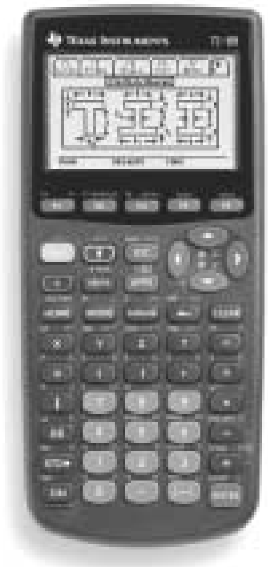
ME•*Pro*®

Mechanical Engineering

User's Manual



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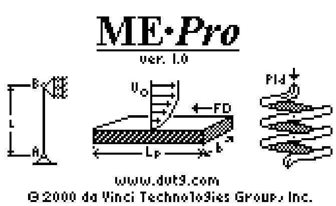

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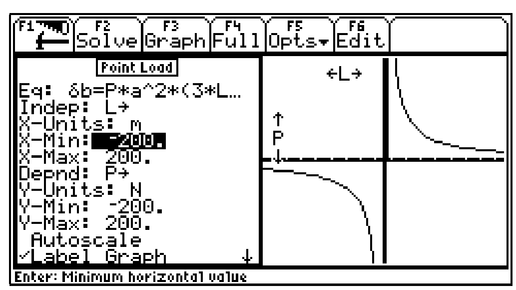

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ME•*Pro*®

A software Application
For TI-89 and TI-92 Plus

User's Guide

August 2000

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Notice

This manual and the examples contained herein are provided "as is" as a supplement to ME•*Pro* application software available from Texas Instruments for TI-89, and TI-92 Plus platforms. **da Vinci Technologies Group, Inc. ("da Vinci") makes no warranty of any kin d with regard to this manual or the accompanying software, including, but not limited to, the implied warranties of merchantability and fitness for a particular purpose**. da Vinci shall not be liable for any errors or for incidental or consequential damages in connection with the furnishing, performance, or use of this manual, or the examples herein.

We welcome your comments on the software and the manual. Forward your comments, preferably by e-mail, to da Vinci at support@dvtg.com.

Acknowledgements

The ME•*Pro* software was developed by Chris Bunsen, Dave Conklin, Michael Conway, Curtis Gammel, and Megha Shyam with the generous support of TI's development team. The user's guide was developed by Michael Conway, Curtis Gammel, Melinda Shaffer, and Megha Shyam. Many helpful comments from the testers at Texas Instruments and other locations during β testing phase is gratefully acknowledged.

Table of Contents

TABLE OF CONTENTS4

CHAPTER 1: INTRODUCTION TO ME•PRO ***12***
- 1.1 Key Features of ME•*Pro* 12
- 1.2 Purchasing, Downloading and Installing ME•*Pro* 13
- 1.3 Ordering a Manual 13
- 1.4 Memory Requirements 13
- 1.5 Differences between TI-89 and TI-92 plus 13
- 1.6 Starting ME•*Pro* 13
- 1.7 How to use this Manual 14
- 1.8 Manual Disclaimer 14
- 1.9 Summary 15

PART I: ANALYSIS 16

CHAPTER 2: INTRODUCTION TO ANALYSIS ***17***
- 2.1 Introduction 17
- 2.2 Features of Analysis 18
- 2.3 Finding Analysis 18
- 2.4 Solving a Problem in Analysis 18
- 2.5 Tips for Analysis 20
- 2.6 Function keys 20
- 2.7 Session Folders, Variable Names 22
- 2.8 Overwriting of variable values in graphing 22
- 2.9 Reserved Variables 22

CHAPTER 3: STEAM TABLES ***23***
- 3.1 Saturated Steam Properties 23
- 3.2 Superheated Steam Properties 23
- 3.3 Air Properties 23
- 3.4 Using Steam Tables 24
- 3.5 Validity Range for Temperature and Pressure 25

CHAPTER 4: THERMOCOUPLES ***26***
- 4.1 Introduction 26
- 4.2 Using the Thermocouples Function 26
- 4.3 Basis for Temperature/Voltage Conversions 27

CHAPTER 5: CAPITAL BUDGETING ***28***
- 5.1 Using Capital Budgeting 28

CHAPTER 6: EE FOR MECHANICAL ENGINEERS ***32***
- 6.1 Impedance Calculations 32
- 6.2 Circuit Performance 33
- 6.3 Wye ↔ Δ Conversion 34

CHAPTER 7: EFFLUX ***36***
- 7.1 Constant Liquid Level 36
- 7.2 Varying Liquid Level 36
- 7.3 Conical Vessel 37
- 7.4 Horizontal Cylinder 38
- 7.5 Large Rectangular Orifice 38
- 7.6 ASME Weirs 39
  - 7.6.1 Rectangular Notch 39
  - 7.6.2 Triangular Weir 39
  - 7.6.3 Suppressed Weir 40
  - 7.6.4 Cipolletti Weir 40

CHAPTER 8: SECTION PROPERTIES ***42***
- 8.1 Rectangle 42

8.2 Hollow Rectangle 43
8.3 Circle........ 43
8.4 Circular Ring 44
8.5 Hollow Circle 45
8.6 1 Section - Uneven........ 45
8.7 I Section - Even 46
8.8 C Section........ 47
8.9 T Section 48
8.10 Trapezoid........ 48
8.11 Polygon 49
8.12 Hollow Polygon........ 50
CHAPTER 9: HARDNESS NUMBER ***52***
9.1 Compute Hardness Number 52
PART II: EQUATIONS……...54
CHAPTER 10: INTRODUCTION TO EQUATIONS ***55***
10.1 Solving a Set of Equations 55
10.2 Viewing an Equation or Result in *Pretty Print* 56
10.3 Viewing a Result in different units 56
10.4 Viewing Multiple Solutions........ 57
10.5 when (…) - conditional constraints when solving equations 58
10.5 Arbitrary Integers for periodic solutions to trigonometric functions........ 58
10.7 Partial Solutions........ 59
10.8 Copy/Paste........ 59
10.9 Graphing a Function........ 59
10.10 Storing and recalling variable values in ME•*Pro*-creation of session folders........ 61
10.11 solve, nsolve, and csolve and user-defined functions (UDF) 61
10.12 Entering a guessed value for the unknown using *nsolve* 61
10.13 Why can't I compute a solution?........ 62
10.14 Care in choosing a consistent set of equations........ 62
10.15 Notes for the advanced user in troubleshooting calculations 62
CHAPTER 11: BEAMS AND COLUMNS ***64***
11.1 Simple Beams........ 64
11.1.1 Uniform Load 64
11.1.2 Point Load 66
11.1.3 Moment Load 68
11.2 Cantilever Beams........ 70
11.2.1 Uniform Load 70
11.2.2 Point Load 71
11.2.3 Moment Load 73
11.3. Columns........ 75
11.3.1 Buckling........ 75
11.3.2 Eccentricity, Axial Load........ 76
11.3.3 Secant Formula 77
11.3.4 Imperfections in Columns 79
11.3.5 Inelastic Buckling 81
CHAPTER 12: EE FOR MES ***83***
12.1 Basic Electricity........ 83
12.1.1 Resistance Formulas........ 83
12.1.2 Ohm's Law and Power 84
12.1.3 Temperature Effect 85
12.2 DC Motors........ 86
12.2.1 DC Series Motor 86
12.2.2 DC Shunt Motor........ 88
12.3 DC Generators........ 90
12.3.1 DC Series Generator 90
12.3.2 DC Shunt Generator........ 91

12.4 AC Motors........ 92
12.4.1 Three ϕ Induction Motor I........ 92
12.4.2 Three ϕ Induction Motor II........ 94
12.4.3 1 Induction Motor........ 96
CHAPTER 13: GAS LAWS........ 98
13.1 Ideal Gas Laws........ 98
13.1.1 Ideal Gas Law........ 98
13.1.2 Constant Pressure........ 99
13.1.3 Constant Volume........ 101
13.1.4 Constant Temperature........ 103
13.1.5 Internal Energy/Enthalpy........ 104
13.2 Kinetic Gas Theory........ 106
13.3 Real Gas Laws........ 108
13.3.1 van der Waals: Specific Volume........ 108
13.3.2 van der Waals: Molar form........ 109
13.3.3 Redlich-Kwong: Sp.Vol........ 110
13.3.4 Redlich-Kwong: Molar........ 112
13.4 Reverse Adiabatic........ 113
13.5 Polytropic Process........ 115
CHAPTER 14: HEAT TRANSFER........ 118
14.1 Basic Transfer Mechanisms........ 118
14.1.1 Conduction........ 118
14.1.2 Convection........ 120
14.1.3 Radiation........ 121
14.2 1 1D Heat Transfer........ 122
14.2.1 Conduction........ 122
14.2.1.1 Plane Wall........ 122
14.2.1.2 Convective Source........ 123
14.2.1.3 Radiative Source........ 125
14.2.1.4 Plate and Two Fluids........ 127
14.2.2 Electrical Analogy........ 128
14.2.2.1 Two Conductors in Series........ 129
14.2.2.2 Two Conductors in Parallel........ 131
14.2.2.3 Parallel-Series........ 132
14.2.3 Radial Systems........ 135
14.2.3.1 Hollow Cylinder........ 135
14.2.3.2 Hollow Sphere........ 136
14.2.3.3 Cylinder with Insulation Wrap........ 137
14.2.3.4 Cylinder - Critical radius........ 139
14.2.3.5 Sphere - Critical radius........ 141
14.3 Semi-Infinite Solid........ 142
14.3.1 Step Change Surface Temperature........ 142
14.3.2 Constant Surface Heat Flux........ 143
14.3.3 Surface Convection........ 145
14.4 Radiation........ 146
14.4.1 Blackbody Radiation........ 146
14.4.2 Non-Blackbody radiation........ 148
14.4.3 Thermal Radiation Shield........ 149
CHAPTER 15: THERMODYNAMICS........ 152
15.1 Fundamentals........ 152
15.2 System Properties........ 153
15.2.1 Energy Equations........ 153
15.2.2 Maxwell Relations........ 155
15.3 Vapor and Gas Mixture........ 157
15.3.1 Saturated Liquid/Vapor........ 157
15.3.2 Compressed Liquid-Sub cooled........ 159

15.4 Ideal Gas Properties 160
15.4.1 Specific Heat 160
15.4.2 Quasi-Equilibrium Compression 162
15.5 First Law 163
15.5.1 Total System Energy 163
15.5.2 Closed System: Ideal Gas 167
15.5.2.1 Constant Pressure 167
15.5.2.2 Binary Mixture 169
15.6 Second Law 171
15.6.1 Heat Engine Cycle 171
15.6.1.1 Carnot Engine 171
15.6.1.2 Diesel Cycle 173
15.6.1.3 Dual Cycle 177
15.6.1.4 Otto Cycle 180
15.6.1.5 Brayton Cycle 184
15.6.2 Clapeyron Equation 186
CHAPTER 16: MACHINE DESIGN 188
16.1 Stress: Machine Elements 188
16.1.1 Cylinders 188
16.1.2 Rotating Rings 189
16.1.3 Pressure and Shrink Fits 190
16.1.4 Crane Hook 192
16.2 Hertzian Stresses 193
16.2.1 Two Spheres 193
16.2.2 Two Cylinders 195
16.3.1 Bearing Life 197
16.3.2 Petroff's law 198
16.3.3 Pressure Fed Bearings 199
16.3.4 Lewis Formula 200
16.3.5 AGMA Stresses 201
16.3.6 Shafts 203
16.3.7 Clutches and Brakes 204
16.3.7.1 Clutches 204
16.3.7.1.1 Clutches 204
16.3.7.2 Uniform Wear - Cone Brake 206
16.3.7.3 Uniform Pressure - Cone Brake 207
16.4 Spring Design 208
16.4.1 Bending 208
16.4.1.1 Rectangular Plate 208
16.4.1.2 Triangular Plate 209
16.4.1.3 Semi-Elliptical 210
16.4.2 Coiled Springs 212
16.4.2.1 Cylindrical Helical - Circular wire 212
16.4.2.2 Rectangular Spiral 213
16.4.3 Torsional Spring 215
16.4.3.1 Circular Straight Bar 215
16.4.3.2 Rectangular Straight Bar 216
16.4.4 Axial Loaded 217
16.4.4.1 Conical Circular Section 217
16.4.4.2 Cylindrical - Helical 219
16.4.4.2.1 Rectangular Cross Section 219
16.4.4.2.2 Circular Cross Section 220
CHAPTER 17: PUMPS AND HYDRAULICS 222
17.1 Basic Definitions 222
17.2 Pump Power 223
17.3 Centrifugal Pumps 225

17.3.1 Affinity Law-Variable Speed ... 225
17.3.2 Affinity Law-Constant Speed ... 226
17.3.3 Pump Similarity ... 227
17.3.4 Centrifugal Compressor ... 228
17.3.5 Specific Speed ... 229
CHAPTER 18: WAVES AND OSCILLATION ... 231
18.1 Simple Harmonic Motion ... 231
18.1.1 Linear Harmonic Oscillation ... 231
18.1.2 Angular Harmonic Oscillation ... 232
18.2 Pendulums ... 233
18.2.1 Simple Pendulum ... 233
18.2.2 Physical Pendulum ... 235
18.2.3 Torsional Pendulum ... 236
18.3 Natural and Forced Vibrations ... 236
18.3.1 Natural Vibrations ... 236
18.3.1.1 Free Vibration ... 236
18.3.1.2 Overdamped Case (ξ>1) ... 238
18.3.1.3 Critical Damping (ξ=1) ... 239
18.3.1.4 Underdamped Case (ξ<1) ... 241
18.3.2 Forced Vibrations ... 244
18.3.2.1 Undamped Forced Vibration ... 244
18.3.2.2 Damped Forced Vibration ... 245
18.3.3 Natural Frequencies ... 247
18.3.3.1 Stretched String ... 247
18.3.3.2 Vibration Isolation ... 248
18.3.3.3 Uniform Beams ... 249
18.3.3.3.1 Simply Supported ... 250
18.3.3.3.2 Both Ends Fixed ... 251
18.3.3.3.3 1 Fixed End / 1 Free End ... 252
18.3.3.3.4 Both Ends Free ... 254
18.3.3.4 Flat Plates ... 255
18.3.3.4.1 Circular Flat Plate ... 255
18.3.3.4.2 Rectangular Flat Plate ... 257
CHAPTER 19: REFRIGERATION AND AIR CONDITIONING ... 259
19.1 Heating Load ... 259
19.2 Refrigeration ... 261
19.2.1 General Cycle ... 261
19.2.2 Reverse Carnot ... 262
19.2.3 Reverse Brayton ... 263
19.2.4 Compression Cycle ... 264
CHAPTER 20: STRENGTH MATERIALS ... 267
20.1 Stress and Strain Basics ... 267
20.1.1 Normal Stress and Strain ... 267
20.1.2 Volume Dilation ... 268
20.1.3 Shear Stress and Modulus ... 269
20.2 Load Problems ... 270
20.2.1 Axial Load ... 270
20.2.2 Temperature Effects ... 271
20.2.3 Dynamic Load ... 272
20.3 Stress Analysis ... 274
20.3.1 Stress on an Inclined Section ... 274
20.3.2 Pure Shear ... 275
20.3.3 Principal Stresses ... 276
20.3.4 Maximum Shear Stress ... 277
20.3.5 Plane Stress - Hooke's Law ... 278
20.4 Mohr's Circle Stress ... 280

20.4 Mohr's Circle Stress 280
20.5 Torsion 281
20.5.1 Pure Torsion 281
20.5.2 Pure Shear 283
20.5.3 Circular Shafts 284
20.5.4 Torsional Member 285
CHAPTER 21: FLUID MECHANICS 288
21.1 Fluid Properties 288
21.1.1 Elasticity 288
21.1.2 Capillary Rise 289
21.2 Fluid Statics 291
21.2 1 Pressure Variation 291
21.2.1.1 Uniform Fluid 291
21.2.1.2 Compressible Fluid 292
21.2.1 Pressure Variation 292
21.2.1.3 Troposphere 292
21.2.1.4 Stratosphere 293
21.2.2.1 Floating Bodies 294
21.2.2.2 Inclined Plane/Surface 296
21.3 Fluid Dynamics 297
21.3.1 Bernoulli Equation 297
21.3.2 Reynolds Number 299
21.3.3 Equivalent Diameter 300
21.3.4 Fluid Mass Acceleration 302
21.3.4.1 Linear Acceleration 302
21.3.4.2 Rotational Acceleration 303
21.4 Surface Resistance 304
21.4.1 Laminar Flow – Flat Plate 304
21.4.2 Turbulent Flow – Flat Plate 306
21.4.3 Laminar Flow on an Inclined Plane 309
21.5 Flow in Conduits 311
21.5.1 Laminar Flow: Smooth Pipe 311
21.5.2 Turbulent Flow: Smooth Pipe 313
21.5.3 Turbulent Flow: Rough Pipe 316
21.5.4 Flow pipe Inlet 319
21.5.5 Series Pipe System 321
21.5.6 Parallel Pipe System 322
21.5.7 Venturi Meter 324
21.5.7.1 Incompressible Flow 324
21.5.7.2 Compressible Flow 327
21.6 Impulse/Momentum 330
21.6.1 Jet Propulsion 330
21.6.2 Open Jet 331
21.6.2.1 Vertical Plate 331
21.6.2.2 Horizontal Plate 332
21.6.2.3 Stationary Blade 333
21.6.2.4 Moving Blade 335
CHAPTER 22: DYNAMICS AND STATICS 338
22.1 Laws of Motion 338
22.2 Constant Acceleration 340
22.2.1 Linear Motion 340
22.2.2 Free Fall 341
22.2.3 Circular Motion 342
22.3 Angular Motion 343
22.3.1 Rolling/Rotation 343
22.3.2 Forces in Angular Motion 345

22.3.3 Gyroscope Motion ... 346
22.4 Projectile Motion ... 347
22.5 Collisions ... 349
22.5.1 Elastic Collisions ... 349
22.5.1.1 1D Collision ... 349
22.5.1.2 2D Collisions ... 350
22.5.2 Inelastic Collisions ... 351
22.5.2.1 1D Collisions ... 351
22.5.2.2 Oblique Collisions ... 352
22.6 Gravitational Effects ... 354
22.6.1 Law of Gravitation ... 354
22.6.2 Kepler's Laws ... 356
22.6.3 Satellite Orbit ... 358
22.7 Friction ... 360
22.7.1 Frictional Force ... 360
22.7.2 Wedge ... 362
22.7.3 Rotating Cylinder ... 363
22.8 Statics ... 364
22.8.1 Parabolic cable ... 364
22.8.2 Catenary cable ... 365
PART III: REFERENCE ... 368
CHAPTER 23: INTRODUCTION TO REFERENCE ... 369
23.1 Introduction ... 369
23.2 Finding Reference ... 369
23.3 Reference Screens ... 370
23.4 Using Reference Tables ... 370
CHAPTER 24: ENGINEERING CONSTANTS ... 372
24.1 Using Constants ... 372
CHAPTER 25: TRANSFORMS ... 374
25.1 Using Transforms ... 374
CHAPTER 26: VALVES AND FITTING LOSS ... 376
26.1 Valves and Fitting Loss Screens ... 376
CHAPTER 27: FRICTION COEFFICIENTS ... 377
27.1 Friction Coefficients Screens ... 377
CHAPTER 28: RELATIVE ROUGHNESS OF PIPES ... 378
28.1 Relative Roughness Screens ... 378
CHAPTER 29: WATER-PHYSICAL PROPERTIES ... 379
29.1 Water-Physical Properties Screens ... 379
CHAPTER 30: GASES AND VAPORS ... 380
30.1 Gases and Vapors Screens ... 381
CHAPTER 31: THERMAL PROPERTIES ... 382
31.1 Thermal Properties Screens ... 382
CHAPTER 32: FUELS AND COMBUSTION ... 383
32.1 Fuels and Combustion Screens ... 383
CHAPTER 33: REFRIGERANTS ... 385
33.1 Refrigerants Screens ... 386
CHAPTER 34: SI PREFIXES ... 387
34.1 Using SI Prefixes ... 387
CHAPTER 35: GREEK ALPHABET ... 388
PART IV: APPENDIX AND INDEX ... 389
APPENDIX A FREQUENTLY ASKED QUESTIONS ... 390
A.1 Questions and Answers ... 390
A.2 General Questions ... 390
A.3 Analysis Questions ... 392
A.4 Equations Questions ... 392
A.5 Graphing ... 395

A.6 Reference........ 396
APPENDIX B WARRANTY, TECHNICAL SUPPORT........ 397
B.1 da Vinci License Agreement........ 397
B.2 How to Contact Customer Support........ 398
APPENDIX C: TI-89 & TI-92 PLUS- KEYSTROKE AND DISPLAY DIFFERENCES........ 399
C.1 Display Property Differences between the TI-89 and TI-92 Plus........ 399
C.2 Keyboard Differences Between TI-89 and TI-92 Plus........ 400
APPENDIX D ERROR MESSAGES........ 404
D.1 General Error Messages........ 404
D.2 Analysis Error Messages........ 405
D.3 Equation Messages........ 405
D.4 Reference Error Messages........ 406
APPENDIX E: SYSTEM VARIABLES AND RESERVED NAMES........ *407*
INDEX........ 408

Chapter 1: Introduction to ME•*Pro*

Thank you for purchasing the ME•*Pro*, a member of the PocketProfessional® Pro software series designed by da Vinci Technologies Group, Inc., to meet the portable computing needs of students and professionals in mechanical engineering. The software is organized in a hierarchical manner so that the topics easy to find. We hope that you will find the ME•*Pro* to be a valuable companion in your career as a student and a professional of mechanical engineering.

Topics in this chapter include:

Key Features of ME•*Pro*
Purchasing, Download and Installing ME•*Pro*
Ordering a Manual
Memory Requirements
Differences between the TI-89 and TI-92 Plus
Starting the ME•*Pro*
How to use this Manual
Manual Disclaimer
Summary

1.1 Key Features of ME•*Pro*

The manual is organized into three sections representing the main menu headings of ME•*Pro*.

Analysis	***Equations***	***Reference***
Steam Tables	Beams and Columns	Engineering Constants
Thermocouples	EE For MEs	Transforms
Capital Budgeting	Gas Laws	Valves/Fitting Loss
EE For MEs	Heat Transfer	Friction Coefficients
Efflux	Thermodynamics	Roughness of Pipes
Section Properties	Machine Design	Water Physical Properties
Hardness Number	Pumps and Hydraulic Machines	Gases and Vapors
	Waves and Oscillation	Thermal Properties
	Refrigeration and Air Conditioning	Fuels and Combustion
	Strength Materials	Refrigerants
	Fluid Mechanics	SI Prefixes
	Dynamics and Statics	Greek Alphabet

These main topic headings are further divided into sub-topics. A brief description of the main sections of the software is listed below:

Analysis: Chapters 2-9

Analysis is organized into 7 topics and 25 sub-topics. The software tools available in this section incorporate a variety of analysis methods used by mechanical engineers. Examples include Steam Tables, Thermocouple Calculations, EE for MEs; Efflux, Section Properties, Hardness Number Computations and Capital Budgeting. Where appropriate, data entered supports commonly used units.

Equations: Chapters 10-22

This section contains over 1000 equations organized under 12 major subjects in over 150 sub -topics. The equations in each sub-topic have been selected to provide maximum coverage of the subject material. In addition, the math engine is able to compute multiple or partial solutions to the equation sets. The computed values are filtered to identify results that have engineering merit. A powerful built-in unit management feature permits inputs in SI or other customary measurement systems. Over 80 diagrams help clarify the essential nature of the problems covered by the equations. Topics covered include, **Beams and Columns; EE for MEs; Gas Laws; Heat Transfer; Thermodynamics; Machine Design; Pumps and Hydraulic Machines; Waves and Oscillation; Strength of Materials; Fluid Mechanics; and, Dynamics and Statics.**

Reference: Chapters 23-25

The *Reference* section contains tables of information commonly needed by mechanical engineers. Topics include, values for Constants used by mechanical engineers; **Laplace and Fourier Transform tables; Valves and Fitting Loss; Friction Coefficient; Roughness of Pipes; Water Physical Properties; Gases and Vapors; Thermal Properties; Fuels and Combustion; Refrigerants; SI prefixes; and the Greek Alphabet.**

1.2 Purchasing, Downloading and Installing ME•*Pro*

The ME•*Pro* software can only be purchased on-line from the Texas Instruments Inc. Online Store at *http://www.ti.com/calc/docs/store.htm.* The software can be installed directly from your computer to your calculator using TI-GRAPH LINK™ hardware and software (sold separately). Directions for purchasing, downloading and installing ME•*Pro* software are available from TI's website.

1.3 Ordering a Manual

Chapters and Appendices of the Manual for ME•*Pro* can be downloaded through TI's Web Store and viewed using the free Adobe Acrobat Reader™ that can be downloaded from *http://www.adobe.com.* Printed manuals can be purchased separately from da Vinci Technologies Group, Inc. by visiting the website *http://www.dvtg.com/ticalcs/docs or calling (541) 754-2860, Extension 100.*

1.4 Memory Requirements

The ME•*Pro* program is installed in the system memory portion of the Flash ROM that is separate from the RAM available to the user. ME•*Pro* uses RAM to store some of its session information, including values entered and computed by the user. The exact amount of memory required depends on the number of user-stored variables and the number of session folders designated by the user. To view the available memory in your TI calculator, use the function [2nd][MEM]. It is recommended that at least 10K of free RAM be available for installation and use of ME•*Pro*.

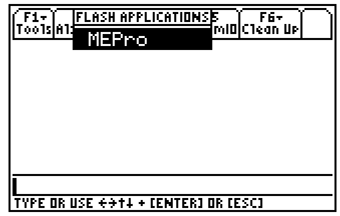
1.5 Differences between TI-89 and TI-92 plus

ME•*Pro* is designed for two models of graphing calculators from Texas Instruments, the TI-92 Plus and the TI-89. For consistency, keystrokes and symbols used in the manual are consistent with the TI-89. Equivalent key strokes for the TI-92 Plus are listed in Appendix D.

1.6 Starting ME•*Pro*

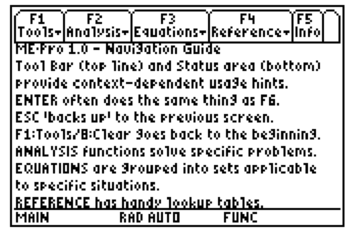
To begin ME•*Pro*, start by pressing the [APPS] key. This accesses a pull down menu. Use the ⊝ key to move the cursor bar to **FlashApps...**. and press [ENTER]. Then move the highlight bar to ME•*Pro* and press the [ENTER] key to get to the home screen of ME•*Pro*. Alternatively, press [◆][APPS]; then, scroll to ME•*Pro* and press the [ENTER] key to get to the home screen of ME•*Pro*.

Pull down Menu for APPS
*(***FlashApps***...option is at the top of the list)*



Pull down Menu on for FlashApps...
*(*ME•*Pro will be in the list)*

The ME•*Pro* home screen is displayed to the right. The tool bar at the top of the screen lists the titles of the main sections of ME•*Pro* which can be activated by pressing the function keys.



F1**: Tools:** Editing features, information about ME•*Pro* in **A: About**.

F2**: Analysis:** Accesses the ***Analysis*** section of the software.

F3**: Equations:** Accesses the ***Equations*** section of the software.

F4**: Reference:** Accesses the ***Reference*** section of the software.

F5**: Info:** Helpful hints on ME•*Pro.*

To select a topic, use the ⊙ key to move the highlight bar to the desired topic and press ENTER , or alternatively type the number next to the item. The **Analysis, Equation** and **Reference** menus are organized in a menu tree of topics and sub-topics. The user can return to a previous level of ME•*Pro* by pressing ESC. You can exit ME•*Pro* at any time by pressing the HOME key. When ME•*Pro* is restarted, the software returns to its previous location in the program.

1.7 How to use this Manual

The manual section, chapter heading and page number appear at the bottom of each page. The first chapter in each of the *Analysis*, *Equations* and *Reference* sections gives an overview of succeeding chapters and introduces the navigation and computation features common to each of the main sections. For example, Chapter 2 explains the basic layout of the *Analysis* section menu and the navigation principles, giving examples of features common to all topics in *Analysis*. Each topic in Analysis has a chapter dedicated to describing its functionality in detail. The titles of these chapters correspond to the topic headings in the software menus. They contain example problems and screen displays of the computed solutions. Trouble-shooting information, commonly asked questions, and a bibliography used to develop the software are provided in appendixes.

1.8 Manual Disclaimer

The calculator screen displays in the manual were obtained during the testing stages of the software. Some screen displays may appear slightly different due to final changes made in the software while the Manual was being completed.

1.9 Summary

The designers of ME•*Pro* invite your comments by logging on to our website at http://www.dvtg.com or by e-mail to improvements@dvtg.com. We hope that you agree we have made complex computations easy with the software by providing the following features:

- *Easy-to-use, menu-based interface.*
- *Computational efficiency for speed and performance.*
- *Helpful-hints and context-sensitive information provided in the status line.*
- *Advanced ME analysis routines, equations, and reference tables.*
- *Comprehensive manual documentation for examples and quick reference.*

Part I: Analysis

Chapter 2: Introduction to Analysis

2.1 Introduction

The analysis section contains subroutines and tools designed to perform specific calculations. Computations include estimating thermodynamic properties of water at different temperature and pressure, in **Steam Tables**, computing fluid flow rates through different shaped orifices in **Efflux**, performing *Wye to Δ circuit conversions* of AC circuits in **EE for MEs**, and evalueating cash flow for different projects in **Capital Budgeting**. The computations are strictly top-down (i.e. the inputs and outputs are generally the same) and the interface for each section guides the user through the solving process. A brief description of some of the different sections in Analysis appear below:

Steam Tables (3 sections): *Saturated Steam, Superheated Steam, Air Properties* computes the thermodynamic parameters of steam including saturated pressure, enthalpy, entropy, internal energy, and specific volume of the liquid and vapor forms of water given entries of temperature and/or pressure. This final topic covered computes the thermodynamic properties of dry air at different temperatures.

