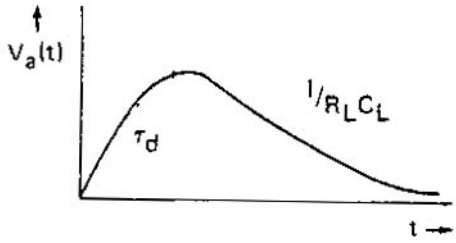

figure 10

the output has a rise time of τ_d with a peak voltage of

$$V_a(max) = \frac{I_o \tau_d}{C_L} \qquad \ldots(22)$$

which represents the total charge in the output pulse being deposited on the capacitor. The pulse decays slowly with a time constant of $R_L\ C_L$, **figure 10**.

(b) $\tau_d \gg R_L C_L$

$$V_a(t) = I_o R_L \exp\text{-}(t/\tau_d) \qquad \ldots(23)$$

Here $V_a(t)$ represents the current pulse itself.

In some measurements the total charge in the pulse is required, in which case the condition $R_L C_L \gg \tau_d$ is chosen. However, the low decay constant $R_L C_L$ presents problems since pile-up occurs unless $R_L C_L \gg N^{-1}$ *(N = pulse rate)*.

A further drawback is that the rise time τ_d prevents the use of this ciruit for fast timing applications.

For most pulse applications a low load impedance is employed.

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