Thermocouples: This tool converts a specified temperature to an *emf* output in millivolts (mV) or from *emf* output millivolts (mV) to a specified temperature. The software supports **T, E, J, K, S, R** and **B type** thermocouples. These computation algorithms result from the IPTS-68 standards adopted in 1968 and modified in 1985.

Capital Budgeting: This section performs analysis of capital expenditure for a project and compares projects against one another. Four measures of capital budgeting are included in this section: Payback period (**Payback**); Net Present Value (**NPV**); Internal Rate of Return (**IRR**); and Profitability Index (**PI**). This module provides the capability of entering, storing and editing capital expenditures for nine different projects. Projects can be graphed on **NPV** vs. **k** scale.

EE for Mechanical Engineers (3 sections): Performs evaluations on three types of circuits: *Impedance calculations; Circuit Performance; Wye↔Δ Circuit conversion* **Impedance Calculations,** computes the impedance admittance of a circuit consisting of a resistor, capacitor and inductor connected in Series or Parallel. **Performance parameters** section computes load voltage and current, complex power delivered, power factor, maximum power available to the load, and the load impedance required to receive the maximum power from a single power source. The final segment of the software converts configurations expressed as a **Wye to** its **Δ** equivalent. It also performs the reverse computation.

Efflux (6 sections): *Constant Liquid level; Varying liquid level; Conical Vessel; Horizontal Cylinder; Large Rectangular Orifice; ASME Weirs (Rectangular notch; Triangular Weir; Suppressed Weir; Cipolletti Weir)* This section contains methods to compute fluid flow via cross sections of different shapes.

Section Properties (12 sections): *Rectangle; Hollow Rectangle; Circle; Circular Ring (Annulus); Uneven I-section; Even I-section; C section; T section; Trapezoid; Polygon (n-sided); Hollow Polygon (n-sided, side thickness)* Computes area moment and location of center of mass for different shaped cross sections. Computed parameters include the cross section area, the polar moment of inertia, the area moment of inertia and radius of gyration on x and y axes.

Hardness Number: A dimensionless number is a measure of the yield of a material from impact. Brinell and Vicker developed two popular methods of measuring the Hardness number. These tests consist of dropping a 10 mm ball of steel with a specified load such as 500 lbf and 3000 lbf. This steel ball results in an indentation in the material. The diameter of indentation indicates of the hardness number using either the Brinell's or Vicker's formulation.

2.2 Features of Analysis

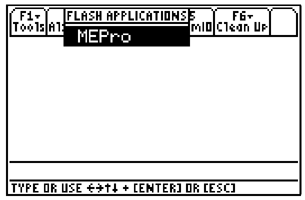
Unit Management: Appropriate unit menus for appending units to variable entries or converting computed results are accessible in most sections.

Numeric Computation – Variable entries must consist of real numbers (unless specified). Algebraic expressions must consist of defined variables so a numeric value can be condensed upon entry.

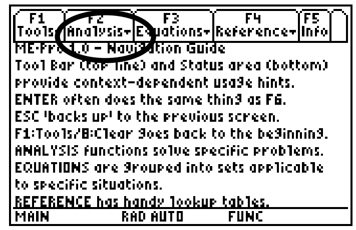
2.3 Finding Analysis

The following panels illustrate how to start ME•*Pro* and locate the **Analysis** section.

1. *Press the* APPS *key in the HOME screen to list the applications stored in your calculator.*

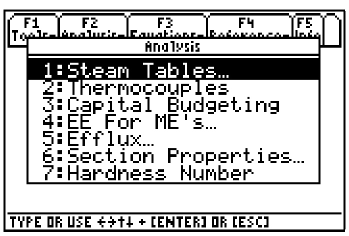


2. *Press* **1:FlashApps** *and press* ENTER *to display the applications stored in the Flash section of memory.*[1]

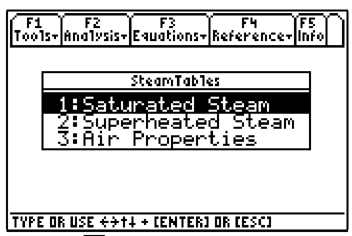


3. *HOME screen of ME•Pro. Analysis is listed as* F2 *on the top function key row.*

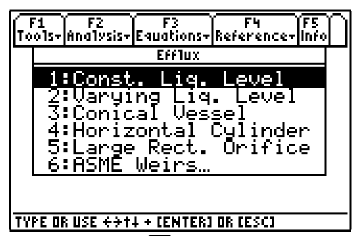
There are seven sections under **Analysis**. To select a topic, use the ⊙ key to move the highlight bar to the desired heading and pressing ENTER, or alternatively type the number next to the item to select. If a topic contains several sections (Steam Tables, EE for MEs, Efflux, Section properties, an ellipsis (…) will appear next to the title (see below).



From the home screen of ME•*Pro* ***Press*** F2 *to display the Analysis menu…*



Press 1 *for topics in* ***Steam Tables****…*



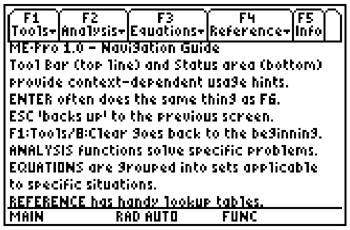
…or, press 5 *for topics in* ***Efflux.***

2.4 Solving a Problem in Analysis

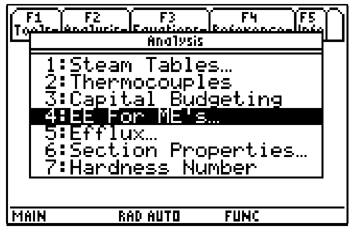
The following example presents some of the navigational features in **Analysis**. This example is drawn from Chapter 6: **EE for MEs**.

Problem - Calculate the performance parameters of a circuit consisting of a current source (10 **-** 5**i*) with a source admittance of .0025 **-** .0012**I*, a load of .0012 + .0034**i*. Display the real result of power in kilowatts.

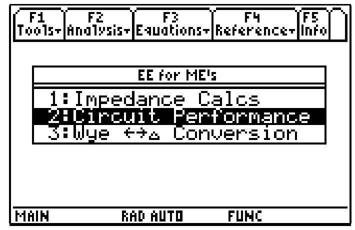
[1] Steps 1 and 2 can be combined by pressing ♦ and APPS.



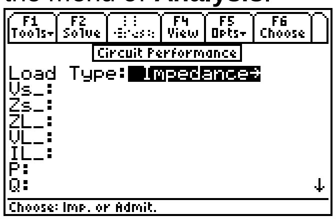
1. *From the home screen of ME• Pro, Press* [F2] to display the menu of **Analysis**.



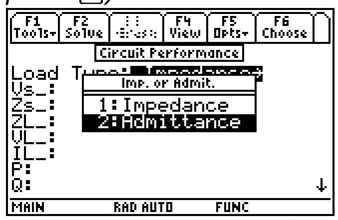
2. *Move the cursor to* **EE for MEs** *and press* [ENTER] *(or press* [4]*).*



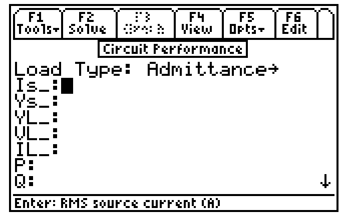
3. *Select* **Circuit Performance** *from the submenu in* **EE for Mes.**



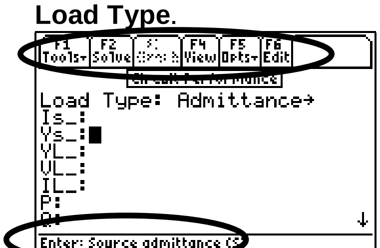
4. *While the cursor is highlighting* **Load Type**, *press the right arrow key,* ⓡ *or* [F6] *to display the menu for* **Load Type**.



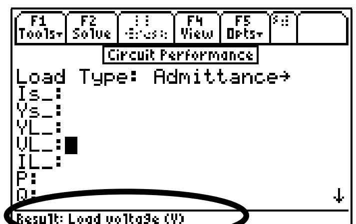
5. *In the menu for* **Load Type** *move the cursor to* **Admittance** *and press* [ENTER].



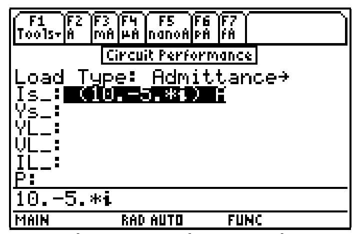
6. **Admittance** *is now selected for* **Load Type** *and the appropriate variables are displayed.*



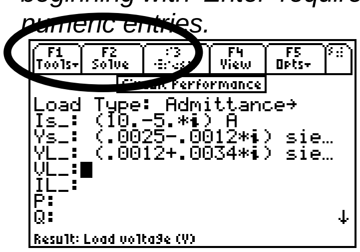
7. *Variable descriptions beginning with 'Enter' require numeric entries.*



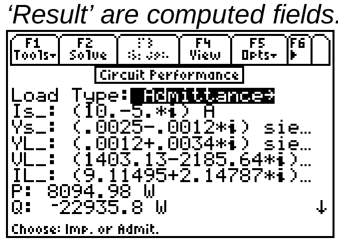
8. *Variable descriptions beginning with the word 'Result' are computed fields.*



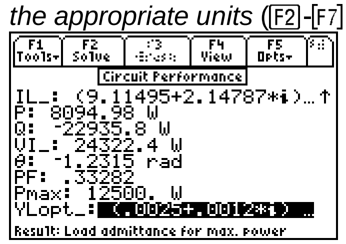
9. *When entering a value, press a function key to add the appropriate units* ([F2]-[F7]).



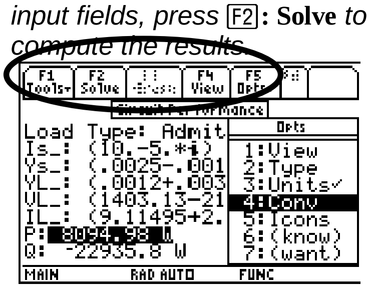
10. *Following entry of all input fields, press* [F2]**: Solve** *to compute the results.*



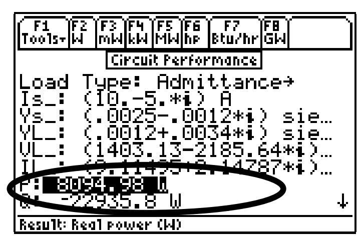
11. *Results: Upper Half*



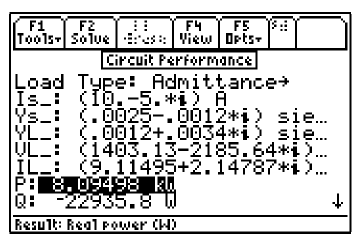
12. *Results: Lower Half*



13. *To display a result in different units, highlight the variable and press* [F5]**:Opts,** *move the cursor to* **4:Conv.**



14. *The unit menu for the variable appears in the top bar. Press the function key corresponding to the desired units.*



15. *The computed value for Real Power,* ***P****, is now displayed in kilowatts (****kW****).*

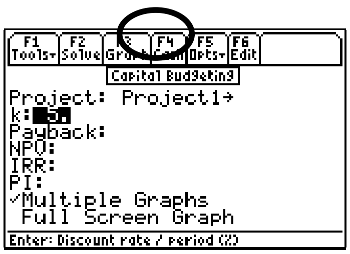
There are two types of interfaces in **Analysis**:

Type 1: Input/Output/Choose Fields (*Steam Tables, Thermocouples, EE for MEs, Efflux, Section Properties, and Hardness Number*). This input form lists the variables for which a numeric entry is required and prompts the user to choose a calculation setting if applicable before computing the results. The entries and results are always displayed in the same screen.

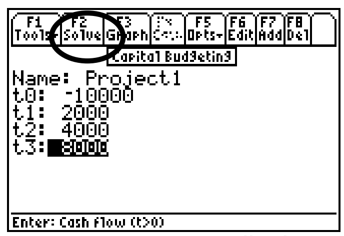
Type 2: Multiple Forms/Graphing (*Capital Budgeting*) This interface includes most of the features of

Type 1 with the additional screens used for entering cash flow for individual projects. The graphing features of the calculator are enabled in this section for visualizing the rate of return (Net Present Value vs. discount rate). An example of this interface is described briefly in this chapter, but in more detail in Chapter 5: Capital Budgeting.

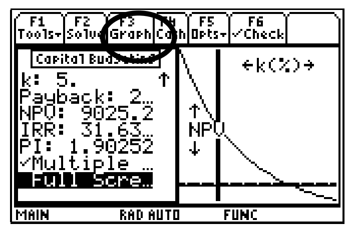
Capital Budgeting allows the user to compare relative financial performance of several projects with relevant data such as **Interest rate or discount rate (k), IRR, NPV, or Payback period**. The screen displays below illustrate the basic user interface.



*Input Screen for **Capital Budgeting**. Press [F4] to display **Cash** Flow for Project 1.*



*A separate screen displays the **Cash** Flow for 'Project 1'. Press [F2]: Solve to revert to the previous screen.*



*Press [F3]: **Graph**. Selecting '**Multiple Graphs**' allows the overlap of plots for different projects (Project 1, Project 2, etc.)*

2.5 Tips for Analysis

The following instructions are useful in the **Analysis** section:

1. If an ellipsis (…) appears at the end of a menu title, a menu of subtopics exists in this section.
2. An arrow '→' to the right of a heading, as in **Load Type**, indicates an additional menu.
3. Variables ending with an underscore '_', such as **Vs_**, **Zs_**, and **IL_**, allow complex values.
4. Descriptions for variables generally appear in the status line when the variable is highlighted.
5. Variables for which an entry is required will have a description prefaced by the word '**Enter**'. Computed variables begin with the word, '**Result**'.
6. To convert values from one unit to another, press [F5]**:Opts,** and **4:Conv** to display the unit menu for the variable at the top of the screen. Press the function key corresponding to the appropriate units.
7. To return to the previous level of ME•*Pro*, press [ESC].
8. To exit ME•*Pro*, press [F1]: Tools and [8]: Clear.
9. To return to ME•*Pro*, press [2nd] [APPS].
10. To toggle between a graph and ME•*Pro* in split-screen mode, press [2nd] [APPS].
11. To remove the split screen in ME•*Pro*. 1) Press [MODE], 2) [F2]: Page 2, 3) (▶): Split Screen App., 4) [1]: Full Screen, 5) [ENTER]: Save.

2.6 Function keys

When **Analysis** functions are selected, the function keys in the tool bar access or activate features, which are specific to the context of the section. They are listed in **Table 2-1**:

Table 2-1 Description of Analysis Function keys

Function Key	Description
[F1]	Labeled "**Tools**" - displays all the functions available on the *TI-89* at the *Home* screen level. These functions are: **1: Open** – This opens an existing folder to store or recall variables used in an ME•*Pro* session. **2: (save as)** – Not active in Analysis. **3: New** – Creates a new folder for storing variable values used in an ME•*Pro* session. **4: Cut** - Removes entered values into the clipboard. Enabled for variables for which the user can enter values. **5: Copy** – Copies a highlighted value into the clipboard. **6: Paste** – Pastes clipboard contents at cursor location. **7: Delete** – Deletes highlighted values. **8: Clear** – Returns to the HOME screen of ME•*Pro.* **9: (format)**-Not active in Analysis. **A: About** - Displays product name and version number.
[F2]	Labeled "**Solve**" - Pressing this key enables the software to begin solving a selected problem and display any resulting output to the user.
[F3]	Labeled "**Graph**" **-** This feature is available in input screens where the solution can be represented in graphical form. A graph can be viewed in the full screen or a split screen mode. This can be performed by pressing [MODE] followed by [F2]. Use [2nd] and [APPS] to toggle between the data entry screen and graph window.
[F4]	Normally labeled as "**View**" **-** This enables the information highlighted by the cursor to be displayed using the entire screen in *Pretty Print* format. In some cases [F4] is labeled as "**Pict**", "**Cash**". "**Pict**" is available in the **Section Properties** or **Hardness Number** sections and when selected displays a diagram to facilitate better understanding of the problem. "**Cash**" is used in **Capital Budgeting** section of the software.
[F5]	Labeled "**Opts**" **-** This key displays a pop up menu listing the options: **1: View -** allows the highlighted item to be viewed using *Pretty Print*. **2: (type) -** Not active **3: Units** – This activates, or deactivates the unit management feature. **4: Conv** – Displays the unit menu for the highlighted variable and allows the conversion of an entry or result into different units. **5: Icons -** Presents a dialog box identifying certain Icons used by the software to display content and context of the information. **These icon systems are only used in equations**. **6: (know)-** Not active **7: Want -** Not active
[F6]	"**Edit**" **-** Brings in a data entry line for the highlighted parameter. "**Choose**" in **Capital Budgeting** enables the user to select from one of nine projects. "√ Check" requesting the user to press this key to select a highlighted parameter for use in an Analysis computation.
[F7]	"**Add**" Adds a cash flow entry for a project in **Capital Budgeting** section.
[F8]	(Not active)

2.7 Session Folders, Variable Names

ME•*Pro* automatically stores its variables in the current folder specified by the user in [MODE] or the HOME screen. The current folder name is displayed in the lower left corner of the screen (default is "Main"). To create a new folder to store values for a particular session of ME•*Pro*, press [F1] **:/TOOLS,** [3] **:/NEW** and type the name of the new folder (see Guidebook for the complete details of creating and managing folders; Chapter 5 for the TI-89 and Chapter 10 for the TI-92 Plus).

There are several ways to display or recall a value:

- The contents of variables in any folder can be displayed using [VAR-LINK], moving the cursor to the variable name and pressing [F6] to display the contents of a particular variable.
- Variables in a current folder can be recalled in the HOME screen by typing the variable name.
- All inputs and calculated results from the *Analysis* and *Equations* section are saved as variable names. Previously calculated, or entered values for variables in a folder are replaced when equations are solved using new values for inputs.

2.8 Overwriting of variable values in graphing

When an equation or analysis function is graphed, ME•*Pro* creates a function for the TI grapher, which expresses the dependent variable in terms of the independent variable. This function is stored under the variable name *pro (x)*. When ME•*Pro*'s equation grapher is executed, values are inserted into the independent variable for *pro (x)* and values for the dependent value are calculated. Whatever values previously existed in either of the dependent and independent variables in the current folder are cleared. To preserve data under variable names, which may conflict with ME•*Pro*'s variables, run ME•*Pro* in a separate folder.

2.9 Reserved Variables

A list of reserved variable names used by the TI operating system, which cannot be used as user variable names or entries are listed in **Appendix F.**

Chapter 3: Steam Tables

Steam properties are a complex function of temperature, pressure, volume, critical temperature, critical pressure, and molecular weight of water. Our software takes into account the ranges of temperature and pressure that results in good fits to data gathered over a long periods in many parts of the world.

Steam Tables offer a collection of programs organized as a powerful computational engine to calculate thermodynamic properties of steam in a user-friendly environment. Calculations of thermodynamic properties are based on standards and conventions adopted by the *International Conventions* covering properties of saturated and superheated steam.

3.1 Saturated Steam Properties

This section computes the properties of saturated steam at a single temperature or pressure.

Variable	Description	Units
Ps	Saturation pressure	MPa
Ts	Saturation temperature	K
Vf	Specific volume – liquid	m^3/kg
Vg	Specific volume – vapor	m^3/kg
Hf	Enthalpy – liquid	kJ/kg
Hfg	Latent heat of vaporization	kJ/kg
Hg	Enthalpy – vapor	kJ/kg
Sf	Entropy – liquid	kJ/(kg·K)
Sfg	S (g) - S (f)	kJ/(kg·K)
Sg	Entropy – vapor	kJ/(kg·K)
UF	Internal energy – liquid	kJ/(kg·K)
UG	Internal energy – vapor	kJ/(kg·K)

3.2 Superheated Steam Properties

The properties of superheated steam require two inputs: temperature and pressure. From the data supplied, the program will compute saturated temperature, specific volume, enthalpy and entropy. The data is displayed in a tabular form.

Variable	Description	Units
Temp	Given temperature	K
Sat Pressure	Given pressure	MPa
Sat Temperature	Corresponding temperature	K
Specific Volume	Specific volume	m^3/kg
Enthalpy	Enthalpy	kJ/kg
Entropy	Entropy	kJ/(kg·K)

3.3 Air Properties

The properties of dry air are computed using the ideal gas law model as the basis. Using temperature as an input, the software computes a variety of parameters including specific heats, enthalpy, entropy, and velocity of sound.

Variable	Description	Units
Temp	Given temperature	K
CP	Specific heat at constant pressure	J/(kg·K)

Variable	Description	Units
CV	Specific heat at constant volume	J/(kg·K)
H	Enthalpy	J/kg
U	Internal energy	J/kg
E	Entropy function	J/(kg·K)
IPR	Isentropic pressure function	unitless
IVR	Isentropic volume function	unitless
G	Specific heat ratio	unitless
A	Speed of sound	m/s

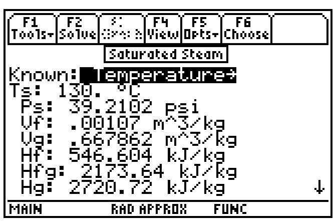
3.4 Using Steam Tables

Once you have selected STEAM TABLES at the main menu, the first screen displays three subtopics - Saturated steam, Superheated steam and Air properties. Selecting "*saturated steam properties*", allows properties to be calculated from user-entered value of temperature or pressure. However, properties for "*superheated steam*" require values for both temperature and pressure. Thermodynamic properties of air are calculated for dry air conditions only.

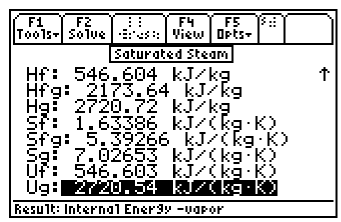
Example 3.1:

Calculate the properties of saturated steam at 130 °C.

Solution - Select the Saturated Steam section. The input screen calls for defining known parameter (temperature or pressure). The default condition is temperature. Move the pointer to the next line and start entering the temperature. The function keys assume unit assignments for the data about to be entered. For our example, enter 130 and press F3 key thereby attaching °C to the value just entered. Press F2 to solve for the thermodynamic parameters.



Upper Display



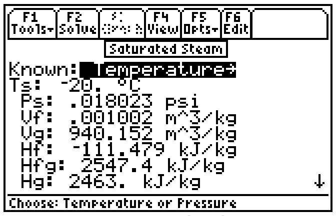
Lower Display

All the calculated parameters are displayed on the screen with SI units attached as shown. If you desire to see the value of saturated pressure parameter Ps, use the ↓ key to move the highlight bar to capture Ps. Press F5 to display a pull down menu of items to select. Select 4 (Conv). This allows other units for Ps such as Pa, kPa, atm, psi, torr attached to F3, F4, F5, F6, and F7 respectively. Pressing F6 converts the value of Ps into the new units of psi. The display is refreshed immediately in the units just selected.

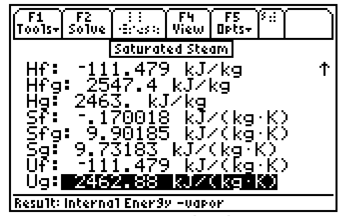
Example 3.2:

Calculate the properties of superheated steam at 125 °C and 20 psi.

Solution - Select the Superheated Steam section. The input screen calls for entering temperature and pressure. Move the pointer to enter the 125 °C and 20 psi for temperature and pressure. Make sure that the appropriate units are attached to the data using the function keys. Press F2 to solve for the thermodynamic parameters.



Upper Display



Lower Display

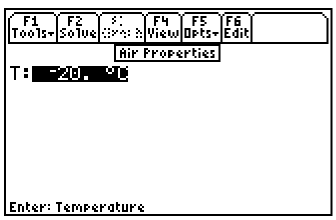
All the calculated parameters are displayed on the screen with units attached as shown. The displayed results could be converted to other units as described in the first example described earlier.

Example 3.3:

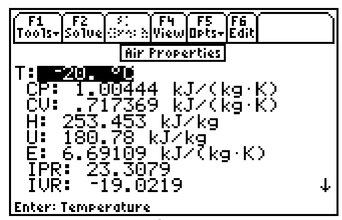
Calculate the properties of dry air to be -20 °C.

Select the Air Properties section. The input screen calls for defining the temperature. Enter a temperature of – 20 °C. When entering -20 °C, be sure to use the unary operator key (-) followed by 20 °C. If you use − key for a negative value this will result in an input error.

Press F2 to solve for the thermodynamic properties of air.



Input Screen



Result screen

All the calculated parameters are displayed on the screen with SI units attached as shown. The parameters computed can be viewed in other units as described in the examples shown here.

3.5 Validity Range for Temperature and Pressure

The computed results are valid only for the following finite ranges of temperature and pressure:

Saturated
Temperature: 273.16 - 647.3 K
Pressure: 0.006113 - 22.08 MPa

Superheated
Saturated temperature
Pressure: 0.006113 - 22.08 MPa

References:

1. Lester Haar, John S. Gallagher and George S. Kell, NBS/NRC Steam Tables, Thermodynamic and Transport Properties for Vapor and Liquid States of Water, Hemisphere Publishing Corporation, Washington, DC.
2. Steam Tables, 1967; Thermodynamic properties of Water and Steam, The Electric Research Association, Edward Arnold Limited, London England, 1967
3. Thomas F. Levine, Jr., and Peter E. Liley, Steam and Gas Tables with Computer Equations, Academic Press, New York, NY, 1984

Chapter 4: Thermocouples

This chapter describes using the software in the **Thermocouples** menu. Thermocouple parameters are calculated for the class of thermocouples in common usage.

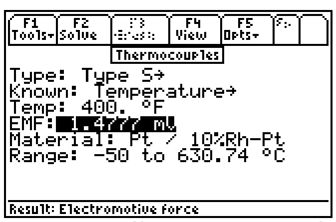
4.1 Introduction

This tool converts a specified temperature to an *emf* output, millivolts (mV), and from an *emf* output, millivolts (mV), to a specified temperature. The software supports **Type T, E, J, K, S, R** and **B** thermocouples. The underlying assumption is a reference temperature of 0°C. These computation algorithms result from the IPTS-68 standards adopted in 1968 and modified in 1985.

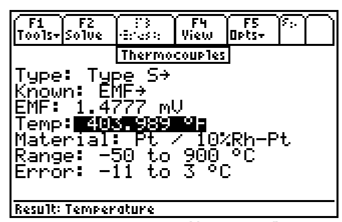
4.2 Using the Thermocouples Function

Select the Thermocouples function from the Analysis menu by highlighting Thermocouples and pressing [ENTER]. This action brings out the main user interface screen. Press ▶ to view the choices available. Seven types of Thermocouples are available for computations. For example, to select a Pt-10%Rh-Pt thermocouple referred to as S Type thermocouple, move the high the highlight bar to Type S and press [ENTER] or press [5]. At this point, the software presents the primary user interface awaiting data entry. At the same time, the material of the thermocouple and the valid range are displayed for reference.

Thermocouple computations involve computing emf available from a known temperature or computing a temperature from an emf. The latter case is by its very nature not as accurate as the first type of computation.



Screen 1 (for temperature)



Screen 2 (for emf)

Example 4.2:

Find the emf for an S type thermocouple at 400 °F. From the value of emf computed, compute the temperature.

Solution 4.2:

Select Type S thermocouple for this problem. For temperature, enter 400, then press [F3]. The computed emf is 1.4777mV. Now return to Known line and select emf for input. Enter 1.4777 mV for emf to get 403.989 °F for temperature.

Notes: The thermocouple emf calculation can be expanded to cover the emf produced by the thermocouple if the reference temperature was different from 0 °C. For example, if the reference temperature was 30 °C instead of 0 °C, you compute the resulting emf in two steps; first find the emf (emf 1) for the temperature desired, say 300 °C, and the emf (emf 0) for the reference temperature. The resulting emf for the new reference temperature of 30 °C is the difference between the two emf's, i.e., "emf 1 **-** emf 0".

4.3 Basis for Temperature/Voltage Conversions

The temperature-to-voltage conversion is based on either a polynomial approximation or a combination of a polynomial coupled with a special sequence. This ensures precise calculations within some prescribed error range. These rages are displayed on each input screen.

References:

1. Robert L. Powell, William J. Hall, Clyde H. Hyink, Larry L. Sparks, George W. Burns, Margaret Scoger and Harmon H. Plumb, Thermocouple Reference Tables based on IPTS-68, NBS Monograph 125, Omega Press, 1975

Chapter 5: Capital Budgeting

This chapter covers the four basic measures of capital budgeting:

- Payback Period
- Net Present Value
- Internal Rate of Return
- Profitability Index

5.1 Using Capital Budgeting

This section performs analysis of capital expenditure for a project and compares projects against one another. Four measures of capital budgeting are included in this section: Payback period (**Payback**); Net Present Value (**NPV**); Internal Rate of Return (**IRR**); and Profitability Index (**PI**). This module provides the capability of entering, storing and editing capital expenditures for nine different projects. The following equations are used in calculations:

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t} - CF_{t=0}$$ **Eq. 1**

$$\sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} - CF_{t=0} = 0$$ **Eq. 2**

$$PI = \frac{\sum_{t=1}^{n} \frac{CF_t}{(1+k)^t}}{CF_{t=0}}$$ **Eq. 3**

CF_t: Cash Flow at time t.

Payback: The number of time periods it takes a firm to recover its original investment.

NPV: The present values of all future cash flows, discounted at the selected rate, minus the cost of the investment.

IRR: The discount rate that equates the present value of expected cash flows to the initial cost of the project.

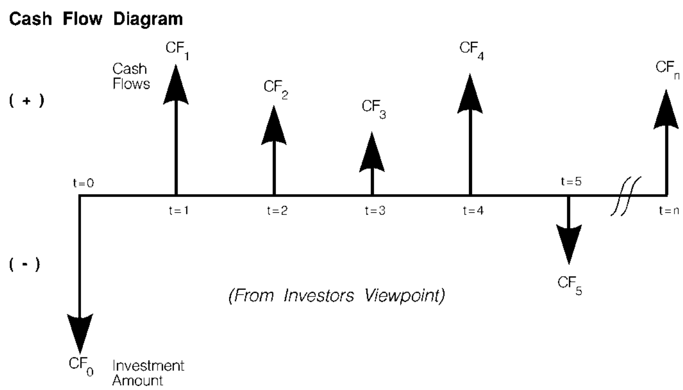
PI: The present value of the future cash flows, discounted at the selected rate, over the initial cash outlay.

Field Descriptions - Input Screen

Project:	(*Project*)	Press [ENTER] to select one of nine unique projects or edit the current name of the project by pressing [F4] for Cash option.
k:	(*Discount Rate per Period in %*)	Enter a real number.
Payback:	(*Payback Period*)	Returns a real number.
npv:	(*Net Present Value*)	Returns a real number.
IRR:	(*Internal Rate of Return*)	Returns a real number (%).
PI:	(*Profitability Index*)	Returns a real number.
Multiple Graphs	(Graph multiple projects simultaneously)	Activation of this feature enables the overlay of each successive graph (projects) on the same axis. Press [ENTER] to activate.
Full Screen Graph	(Graph on full or split screen?)	Press [ENTER] to activate.

Field Descriptions - Project Edit Screen

NAME:	(*Project Name*)	Enter the name of the project.
t0:	(*Investment at t=0*)	Enter a real number.
t1:	(*Cash flow at t=1*)	Enter a positive or negative real number.
tn:	(*Cash flow at t=n*)	Enter a positive or negative real number.

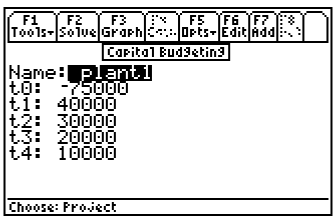


Example 5.1:

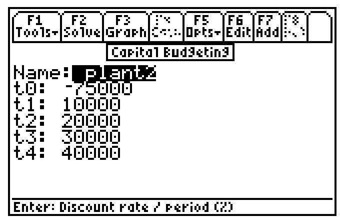
The following projects have been proposed by ACME Consolidated Inc. What are the Payback period, Net Present Value, Internal Rate of Return, and Profitability Index of each project? Which is the more viable project?

Table 5-1 Cash Flow for two projects

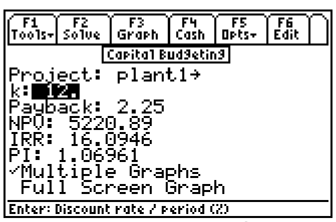
Name of Project: **Investment Outlay:** **Cost of Capital:**	**Plant 1** **$75,000** *(at t=0)* **12%**	**Plant 2** **$75,000** *(at t=0)* **12%**
Year	Net Cash Flow ($)	Net Cash Flow ($)
0	-75,000	-75,000
1	40,000	10,000
2	30,000	20,000
3	20,000	30,000
4	10,000	40,000



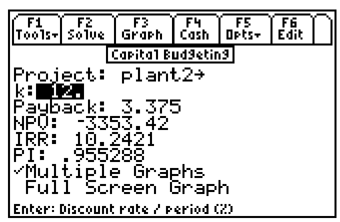
Cash Flow Input: plant1



Cash Flow Input: plant2



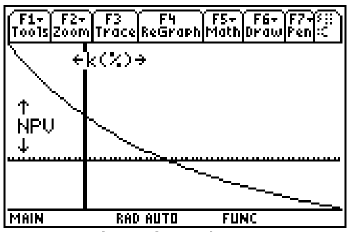
Output Screen: plant1



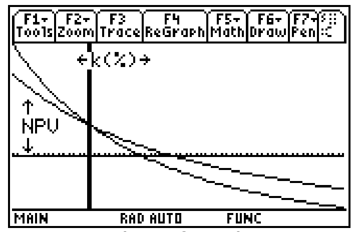
Output Screen: plant2

1. With the highlight bar on the **Project** field, press [ENTER] to select a project to edit. Select a project that has not been used. Note: this example uses projects 1 and 2. Press [ENTER] to return to the **Capital Budgeting** screen.
2. Press [F4] to select **Cash** option; enter the project edit screen; and, edit the cash flows.
3. Enter “plant1” in the **Name** field. **Note:** Cash flow data for this project will be stored in a variable of this name. Therefore the entered name must begin with a letter; be no more than 8 characters in length; and, contain no embedded spaces.
4. Press [F7] 5 times to add 5 time points and enter the cash flows at each time point from the table on the previous page. When finished, your screen should look like the project edit screen above. Be sure to enter 75,000 as a negative number for **t0**. Press [ESC] to save your changes and return to the **Capital Budgeting** screen.
5. Enter 12 for **k**.
6. Press [F2] to calculate **Payback**, **NPV**, **IRR**, and **PI**.
7. Move the highlight bar to **Multiple Graphs** and press [ENTER] to enable overlaying of successive graphs of each project.
8. Press [F3] to graph the curvilinear relationship between the **Net Present Value** and the **Discount Rate**.
9. Press [2nd] followed by [APPS] to enable the graph editing toolbar.

10. The curve indicates where k=0, the **Net Present Value** is simply cash inflows minus cash outflows. The IRR % is shown at the point where NPV=0. Using the built-in graphing capabilities of the TI 89, you can trace the graph to find the values of these two points. The TI 89 will give you the exact coordinates of any point along the graph. Press [◆] followed by [APPS] to return to the **Capital Budgeting** screen.
11. Repeating steps 1 through 9 for the second project, under the **Project** field, "plant2" and input the values in Table 5-1. Activating the **Multiple Graph** feature enables a simultaneous plot of the two projects. This will overlay a second graph on top of a previously plotted function. First plot *plant1*. After graphing, plot *plant2*. The first curve to appear, is *plant1*, the second is *plant2*. The most viable project in terms of discounted cash flows, in this example, is the one with the highest curve.
12. Pressing [MODE] [F2] (▶) [1] [ENTER] resets the display to full screen.



Plot of Project 1



Overlay of Project 2

Chapter 6: EE for Mechanical Engineers

This chapter describes the software in the **AC Circuits** section and is organized under three topics. These topics form the backbone of circuit calculations of interest to mechanical engineers.

- Impedance Calculations
- Wye ↔ Δ Conversion
- Circuit Performance

6.1 Impedance Calculations

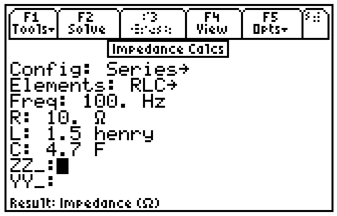
The **Impedance Calculations** topic computes the impedance and admittance of a circuit consisting of a resistor, capacitor and inductor connected in Series or Parallel. The impedance and admittance values are displayed to the user in real or complex form.

Field Descriptions

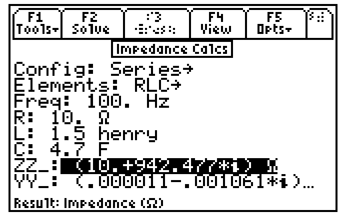
CONFIG:	*Circuit Configuration*	(Press [ENTER] and select **Series** or **Parallel** configuration by using ◯**.** After choosing**,** press [ENTER] to display the input screen updated for the new configuration..
Elements:	*Element Combination*	Press [ENTER] to display the following circuit elements: L, C, RL, RC, LC and RLC The choice of elements determines which of the input fields are available.
fr:	*Frequency in Hz*	Enter a real positive number.
R:	*Resistance in ohms; only appears if* RL, RC *or* RLC *is chosen in* **Elements** *field*	Enter a real positive number.
L:	*Inductance in Henry; only appears if* L, RL, LC *or* RLC *is chosen in* **Elements** *field*	Enter a real positive number.
C:	*Capacitance in Farads; only appears if C*, LC, RC *or* RLC *is chosen in* **Elements** *field*	Enter a real positive number.
ZZ_:	*Impedance in ohms*	Returns a real or complex number.
YY_:	*Admittance in Siemens*	Returns a real or complex number.

Example 6.1:

Compute the impedance of a series RLC circuit consisting of a 10-ohm resistor, a 1.5 Henry inductor and a 4.7 Farad capacitor at a frequency of 100 Hz.



Input Screen



Output Screen

1. Choose Series for **Config** and RLC for **Elements** using the procedure described above.
2. Enter 100 Hz for **Freq**.
3. Enter 10 Ω for **R**; 1.5 Henry for **L**; and 4.7 F for **C**.
4. Press [F2] to calculate **ZZ_** and **YY_**.
5. The output screen shows the results of computation.

6.2 Circuit Performance

This section shows how to compute the circuit performance of a simple load connected to a voltage or current source. Performance parameters include load voltage and current, complex power delivered, power factor, maximum power available to the load, and the load impedance required to receive the maximum power.

Field Descriptions - Input Screen

Load Type:	*Type of Load*	Press [ENTER] to select load impedance (Z) or admittance (Y). This will determine whether the remaining fields **Vs_**, **Zs_**, and **ZL_** or Is_, **Ys_**, and **YL_** are displayed, respectively.
Vs_:	*rms Source Voltage in V*	A real or complex number.
Zs_	*Source Impedance in Ω*	A real or complex number.
ZL_:	*Load Impedance in Ω*	A real or complex number.
Is_:	*rms Source Current in A*	A real or complex number.
ys_:	*Source Admittance in Siemens*	Enter a real or complex number.
yl_:	*Load Admittance in Siemens*	Enter a real or complex number.

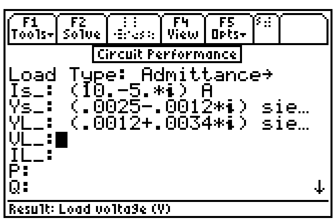
Field Descriptions - Output Screen

Vl_:	*Load Voltage in V*	Returns a real, complex number.
il_:	*Load Current in A*	Returns a real, complex number.
P:	*Real Power in W*	Returns a real number.
Q:	*Reactive Power in W*	Returns a real number.

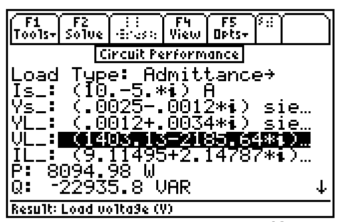
VI_:	*Apparent Power in W*	Returns a complex number.
θ:	*Power factor Angle in degrees or radian, determined by the* [MODE] *setting*	Returns a real number.
PF:	*Load Power Factor*	Returns a real number.
Pmax:	*Maximum Power Available in W*	Returns a real number.
Zlopt_:	*Load Impedance for Maximum Power in Ω - if* Impedance *is chosen for* Load Type *at the input screen*	Returns a real, complex number.
Ylopt_:	*Load Admittance for Maximum Power in Siemens - if* Admittance, *is chosen for* Load Type *at the input screen*	Returns a real, complex number.

Example 6.2:

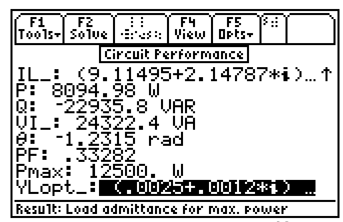
Calculate the performance parameters of a circuit consisting of a current source (10 - 5**i*) with a source admittance of .0025 - .0012**I*, and a load of .0012 + .0034**i*.



Input Screen



Output: Upper Half



Output: Lower Half

1. Choose Admittance for Load Type.
2. Enter the value 10 - 5*i A for Is_.
3. Enter the value .0025 - .0012* *i* siemens for Ys_, and .0012 + .0034* *i* siemens for a load of YL_.
4. Press [F2] to calculate the performance parameters.
5. The input and results of computation are shown above.

6.3 Wye ↔ Δ Conversion

The Wye ↔ Δ Conversion converts three impedances connected in Wye or Δ form to its corresponding Δ or Wye form, i.e., Wye ↔ Δ or Δ ↔ Wye

Input Fields -

Input Type: Selection choices are Δ→Wye or Wye→Δ. This determines whether the next 3 fields (input fields) accept Δ or Wye Impedances.

ZZA_: *Δ Impedance* Real or complex number.
ZZB_: *Δ Impedance* Real or complex number.
ZZC_: *Δ Impedance* Real or complex number.

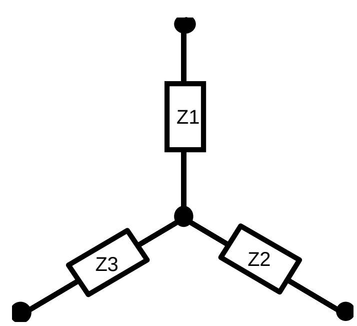


Fig. 6.1 Wye Network

ZZ1_: *Y Impedance* Real or complex number.
ZZ2_: *Y Impedance* Real or complex number.

ZZ3_: *Y Impedance* Real or complex number.

Result Fields

ZZA_: (*Δ Impedance*) Real or complex number.
ZZB_: (*Δ Impedance*) Real or complex number.
ZZC_: (*Δ Impedance*) Real or complex number.
ZZ1_: (*Y Impedance*) Real or complex number.
ZZ2_: (*Y Impedance*) Real or complex number.
ZZ3_: (*Y Impedance*) Real or complex number.

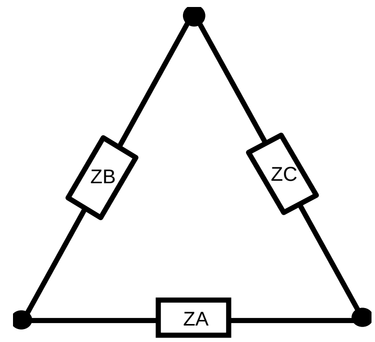
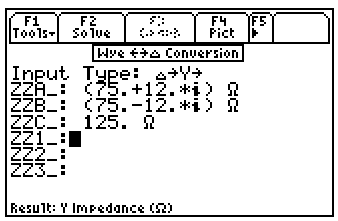


Fig. 6-2 Δ Network

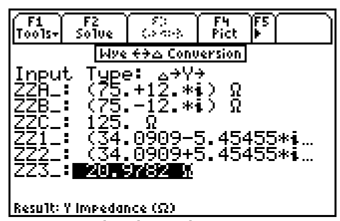
Example 6.3:

Compute the Wye impedance equivalent of a Δ network with impedances 75+12**i*, 75-12**i*, and 125 ohms.

1. Select Δ →Y for Input Type. .
2. Enter the values 75+12**i Ω*, 75 -12**i Ω*, 125 *Ω* for **ZZA_, ZZB_** and **ZZC_** respectively**.**
3. Press F2 to calculate **ZZ1_, ZZ2_** and **ZZ3_.**



Input Parameters



Calculated Output

The computed results are:

ZZ1_: 34.0909 **-** 5.45455· ***i*** *Ω*
ZZ2_: 34.0909 + 5.45455· ***i*** *Ω*
ZZ3_*:* 20.9782 *Ω*

References:

1. Sanford I. Heisler, The Wiley Engineer's Desk Reference, A concise guide for the Professional Engineer, John Wiley and Sons, New York, NY, 1984
2. James W. Nilsson, Electric Circuits, 2nd Edition, Addison-Wesley Publishing Company, Reading MA, 1987 and later editions.

Chapter 7: Efflux

This section of Analysis contains methods to compute fluid flow via cross sections of different shapes. Many of these formulas have been derived empirically over many years of experimental observations.

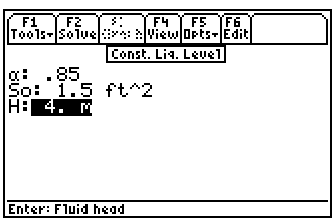
- Constant Liquid Level
- Varying Liquid Level
- Conical Vessel
- Horizontal Cylinder
- ASME Weirs

7.1 Constant Liquid Level

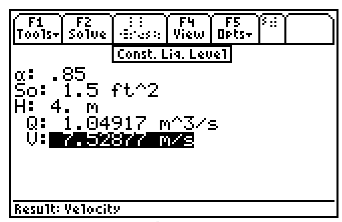
This portion of the software computes fluid discharge from an opening of cross sectional area **So** (m^2), and a constant head **H** (m). The discharge coefficient α (unitless) is a unique number (a number less than 1) accounting for the shape of the cross section, edge-rounding effects, and turbulence effects. Upon receiving these inputs, the software computes the volume discharge, **Q**, (m^3/s) and the discharge velocity **V** (m/s).

Example 7.1:

Find the discharge parameters for a 1.5 sqft opening subject to a head of 4_m. Assume the discharge coefficient to be 0.85.



Input Screen



Result Screen

To solve the problem, enter values for α = 0.85, **H** = 4 m and **so** = 1.5 ft^2 using the appropriate unit keys accessible during data entry. After all the input variables have been entered, press [F2] key to get the following results.

Given:
α = 0.85
So = 1.5 ft^2
H = 4 m

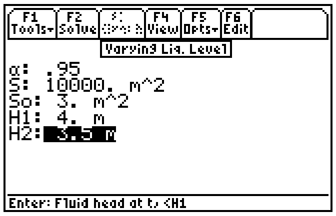
Results
Q = 1.04917 m^3/s
V = 7.52877 m/s

7.2 Varying Liquid Level

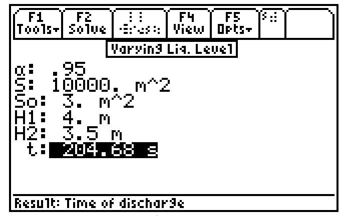
This segment of the software considers fluid discharge from a tank of cross sectional area **S** (m^2), an opening of cross sectional area **So** (m^2) where the head has dropped from **H1** (m) to **H2** (m). The discharge coefficient α (unitless) is a unique number (a number less than 1) accounting for the shape of the cross section, edge-rounding effects, and turbulence effects. Upon receiving these inputs, the software computes the time **t** (s) taken to see the drop in head height.

Example 7.2:

A large reservoir has an area of 10000 m^2 has head level of 4_m that needs to brought down by 0.5_m. The drainage opening is 3 m^2 and has a discharge coefficient of 0.95. How long will this process take?



Input Screen



Result Screen

To solve the problem, enter values for **α** = 0.95, **H1** = 4 m, **H2** = 3.5 m and **So** = 3 m^2 using the appropriate unit keys accessible when data is being entered. After all the input variables have been entered, press [F2] key to get the results.

Given:
α = 0.95
S = 10000 m^2
So = 3 m^2
H1 = 4 m
H2 = 3.5 m

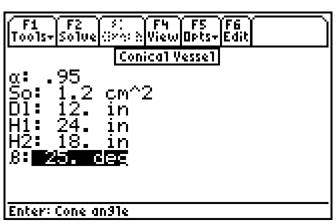
Results
t = 204.68 s

7.3 Conical Vessel

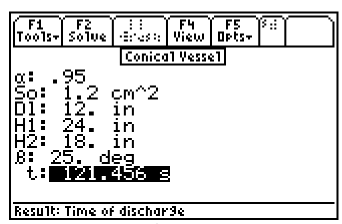
This segment considers fluid discharge from a container that conical in shape. The parameters specified for the computations include cross sectional area, **So** (m^2), and a fluid head **H1** (m) at t=0 and **H2** (m) the fluid head at t. The discharge coefficient **α** (unitless) is a unique number (a number less than 1) accounting for the shape of the cross section, edge-rounding effects, turbulent effects, and **β** (rad) refers to the cone angle. Upon receiving these inputs, the software computes the time for the head to \drop from **H1** to **H2**.

Example 7.3:

Find the time taken to discharge water from a 25 ° conical vessel with a discharge coefficient of 0.95 for a 1.2 cm^2 opening. Consider the head drops from 24 in to 18 in. The base of the cone has a diameter **Dl** of 12 in.



Input Screen



Result Screen

To solve the problem, enter values for **α** = 0.95, **H1** = 24 in, **H2** =18 in, and **So** = 1.2 cm^2 using the appropriate unit keys accessible when data is being entered. After all the input variables have been entered, press [F2] key to get the results.

Given:
α = 0.95
So = 1.2 cm^2
Dl = 12 in
H1 = 24 in
H2 = 18 in
β = 25 deg

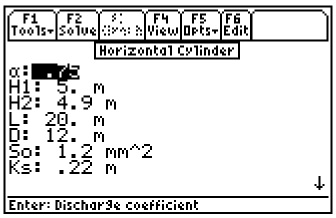
Results
t = 121.456 s

7.4 Horizontal Cylinder

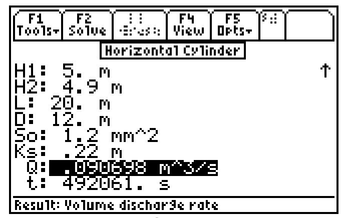
This segment of the software computes fluid discharge from a cylindrical tank of diameter **D** (m) and length **L** (m). The parameters specified for the computations include an opening for discharge with an area **So** (m^2), and a fluid head **H1** (m) at **t**=0 and **H2** (m) the fluid head at **t**. In addition, the discharge coefficient α (unitless) is a unique number (a number less than 1) accounting for the shape of the cross section, edge-rounding effects, and turbulent effects, along with **Ks** (unitless), and a space factor coefficient.

Example 7.4:

Find the time to oil from a cylindrical tank 12 m in diameter and 20 m long vessel with a space factor of 0.8 and orifice discharge coefficient of 0.75. How long does it take for the oil to drop down 10 cm from an initial height of 5 m. Assume the orifice opening to be 1.2 mm^2.



Input Screen



Result Screen

To solve the problem, enter values for α = 0.55, **Ks** = 0.8 m, **H1** = 5 m, **H2** = 3.5 m, **L** = 20 m, and **So** = 1.2 cm^2 using the appropriate unit keys accessible when data is being entered. After all the input variables have been entered, press [F2] key to get the results.

Given:
α = 0.75
H1 = 5 m
H2 = 4.9 m
L = 20 m
D = 12 m
So = 1.2 mm^2
Ks = 0.22 m

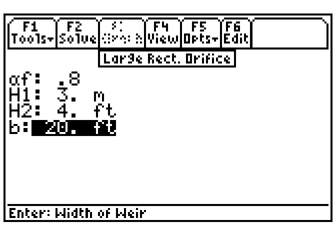
Results
Q = .090698 _m^3/s
t = 492061 s

7.5 Large Rectangular Orifice

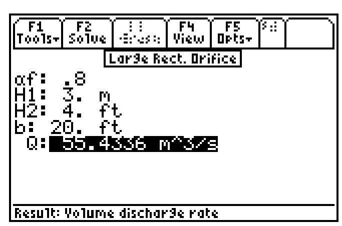
This segment of the software computes fluid discharge using a large rectangular orifice. The weir has a width of **b** (m). The system has a discharge coefficient αf, fluid heights **H1** (m) and **H2** (m) at the beginning and end.

Example 7.5:

Find the time to water to drop head from 3 m to 4 ft, for a weir 20 ft wide. Assume discharge coefficient of 0.8.



Input Screen



Result Screen

To solve the problem, enter values for **αf** = 0.8, **H1** = 3 m, **H2** = 4 ft, and **b** = 20 ft. Enter all the inputs. After all the input variables have been entered, press [F2] key to get the results.

Given:
αf = 0.8
H1 = 3 m
H2 = 4 ft
b = 20 ft

Results
Q = 55.4336 m^3/s

7.6 ASME Weirs

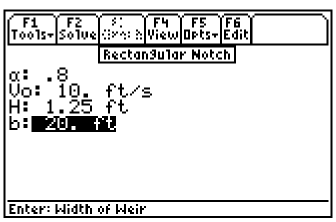
Weirs are useful devices to measure flow of liquids in open channels. A large number of empirical formulas have been developed in the engineering literature each with its own limitations. A few representative samples are included in this software.

7.6.1 Rectangular Notch

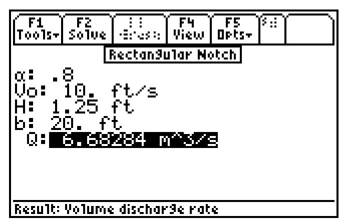
A rectangular weir has a width of **b** (m), initial static head of **H** (m), and a velocity of **Vo** (m/s). Assume a discharge coefficient **α** (unitless).

Example 7.6.1:

A rectangular weir is 20 ft wide, has water flowing over it at a velocity of 10 ft/s, a static head of 1.25 ft., and a discharge coefficient of .8. Find the discharge for the system.



Input Screen



Result Screen

Given:
α = 0.8
Vo = 10 ft/s
H = 1.25 ft
b = 20 ft

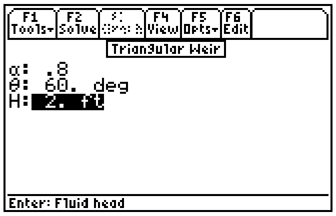
Results
Q = 6.68284 m^3/s

7.6.2 Triangular Weir

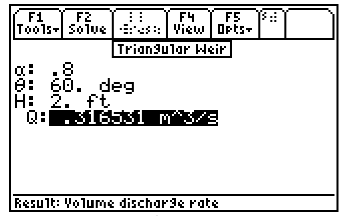
A triangular weir has a water height **H** (m), a discharge coefficient with a static head of **H** (m) and a velocity of **Vo** (m/s). Assume a discharge coefficient **α** (unitless).

Example 7.6.2:

A triangular weir has an angle of 60 deg, a discharge coefficient of 0.8 and a 2 ft water height. Find the discharge for this condition.



Input Screen



Result Screen

Given:
α = 0.8
θ = 60 deg
H = 2 ft

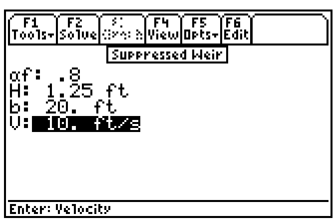
Results
Q = .316531m^3/s

7.6.3 Suppressed Weir

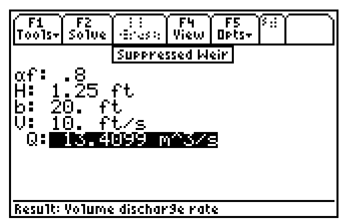
A suppressed weir helps measure flow depending upon the height above the weir. The system has a coefficient of discharge **αf**, a height **H** (m) over the weir, weir of width **b** (m), with an initial a velocity of **Vo** (m/s). Assume a discharge coefficient **α** (unitless).

Example 7.6.3:

A rectangular weir is 20 ft wide, has water flowing over it at a velocity of 10 ft/s, a static head of 1.25 ft., and the discharge coefficient is .8. Find the discharge for the system.



Input Screen



Result Screen

Given:
αf = 0.8
H = 1.25 ft
b = 20 ft
Vo = 10 ft/s

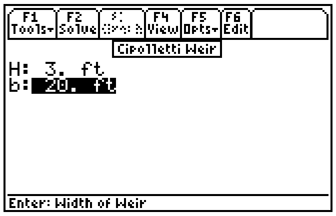
Results
Q = 13.4099 m^3/s

7.6.4 Cipolletti Weir

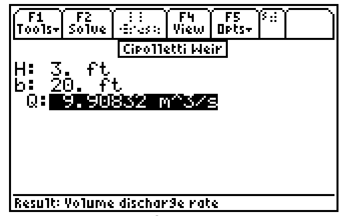
A Cipolletti weir of width **b** (ft) and static head of **H** (ft) computes discharge in ft^3/s. The formulas used here are determined by experimental observation.

Example 7.6.4:

A rectangular weir is 20 ft wide, has water flowing over it with a 3 ft head. Find the discharge for the system.



Input Screen



Result Screen

Given:
H = 3 ft
b = 20 ft

Results
Q = 9.90832 m^3/s

References:

1. Eugene A Avallone and Theodore Baumeister, III, General Editors, 9th Edition, Mark's Standard Handbook for Mechanical Engineers, McGraw-Hill Book Company, New York, NY
2. Ranald Giles, Fluid Mechanics and Hydraulics, 2nd Edition, Schaum's Outline Series, McGraw-HillBook Company, New York, NY, 1962
3. Michael R. Lindeburg, Mechanical Engineering Reference Manual, 8th Edition, Professional Publications Inc., Belmont, CA, 1990

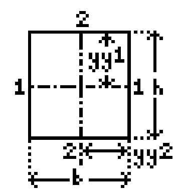
Chapter 8: Section Properties

This section of the software computes properties commonly associated with sections. These properties include calculating the area of cross-section, location of the center of mass from the vertical and horizontal axes. Computed area moments, **I11** (m^4) and **I12** (m^4), reflect their value with reference to vertical and horizontal axes. In cases where it is meaningful, the polar area moment is computed along with radius of gyration. Twelve standard cross-sections are found in this section. A pictorial description is included where possible.

- **Rectangle**
- **Circle**
- **Hollow Circle**
- **I Section - Even**
- **T Section**
- **Polygon**
- **Hollow Rectangle**
- **Circular Ring**
- **I-Section uneven**
- **C-Section**
- **Trapezoid**
- **Hollow Polygon**

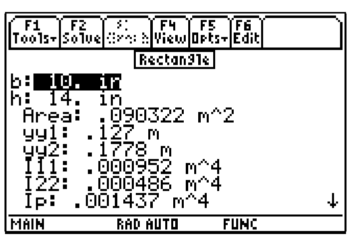
8.1 Rectangle

The input screen for the rectangle requires the user to enter values of base, **b** (m), and height, **h** (m). In an illustrative example, we use a value of 10 inches for the base and 14 inches for the height. The results are displayed in SI units, however, they can be converted to different units by highlighting the values with the cursor, pressing F5**:** **Opts,** 4**: Conv** and pressing a function key (F1 **-** F7) to display the desired units.

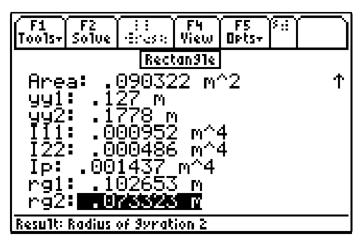


Entered values

Variable	Description	Value
b	Base	10 in
h	Height	14 in



Upper Display



Lower Display

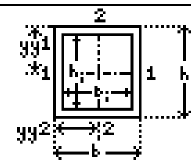
Solution - To enter the value of 10 inches for the base, at the data input screen, move the highlight bar to **b** and press ENTER. Type in 10 and press F4 to append the inch units to the value. Enter a value of 14 in for **h** in a similar manner. Press F2 to compute the results.

Computed Results

Variable	Description	Value
Area	Area	.090322 m^2
yy1	Distance of center of mass from axis 1	.127 m
yy2	Distance of center of mass from axis 2	.1778 m
I11	Area moment inertia axis 11	.000952 m^4
I22	Area moment inertia axis 12	.000486 m^4
Ip	Polar area moment	.001437 m^4

Variable	Description	Value
rg1	Radius of gyration 1	.102653 m
rg2	Radius of gyration 2	.073323 m

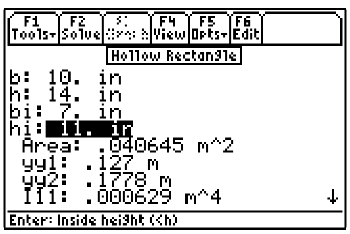
8.2 Hollow Rectangle



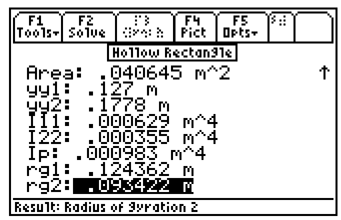
The input screen for the hollow rectangle requires the user to enter outer and inner values of the base **b** and **bi** along with outer and inner heights **h** and **hi**. In an illustrative example, we use a value of 10 inches for the base and 14 inches for the height. The wall thickness is 1.5 in.

Entered values

Label	Description	Value
b	Base	10 in
h	Height	14 in
bi	Inside base (<b)	7 in
hi	Inside Height (<h)	11 in



Upper Display



Lower Display

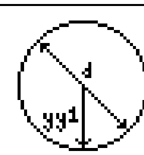
To enter the value of 10 inches for the base, move the scroll bar to **b** and press [ENTER]. Type in 10 and press [F4] to append the inch units to the value. In a like manner, enter **7 in** for **bi**, **14 in** for **h** and **11 in** for **hi.** Press [F2] to begin the calculations. The calculated results are shown below:

Computed Results

Label	Description	Value
Area	Area	0.040645 m^2
yy1	Distance of center of mass from axis 1	0.127 m
yy2	Distance of center of mass from axis 2	0.1778 m
I11	Area moment inertia axis 11	0.000629 m^4
I22	Area moment inertia axis 12	0.000355 m^4
Ip	Polar area moment	0.000983 m^4
rg1	Radius of gyration 1	0.124362 m
rg2	Radius of gyration 2	0.093422 m

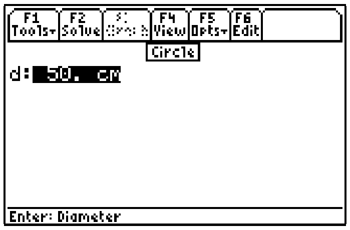
8.3 Circle

The input screen for the circle requires the user to enter a value of diameter **d.** In an illustrative example, we use a value of 50 cm for the diameter.

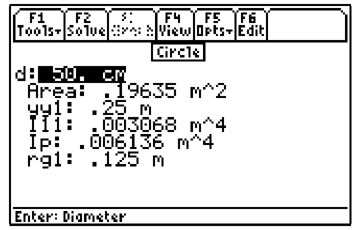


Entered values

Label	Description	Value
d	Diameter	50 cm



Entered Values



Computed results

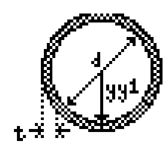
To enter the value of 50 cm for the diameter, match the scroll bar to **d** and press [ENTER]. Press [F2] to begin the calculations. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	0.19635 m^2
yy1	Distance of center of mass from axis 1	0.25 m
I11	Area moment inertia axis 11	0.003068 m^4
Ip	Polar area moment	0.006136 m^4
rg1	Radius of gyration 1	0.125 m

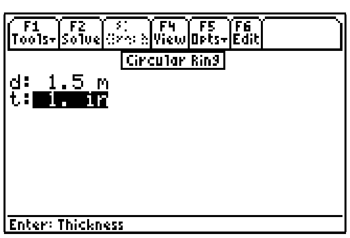
8.4 Circular Ring

The input screen for a circular ring requires the user to enter values of diameter **d** and thickness **t**. In an illustrative example, we use a value of 1.5 m for a wagon wheel with a ring thickness of 1 in.

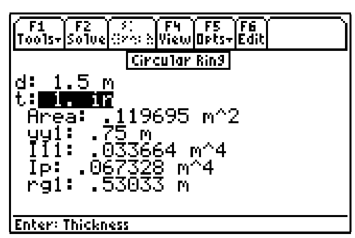


Entered values

Label	Description	Value
d	Diameter	1.5 m
t	Thickness	1 in



Entered Values



Computed results

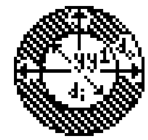
To enter the value of 1.5 m for the diameter **d**, at the data input screen match the scroll bar to **d** and press [ENTER]. Type in 1.5 and press [F2] to append the meter, **m**, and units to the value. Enter a value of 1 in for **t** in a similar manner. Press [F2] to begin the calculations. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	0.119695 m^2
yy1	Distance of center of mass from axis 1	0.75 m
I11	Area moment inertia axis 11	0.033664 m^4

Label	Description	Value
Ip	Polar area moment	0.067328 m^4
rg1	Radius of gyration 1	0.53033 m

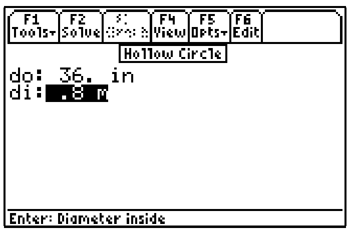
8.5 Hollow Circle



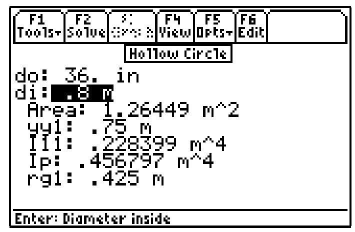
The input screen for the hollow circle requires the user to enter values of outer and inner diameters **do** and **di**. As an illustrative example, we use a value of 36 inches for the outer diameter and an inner diameter of 0.8 m.

Entered values

Label	Description	Value
do	Diameter	36 in
di	Thickness	0.8 m



Entered Values



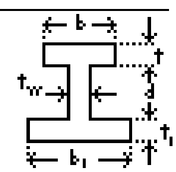
Computed results

To enter the value of 36 inches for the outer diameter, at the data input screen match the scroll bar to **do** and press [ENTER]. Type in 36 and press [F4] to append the inch units to the value. Enter a value of .8 m for **di** in a similar manner. Press [F2] to begin the calculations. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	1.26449 m^2
yy1	Distance of center of mass from axis 1	0.75 m
I11	Area moment inertia axis 11	0.228399 m^4
Ip	Polar area moment	0.456797 m^4
rg1	Radius of gyration 1	0.425 m

8.6 1 Section - Uneven

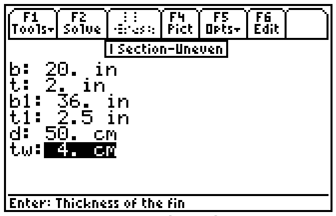


The input screen for an uneven I-Section consists of 6 variables needing user input. Pressing [F4] allows a pictorial representation of T- and I-Beams. As shown in the picture shown here **b** and **b1** represent the widths of top and bottom flanges respectively while **t** and **t1** reflect the thickness of top and bottom flanges. The height and thickness of the fin connecting the top and bottom flanges is represented by **d** and **tw**.

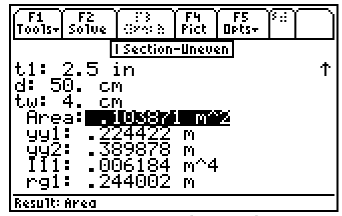
As an illustrative example, we use an I-Beam; we use a top flange width of 20 in and 2 in thickness followed by 36 in width and 2.5 in thickness. The fin is 50 cm in height and 4 cm in thickness.

Entered values

Label	Description	Value
b	Width of top flange	20 in
t	Thickness of top flange	2 in
b1	Width of bottom flange	36 in
t1	Thickness of bottom flange	2.5 in
d	Height of fin	50 cm
tw	Thickness of the fin	4 cm



Entered Values



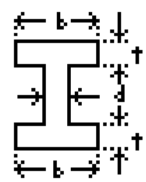
Computed results

To enter the value of 20 inches for the base, at the data input screen match the scroll bar to **b** and press ENTER. Type in 20 and press F4 to append the inch units to the value. Enter the remaining values in a similar fashion. Press F2 to begin the calculating process. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	.103871 m^2
yy1	Distance of center of mass from axis 1	.224422 m
yy2	Distance of center of mass from axis 2	.389878 m
I11	Area moment inertia axis 11	.006184 m^4
rg1	Radius of gyration 1	.244002 m

8.7 I Section - Even

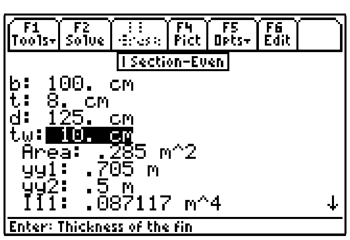


The input screen for an even I-Section, (sometimes can be looked upon and a H-Section turned on its side), consist of four inputs the top and bottom flange width **b** and **t,** and height **d** of the fin and its thickness **tw.** Pressing F4 reveals a picture on an even I-Section.

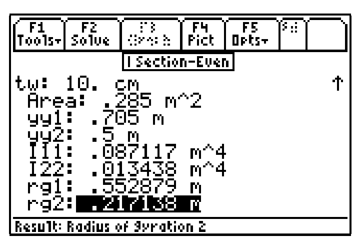
As an example, we use an even I-Section with a flange width of 100 cm and a thickness of 8 cm, while the fin has a height of 125 cm and a thickness of 10 cm.

Entered values

Label	Description	Value
b	Width of top flange	100 cm
t	Thickness of top flange	8 cm
d	Height of fin	125cm
tw	Thickness of the fin	10 cm



Upper Display



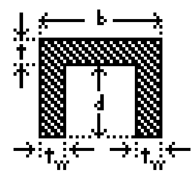
Lower Display

To enter the value of 100 cm for the flange width, at the data input screen match the scroll bar to **b** and press ENTER. Type in 100 and press F5 to append the cm units to the value. Use this procedure to enter 8 cm for **t**, 125 cm for **d** and 10 cm for **tw**. Press F2 to begin the calculations. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	0.285 m^2
yy1	Distance of center of mass from axis 1	0.705 m
yy2	Distance of center of mass from axis 2	0.5 m
I11	Area moment inertia axis 11	0.087117 m^4
I22	Area moment inertia axis 12	0.013438 m^4
rg1	Radius of gyration 1	0.552879 m
rg2	Radius of gyration 2	0.217138 m

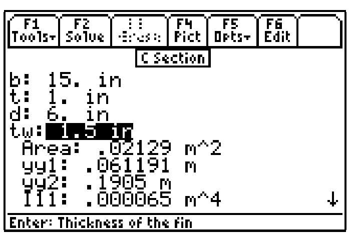
8.8 C Section



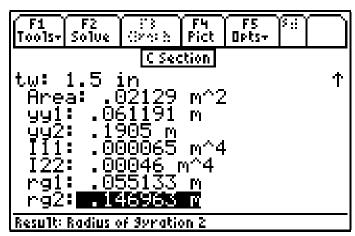
The input screen for the C Section, width of flange **b** and thickness **t**, the fins have a height **d** and thickness **tw**. A picture of C Section can be viewed by pressing [F4]. As an example, we will compute the properties of a C Section with a 15 in flange with a thickness of 1 in, and the fin has a height of 6 in and a thickness of 1.5 in.

Entered values

Label	Description	Value
b	Width of top flange	15 in
t	Thickness of top flange	1 in
d	Height of fin	6 in
tw	Thickness of the fin	1.5 in



Upper Display



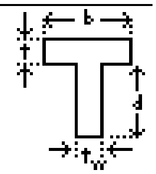
Lower Display

To enter the value of 15 in for the flange width at the data input screen match the scroll bar to **b** and press [ENTER]. Type in 15 and press [F4] to append the inch units to the value. Enter the values for the rest of the parameters in a similar manner. Press [F2] to begin the calculations. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	0.02129 m^2
yy1	Distance of center of mass from axis 1	0.061191 m
yy2	Distance of center of mass from axis 2	0.1905 m
I11	Area moment inertia axis 11	0.000065 m^4
I22	Area moment inertia axis 12	0.00046 m^4
rg1	Radius of gyration 1	0.055133 m
rg2	Radius of gyration 2	0.146963 m

8.9 T Section

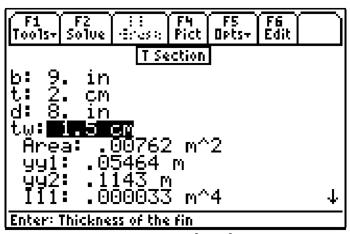


The input screen for a T Section requires the user to enter values of top flange width **b** and thickness **t**, along height of the fin and its thickness **d** and **tw**.

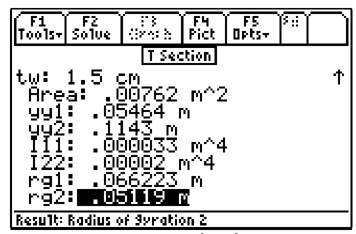
As an example, we compute the properties of a T Section 9 in wide flange with a thickness of 2 cm, and a fin of height 8 in and thickness 1.5 cm.

Entered values

Label	Description	Value
b	Width of top flange	9 in
t	Thickness of top flange	2 cm
d	Height of fin	8 in
tw	Thickness of the fin	1.5 cm



Upper Display



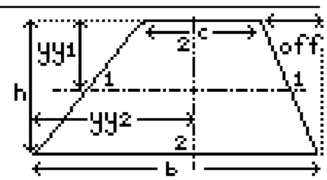
Lower Display

To enter the value of 9 in for the base, at the data input screen match the scroll bar to **b** and press [ENTER]. Type in 9 and press [F4] to append the inch units to the value. Enter the other values in a similar fashion. Press [F2] to begin the calculations. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	0.00762 m^2
yy1	Distance of center of mass from axis 1	0.05464 m
yy2	Distance of center of mass from axis 2	0.1143 m
I11	Area moment inertia axis 11	0.000033 m^4
I22	Area moment inertia axis 12	0.00002 m^4
rg1	Radius of gyration 1	0.066223 m
rg2	Radius of gyration 2	0.05119 m

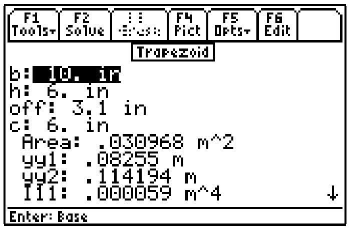
8.10 Trapezoid



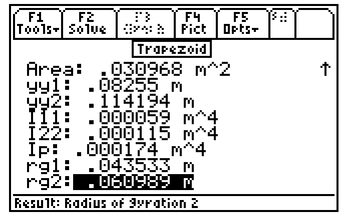
The input screen for a trapezoid requires the user to enter value of base **b**, height **h**, and offset **off,** and top width **c**. In an illustrative example, we use a value of 10 in for the base and 6 in for top, a height of 6 in and an offset of 3.1 in.

Label	Description	Value
b	Base length	10 in
h	Height	6 in
off	Offset	3.1 in
c	Top	6 in

By pressing [F4] you can access a schematic of the trapezoid. To enter the value of 10 inches for the base, at the data input screen match the scroll bar to **b** and press [ENTER]. Type in 10 and press [F4] to append the inch units to the value. Enter the values for other parameters in a like manner. Press [F2] to begin the calculations and display the results. The calculated results are listed below.



Upper Display



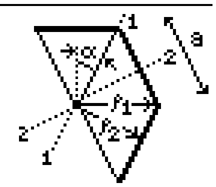
Lower Display

Computed Results

Label	Description	Value
Area	Area	0.030968 m^2
yy1	Distance of center of mass from axis 1	0.08255 m
yy2	Distance of center of mass from axis 2	0.114194 m
I11	Area moment inertia axis 11	.000059 m^4
I22	Area moment inertia axis 12	0.000115 m^4
Ip	Polar area moment	0.000174 m^4
rg1	Radius of gyration 1	0.043533 m
rg2	Radius of gyration 2	0.060989 m

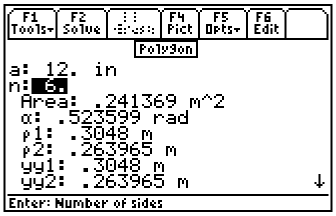
8.11 Polygon

A solid n-sided polygon with side a forms the basis of this segment. The input screen requires the user to enter values of side **a,** and number of sides **n**. In an illustrative example, we use a value of 12 inches for the side of a 6-sided polygon. A cross section of the polygon is shown in the screen display here.

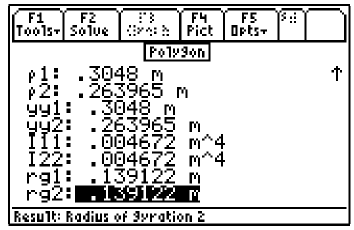


Entered values

Label	Description	Value
a	Side Length	12 in
n	Number of sides	6



Upper Display



Lower Display

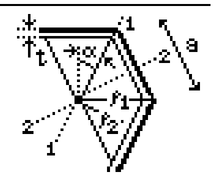
To enter the value of 12 inches for the base, at the data input screen match the scroll bar to **b** and press [ENTER]. Type in 10 and press [F4] to append the inch units to the value. Enter the number of sides in a similar manner. Press [F2] to begin the calculations and display results. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	.241369m^2
α	Angle	.523599 rad
ρ1	Radius to point	0.3048 m
ρ2	Radius to line	0.263965 m
yy1	Distance of center of mass from axis 1	0.3048 m
yy2	Distance of center of mass from axis 2	0.263965 m
I11	Area moment inertia axis 11	0.004672 m^4
I22	Area moment inertia axis 12	0.004672 m^4
rg1	Radius of gyration 1	0.139122 m
rg2	Radius of gyration 2	0.139122 m

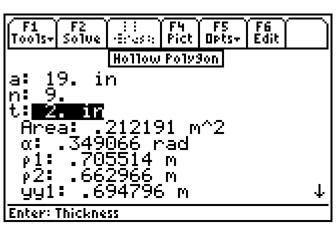
8.12 Hollow Polygon

A hollow n-sided polygon with side a forms the basis of this segment. The input screen requires the user to enter values of side **a**, wall thickness **t** and number of sides **n**. In an illustrative example, we use a value of 19 inches for the side, and a thickness of 2 in of a 9-sided polygon. A cross section of the polygon is shown in the screen display here.

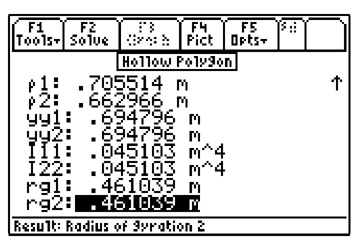


Entered values

Label	Description	Value
a	Side Length	19 in
n	Number of sides	9
t	Thickness	2 in



Upper Display



Lower Display

To enter the value of 19 inches for the base, at the data input screen match the scroll bar to **b** and press [ENTER]. Type in 10 and press [F4] to append the inch units to the value. Enter the number of sides in a similar manner. Press [F2] to begin the calculations and display results. The calculated results are listed below.

Computed Results

Label	Description	Value
Area	Area	0.212191 m^2
α	Angle	.349066 rad
$\rho1$	Radius to point	0.705514 m
$\rho2$	Radius to line	0.662966 m
yy1	Distance of center of mass from axis 1	0.694796 m
yy2	Distance of center of mass from axis 2	0.694796 m
I11	Area moment inertia axis 11	0.045103 m^4
I22	Area moment inertia axis 12	0.045103 m^4
rg1	Radius of gyration 1	0.461039 m
rg2	Radius of gyration 2	0.461039 m

References:

1. Warren C. Young, Roark's Formulas for Stress and Strain, 6th Edition, McGraw-Hill Book Company, New York, NY, 1989.

Chapter 9: Hardness Number

Brinnell and Vicker developed two popular methods of measuring the Hardness number. Brinell composed his tests by dropping a ball of steel onto slab of finite thickness with standard loads such as 500 lbf and 3000 lbf. This steel ball results in an indentation in the material. Vicker had a similar principle to Brinell; instead of steel ball he used a diamond in the shape of square pyramid. By measuring the diameter of indentation, **di**, for Brinells test and the diagonal length of the impression, **di,** for Vickers test, the hardness number is computed. By long standing convention, the diameter **di** is measured in millimeters.

9.1 Compute Hardness Number

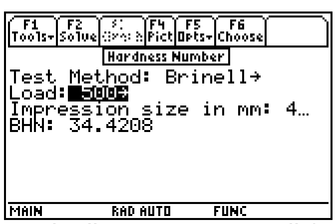
The tests to measure the hardness number allows a 10 mm diameter steel ball or 10mm diagonal length pyramid to be dropped onto the material surface with a effective force of 500 lbf or 3000 lbf. The formulas used to compute the hardness number for by Brinell's and Vicker's methods give slightly different results, but do converge in several areas of the yield curves.

Upon selecting this topic, the input screen (shown here) presents four options to the user. You can choose Brinell's method at 500 lbf or 3000 lbf or Vickers' method for the same two load values.

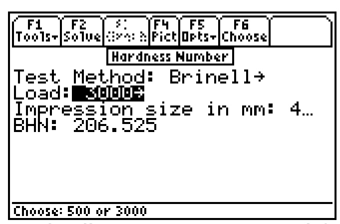
Upon making the selection, enter the measured data for **di** by aligning the highlight bar to **Impression size in mm.** This number should be less than 10mm. After entering numbers for di**,** press ENTER to start the solving process. At the end of the computation, the results are displayed on the screen as shown in the example below.

Example 9.1:

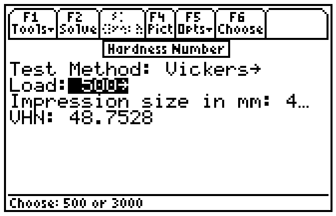
We choose an example of 4.2mm as the measured value of the indentation di and compute BHN and VHN for both 500-lbf loads and the 3000-lbf loads. Input and computed results are shown below.



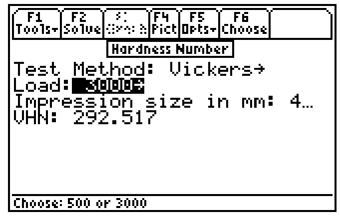
Brinell's output at 500 lbf



Brinell's output at 3000 lbf



Vicker's output at 500 lbf



Vicker's output at 3000 lbf

Given:

di = 4.2 mm

Results:

Test	500 lbf	3000 lbf
Brinnell Test	34.4208	206.525
Vicker Test	48.7528	292.517

Part II: Equations

Chapter 10: Introduction to Equations

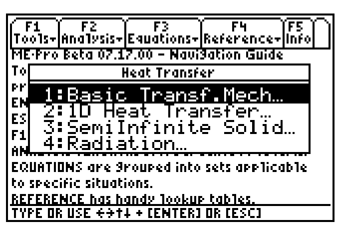
The *Equations* section of ME•*Pro* contains over 1000 equations organized into 12 topic and 185 sub-topic menus.

- The user can select to solve equation sets in a particular sub-topic, display all the variables used in the set of equations, enter the values for the known variables and solve for the unknowns.
- The equations in each sub-topic can be solved individually, collectively or as a sub-set.
- A unit management feature allows easy entry and display of results.
- Each equation can be graphed to examine the relationship between to variables in an equation.
- Multiple and partial solutions are possible.
- Specific parametric information about a particular variable can be displayed by highlighting the variable, press [F5] and [2]**/Type:** to show a brief description of a variable and its entry parameters.
- The input form accepts valuables for variables that have physical meaning only. For example only positive real values are accepted for variables such as radius, thickness etc.

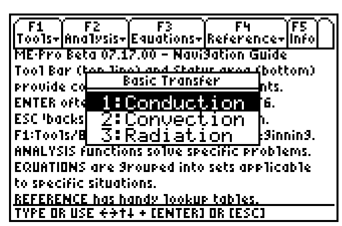
10.1 Solving a Set of Equations

- Equations are accessed from the main level of the ME•*Pro* by pressing function key [F3] labeled **"Equations."** A pull-down menu listing all the main categories is displayed as shown in the screen display below.
- An arrow to the left of the bottom topic '↓'indicates more items are listed. Pressing [2nd] ⊙ jumps to the bottom of the menu list.
- Scroll the highlight bar to an item using the arrow key ⊙ and press [ENTER], or type the subject number appearing next to subject heading (*Heat Transfer* is selected for this example).
- A dialog box presents more subjects (sub-topics) under the topic heading. For example selecting the 1st item (i.e., *Basic Transfer Mechanisms*) displays a list of subtopics (*Conduction, Convection, Radiation*).   Select *Conduction* to display a set of equations for this section. **A complete list of topics in the equation section is listed at this end of the chapter.**
- Use the arrow key ⊙ to move the highlighter and press [ENTER] to select an equation, pressing [F2] selects all equations in the set. A selected equation is marked with a check (√).
- Press [F2] to display all of the variables in the selected equations. A brief description of each variable will appear in the status line at the bottom of the screen.
- Enter values for the known parameters, selecting appropriate units for each value using the toolbar menu.
- Press [F2] to compute values for the unknown parameters.
- Entered and calculated values are distinguished in the display; '▪' for entered values and '◆' for computed results.

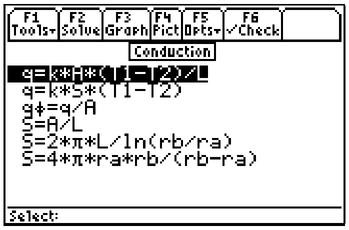
1. Pressing [F3] displays the Equations menu. Press [4] to select 'Heat Transfer'.



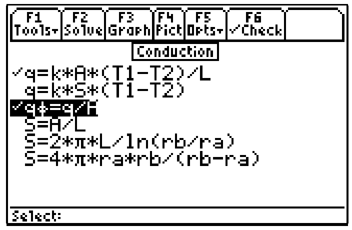
2. Press [1] to display the menu for 'Basic Transfer Mechanisms'.



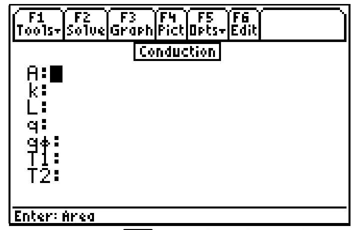
3. Press [1] to display the equations for 'Conduction'.



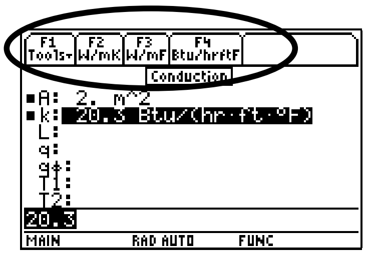
4. *Equations found under conduction.*



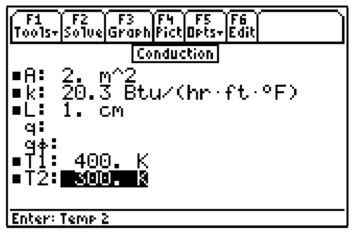
5. *Select the equations for a calculation by moving the cursor to each equation and pressing* [ENTER]*. (Check mark '√' appears when selected).*



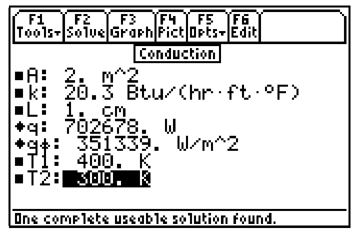
6. *Press* [F2] *to display all the variables in the selected equations. Variable description appears on the status line at the bottom of the screen.*



7. *Enter values for each known variable. Append the units by pressing the function key corresponding to the desired units.*



8. *Once all known variable values are entered, press* [F2] *to solve for the unknowns.*

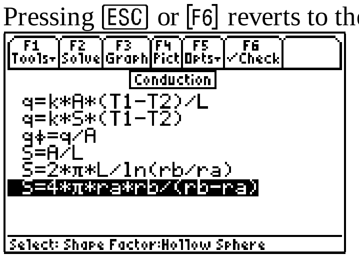


9. *Note: Computed results '♦' are distinguished from entered values '▪'.*

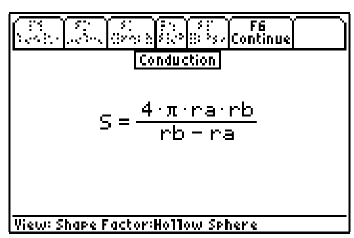
Note: Only values designated as *known* '▪' will be used in a computation. Results displayed from an earlier calculation will not be automatically used unless designated by the user by selecting the variable and pressing [ENTER]. Press [F2] to compute a new result for any input that is changed.

10.2 Viewing an Equation or Result in *Pretty Print*

Sometimes equations and calculated results exceed the display room of the calculator. The TI-89 and TI 92 Plus include a built-in equation display feature called *Pretty Print* which is available in many areas of ME•*Pro* and can be activated by highlighting a variable or equation and pressing the right arrow key ⊳ or pressing the [F4] function key when it is designated as *View*. The object can be scrolled using the arrow keys ⊲⊳. Pressing [ESC] or [F6] reverts to the previous screen.



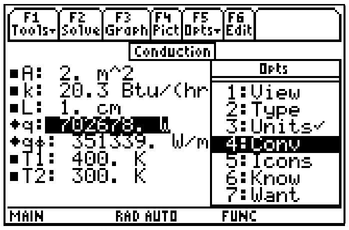
1. *To view an equation in, Pretty Print, highlight and press* [F4] *or* ⊳.



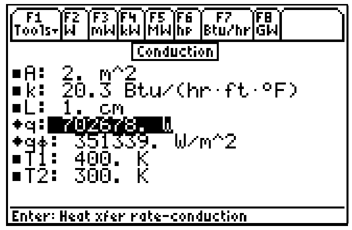
2. *Scroll features, using the arrow keys* ⊲⊳*, enable a complete view of a large object.*

10.3 Viewing a Result in different units

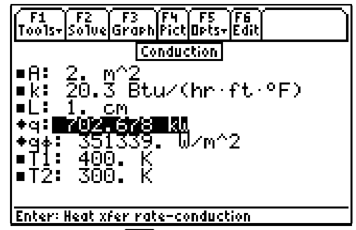
To view a calculated result in units which are different from what is displayed, highlight the variable, press [F5]**/Options** and [4]**/Conv** to display the unit tool bar at the top of the screen. Press the appropriate function key to convert the result to the desired units.



1. *Highlight the result to be converted **q**. Press* F5 *to display the Options menu, press* 4*: conv.*



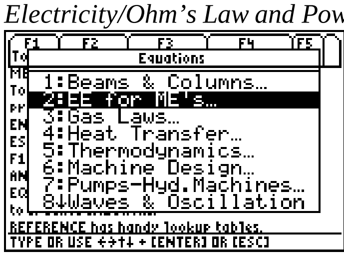
2. *The unit menu tool bar is now displayed.*



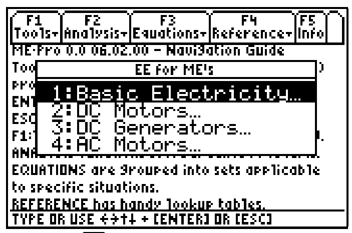
3. *Press* F4 *to convert the result of **q** from W to kW.*

10.4 Viewing Multiple Solutions

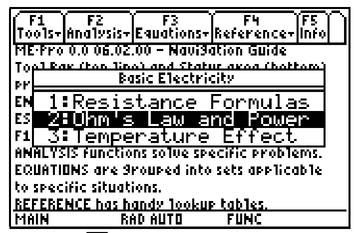
When multiple solutions exist, the user is prompted to select the number of a series of computed answers to be displayed. To view additional solutions, press F2 to repeat the calculation and enter another solution number. The user will need to determine which result is most useful to the application. The following example is taken from **Ohm's Law and Power** in the **EE for MEs** section. *Equations/EE for MEs/Basic Electricity/Ohm's Law and Power*.



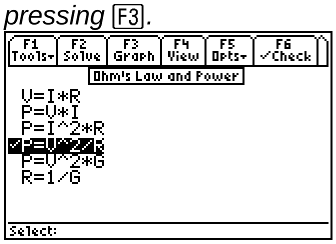
1. *Return to the HOME screen of ME•Pro (*F1*: Tools,* 8*: Clear or press* ESC *repeatedly) and access the equations section by pressing* F3*.*



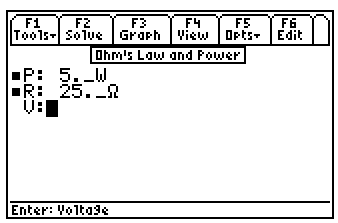
Press 1 *for 'Basic Electricity'.*



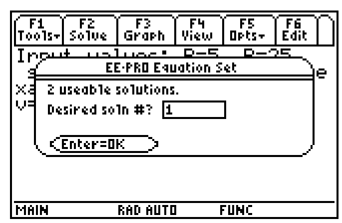
Press 2 *for 'Ohm's Law and Power.'*



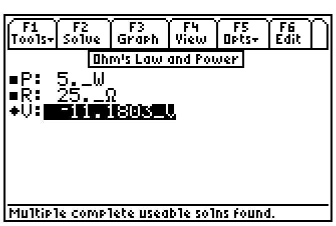
1. *Select an equation using highlighting the cursor bar and to display variables.*



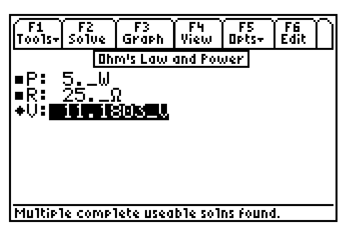
2. *Enter known values for each variable using the tool bar to designate units. Press* F2 *to compute the results.*



3. *If multiple solutions exist, a dialogue box will appear requesting the user to enter the number of a solution to view.*



Solution 1: To view another solution, press F2 *to repeat the calculation and enter the number of another solution to be viewed.*

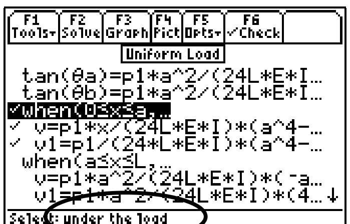


Solution 2: Enter a new number for each 'solve' to display a series of multiple solutions.

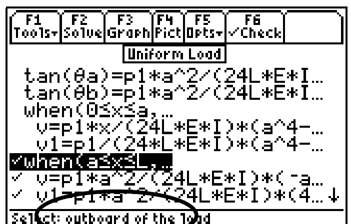
10.5 When (…) - conditional constraints when solving equations

In several sections of ME•*Pro*, equations are limited to certain variable ranges. An example can be found in *Beams and Columns/Simple Beams/Uniform Load (Chapter 11.1.1):* A beam, experiencing a uniform load along a distance, **a** (m), from one end, has a deflection, **v** (m), and slope of deflection, **v1**, located at a distance, **x** (m), from the end of the beam. Two sets of unique equations compute **v** and **v1** depending on whether the condition is **x≤a** or **x≥a**. The conditions for the equations appear in the **'when'** clauses precede the equations. ME•*Pro* allow's the selection of equations under a single **'when'** clause since the conditions are generally exclusive to each other[2].

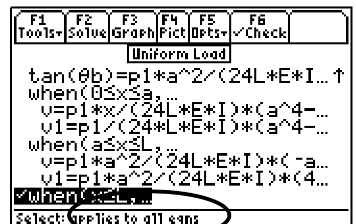
> Note: **The 'when' clauses do not serve any mathematical function in the solving process or for screening variable entry; they are only a guide for selecting equations for a specific circumstance**. Additional information for a **'when'** clause appears in the status line while it is highlighted.



All equations following a highlighted '**when**' heading are selected when [ENTER] is pressed.



Only equations under a single '**when**' heading can be selected at a time.



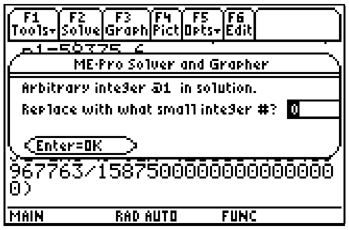
Description of constraint appears in the status line at the bottom of the screen.

Some equation sets do not form a consistent set, which can be solved together. An example occurs in *Equations/Fluid Mechanics/Fluid Dynamics/Equivalent Diameter* (see **Chapter 21.3.3**), where each equation represents fluid flow through a different-shaped cross-section. In such a case, the specific conditions for each equation appear on the status line.

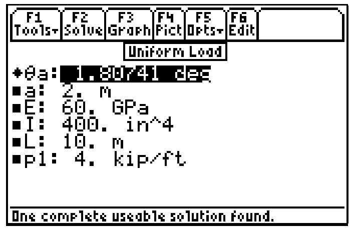
10.5 Arbitrary Integers for periodic solutions to trigonometric functions

When an angle value is being computed in a trigonometric function such as tangent, cosine and sine, ME•*Pro* may prompt an entry for an arbitrary integer (-2, -1, 0, 1, 2…) before displaying a solution. Solutions for angles inside of trigonometric functions are generally periodic, however the solution, which is most often sought, is the principal solution. The principal solution, P, in a periodic trigonometric function, **trig (…)**, is **P=trig ($\theta + a \cdot n \cdot \pi$)** where n is the arbitrary integer and **a=1** for **tan (…), a=2** for **sin (…)** and **cos (…). Selecting the arbitrary integer to be 0** gives the principal solution.

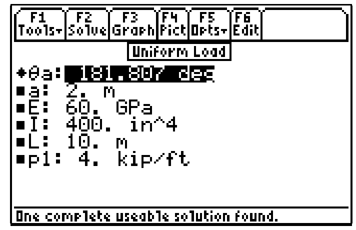
[2] In at least one known case (*Beams and Columns/Simple Beams/Point load*), conditions in more than one **when (…)** statement can occur simultaneously. A work around is to solve the equation set in two steps, using equations under a single **when (…)** heading at a time and designating the results from one calculation as the input into the second.



Dialog box for entering an arbitrary integer for a periodic solution.



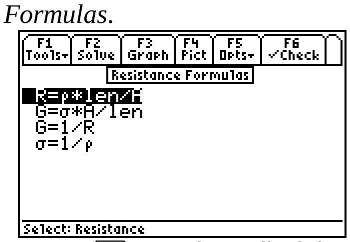
If '0' is entered, the principal solution (P) for an angle is displayed.



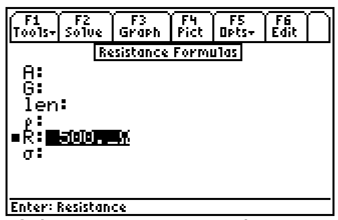
If '1' is entered, a non-principal solution (P+180°) is displayed.

10.7 Partial Solutions

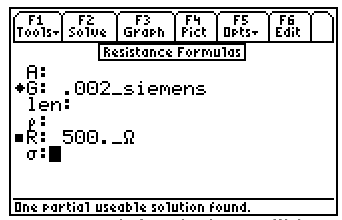
"*One partial useable solution found" or "Multiple partial solutions found."* will be displayed in the status line if values for one or more variables in the selected equation set cannot be computed. This situation can occur if there are more unknowns than equations in the selected set, the entered values do not form consistent relationships with the selected equations, or if the selected equations do not establish a closed form relationship between all of the entered values and the unknowns. In such a case, only the calculated variables will be displayed. This section appears in *Equations/EE for MEs/Basic Electricity/Resistance Formulas*.



Press [F2] to select all of the equations in Resistance Formulas.



If there are more unknowns Than selected equations or Relationships between Variables are not established From the selected equations...



...a partial solution will be displayed if one or more of the unknown variables can be computed from the entered inputs.

10.8 Copy/Paste

A computed result and it's expressed units can be copied and pasted to an appropriate part of the TI operating system using [F1]**: Tools-5: Copy** key sequence to copy a value and [F1]**: Tools-6: Paste**. In a few cases, The TI-89 and TI-92 operating systems, and ME•*Pro* use slightly different conventions for displaying units. The unit system in ME•*Pro* is designed to conform to the convention established by SI, however, in order to CUT and PASTE a value and units from ME•*Pro* to another area of the TI operating system, ME•*Pro* must insert extra characters in the units to match TI's syntax. **The COPY/PASTE function can only work INSIDE of ME•*Pro* if the unit feature has been deactivated** (press [F5]**:Opts,** [3]**: Units** to toggle the unit feature on or off).

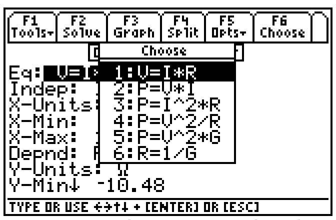
10.9 Graphing a Function

The relationship between two variables in an equation can be graphed on a real number scale if the other variables in the equation are defined.

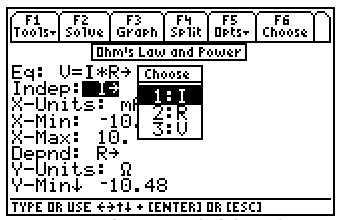
- After solving an equation, or entering values for the *non*-x, y variables in the equation to be plotted, press [F3] **/Graph** to display the graph settings.
- Highlight **Eq:** and press [ENTER] to select the equation from the list to graph.
- Use the same steps as above to select the independent and dependent variables (**Indep:** and **Depnd:**) from the equation.
- **Note: all pre-existing values stored in the variables used for Indep: and Depnd : will be cleared when the graphing function is executed.**

- The graphing unit scale for each variable reflects the settings in the *Equations* section of ME•*Pro*.
- Scrolling down the list, specify the graphing ranges for the x and y variables, whether to graph in full or split screen modes, automatically scale the graph to fit the viewing area, and label the graph.
- Press [F3] to graph the function.

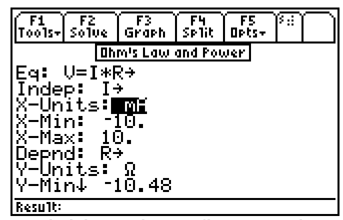
Once the graph command has been executed, ME•*Pro* will open a second window to display the plot. All of the TI graphing features are available and are displayed in the toolbar, including *Zoom* [F2], *Trace* [F3], *Math* [F5], etc. All the tools from TI graphics engine are now available to theuser. If the split-screen graphing mode is activated, the user can toggle between the ME•*Pro* graph dialogue display and the TI graph by pressing [2nd] [APPS]. If the full-screen graphing mode is activated, the user can switch between ME•*Pro* and the graph by pressing [APPS] 4:Graph or A: ME•*Pro.* To remove the split screen after graphing, you will need to change the display settings in the MODE screen of the calculator. To do this: 1) Press [MODE] 2) Press [F2]**: Page 2**; 3) move the cursor to **Split Screen**. 4) Press the right arrow key ▶ to display a pop-up menu. 5) Select FULL. 6) Press [ENTER] twice.



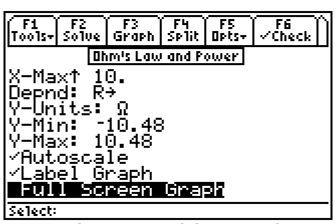
1*. Graph an equation by pressing [F3]. Press [ENTER] to choose an equation.



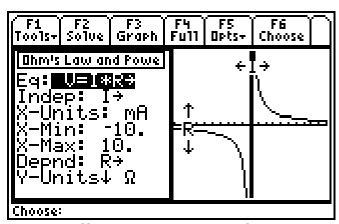
2. Select variables for Independent (x) and Dependent (y) variables.



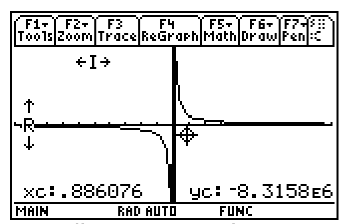
Variable units reflect settings in ME•Pro.



4. Select graphing options by pressing [ENTER].



5. Split Screen Mode: Toggle Between graph and settings by pressing [2nd] and [APPS].



6: Full Screen Mode: Press [◆] and [APPS] to return to ME•Pro.

***Before graphing an equation, be sure to specify values for variables in an equation, which are not going to be used as x, and y variables.**

Note: If an error is generated when attempting to graph, be sure that all of the variables in the graphed equation, which are not specified as the independent, and dependent variables have known values. In the ME•*Pro* window, press [ESC] to view the equations in the sub-topic, select the equation to be graphed and press [F2] to display the list of variables in the equation and enter values. Only the dependent (y) and independent (x) variables do not have to contain specified values. Press [F3]to display the graph dialogue and repeat the above steps to graph the function.

10.10 Storing and recalling variable values in ME•*Pro*-creation of session folders

ME•*Pro* automatically stores its variables in the current folder specified by the user in [MODE] or the HOME screens. The current folder name is displayed in the lower left corner of the screen (default is “Main”). To create a new folder to store values for a particular session of ME•*Pro*, press [F1]**:/TOOLS,** [3]**:/NEW** and type the name of the new folder.

There are several ways to display or recall a value:

- The contents of variables in **any** folder can be displayed using the [VAR-LINK], moving the cursor to the variable name and pressing [F6] to display the contents of a particular variable.
- Variables in a current folder can be recalled in the HOME screen by typing the variable name.
- Finally, values and units can be copied and recalled using the [F1]**/Tools** [5]**: COPY** and [6]**: PASTE** feature.

All inputs and calculated results in *Analysis* and *Equations* are saved as variable names. Previously calculated, or entered values for variables in a folder are replaced when equations are solved using new values for inputs. To preserve data under variable names, which may conflict with ME•*Pro* variables, run ME•*Pro* in a separate folder.

10.11 solve, nsolve, and csolve and user-defined functions (UDF)

When a set of equations is solved in ME•*Pro*, three different functions in the TI operating system (solve, numeric solve, and complex solve) are used to find the most appropriate solution. In a majority of cases, the entered values are adequate to find numeric solutions using either *solve*, or *csolve* functions. However, there are a few instances when UDFs external to equations are incorporated into the solving process. User defined functions which appear ME•*Pro* are the error functions **erfc (x)** and **erf (x).**

When all the inputs to a UDF are known, *solve* or *csolve* passes a computed result to the equation: however, if the unknown variable is an input to the UDF, *solve* or *csolve* are unable to isolate the variable in an explicit form, and the operating system resorts to using *nsolve* which initiates a trial and error iteration until the solution converges. It should be noted that the solution generated by *nsolve is not* guaranteed to be unique (i.e. this solving process cannot determine if multiple solutions exist.).

Because nsolve is used, an equation containing a user-defined function (UDF) cannot be graphed when the dependent variable is contained in the UDF.

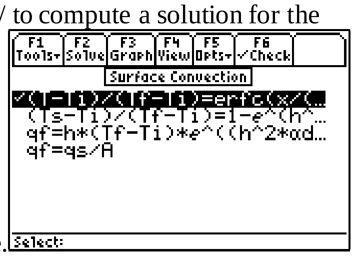
Table 15-1 User Defined Functions[2]

User-defined Function	***Topic***	***Sub-topic***
erf(x, αd, time)	Heat Transfer	Step Change in Surface Temperature
erfc (x, αd, time)	Heat Transfer	Constant Surface Heat Flux
erfc (x, αd, h, k, time)	Heat Transfer	Surface Convection

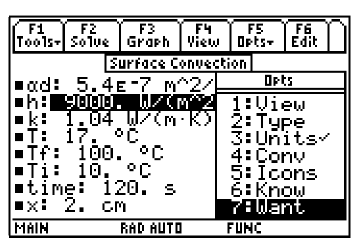
10.12 Entering a guessed value for the unknown using *nsolve*

To accelerate the converging process and, if multiple solutions exist, enhance the possibility that*nsolve* resolves the correct solution, the user can enter a guessed value for the unknown which nsolve will use as an initial value in its solving process.

- Enter guessed a value for the variable in the input dialogue.
- Press [F5]/Opts, [7]/Want.
- Press [F2]/ to compute a solution for the variable.

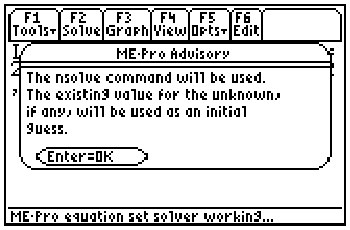


erfc(h) is a user-defined function (UDF) that appears in the ‘Surface Convection’ topic of ‘Heat



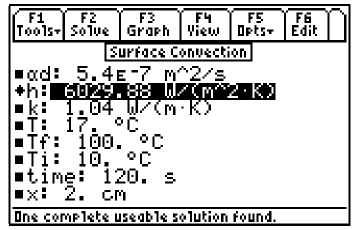
Only one variable in a user-defined function can be specified as an unknown.

Transfer'.



ME•Pro posts a notice if the nsolve routine is used.

An initial value for the unknown can be specified for the n-solve process [F5]**:Opts,** [7]**: Want.**



The user can enter a value for the unknown and designate it as a guessed value to accelerate the nsolve convergence process.

10.13 Why can't I compute a solution?

If a solution is unable to be computed, you might check the following:

1. Are there at least as many equations selected, as there are unknown parameters?
2. Are the entered values or units for the known parameters reasonable for a specific case?
3. Are the selected equations consistent in describing a particular case? (For example, certain equations used in the calculation of equivalent circular cross-section diameters in *Fluid Mechanics/Fluid Dynamics/Equivalent Diameter*, are only valid for certain shaped cross sections). Check the headings displayed at the bottom of the screen while the equation is highlighted to determine if special restrictions for a particular equation (set) apply.

10.14 Care in choosing a consistent set of equations

The success in obtaining a useful solution, or a solution at all is strongly dependent on an insight into the problem and care in choosing equations, which describe consistent relationships between the parameters.

The following steps are recommended:

- Read the description of each set of equations in a topic to determine which subsets of equations in a series are compatible and consistent in describing a particular case. Some restrictions or special conditions for an equation or set of equations are listed in the status line while the equation or '**when**' conditions are highlighted.
- Select the equations from a subset, which describe the relationships between all of the known and unknown parameters.
- As a rule of thumb, select as many equations from the subset as there are unknowns to avoid redundancy or over-specification. The equations have been researched from a variety of sources and use slightly different approximation techniques. Over-specification (selecting too many equations) may lead to an inability of the equation solver to resolve slight numerical differences in different empirical methods of calculating values for the same variable.

10.15 Notes for the advanced user in troubleshooting calculations

When there are no solutions possible, ME•*Pro* provides important clues via the variables, **meinput, meprob, means,** and **meanstyp**. These variables are defined during the equation setup process by the built-in multiple equation solver. ME•*Pro* saves a copy of the problem, its inputs, its outputs, and a characterization of the type of solution in the user variables **meprob, meinput, means,** and **meanstyp**. For the developer who is curious to know exactly how the problem was entered into the multiple equation solver, or about what the multiple equation solver returned, and to examine relevant strings. The contents of these variables may be viewed and examined by using [VAR-LINK]. Press [VAR-LINK] ([2nd] followed by [−]), scroll to the variable name in the current folder and press [F6] to view the contents of the variable. The string may be recalled to the status line of the home screen, modified and re-executed, if desired. **If no solutions are possible when one should be displayed, try clearing the variables in the current folder, or opening a new folder**.

Chapter 11: Beams and Columns

This chapter covers the details found in the Beams and Columns section. Three broad areas of common interest are covered. They are:

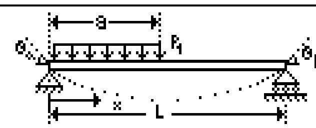
- **Simple Beams**
- **Cantilever Beams**
- **Columns**

11.1 Simple Beams

Structural members designed to resist forces acting perpendicular to its longitudinal axis are defined as Beams. The simplest types of beams can be thought of "planar structures" wherein all the deflections occur in the same plane. The essential features of a Simple Beam, or Simply Supported Beam, are a pin support is placed at one end and a roller support at the other. The equations used here cover Simple Beams where the rollers support are always at the *right end* of the beam.

Three types of loads are considered here. They are **Uniform load, Point load** and **Moment load**. Each of these load types has unique characteristics and is illustrated in the software.

11.1.1 Uniform Load



The equation set here covers problems associated with a uniformly distributed load, **p1** (N/m), covering a distance, **a** (m), from the left end. **Equations 1** and **2** yield the slopes at the left and right ends of the beam, **θa** (rad), and, **θb** (rad), respectively. To compute the deflection, **v** (m), and slope, **v1,** at any location, **x** (m), from the left, use **equations 3** and **4** when **x≤a**. When, **x,** lies between, **a.** and the beam length, **L** (m), use **equations 5** and **6** to compute, **v,** and **v1**. **E** (Pa), the modulus of elasticity and **I** (m^4), the area moment of inertia, represent the material properties of the beam.

$$\tan(\theta a) = \frac{p1 \cdot a^2}{24 \cdot L \cdot E \cdot I} \cdot (2 \cdot L - a)^2$$ **Eq. 1**

$$\tan(\theta b) = \frac{p1 \cdot a^2}{24 \cdot L \cdot E \cdot I} \cdot (2 \cdot L^2 - a^2)$$ **Eq. 2**

When 0 ≤ x ≤ a, the following two equations are applicable

$$v = \frac{p1 \cdot x}{24 \cdot L \cdot E \cdot I} \cdot (a^4 - 4 \cdot a^3 \cdot L + 4 \cdot a^2 \cdot L^2 + 2 \cdot a^2 \cdot x^2 - 4 \cdot a \cdot L \cdot x^2 + L \cdot x^3)$$ **Eq. 3**

$$v1 = \frac{p1}{24 \cdot L \cdot E \cdot I} \cdot (a^4 - 4 \cdot a^3 \cdot L + 4 \cdot a^2 \cdot L^2 + 6 \cdot a^2 \cdot x^2 - 12 \cdot a \cdot L \cdot x^2 + 4 \cdot L \cdot x^3)$$ **Eq. 4**

When a ≤ x ≤ L, the following two equations are applicable

$$v = \frac{p1 \cdot a^2}{24 \cdot L \cdot E \cdot I} \cdot \left(-a^2 \cdot L + 4 \cdot L^2 \cdot x + a^2 \cdot x - 6 \cdot L \cdot x^2 + 2 \cdot x^3\right) \quad \textbf{Eq. 5}$$

$$v1 = \frac{p1 \cdot a^2}{24 \cdot L \cdot E \cdot I} \cdot \left(4 \cdot L^2 + a^2 - 12 \cdot L \cdot x + 6 \cdot x^2\right) \quad \textbf{Eq. 6}$$

When 0 ≤ x ≤ L, applies to all equations

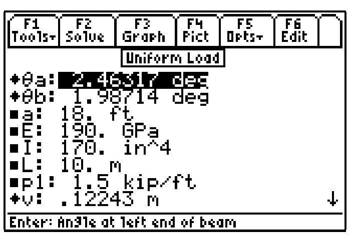
The variable names, description and applicable default units used in these equations are listed below.

Variable	**Description**	**Units**
θa	Angle at left (fixed) end	rad
θb	Angle at right (roller supported) end	rad
a	Load location from left (fixed) end	m
E	Young's modulus	Pa
I	Area moment of inertia	m^4
L	Length	m
p1	Load/unit length	N/m
v	Beam deflection	m
v1	Slope of deflection	unitless
x	Dist. from left end	m

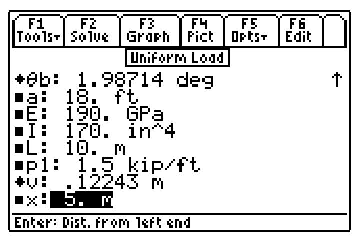
Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 11.1.1:

A simple beam, 10 meters long, is subject to a uniform load of 1.5 kips/ft, spanning 18 feet from the left end. Find the slopes at left and right ends and deflection at mid-point of the beam. Assume that the Young's modulus of the beam material is 190 GPa and that the area moment is 170 in^4.



Upper Display



Lower Display

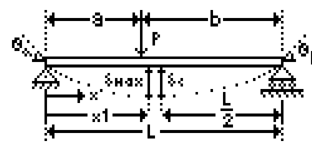
Solution – Select the **first three equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. **Select 0** to compute the principal solution). The entries and results are shown in the screen displays above.

Given	**Solution**
a = 18 ft	θa = 2.46317 °
E = 190 GPa	θb = 1.98714 °
I = 170 in^4	v = .12243 m
L = 10 m	

Given
p1 = 1.5 kip/ft
x = 5 m

Solution

11.1.2 Point Load



This equation set covers problems associated with a point load, **P** (N), located a distance, **a** (m), from the left (fixed) end of a simple beam. **Equations 1** and **2** yield the slope angles, **θa** (rad), **θb** (rad) at the left and right ends of the beam, respectively. **Equation 3** computes the length of the beam, **L** (m), in terms of the load distance from the left, **a** (m), and right, **b** (m) ends. To compute the deflection, **v** (m), and slope **v1** at any location, **x** (m), from the left side, use **equations 4** and **5** when **0≤x≤a.** **Equation 6** yields **δc** (m), the deflection at the center of the beam when **a≥b**. **Equation 7** computes **x1** (m), the location of maximum deflection, **δmax** (m), calculated by **equation 8**. The material properties of the beam are represented by, **E** (Pa) the modulus of elasticity, **I** (m^4) the area moment of inertia, and **L** (m), the length of the beam.

Eq. 1

$$\tan(\theta a) = \frac{P \cdot a \cdot b \cdot (L + b)}{6 \cdot L \cdot E \cdot I}$$

Eq. 2

$$\tan(\theta b) = \frac{P \cdot a \cdot b \cdot (L + a)}{6 \cdot L \cdot E \cdot I}$$

Eq. 3

$$L = a + b$$

When 0 ≤ x ≤ a, the following two equations are applicable

Eq. 4

$$v = \frac{P \cdot b \cdot x}{6 \cdot L \cdot E \cdot I} \cdot \left(L^2 - b^2 - x^2\right)$$

Eq. 5

$$v1 = \frac{P \cdot b}{6 \cdot L \cdot E \cdot I} \cdot \left(L^2 - b^2 - 3 \cdot x^2\right)$$

When a ≥ b, the following two equations are applicable

Eq. 6

$$\delta c = \frac{P \cdot b \cdot \left(3 \cdot L^2 - 4 \cdot b^2\right)}{48 \cdot E \cdot I}$$

Eq. 7

$$x1 = \sqrt{\frac{L^2 - b^2}{3}}$$

Eq. 8

$$\delta \max = \frac{P \cdot b \cdot \left(L^2 - b^2\right)^{1.5}}{243^{.5} \cdot L \cdot E \cdot I}$$

When 0 ≤ x ≤ L, applies to all equations

The variable names, description and applicable default units used in these equations are listed below.

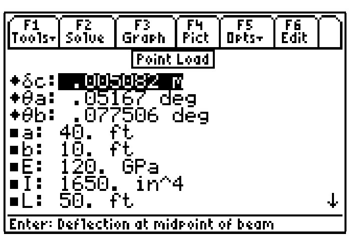
Variable	Description	Units
δc	Deflection at mid point	m
δmax	Maximum deflection at x1	m
θa	Angle at left (fixed) end	rad
θb	Angle at right (roller supported) end	rad
a	Load location from left (fixed) end	m
b	Dist. from right (roller supported) end	m
E	Young's modulus	Pa
I	Area moment	m^4
L	Length	m
P	Point load	N
v	Beam deflection	m
v1	Slope of deflection	unitless
x	Distance from left end	m
xl	Maximum deflection location	m

Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

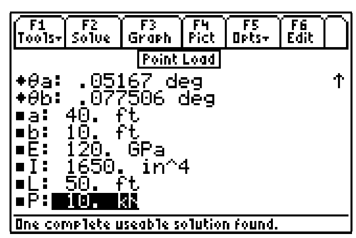
Example 11.1.2:

A simple beam, 50 ft long, is subject to a point load of 4000 lbf located 10 feet from the left end and a second load of 10 kN located 10 feet from the right end. Find the deflection at mid-beam and the slope at both ends. Assume that the Young's modulus of the beam material is 120 GPa, and the area moment is 1650 in^4.

Solution - The problem is solved in two stages. First, compute the slopes at both ends, and the deflection at the center for the load of 10 kN. Repeat the calculations using the second load and add the two computed values using the superposition to calculate the final result. Select an arbitrary integer of 0 to compute the principal solution (the principal solution, P, in a periodic trigonometric function, trig (…), is P=trig (θ + n·π) and n is the arbitrary integer).



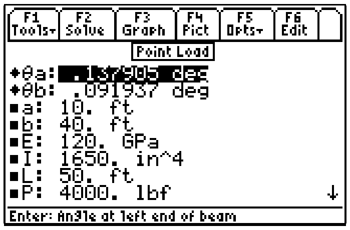
First load: Upper Display



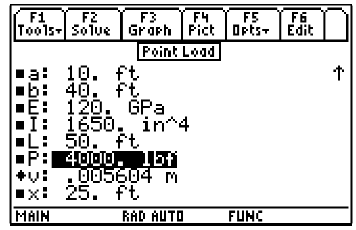
First load: Lower Display

Use Equations 1, 2 and 6 to calculate the results from the first load.

Given	**Solution**
a = 40 ft	δc = .005082 m
b = 10 ft	θa = .05167 deg
E = 120 GPa	θb = 0.077506 deg
I = 1650 in^4	
L = 50 ft	
P = 10 kN	



Second load: Upper Display



Second load: Lower Display

Use Equations 1, 2 and 4 to calculate the results from the second load.

Given

a = 10ft
b = 40 ft
E = 120 GPa
I = 1650 in^4
L = 50 ft
P = 4000 lbf
x = 25 ft

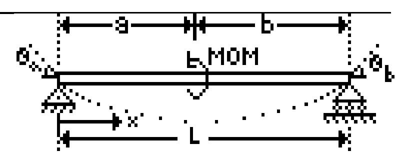
Solution

θa = .137905 deg
θb = .091937 deg
v =.005604 cm

The final results of the two loads are obtained by invoking the super position principle; thus add the results of the two sets of calculations.

Solution	**θa**	**θb**	**δc/v**
First load	.05167 deg	.077506 deg	.005082 m
Second load	.091937 deg	.13905 deg	.005604 m
Total	**.143607 deg**	**.216556 deg**	**.010686 m**

11.1.3 Moment Load



This equation set covers problems associated with a moment load, **MOM** (N·m), applied at a distance, **a** (m), from the left end. **Equations 1** and **2** calculate the slope angles, **θa** (rad), and, **θb** (rad), at the left and right ends of the beam, respectively. The deflection distance, **v** (m), and slope, **v1**, at distance, **x** (m), from the left side of the beam are computed by **Equations 3** and **4**. The material properties of the beam are represented by, **E** (Pa) the modulus of elasticity, **I** (m^4) the area moment of inertia, and, **L** (m) the length of the beam.

Eq. 1

$$\tan(\theta a) = \frac{MOM}{6 \cdot L \cdot E \cdot I} \cdot \left(6 \cdot a \cdot L - 3 \cdot a^2 - 2 \cdot L^2\right)$$

Eq. 2

$$\tan(\theta b) = \frac{MOM}{6 \cdot L \cdot E \cdot I} \cdot \left(3 \cdot a^2 - L^2\right)$$

When 0 ≤ x ≤ a, the following two equations are applicable

Eq. 3

$$v = \frac{MOM \cdot x}{6 \cdot L \cdot E \cdot I} \cdot \left(6 \cdot a \cdot L - 3 \cdot a^2 - 2 \cdot L^2 - x^2\right)$$

Eq. 4

$$v1 = \frac{MOM}{6 \cdot L \cdot E \cdot I} \cdot \left(6 \cdot a \cdot L - 3 \cdot a^2 - 2 \cdot L^2 - 3 \cdot x^2\right)$$

When 0 ≤ x ≤ L, applies to all equations

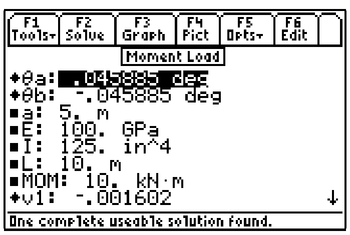
The variable names, description and applicable default units used in the equations above are listed below.

Variable	Description	Units
θa	Angle at left (fixed) end	rad
θb	Angle at right (roller supported) end	rad
a	Distance of load from left (fixed) end	m
E	Young's modulus	Pa
I	Area moment of inertia	m^4
L	Length	m
MOM	Moment applied to beam	N·m
v	Beam deflection	m
v1	Slope of deflection	unitless
x	Distance from left (fixed) end	m

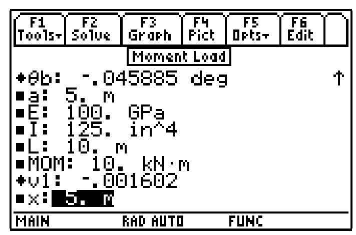
Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 11.1.3:

A simple beam, 10 meters long, is subject to a moment load of 10kN·m at the middle point of the beam. Find the slopes at left and right ends of the beam, and deflection at mid point of the beam. Assume that the Young's modulus of the beam material is 100 GPa, and the area moment to be 125 in^4.



Upper Display



Lower Display

Solution – Select the **first, second**, and **fourth equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. **Select an arbitrary integer of 0** to compute the principal solution (the principal solution, P, in a periodic trigonometric function, trig (…), is P=trig (θ + n·π) and n is the arbitrary integer). The entries and results are shown in the screen displays above.

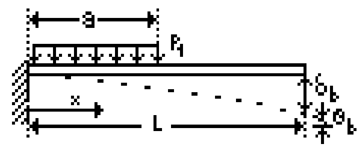
Given	Solution
a = 5 m	θa = .045885 deg
E = 100 GPa	θb = -.046885 deg
I = 125 in^4	v1 = -.001602
L = 10 m	
MOM = 10 kN·m	
x = 5 m	

11.2 Cantilever Beams

Structural members designed to resist forces acting perpendicular to its longitudinal axis are defined as Beams. The essential features of a Cantilever Beam are that it has a fixed support at one end and a free at the other end with no support. The fixed end of the beam is inflexible and does not incur any bending, thus ensuring the deflection and slope to be zero at this end.

Three types of loads are considered here. They are **Uniform load, Point load**, and **Moment load**. Each of these load types has unique characteristics and is illustrated in the software.

11.2.1 Uniform Load



This equation set covers problems associated with a uniformly distributed load, **p1** (N/m), over a distance, **a** (m), from the left (fixed) end. **Equations 1** and **2** yield the deflection, **δb** (m), and slope angle, **θb** (rad), at the right (free) end of the beam. Deflection **v** (m), and slope **v1,** at any location **x** (m), away from the left can be computed by using **equations 3** and **4,** when **0≤x≤a**. When **x** lies between **a,** and beam length **L**, use **equations 5** and **6** for the same properties. The material properties of the beam are represented by, the modulus of elasticity **E** (Pa), and, the area moment of inertia **I** (m^4).

Eq. 1

$$\delta b = \frac{p1 \cdot a^3}{24 \cdot E \cdot I} \cdot (4 \cdot L - a)$$

Eq. 2

$$\tan(\theta b) = \frac{p1 \cdot a^3}{6 \cdot E \cdot I}$$

When 0 ≤ x ≤ a, the following two equations are applicable

Eq. 3

$$v = \frac{p1 \cdot x^2}{24 \cdot E \cdot I} \cdot (6 \cdot a^2 - 4 \cdot a \cdot x + x^2)$$

Eq. 4

$$v1 = \frac{p1 \cdot x}{6 \cdot E \cdot I} \cdot (3 \cdot a^2 - 3 \cdot a \cdot x + x^2)$$

When a ≤ x ≤ L, the following two equations are applicable

Eq. 5

$$v = \frac{p1 \cdot a^3}{24 \cdot E \cdot I} \cdot (4 \cdot x - a)$$

Eq. 6

$$v1 = \frac{p1 \cdot a^3}{6 \cdot E \cdot I}$$

When 0 ≤ x ≤ L, applies to all equations

The variable names, description and applicable default units used in these equations are listed below.

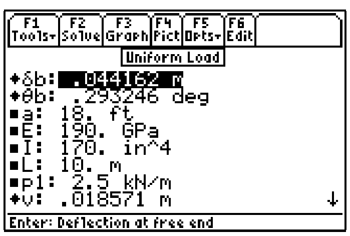
Variable	**Description**	**Units**
δb	Deflection at right (free) end	m

Variable	**Description**	**Units**
θb	Slope at right (free) end	unitless
a	Distance of load from left (fixed) end	m
E	Young's modulus	Pa
I	Area moment of inertia	m^4
L	Length	m
p1	Load/unit length	N/m
v	Beam deflection	m
v1	Slope of deflection	unitless
x	Distance from left (fixed) end	m

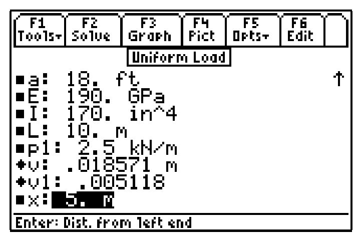
> **Caution:** Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 11.2.1:

A cantilever beam, 10 meters long, is subject to a uniform load of 2.5 kN/m, 18 ft from the fixed end. Find the slope and deflection at the free end of the beam, as well as at the mid-point of the beam. Assume that the Young's modulus of the beam material is 190 GPa and that the area moment is 170 in^4.



Upper Display

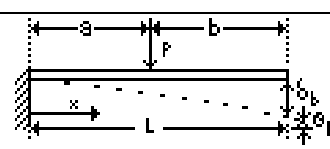


Lower Display

Solution – Select **equations 1, 2, 5** and **6** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. **Select the integer of 0** to compute the principal solution. The entries and results are shown in the screen displays above.

Given	**Solution**
a = 18 ft	δb = .044162 m
E = 190 GPa	θb = .293246 deg
I = 170 in^4	v = .018571 m
L = 10 m	v1 = .005118
p1 = 2.5 kN/m	
x = 5 m	

11.2.2 Point Load



This equation set covers problems associated with a point load, **P** (N), located a distance, **a** (m), from the left (fixed) end. **Equations 1** and **2** yield the deflection, **δb** (m), and slope angle, **θb** (rad), at the right (free) end of the beam. To compute the deflection, **v** (m), and slope, **v1**, at a location **x** (m) from the left end, use **equations 3** and **4**, when

x occurs before the location of the load, **a** (m) (**0≤x≤a)**, and **equations 5** and **6** when **x** occurs after the load (**a≤x≤L**). The material propertie of the beam is represented by **E** (Pa), the modulus of elasticity by E (Pa), the area moment of inertiaby, **I** (m^4), and beam length, **L** (m).

Eq. 1

$$\delta b = \frac{P \cdot a^2 \cdot (3 \cdot L - a)}{6 \cdot E \cdot I}$$

Eq. 2

$$\tan(\theta b) = \frac{P \cdot a^2}{2 \cdot E \cdot I}$$

When 0 ≤ x ≤ a, the following two equations are applicable

Eq. 3

$$v = \frac{P \cdot x^2}{6 \cdot E \cdot I} \cdot (3 \cdot a - x)$$

Eq. 4

$$v1 = \frac{P \cdot x}{2 \cdot E \cdot I} \cdot (2 \cdot a - x)$$

When a ≤ x ≤ L, the following two equations are applicable

Eq. 5

$$v = \frac{P \cdot a^2 \cdot (3 \cdot x - a)}{6 \cdot E \cdot I}$$

Eq. 6

$$v1 = \frac{P \cdot a^2}{2 \cdot E \cdot I}$$

When 0 ≤ x ≤ L, applies to all equations

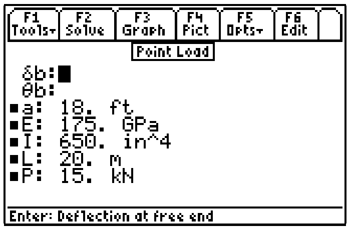
The variable names, description and applicable default units used in these equations are listed below.

Variable	**Description**	**Units**
δb	Deflection at left (fixed) end	m
θb	Angle at right (free) end of beam	rad
a	Distance of load from left (fixed) end	m
E	Young's modulus	Pa
I	Area moment	m^4
L	Length	m
P	Load	N
v	Beam deflection	m
v1	Slope of deflection	unitless
x	Distance from left (fixed) end	m

Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 11.2.2:

A cantilever beam, 20 m long, is subject to a 15 kN point load located 18 feet from the fixed end. Find the slope and deflection at the free end of the beam. Assume that the Young's modulus of the beam material is 175 GPa, and that the area moment is 650 in ^{4}.



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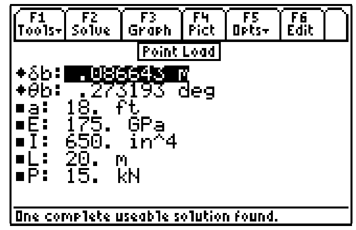


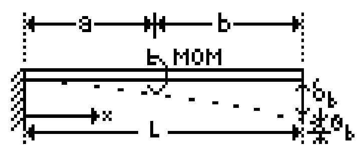
Computed results

Solution – Select the **first two equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
a= 18 ft
E = 175 GPa
I = 650 in^4
L = 20 m
P = 15 kN

Solution
δb = .086643 m
θb = .273193 deg

11.2.3 Moment Load



This equation set covers problems associated with a moment load, **MOM** (N·m), applied at a distance **a** (m), from the left (fixed) end. **Equations 1** and **2** calculate the deflection, **δb** (m), and slope, **θb** (rad), at the right (free) end. The deflection and slope, **v** (m) and **v1**, at a distance, **x** (m), from the left (fixed) side of the beam are defined by **equations 3** and **4** for the case when **0≤x≤a**, and **equations 5** and **6** when **a<x<L**. The materials properties of the beam are represented by **E** (Pa), the modulus of elasticity; **I** (m^4), the area moment of inertia; and **L** (m), the length of the beam.

Eq. 1

$$\delta b = \frac{MOM \cdot a}{2 \cdot E \cdot I} \cdot (2 \cdot L - a)$$

Eq. 2

$$\theta b = \frac{MOM \cdot a}{E \cdot I}$$

When 0 ≤ x ≤ a, the following two equations are applicable

Eq. 3

$$v = \frac{MOM \cdot x^2}{2 \cdot E \cdot I}$$

Eq. 4

$$v1 = \frac{MOM \cdot x}{E \cdot I}$$

When a ≤ x ≤ L, the following two equations are applicable

Eq. 5

$$v = \frac{MOM \cdot a}{2 \cdot E \cdot I} \cdot (2 \cdot x - a)$$

Eq. 6

$$v1 = \frac{MOM \cdot a}{E \cdot I}$$

When 0 ≤ x ≤ L, applies to all equations

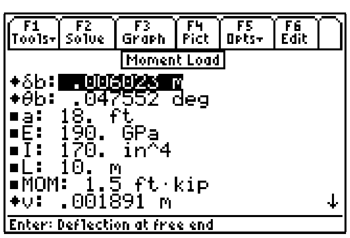
The variable names, description and applicable default units used in these equations are listed below.

Variable	Description	Units
δb	Deflection at right (free) end	m
θb	Angle at right (free) end	rad
a	Distance of load from left (fixed) end	m
E	Young's modulus	Pa
I	Area moment	m^4
L	Length	m
MOM	Applied moment	N·m
v	Beam deflection	m
v1	Slope of deflection	unitless
x	Distance from left (fixed) end	m

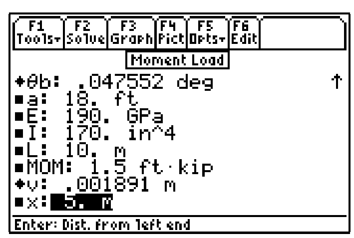
Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 11.2.3:

A simple beam, 10 meters long, is subject to a moment load of 1.5 ft·kip, 18 feet from the fixed end. Find the slope and deflection at the right end of the beam, and the deflection at mid point of the beam. Assume that the Young's modulus of the beam material is 190 GPa, and that the area moment is 170 in^4.



Upper Display



Lower Display

Solution – Since the load occurs to the right of the midpoint of the beam (**x=L/2<a<L**, since x=5 m = 16.4 ft <18 ft=**a**) use **equations 1, 2,** and **3,** to solve this problem. Select these equations and press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. **Select an arbitrary integer of 0** to compute the principal solution. The entries and results are shown in the screen displays above.

Given
a = 18 ft
E = 190 GPa
I = 170 in^4
L = 10 m
MOM = 1.5 ft·kip
x =5 m

Solution
δb = .006023 m
θb = .047552 deg
v = .001891 m

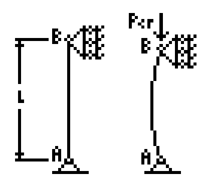
11.3. Columns

Structures fail a variety of ways depending upon material properties, loads and conditions of support. In this section, a variety of failures of columns will be considered. For our purpose, we define a column to be long slender structural members loaded axially in compression.

Five type of problems are considered **- Buckling**, **Eccentric Axial load**, **Secant formula**, **Column imperfections**, and **Inelastic buckling**.

11.3.1 Buckling

These four equations give an insight into the critical parameters for designing columns. **Equations 1** and **2** compute the critical load, **Pcr** (N), in terms of the cross-sectional area of the column, **Area** (m^2), the modulus of elasticity, **E** (Pa), the column length, **L** (m), radius of gyration, **r** (m), and the area moment of inertia, **I** (m^4). The compressive stress, **σcr** (Pa), is calculated from **equation 3** and the radius of gyration **r** is computed in **equation 4**.



$$Pcr = \frac{\pi^2 \cdot E \cdot Area}{\left(\frac{Ke \cdot L}{r}\right)^2}$$ **Eq. 1**

$$Pcr = \frac{\pi^2 \cdot E \cdot I}{(Ke \cdot L)^2}$$ **Eq. 2**

$$\sigma cr = \frac{Pcr}{Area}$$ **Eq. 3**

$$r = \sqrt{\frac{I}{Area}}$$ **Eq. 4**

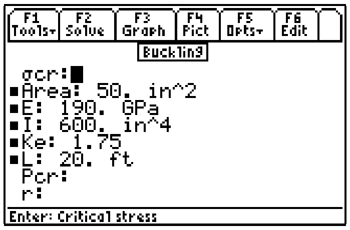
The variable names, description and applicable default units used in these equations are listed below.

Variable	**Description**	**Units**
σcr	Critical stress	Pa
Area	Area	m^2
E	Young's modulus	Pa
I	Area moment	m^4
Ke	Effective length factor	unitless

Variable	**Description**	**Units**
L	Length	m
Pcr	Critical load	N
r	Radius of gyration	m

Example 11.3.1:

A steel column, with an area of 50 in^2 and 20 ft long, has a modulus of elasticity of 190 GPa. The effective length factor for this column is 1.75, and the area moment is 600 in^4. Find the critical load, the radius of gyration and critical stress.



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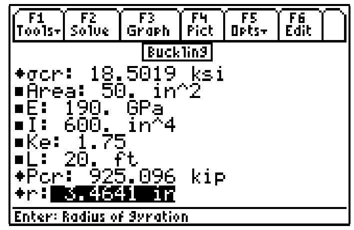


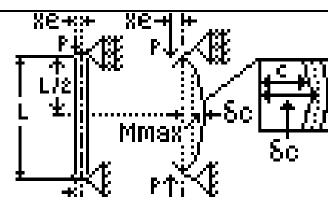
Computed results

Solution – Select **all of the equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
Area = 50 in^2	σcr = 18.5019 ksi
E = 190 GPa	Pcr = 925.096 kip
I = 600 in^4	r = 3.4641 in
Ke = 1.75	
L = 20 ft	

11.3.2 Eccentricity, Axial Load

An eccentric load refers to the situation where the point load, **P** (N), is not located at the center of mass of the area, but is offset by a distance, **xe** (m). **Equation 1** computes the deflection of the column at the mid-point, **δc** (m). **Equation 2** calculates the maximum bending moment, **Mmax** (N·m), for the specified eccentric load. The location of **Mmax** is the same as the location of maximum deflection (typically at half the height of the column, **L/2**).



$$\delta c = \frac{P \cdot xe \cdot L^2}{8 \cdot E \cdot I}$$ **Eq. 1**

$$Mmax = \frac{\mathrm{P} \cdot xe}{\cos\left(\frac{\mathrm{k} \cdot \mathrm{L}}{2}\right)}$$ **Eq. 2**

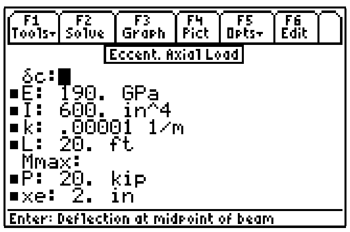
The variable names, description and applicable default units used in these equations are listed below.

Variable	**Description**	**Units**
δc	Deflection at mid-point	m

Variable	Description	Units
E	Young's modulus	Pa
I	Area moment of inertia	m^4
k	Stiffness	1/m
L	Length	m
Mmax	Maximum bending moment	N·m
P	Point load	N
xe	Eccentricity offset	m

Example 11.3.2:

A 20 kip point load on the column described in Example 11.3.1 is offset 2 inches from the column's central axis. Find the buckling at the center of the column and the maximum moment if the stiffness is 1×10^{-5} 1/m.



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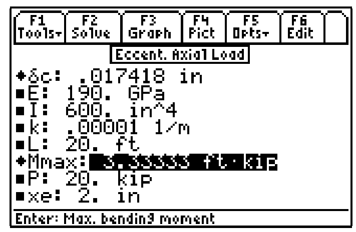
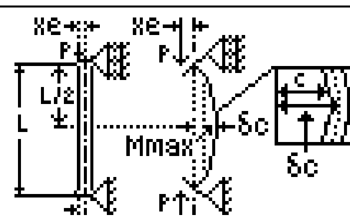


Computed results

Solution – Select **both equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
E = 190 GPa	δc = .017418 in
I = 600 in^4	Mmax = 3.33333 ft·kip
k = .00001 1/m	
L = 20 ft	
P = 20 kip	
xe = 2 in	

11.3.3 Secant Formula



The equations in this section focus on further analysis of eccentric loads. **Equation 1** computes the radius of gyration, **r** (m), from the area moment **I** (m^4), and the cross sectional area, **Area** (m^2). **Equations 2** and **3** show two ways to calculate maximum stress, σmax (Pa), from known properties of the column including length, **L** (m); load, **P** (N); **Area**; and the modulus of elasticity, **E** (Pa). Two key ratios, the eccentricity ratio, **ecr** and the slenderness ratio, **sr**, are introduced in **equations 4** and **5**. The eccentric offset, **xe** (m), is distance of the load from the central longitudinal axis of the column and **c** (m) is the distance from the axis to the concave side of the column at the location of maximum deflection.

Eq. 1

$$r = \sqrt{\frac{I}{Area}}$$

$$\sigma max = \frac{P}{Area} + \frac{Mmax}{S} \qquad \textbf{Eq. 2}$$

$$\sigma max = \frac{P}{Area} \cdot \left(1 + \frac{xe \cdot c}{r^2} \cdot \frac{1}{\cos\left(\frac{L}{2 \cdot r} \cdot \sqrt{\frac{P}{E \cdot Area}} \right)} \right) \qquad \textbf{Eq. 3}$$

$$ecr = \frac{xe \cdot c}{r^2} \qquad \textbf{Eq. 4}$$

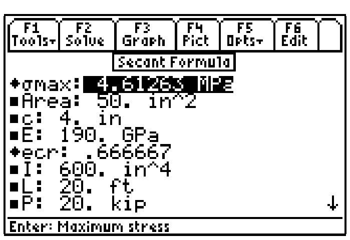
$$sr = \frac{L}{r} \qquad \textbf{Eq. 5}$$

The variable names, description and applicable default units used in these equations are listed below.

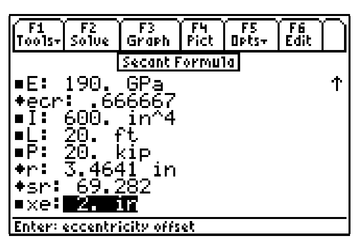
Variable	**Description**	**Units**
σmax	Maximum stress	Pa
Area	Area	m^2
c	Distance of centroid to column edge	m
E	Young's modulus	Pa
ecr	Eccentricity ratio	unitless
I	Area moment of inertia	m^4
L	Length	m
Mmax	Maximum bending moment	N·m
P	Point load	N
r	Radius of gyration	m
S	Section modulus	m^3
sr	Slenderness ratio	unitless
xe	Eccentric offset	m

Example 11.3.3:

The column described in example 11.3.1 and 11.3.2 has a 20 kip axial load located at distance of 4 inches from the concave extremum. Find the maximum stress, radius of gyration and the slenderness ratio.



Upper Display

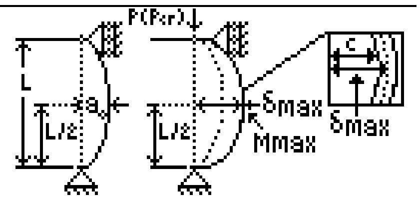


Lower Display

Solution – Select **equations 1, 3, 4,** and **5** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
Area = 50 in^2	σmax = 4.61263 MPa
c = 4 in	ecr = .666667
E = 190 GPa	r = 3.4641 in
I = 600 in^4	sr = 69.282
L = 20 ft	
P = 20 kip	
xe = 2 in	

11.3.4 Imperfections in Columns



The approaches used in analyzing the impact of imperfections in columns are defined by the seven equations here. A column having a length, **L** (m), and initial deflection, **ao** (m), from the centroid (central longitudinal) axis is considered. The ratio **α** (unitless), of the applied load, **P** (N), to critical load, **Pcr** (N), for the column is calculated in **equation 1**. An alternate way to compute **α** is using **equation 2**, which uses area moment **I** (m^4), column length **L** (m), applied load **P**; and modulus of elasticity, **E** (Pa). **Equations 3** and **4** compute maximum bending moment **Mmax** (N·m) and the maximum deflection, **δmax** (Pa), at the mid-point of the column. The **three remaining equations** solve for the maximum shear stress, **σmax** (Pa), using alternate methods depending on which variables are known. The variable, **r** (m), is the radius of gyration and, **c** (m), is the horizontal distance from the centroid axis to the furthest point on the concave side of the column.

Eq. 1
$$\alpha = \frac{P}{Pcr}$$

Eq. 2
$$\alpha = \frac{P \cdot L^2}{\pi^2 \cdot E \cdot I}$$

Eq. 3
$$M\max = \frac{P \cdot ao}{1-\alpha}$$

Eq. 4
$$\delta\max = \frac{ao}{1-\alpha}$$

Eq. 5
$$\sigma\max = \frac{P}{Area} + \frac{M\max \cdot c}{I}$$

Eq. 6
$$\sigma\max = \frac{P}{Area} \cdot \left(1 + \frac{ao \cdot c}{r^2 \cdot (1-\alpha)}\right)$$

Eq. 7

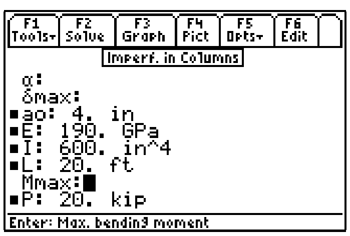
$$\sigma max = \frac{P}{Area} \cdot \left(1 + \frac{\frac{ao \cdot c}{r^2}}{1 - \frac{P}{\pi^2 \cdot E \cdot Area} \cdot \left(\frac{L}{r}\right)^2} \right)$$

The variable names, description and applicable default units used in the equations above are listed below.

Variable	**Description**	**Units**
α	Ratio	unitless
δmax	Maximum deflection due to load	m
σmax	Maximum stress	Pa
ao	Deflection w/o load	m
Area	Area	m^2
c	Centroid offset extremum	m
E	Young's modulus	Pa
I	Area moment of inertia	m^4
L	Length	m
Mmax	Maximum bending moment	N·m
P	Point load	N
Pcr	Critical load	N
r	Radius of gyration	m

Example 11.3.4:

Find the maximum deflection for the column described in example 11.3.1 and 11.3.2. Assume an initial deflection of 4 inches exists at the mid-point of this column.



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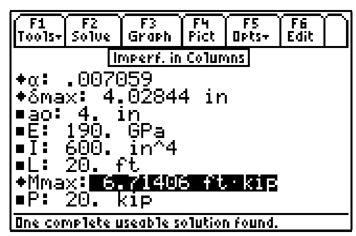


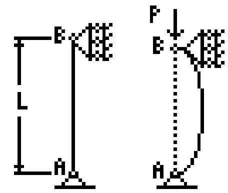
Computed results

Solution – Select **equations 2, 3,** and **4** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
ao = 4 in	α = .007059
E = 190 GPa	δmax = 4.02844 in
I = 600 in^4	Mmax = 6.71406 ft·kip
L = 20 ft	
P = 20 kip	

11.3.5 Inelastic Buckling

Critical load calculations for inelastic buckling are considered in this section. **Equation 1** computes the tangential modulus load **Pt** (N) given the column length **L** (m), the area moment **I** (m^4), and tangent modulus **Et** (Pa). The corresponding critical stress, **σt** (Pa), is computed in **equation 2** in terms of **Et**, column length, **L** (m), and the radius of gyration, **r** (m). **Equations 3** and **4** define a reduced modulus of elasticity, **Er**, and a reduced stress, **σr** (Pa), in terms of the known quantities **Et**, **L**, **r** and the Young's modulus of elasticity, **E** (Pa).



$$Pt = \frac{\pi^2 \cdot Et \cdot I}{L^2}$$ **Eq. 1**

$$\sigma t = \frac{\pi^2 \cdot Et}{\left(\frac{L}{r}\right)^2}$$ **Eq. 2**

$$Er = \frac{4 \cdot E \cdot Et}{\left(E^{.5} + Et^{.5}\right)^2}$$ **Eq. 3**

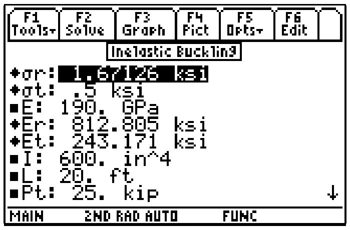
$$\sigma r = \frac{\pi^2 \cdot Er}{\left(\frac{L}{r}\right)^2}$$ **Eq. 4**

The variable names, description and applicable default units used in these equations are listed below.

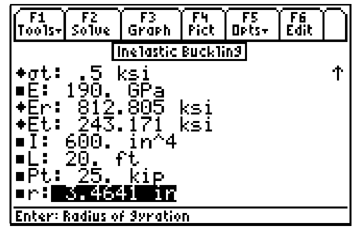
Variable	**Description**	**Units**
σr	Reduced stress	Pa
σt	Tangential stress	Pa
E	Young's modulus	Pa
Er	Reduced modulus of elasticity	Pa
Et	Tangent modulus of elasticity	Pa
I	Area moment of inertia	m^4
L	Length	m
Pt	Tangential load	N
r	Radius of gyration	m

Example 11.3.5:

A 25 kip tangential load is applied to the column having the computed properties in example 11.3.1. Find the tangential modulus and stress, in addition to the reduced modulus and stress.



Upper display



Lower display

Solution – Select **all of the equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
E = 190 GPa	σr = 1.67126 ksi
I = 600 in^4	σt = .5 ksi
L = 20 ft	Er = 812.805 ksi
Pt = 25 kip	Et = 243.171 ksi
r = 3.4641 in	

References:

1. Mechanics of Materials, 3rd Edition, (1990) *James M Gere and Stephen P. Timoshenko*, PWS Kent Publishing Company, Boston, MA Specific sections from Chapter 9 and Appendix G.

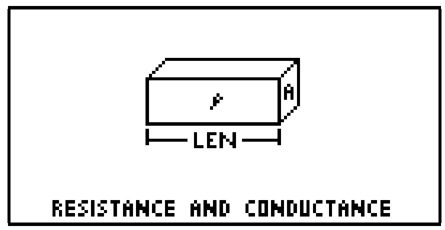
Chapter 12: EE for MEs

This portion of the software deals common electrical engineering problems encountered by mechanical engineers. Four sections form the core of the topic.

- Basic Electricity
- DC Generators
- DC Motor
- AC Motors

12.1 Basic Electricity

12.1.1 Resistance Formulas



Four equations in this section show the basic relationships between resistance and conductance. The **first equation** links the resistance, **R** (Ω), of a bar with a length, **len** (m), with a uniform cross-sectional area, **A** (m^2), with a resistivity, **ρ** (Ω·m). The **second equation** defines the conductance, **G** (siemen), of the same bar in terms of conductivity, **σ** (Sm/m), **len** (m), and **A.** The **3rd** and **4th equations** show the reciprocity of conductance **G** resistance **R** as well as the resistivity **ρ** and conductivity **σ**.

$$R = \frac{\rho \cdot len}{A} \qquad \textbf{Eq. 1}$$

$$G = \frac{\sigma \cdot A}{len} \qquad \textbf{Eq. 2}$$

$$G = \frac{1}{R} \qquad \textbf{Eq. 3}$$

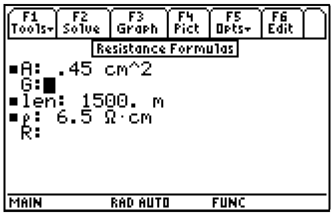
$$\sigma = \frac{1}{\rho} \qquad \textbf{Eq. 4}$$

The variable names, description and applicable default units used in the equations above are listed below.

Variable	**Description**	**Unit**
A	Area	m^2
G	Conductance	Siemens
Len	Length	m
ρ	Resistivity	Ω·m
R	Resistance	Ω
σ	Conductivity	Siemens/m

Example 12.1.1:

A copper wire 1500 m long has a resistivity of 6.5 Ω·cm and a cross sectional area of 0.45 cm^2. Compute the resistance and conductance.



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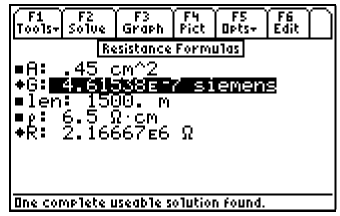


Computed results

Solution - Examining the problem, two clear choices are evident. Either **equations 1, 2** and **4 or 1** and **3** can be used to find the solution required. The second choice of equations will be used in this example. All the equations can be viewed at the equation screen wherein the two equations 1 and 3 can be selected by using the ▼ key to highlight the desired equation and pressing [ENTER]. Once both equations have been selected, press [F2] to display all the variables in the selected equation set. The software is now ready for receiving the input variables. Use the ▼ key to move the highlight bar to the variable that needs input. Type the value for the variable and press [ENTER]. Repeat to enter all the known variables and press [F2] to solve the selected equation set. The computed results are shown in the screen display shown here.

Given	**Solution**
A=.45 cm^2	G=4.61538E-7 Siemens
len=1500 m	R=2.16667E6 Ω
ρ=6.5 Ω·cm	

12.1.2 Ohm's Law and Power

The fundamental relationships between voltage, current and power are presented in this section. The **first equation** is the classic Ohm's Law, computes the voltage, **V** (V), in terms of the current, **I** (A)**,** and the resistance, **R** (Ω). The **next four equations** describe the relationship between power dissipation, **P** (W)**,** voltage, **V,** current, **I,** resistance, **R,** and conductance, **G** (siemens). The final equation represents the reciprocity between resistance, **R**, and conductance, **G**.

$$V = I \cdot R \qquad \textbf{Eq. 1}$$

$$P = V \cdot I \qquad \textbf{Eq. 2}$$

$$P = I^2 \cdot R \qquad \textbf{Eq. 3}$$

$$P = \frac{V^2}{R} \qquad \textbf{Eq. 4}$$

$$P = V^2 \cdot G \qquad \textbf{Eq. 5}$$

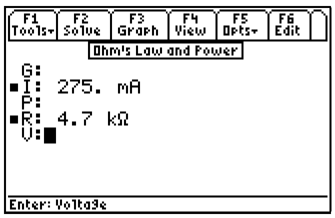
$$R = \frac{1}{G} \qquad \textbf{Eq. 6}$$

The variable names, description and applicable default units used in the equations above are listed below.

Variable	Description	Unit
G	Conductance	Siemens
I	Current	A
P	Power	W
R	Resistance	Ω
V	Voltage	V

Example 12.1.2:

A 4.7 kΩ load carries a current of 275 ma. Calculate the voltage across the load, power dissipated and load conductance.



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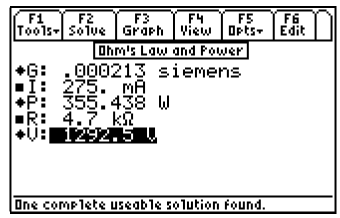


Computed results

Solution - Upon examining the problem, several choices are noted. Either **Equations 1, 2 and 6;** or **1, 2, 3** and **5;** or **2, 3** and **6;** or **1, 2** and **5;** or, **all** the **equations**. Choose the last option, press [F2] to open the input screen, enter all the known variables and press [F2] to solve.

Given	Solution
I = 275 mA	G = .000213 siemens
R = 4.7 kΩ	P = 355.438 W
	V = 1292.5_V

12.1.3 Temperature Effect

This equation models the effect of temperature on resistance. Electrical resistance changes from **RR1** (Ω) to **RR2** (Ω) when the temperature change from **T1** (K) to **T2** (K) is modified by the temperature coefficient of resistance α (1/K)**.**

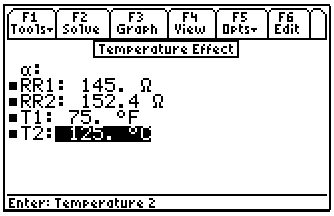
$$RR2 = RR1 \cdot (1 + \alpha (T2 - T1)) \qquad \textbf{Eq. 1}$$

The variable names, description and applicable default units used in the equations above are listed below.

Variable	Description	Unit
α	Temperature coefficient	1/K
RR1	Resistance, T1	Ω
RR2	Resistance, T2	Ω
T1	Temperature 1	K
T2	Temperature 2	K

Example 12.1.3:

A 145 Ω resistor at 75 °F reads 152.4 Ω at 125 ºC. Find the temperature coefficient of resistance.



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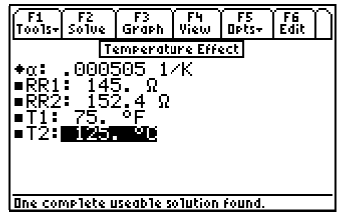


Computed results

Solution - Press [F2] to display the input screen. Enter the variable values and press [F2] to solve for the unknown variable.

Given
RR1=145 Ω
RR2 = 152.4 Ω
T1=75 ºF
T2=125_ºC

Solution
α= .000505 1/K

12.2 DC Motors

12.2.1 DC Series Motor

These eight equations describe the performance characteristics of a series DC motor. The **first equation** links the terminal voltage, **Vt** (V), to the back emf, **Ea** (V), defined by the **third equation** and the I·R drop due to armature resistance, **Ra** (Ω), adjustable resistance, **Rd** (Ω), and series resistance **Rs**, (Ω). The **second equation** calculates the load torque, **TL** (N·m), with the machine constant **Ke**, flux, **ϕ** (Wb), load current, **IL** (A), and the torque loss, **Tloss** (N·m). The **third equation** defines the back emf in the armature, **Ea** (V), in terms of **Ke**, **ϕ**, and mechanical frequency **ωm** (rad/s). The **fourth equation** shows torque generated at the rotor due the magnetic flux, **ϕ** and current **IL**. The **sixth equation** computes the torque generated **T** as the sum of load torque **TL** and lost torque **Tloss**. The **last two equations** show the connection between **Ke**, **ϕ**, a field constant **Kef** (Wb/A), load current **IL**, and torque **T**.

$$Vt = Ke \cdot \phi \cdot \omega m + (Ra + Rs + Rd) \cdot IL \quad \textbf{Eq. 1}$$

$$TL = Ke \cdot \phi \cdot IL - Tloss \quad \textbf{Eq. 2}$$

$$Ea = Ke \cdot \omega m \cdot \phi \quad \textbf{Eq. 3}$$

$$T = Ke \cdot \phi \cdot IL \quad \textbf{Eq. 4}$$

The **fifth** equation shows a reciprocal quadratic link between **ωm**, **Vt**, **Ke**, **ϕ**, **Ra**, **Rs**, **Rd**, and torque **T** (N·m).

$$\omega m = \frac{Vt}{Ke \cdot \phi} - \frac{(Ra + Rs + Rd) \cdot T}{(Ke \cdot \phi)^2} \quad \textbf{Eq. 5}$$

$$T = Tloss + TL \quad \textbf{Eq. 6}$$

$$Ke \cdot \phi = Kef \cdot IL \qquad \textbf{Eq. 7}$$

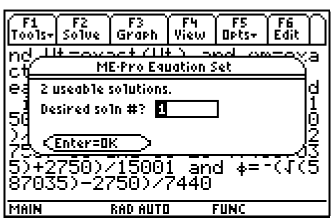
$$T = Kf \cdot IL^2 \qquad \textbf{Eq. 8}$$

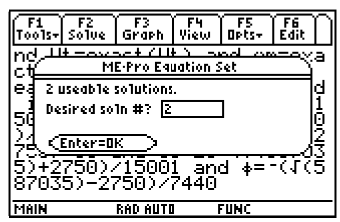
The variable names, description and applicable default units used in the equations above are listed below.

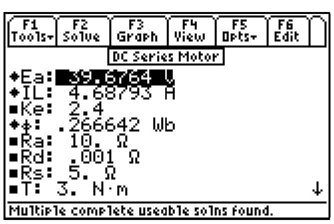
Variable	**Description**	**Unit**
Ea	Average emf induced in the armature	V
IL	Load current	A
Ke	Machine constant	Unitless
Kef	Field coefficient	Wb/A
ϕ	Flux	Wb
Ra	Armature resistance	Ω
Rd	Adjustable resistance	Ω
Rs	Series field resistance	Ω
T	Internal torque	N·m
TL	Load torque	N·m
Tloss	Torque loss	N·m
Vt	Terminal voltage	V
ωm	Mechanical radian frequency	rad/s

Example 12.2.1:

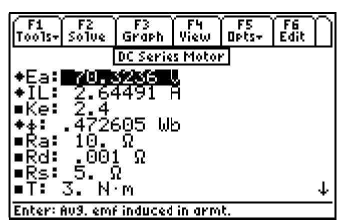
A series motor, with a machine constant of 2.4 and rotating at 62 rad/s, is supplied with a terminal voltage of 110 V and produces a torque of 3 Nm. The armature resistance is 10 Ω, the series resistance is 5 Ω, and the adjustable resistance is 0.001 Ω. Find the average voltage induced in the armature, the flux, and the load current.



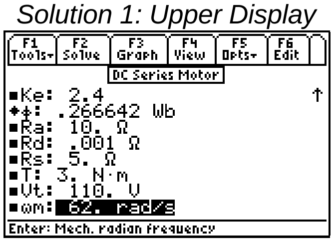




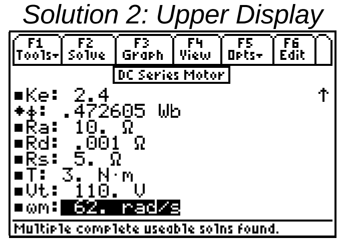
Solution 1: Upper Display



Solution 2: Upper Display



Solution 1: Lower Display



Solution 2: Lower Display

Solution - The **first, third** and **fifth equations** are needed to compute a solution. Select these by highlighting and pressing [ENTER]. Press [F2] to display the input screen, enter all the known variables and press [F2] to solve the selected equation set. There are two possible solutions for this example. Type the number of the solution set to be viewed and press [ENTER] twice. To view another solution set, press [F2] to and select another number. The computed results are shown in the screen displays above.

Given	**Solution**
Ke = 2.4	Ea = 39.6764 V (70.3236 V)
Ra = 10 Ω	IL = 4.68793 A (2.64491 A)
Rd = .001 Ω,	ϕ = .266642 Wb (.472605 Wb)
Rs = 5. Ω	
T = 3. N·m	
Vt = 110. V	
ωm = 62. rad/s	

12.2.2 DC Shunt Motor

These seven equations describe the principal characteristics of a DC shunt motor. The **first equation** expresses the terminal voltage, **Vt** (V), in terms of the field current, **IIf** (A), and field resistance, **Rf** (Ω), along with the external field resistance, **Re** (Ω). The **second equation** defines the terminal voltage, **Vt** (V), in terms of the back emf (expressed in terms of the machine constant, **Ke**, flux swept, **ϕ** (Wb), angular velocity, **ωm** (rad/s), and the IR drop in the armature circuit.

$$Vt = (Re + Rf) \cdot IIf \qquad \textbf{Eq. 1}$$

$$Vt = Ke \cdot \phi \cdot \omega m + Ra \cdot Ia \qquad \textbf{Eq. 2}$$

The **third equation** refers to the torque available at the load, **TL** (N·m), due to the current, **Ia** (A), in the armature minus the loss of torque, **Tloss** (N·m), due to friction and other reasons.

$$TL = Ke \cdot \phi \cdot Ia - Tloss \qquad \textbf{Eq. 3}$$

The **fourth equation** gives the definitive re lationship between the back emf **Ea** (V), **Ke**, **ϕ** (Wb), and **ωm** (rad/s).

$$Ea = Ke \cdot \omega m \cdot \phi \qquad \textbf{Eq. 4}$$

The **fifth** equation displays the reciprocal quadratic relationship between **ωm**, **Vt**, **K**, **ϕ**, armature resistance, **Ra** (Ω), adjustable resistance, **Rd** (Ω), and **T** (N·m).

$$\omega m = \frac{Vt}{Ke \cdot \phi} - \frac{(Ra + Rd) \cdot T}{(Ke \cdot \phi)^2}$$ **Eq. 5**

The **last two equations** compute torque **T** in terms of **Tloss**, load torque **TL**, flux **ϕ**, **Ia** (A), and **Ke**.

$$T = Tloss + TL$$ **Eq. 6**

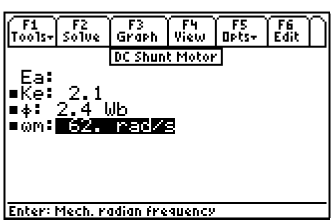
$$T = Ke \cdot \phi \cdot Ia$$ **Eq. 7**

The variable names, description and applicable default units used in the equations above are listed below.

Variable	Description	Unit
Ea	Average emf induced in armature	V
Ia	Armature current	A
Iif	Field current	A
Ke	Machine constant	Unitless
ϕ	Flux	Wb
Ra	Armature resistance	Ω
Rd	Adjustable resistance	Ω
Re	Ext. shunt resistance	Ω
Rf	Field coil resistance	Ω
T	Internal torque	N·m
TL	Load torque	N·m
Tloss	Torque loss	N·m
Vt	Terminal voltage	V
ωm	Mechanical radian frequency	rad/s

Example 12.2.2:

Find the back emf for a motor with a machine constant of 2.1, rotating at 62 rad/s in a flux of 2.4 Wb.



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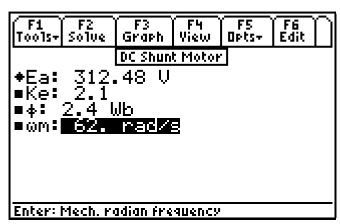


Calculated Results

Solution - Use the **fourth equation** to solve this problem. Select the equation with the cursor bar and press [ENTER]. Press [F2] to display the input screen, enter all the known variables and press [F2] to solve the selected equation. The computed result is shown in the screen display above.

Given	Solution
Ke=2.1	Ea=312.48 V
ϕ=2.4 Wb	
ωm=62. rad/s	

12.3 DC Generators

12.3.1 DC Series Generator

The two equations in this section describe the properties of a series DC generator. The **first equation** specifies the field current, **IIf** (A), and the armature current, **Ia** (A), to be the same. The **second equation** computes the terminal voltage, **Vt** (V), in terms of the induced emf, **Ea** (V), load current, **IL** (A), armature resistance, **Ra** (Ω), and series field windings, **Rs** (Ω).

$$Ia = IIf \qquad \textbf{Eq. 1}$$

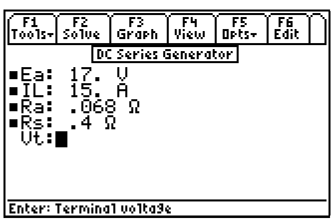
$$Vt = Ea - (Ra + Rs) \cdot IL \qquad \textbf{Eq. 2}$$

The variable names, description and applicable default units used in the equations above are listed below.

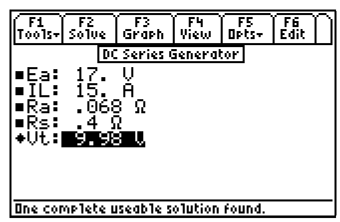
Variable	**Description**	**Unit**
Ea	Average emf induced in armature	V
Ia	Armature current	A
IIf	Field current	A
IL	Load current	A
Ra	Armature resistance	Ω
Rs	Series field resistance	Ω
Vt	Terminal voltage	V

Example 12.3.1:

Find the terminal voltage of a series generator with an armature resistance of 0.068 Ω and a series resistance of 0.40 Ω. The generator delivers a 15 A load current from a generated voltage of 17 V.



Entered Values



Calculated Results

Solution - Use the **second equation** to solve this problem. Select this with the highlight bar and press ENTER. Press F2 to display the input screen, enter all the known variables and press F2 to solve the selected equation. The computed result is shown in the screen display above.

Given	**Solution**
Ea=17. V	Vt = 9.98 V
IL=15. A	
Ra=.068 Ω	
Rs=.4_Ω	

12.3.2 DC Shunt Generator

The **first equation** in this section expresses the induced armature voltage, **Ea** (V), in terms of the machine constant, **Ke**, the mechanical angular frequency, **ωm** (rad/s), and flux, **ϕ** (Wb).

$$Ea = Ke \cdot \omega m \cdot \phi \qquad \textbf{Eq. 1}$$

The **second equation** defines terminal voltage, **Vt** (V), in terms of the field current, **IIf** (A), external resistance, **Re** (Ω), and field coil resistance, **Rf** (Ω). The **third equation** computes **Vt** in terms of load current, **IL** (A), and load resistance, **Rl** (Ω). The **fourth equation** expresses **Vt** as the induced emf, **Ea** (V), minus armature IR drop, **Ra·Ia**.

$$Vt = (\mathrm{Re} + Rf) \cdot IIf \qquad \textbf{Eq. 2}$$

$$Vt = IL \cdot Rl \qquad \textbf{Eq. 3}$$

$$Vt = Ea - Ra \cdot Ia \qquad \textbf{Eq. 4}$$

The armature current, **Ia** (A), is the sum of the load current **IL** and field current **IIf** in the **fifth equation**.

$$Ia = IL + IIf \qquad \textbf{Eq. 5}$$

The **final equation** is an alternate form of expression for **Ea**.

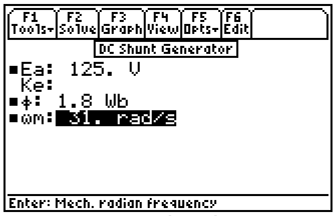
$$Ea = Ra \cdot Ia + (\mathrm{Re} + Rf) \cdot IIf \qquad \textbf{Eq. 6}$$

The variable names, description and applicable default units used in the equations above are listed below.

Variable	**Description**	**Unit**
Ea	Average emf induced in armature	V
Ia	Armature current	A
IIf	Field current	A
IL	Load current	A
Ke	Machine constant	unitless
ϕ	Flux	Wb
Ra	Armature resistance	Ω
Re	Ext. shunt resistance	Ω
Rf	Field coil resistance	Ω
Rl	Load resistance	Ω
Vt	Terminal voltage	V
ωm	Mechanical radian frequency	rad/s

Example 12.3.2:

Find the machine constant of a shunt generator running at 31 rad/s and producing 125 V with a 1.8 Wb flux.



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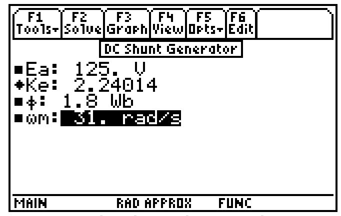


Calculated Results

Solution - Use the **first equation** to solve this problem. Select this by pressing [ENTER]. Press [F2] to display the input screen, enter all the known variables and press [F2] to solve the selected equation. The computed result is shown in the screen display above.

Given	**Solution**
Ea= 125. V	Ke=2.24014
ϕ=1.8 Wb	
ωm=31. rad/s	

12.4 AC Motors

12.4.1 Three ϕ Induction Motor I

These eleven equations define the relationships amongst key variables used in evaluating the performance of an induction motor. The **first equation** expresses the relationship between the radian frequency induced in the rotor, **ωr** (rad/s), the angular speed of the rotating magnetic field, of the stator **ωs** (rad/s), number of poles, **p**, and the mechanical angular speed, **ωm** (rad/s).

$$\omega r = \omega s - \frac{p}{2} \cdot \omega m \qquad \textbf{Eq. 1}$$

The **second, third** and **fourth equations** describe the slip, **s**, using **ωr**, **ωs**, **ωm**, **p**, the induced rotor power per phase, **Pr** (W), and the power transferred to the rotor per phase, **Pma** (W).

$$s = 1 - \frac{p}{2} \cdot \frac{\omega m}{\omega s} \qquad \textbf{Eq. 2}$$

$$\frac{\mathrm{Pr}}{Pma} = s \qquad \textbf{Eq. 3}$$

$$\omega r = s \cdot \omega s \qquad \textbf{Eq. 4}$$

Pma is defined in the **fifth equation** in terms of the rotor current, **Ir** (A), and the rotor phase voltage, **Ema** (V).

$$Pma = 3 \cdot Ir \cdot Ema \qquad \textbf{Eq. 5}$$

The **sixth** and **seventh equations** account for the mechanical power, **Pme** (W),in terms of **p**, **ωm**, **ωs**, **Pma**, and torque, **T** (N·m).

$Pme = 3 \cdot \frac{p}{2} \cdot \frac{\omega m}{\omega s} \cdot Pma$	**Eq. 6**
$Pme = T \cdot \omega m$	**Eq. 7**

The **eighth equation** expresses torque in terms of **p**, **Pma**, and **ωs**.

$T = 3 \cdot \frac{p}{2} \cdot \frac{Pma}{\omega s}$	**Eq. 8**

The **last three equations** show an equivalent circuit representation of induction motor action and links the power, **Pa** with rotor resistance, **Rr** (Ω), rotor current, **Ir**, slip **s**, rotor resistance per phase, **RR1** (Ω), and the machine constant, **KeM**.

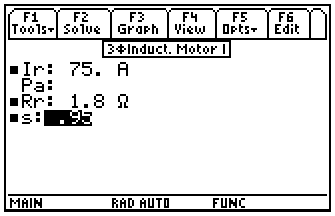
$Pma = Rr \cdot Ir^2 + \frac{1-s}{s} \cdot Rr \cdot Ir^2$	**Eq. 9**
$Pa = \frac{1-s}{s} \cdot Rr \cdot Ir^2$	**Eq. 10**
$Rr = \frac{RR1}{KeM^2}$	**Eq. 11**

The variable names, description and applicable default units used in the equations above are listed below.

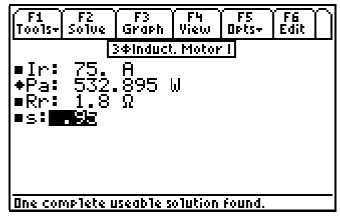
Variable	**Description**	**Unit**
Ema	Phase voltage	V
Ir	Rotor current per phase	A
KeM	Induction motor constant	unitless
p	# poles	unitless
Pa	Mechanical power available	W
Pma	Power in rotor per phase	W
Pme	Mechanical power	W
Pr	Rotor power per phase	W
RR1	Rotor resistance per phase	Ω
Rr	Equivalent rotor resistance	Ω
s	Slip	unitless
T	Internal torque	N·m
ωm	Mechanical radian frequency	rad/s
ωr	Electrical rotor speed	rad/s
ωs	Electrical stator speed	rad/s

Example 12.4.1:

Find the mechanical power for an induction motor with a slip of 0.95, a rotor current of 75 A, and a resistance of 1.8 Ω.



Input Values



Calculated Results

Solution - Choose **equation ten** to compute the solution. Select by highlighting and pressing ENTER. Press F2 to display the input screen, enter all the known variables and press F2 to solve the equation

Given
Ir = 75. A
Rr = 1.8 Ω
s = .95

Solution
Pa = 532.895 W

12.4.2 Three φ Induction Motor II

These equations are used to perform equivalent circuit analysis for an induction motor. The **first equation** shows the power in the rotor per phase, **Pma** (W), defined in terms of the rotor current, **Ir** (A), rotor resistance, **Rr** (Ω), and slip **s**.

$$Pma = \frac{Rr}{s} \cdot Ir^2 \qquad \textbf{Eq. 1}$$

The **second equation** shows the expression for torque, **T** (N·m), in terms of poles **p**, **Pma** and radian frequency of the induced voltage in the stator, **ωs** (rad/s). The **third equation** is an alternate representation of torque in terms of the applied voltage, **Va** (V), stator resistance, **Rst** (Ω), **Rr** (Ω), inductive reactance **XL** (Ω), and **ωs** (rad/s).

$$T = \frac{3}{2} \cdot p \cdot \frac{Pma}{\omega s} \qquad \textbf{Eq. 2}$$

$$T = \frac{3}{2} \cdot \frac{p}{\omega s} \cdot \frac{Rr}{s} \cdot \frac{Va^2}{\left(Rst + \frac{Rr}{s}\right)^2 + XL^2} \qquad \textbf{Eq. 3}$$

The **fourth equation** computes **Tmmax** (N·m) represents the maximum positive torque available at the rotor, given the parameters of the induction motor stator resistance, **Rst**, **XL**, **Va**, **p**, and **ωs**.

$$Tm\max = \frac{3}{4} \cdot \frac{p}{\omega s} \cdot \frac{Va^2}{\sqrt{Rst^2 + XL^2} + Rst} \qquad \textbf{Eq. 4}$$

The maximum slip, **sm**, in the **fifth equation** represents the condition when dT/ds=0.

$$sm = \frac{Rr}{\sqrt{Rs^2 + XL^2}} \qquad \textbf{Eq. 5}$$

The **sixth equation** defines the so-called breakdown torque, **Tgmax** (N·m), of the motor. The **final equation** relates, **Rr** (Ω), with machine constant, **KeM**, and the rotor resistance per phase, **RR1** (Ω).

$$Tg\max = -\frac{3}{4} \cdot \frac{p}{\omega s} \cdot \frac{Va^2}{\sqrt{Rs^2 + XL^2} - Rst} \qquad \textbf{Eq. 6}$$

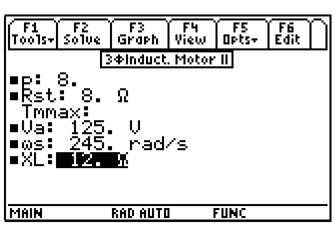
$$Rr = \frac{RR1}{KeM^2} \qquad \textbf{Eq. 7}$$

The variable names, description and applicable default units used in the equations above are listed below.

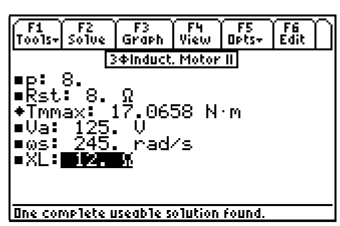
Variable	Description	Unit
Ir	Rotor current per phase	A
KeM	Induction motor constant	unitless
p	# poles	unitless
Pma	Power in rotor per phase	W
RR1	Rotor resistance per phase	Ω
Rr	Equivalent rotor resistance	Ω
Rs	Series field resistance	Ω
Rst	Stator resistance	Ω
S	Slip	unitless
Sm	Maximum slip	unitless
T	Internal torque	N·m
Tgmax	Breakdown torque	N·m
Tmmax	Maximum positive torque	N·m
Va	Applied voltage	V
ωs	Electrical stator speed	rad/s
XL	Inductive reactance	Ω

Example 12.4.2:

An applied voltage of 125 V is applied to an eight-pole motor rotating at 245 rad/s. The stator resistance and reactance is 8 and 12 Ω respectively. Find the maximum torque.



Input Values



Calculated Results

Solution - Use the **fourth equation** to compute the solution. Select by moving the cursor bar, highlighting, and pressing [ENTER]. Press [F2] to display the input screen, enter all the known variables and press [F2] to solve the equation.

Given
p = 8
Rst = 8. Ω
Va = 125. V
ωs = 245. rad/s
XL = 12._ Ω

Solution
Tmmax = 17.0658 N·m

12.4.3 1 Induction Motor

These three equations describe the properties of a single-phase induction motor. The **first equation** defines the slip for forward flux **sf** with respect to the forward rotating flux, ϕ (Wb). The radian frequency of induced is current in the stator, **ωs** (rad/s). Other variables of consequence include the number of poles, **p,** and the angular mechanical speed of the rotor **ωm** (rad/s). The **final two equations** represent the forward and backward torques, **Tf** (N·m) and **Tb** (N·m), for the system with respect to **sf,** the number of poles **p**, the electrical stator speed, **ωs** (rad/s), the equivalent rotor resistance, **Rr** (Ω), and the currents, **Isf** (A) and **Isb** (A). The forward torque, **Tf**, is given by the power dissipated in the fictitious rotor resistor.

$$sf = 1 - \frac{p}{2} \cdot \frac{\omega m}{\omega s}$$ **Eq. 1**

$$Tf = \frac{p}{2} \cdot \frac{1}{\omega s} \cdot \frac{Isf^2 \cdot Rr}{2 \cdot sf}$$ **Eq. 2**

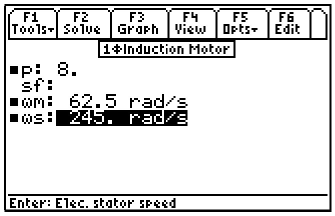
$$Tb = -\frac{p}{2} \cdot \frac{1}{\omega s} \cdot \frac{Isb^2 \cdot Rr}{2 \cdot (2 - sf)}$$ **Eq. 3**

The variable names, description and applicable default units used in the equations above are listed below.

Variable	**Description**	**Unit**
Isb	Backward stator current	A
Isf	Forward stator current	A
p	# poles	unitless
Rr	Equivalent rotor resistance	Ω
sf	Slip for forward flux	unitless
Tb	Backward torque	N·m
Tf	Forward torque	N·m
ωm	Mechanical radian frequency	rad/s
ωs	Electrical stator speed	rad/s

Example 12.4.3:

Find the forward slip for an eight-pole induction motor with a stator frequency of 245 rad/s, and a mechanical radian frequency of 62.5 rad/s.



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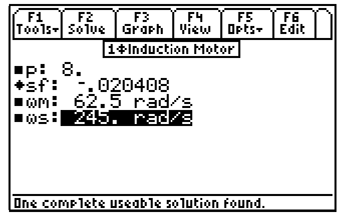


Calculated Results

Solution - The **first equation** is needed to compute the solution. Select by highlighting and pressing [ENTER]. Press [F2] to display the input screen, enter all the known variables and press [F2] to solve the equation.

Given	**Solution**
p = 8	sf = -.020408
ωm = 62.5_rad/s	
ωs = 245._rad/s	

References:

1. Slemon G. R., and Straughen, A., *Electric Machines*, Addison-Wesley, Reading, MA 1980
2. Stevenson Jr., William D., *Elements of Power Systems Analysis*, McGraw-Hill International, New York, 1982
3. Wildi, Theodore, *Electrical Power Technology*, John Wiley and Son, New Jersey, 1981

Chapter 13: Gas Laws

This section computes properties and state changes for ideal and real gases with an emphasis on ideal gases. Selected topics for real gases include van der Waals, and Redlich -Kwong models. The ideal gas constant, **Rm,** 8.31451 J/(mol·K) is automatically inserted into all calculations and does not appear in the list of variables in the calculator.

- **Ideal Gas Laws**
- **Real Gas Laws**
- **Polytropic Process**
- **Kinetic Gas Theory**
- **Reverse Adiabatic**

13.1 Ideal Gas Laws

13.1.1 Ideal Gas Law

The ideal gas law approximates, to a high degree, the actual properties of a gas at high temperature or low pressure. **Equations 1** and **2** define the molar volume, **vm** (m^3/mol)**,** and specific volume, **vs** (m^3/kg)**,** of the gas in terms of the number of moles, **N** (mol), of gas or total mass, **m** (kg), occupying a volume, **V** (m^3). **Equations 3, 4** and **5** are three alternate forms of the ideal gas relationship between pressure, **p** (Pa), volume, **V,** temperature, **T** (K), molecular weight, **MWT** (kg/mol), and specific volume, **vs**. The **last equation** computes molar mass, **MWT,** in terms of **m** and **N**.

$$vm = \frac{V}{N}$$ **Eq. 1**

$$vs = \frac{V}{m}$$ **Eq. 2**

$$p \cdot vs = \frac{Rm}{MWT} \cdot T$$ **Eq. 3**

$$p \cdot V = N \cdot Rm \cdot T$$ **Eq. 4**

$$p \cdot vm = Rm \cdot T$$ **Eq. 5**

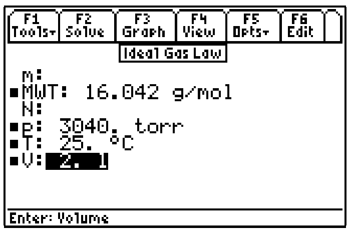
$$MWT = \frac{m}{N}$$ **Eq. 6**

Variable	Description	Units
m	Mass	kg
MWT	Molar mass	kg/mol
N	No. moles	mol
p	Pressure	Pa
Rm	Molar gas constant	8.3145 J/(mol·K)
T	Temperature	K

Variable	Description	Units
V	Volume	m^3
vm	Molar volume	m^3/mol
vs	Specific volume	m^3/mol

Example 13.1.1:

A 2-liter container is filled with methane (molecular mass = 16.042 g/mol) to a pressure of 3040 torr at room temperature (25^{o}C). Calculate the number of moles and the total mass of methane.



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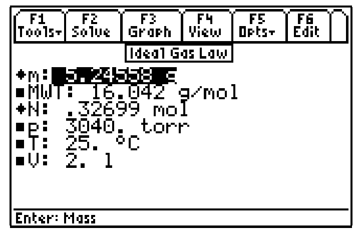


Computed results

Solution - Select the **fourth** and **sixth equations** to solve this problem. All the equations can be viewed at the equation screen wherein the two equations 4 and 6 can be selected by using the ↓ key to highlight the desired equation and pressing [ENTER]. Once both equations have been selected, press [F2] to display all the variables in the selected equation set. Use the ↓ key to move the highlight bar to the variable needing data. Enter the value for the variable then press [ENTER]. Repeat this until values for all known variables have been entered. Press [F2] to solve the selected equation set. The computed results are shown in the screen display shown here.

Given	Solution
MWT = 16.042 g/mol	m = 5.24558 g
p=3040 torr	N = .32699 mol
T=25. ^{o}C	
V=2. l	

13.1.2 Constant Pressure

The following equations describe the changes of the state for a fixed quantity gas at constant pressure, **p. Equation 1** describes Charles's Law- volume, **V** (m^3)**,** of a fixed amount of gas is directly proportional to the absolute temperature, **T** (K)**. Equations 2** and **3** express relationship between W12 (J) in terms of pressure and volume or moles and temperatures change respectively. **Equations 4** and **5** calculate the change in total entropy, **S21** (J/K), and mass-specific entropy (entropy per unit mass), **ss21** (J/(kg·K)), due to change in temperature, **T2-T1** (K). **Equation 6** calculates the change in entropy per mole of gas, **sm21** (J/(mol·K))**,** from **ss21**, and molecular mass, **MWT** (kg/mol). **Equations 7** and **8** compute the transfer of heat to the system, **Q12** (J), due to expansion under constant pressure. **Equations 9** and **10** describe the relationships between the specific heat ratio **k,** to the specific heats constant volume, **cv (**J/(kg·K)), and constant pressure, **cp** (J/(kg·K)). **Equation 11** relates molecular weight, **MWT** (kg/mol), to the number of moles, **N** (mol) and total mass of the gas, **m** (kg).

$$\frac{V2}{V1} = \frac{T2}{T1} \quad \textbf{Eq. 1}$$

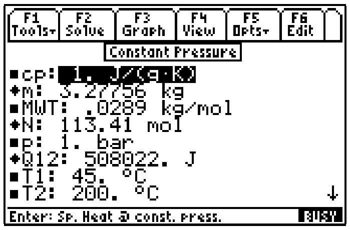
$$W12 = -p \cdot (V2 - V1) \quad \textbf{Eq. 2}$$

$W12 = -N \cdot Rm \cdot (T2 - T1)$	**Eq. 3**
$S21 = m \cdot cp \cdot \ln\left(\frac{T2}{T1}\right)$	**Eq. 4**
$ss21 = cp \cdot \ln\left(\frac{T2}{T1}\right)$	**Eq. 5**
$sm21 = MWT \cdot ss21$	**Eq. 6**
$Q12 = m \cdot cp \cdot (T2 - T1)$	**Eq. 7**
$Q12 = \frac{k \cdot W12}{k - 1}$	**Eq. 8**
$cp = cv + \frac{Rm}{MWT}$	**Eq. 9**
$k = \frac{cp}{cv}$	**Eq. 10**
$MWT = \frac{m}{N}$	**Eq. 11**

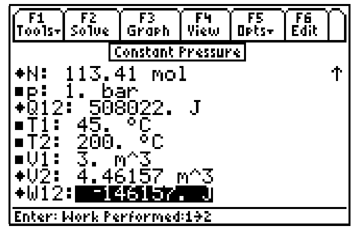
Variable	**Description**	**Units**
cp	Specific Heat-constant pressure	J/(kg·K)
cv	Specific Heat-constant volume	J/(kg·K)
k	Specific Heat Ratio	unitless
m	Mass	kg
MWT	Molar Mass.	kg/mol
N	No. moles	mol
p	Pressure	Pa
Q12	Heat Transfer: 1→2	J
Rm	Molar Gas constant	8.3145 J/(mol·K)
sm21	Entropy Change-mole: 1→2	J/(mol·K)
ss21	Entropy Change-mass: 1→2	J/(kg·K)
S21	Entropy Change: 1→2	J/K
T1	Initial Temperature: 1	K
T2	Final Temperature: 2	K
V1	Initial Volume	m^3
V2	Final Volume	m^3
W12	Work Performed: 1→2	J

Example 13.1.2:

Dry air has a molecular mass of 0.0289 kg/mol; see **mwa** in *Reference/Engineering Constants*, and a specific heat of 1.0 J/(g·K) at constant pressure of 1 bar in the temperature range of 200-500K. Air in a 3-m3 cylinder performs work on a frictionless piston, exerting a constant pressure of 1 bar. A heating element increases the temperature of the air from 45°C to 200°C. Assuming the air has ideal gas behavior, calculate the change in volume, the work performed on the piston, and the heat absorbed by the gas due to heating.



Upper Display



Lower Display

Solution - Select **equations, one, two, three, seven** and **eleven.**

Given:
cp = 1 J/(g·K)
MWT = 0.0289 kg/mol
p = 1 bar
T1 = 45 °C
T2 = 200 °C
V1 = 3 m^3

Solution:
m = 3.27756 kg
N = 113.41 mol
Q12 = 508022 J
V2 = 4.46157 m^3
W12 = -146157 J

13.1.3 Constant Volume

The following equations describe the changes of state for a fixed quantity gas under constant volume, **V (m^3),** conditions. **Equation 1** often called Gay-Lussac's Law, states that the pressure of a fixed quantity of gas is directly proportional to the absolute temperature under conditions of constant volume. **Equations 2** and **3** compute heat, **Q12** (J), absorbed by the constant volume system. **The fourth** and **fifth** equations compute the change in total entropy, **S21** (J/K), and mass-specific entropy (entropy per unit mass), **ss21 (**J/(kg·K)), due to change in temperature, **T2-T1** (K) under conditions of constant volume. **Equation 6** calculates the change entropy per mole of gas, **sm21** J/(mol·K), from **ss21** and molar mass, **MWT** (kg/mol). **Equations 7** and **8** compute the specific heat ratio, **k**, from the specific heat for constant volume, **cv** J/(kg·K), and constant pressure, **cp** (J/(kg·K)). The **last equation** relates the molar mass, **MWT**, to the number of moles, **N** (mol), and total mass of the gas, **m** (kg).

$$\frac{p2}{p1} = \frac{T2}{T1}$$ **Eq. 1**

$$Q12 = m \cdot cv \cdot (T2 - T1)$$ **Eq. 2**

$$Q12 = \frac{V \cdot (p2 - p1)}{k - 1}$$ **Eq. 3**

$$S21 = m \cdot cv \cdot \ln\left(\frac{T2}{T1}\right)$$ **Eq. 4**

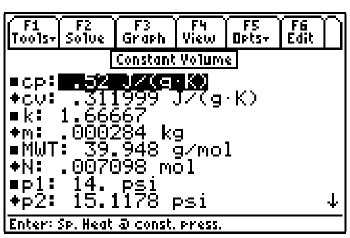
$$ss21 = cv \cdot \ln\left(\frac{T2}{T1}\right)$$ **Eq. 5**

$sm21 = MWT \cdot ss21$	**Eq. 6**
$cp = cv + \dfrac{Rm}{MWT}$	**Eq. 7**
$k = \dfrac{cp}{cv}$	**Eq. 8**
$MWT = \dfrac{m}{N}$	**Eq. 9**

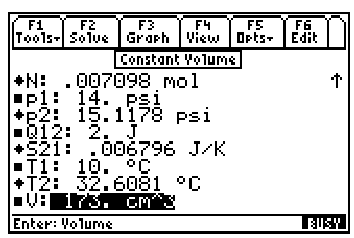
Variable	**Description**	**Units**
cp	Specific heat-constant pressure	J/(kg·K)
cv	Specific heat @ constant volume	J/(kg·K)
k	Specific heat ratio	unitless
m	Mass	kg
MWT	Molar mass	kg/mol
N	No. moles	mol
p1	Initial pressure	Pa
p2	Final pressure	Pa
Rm	Molar gas constant	8.3145 J/(mol·K)
Q12	Heat transfer	J
S21	Entropy change: 1→2	J/K
sm21	Entropy change-mole: 1→2	J/(mol·K)
ss21	Entropy change-mass: 1→2	J/(kg·K)
T1	Initial temperature	K
T2	Final temperature	K
V	Volume	m^3

Example 13.1.3:

An electric current transfers 2 J as heat to argon contained in a 173 cm^3 neon sign. The noble gas is initially charged to a pressure of 14 psi at 10°C. Argon has a molar mass of 39.948 g/mol, a specific heat at constant pressure of 0.52 J/(g·K) (*see Reference/Thermal Properties/Cp Liquids and Gases*). For an ideal gas, the molar specific heat ratio is 5/3. Calculate the specific heat at constant volume, the final temperature and pressure, the total mass of argon and the total increase in entropy.



Upper Display



Lower Display

Solution – The molar specific heat ratio is the same as the mass specific heat ratio k. Select **equations one, two, three, four, eight** and **nine** to solve this problem

Given	**Solution**
cp = .52 J(g·K)	cv = .311999 J/(g·K)
k = 5/3	m = .000284 kg

MWT = 39.948 g/mol
p1 = 14 psi
Q12 = 2 J
T1 = 10 ^oC
V = 173 cm^3

N = 0.007098 mol
p2 = 15.1178 psi
S21 = .006796 J/K
T2 = 32.6081 ^oC

13.1.4 Constant Temperature

Often called Boyle's Law, the **first equation** shows that the pressure of a fixed quantity of gas is inversely proportional to the volume if the temperature,**T** (K), remains constant. **Equations 2** and **3** calculate the energy leaving the system in the form of work, **W12** (J), for a reversible isothermal expansion between initial volume, **V1** (m^3), and final volume, **V2** (m^3). **Equation 4** displays the heat transferred to a system, **Q12** (J), undergoing reversible expansion is converted to work, **W12** (J), leaving the gas system during an isothermal process. **Equation 5** computes the total entropy change, **S12** (J/K), from the heat transferred to the system, **Q12** (J), at temperature, **T** (K). **Equations 6, 7** and **8** compute the entropy per mass (specific), **ss21** (J/(kg·K)), and entropy per mole, **sm21 (**J/(mol·K)). The **last equation** computes the molar mass, **MWT** (kg/mol), from the number of moles, **N** (mol), and total mass of the gas, **m** (kg).

$$\frac{p2}{p1}=\frac{V1}{V2}$$ **Eq. 1**

$$W12=-N\cdot Rm\cdot T\cdot \ln\left(\frac{V2}{V1}\right)$$ **Eq. 2**

$$W12=-p1\cdot V1\cdot \ln\left(\frac{V2}{V1}\right)$$ **Eq. 3**

$$Q12=-W12$$ **Eq. 4**

$$S21=\frac{Q12}{T}$$ **Eq. 5**

$$ss21=Rm\cdot T\cdot \ln\left(\frac{V2}{V1}\right)$$ **Eq. 6**

$$ss21=\frac{S21}{m}$$ **Eq. 7**

$$sm21=MWT\cdot ss21$$ **Eq. 8**

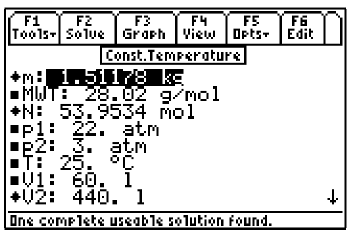
$$MWT=\frac{m}{N}$$ **Eq. 9**

Variable	**Description**	**Units**
m	Mass	kg
MWT	Molar Mass	kg/mol
N	No. moles	mol

Variable	Description	Units
p1	Initial Pressure	Pa
p2	Final Pressure	Pa
Q12	Heat Transfer	J
Rm	Molar Gas constant	8.3145 J/(mol·K)
sm21	Entropy Change-mole: $1 \rightarrow 2$	J/(mol·K)
ss21	Entropy Change-mass: $1 \rightarrow 2$	J/(kg·K)
S21	Entropy Change: $1 \rightarrow 2$	J/K
T	Temperature	K
V1	Initial Volume	m^3
V2	Final Volume	m^3
W12	Work Performed: $1 \rightarrow 2$	J

Example 13.1.4:

A 60-liter tank of compressed nitrogen gas, measured at 22 atm relative to atmospheric pressure, is used to inflate a large Stay Puft marshmallow balloon figure. Following inflation, the equilibrium pressure between the marshmallow man and the tank is 3 atm, relative to the atmospheric pressure. Assuming a reversible expansion occurs at a constant temperature of 25°C, what is the volume of the marshmallow figure? How much work was performed to inflate the figure? What was the total mass nitrogen initially in the tank? The molar mass of nitrogen is 28.02 g/mol.



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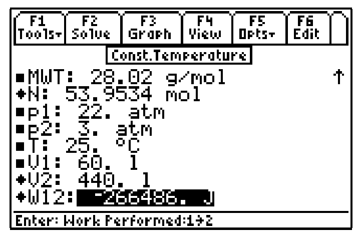

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Solution – Select **equations one, two, three** and **nine**. The volume of the inflatable figure is the difference between V2-V1 (380 l).

Given	Solution
MWT = 28.02 g/mol	m = 1.51178 kg
p1 = 22 atm	N = 53.9534 mol
p2 = 3 atm	V2 = 440 l
T = 25°C	W12 = -266486 J (energy leaving the system as work)
V1 = 60 l	

13.1.5 Internal Energy/Enthalpy

These equations compute changes in entropy, enthalpy and internal energy for ideal gases between initial and final states. **Equation 1** computes the change in internal energy **us21** (J/kg), of an ideal gas from change in temperature **T2-T1** (K)**,** and specific heat at constant volume **cv** (J/(kg·K)). The **second equation** computes **us21** from changes in pressure, **p2-p1** (Pa), and specific volume, **vs2-vs1** (m^3/kg), and from the specific heat ratio **k**. **Equations 3** and **4** compute the change in specific enthalpy, **hs21** (J/kg), from the change in temperature, **T2-T1,** and specific heat at constant pressure, **cp (**J/(kg·K)), or from the molar specific heat ratio **k** and changes in specific volume, **vs2-vs1,** and pressure, **p2-p1**. **Equations 5, 6** and **7** calculate the change in entropy per kilogram of substance, **ss21** (J/(kg·K)), from a combination of temperature, specific volume, or pressure changes. **Equations 8, 9** and **10** compute the molar versions of enthalpy difference, **hm21** (J/mol), internal energy difference, **um21** (J/mol), and difference in entropy, **sm21** (J/(mol·K)), from their mass equivalents **hs21**, **us21** and **ss21**. **Equation 10** describes the relationship between specific heats at constant pressure, **cp**, and constant volume, **cv**, for an ideal gas. The **equation 11** calculates **k,** the ratio of specific heat at constant pressure, **cp**, and the specific heat at constant volume, **cv**. The **last two equations** show the ideal gas law for the initial (1) and final (2) states.

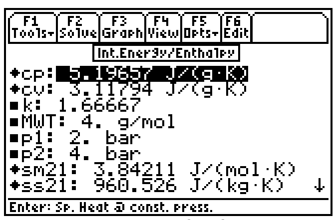
$us21 = cv \cdot (T2 - T1)$	**Eq. 1**
$us21 = \dfrac{p2 \cdot vs2 - p1 \cdot vs1}{k - 1}$	**Eq. 2**
$hs21 = cp \cdot (T2 - T1)$	**Eq. 3**
$hs21 = \dfrac{k \cdot (p2 \cdot vs2 - p1 \cdot vs1)}{k - 1}$	**Eq. 4**
$ss21 = cv \cdot \ln\left(\dfrac{T2}{T1}\right) + \dfrac{Rm}{MWT} \cdot \ln\left(\dfrac{vs2}{vs1}\right)$	**Eq. 5**
$ss21 = cp \cdot \ln\left(\dfrac{T2}{T1}\right) - \dfrac{Rm}{MWT} \cdot \ln\left(\dfrac{p2}{p1}\right)$	**Eq. 6**
$ss21 = \left(cp \cdot \ln\left(\dfrac{vs2}{vs1}\right) + cv \cdot \ln\left(\dfrac{p2}{p1}\right)\right)$	**Eq. 7**
$hm21 = MWT \cdot hs21$	**Eq. 8**
$um21 = MWT \cdot us21$	**Eq. 9**
$sm21 = MWT \cdot ss21$	**Eq. 10**
$cp = cv + \dfrac{Rm}{MWT}$	**Eq. 11**
$k = \dfrac{cp}{cv}$	**Eq. 12**
$p1 \cdot vs1 = \dfrac{Rm}{MWT} \cdot T1$	**Eq. 13**
$p2 \cdot vs2 = \dfrac{Rm}{MWT} \cdot T2$	**Eq. 14**

Variable	**Description**	**Units**
cp	Specific Heat -const. pressure	J/(kg·K)
cv	Specific Heat-const. volume	J/(kg·K)
hs21	Enthalpy Change-mass: 1→2	J/kg

Variable	Description	Units
hm21	Enthalpy Change-molar: 1→2	J/mol
k	Specific Heat Ratio	unitless
MWT	Molecular Weight	kg/mol
p1	Initial Pressure	Pa
p2	Final Pressure	Pa
Rm	Molar Gas constant	8.3145 J/(mol·K)
sm21	Entropy Change-mole: 1→2	J/(mol·K)
ss21	Entropy Change-mass: 1→2	J/(kg·K)
T1	Initial Temperature	K
T2	Final Temperature	K
um21	Internal energy change-molar: 1→2	J/mol
us21	Internal energy change-mass: 1→2	J/kg
vs1	Initial Specific Volume	m^3/kg
vs2	Final Specific Volume	m^3/kg

Example 13.1.5:

Helium (molar mass = 4 g/mol) is compressed adiabatically from a pressure of 2 bars to a pressure of 4 bars. The temperature increases from 300 K to 476.22 K. What are the initial and final values for specific volume, and the change in entropy per mass, and entropy per mole? Since helium is an ideal monatomic, gas the specific heat ratio **k** is equivalent to 5/3.



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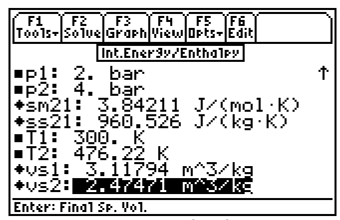

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Solution – Select **equations 6**, and **10** through **14** to solve this problem

Given	Solution
k = 5/3	cp = 5.19657 J/(g·k)
MWT = 4 g/mol	cv = 3.11794 J/(g·k)
p1 = 2 bar	sm21 = 3.842121 J/(mol·k)
p2 = 4 bar	ss21 = 960.526 J/(g·k)
T1 = 300 K	vs1 = 3.11794 m^3/kg
T2 = 476.22 K	vs2 = 2.47471 m^3/kg

13.2 Kinetic Gas Theory

These two equations calculate the velocity and distribution for molecules in a gas phase. The **first equation** computes **vrms** (m/s), the root mean speed for molecules in an ideal gas, as a function of mass temperature, **T** (K), and molecular weight, **MWT** (kg/mol). The **second equation** calculates the fraction of molecules, **f**, in a gas sample having velocities in the range, **vel ± Δv/2** (m/s). The second equation is often referred to as the *Maxwell distribution of speeds*.

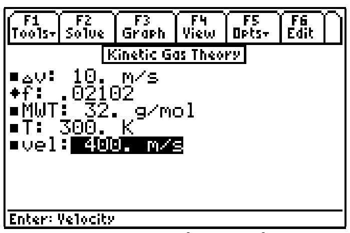
$$vrms = \sqrt{\frac{3 \cdot Rm \cdot T}{MWT}} \qquad \textbf{Eq. 1}$$

$$f = 4 \cdot \pi \cdot \left(\frac{MWT}{2 \cdot \pi \cdot Rm \cdot T} \right)^{\frac{3}{2}} \cdot vel^2 \cdot e^{-\frac{MWT \cdot vel^2}{2 \cdot Rm \cdot T}} \cdot \Delta v \qquad \textbf{Eq. 2}$$

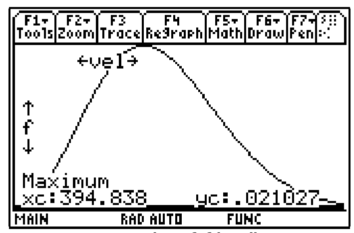
Variable	**Description**	**Units**
Δv	Velocity range	m/s
f	Distribution function	Unitless
MWT	Molecular weight	kg/mol
Rm	Molar gas constant	8.3145 J/(mol·K)
T	Temperature	K
vel	Velocity	m/s
vrms	RMS velocity	m/s

Example 13.2:

A container is filled with molecular oxygen (MWT = 32 g/mol) at 300 K. What fractions of molecules have velocities in the range of 395-405 m/s? Plot the distribution function as a function of velocity. Find the velocity where the maximum fraction occurs.



Computed Results



Graph of f(vel).

Solution – Select the **second equation**. Enter the known values and compute **f**. To graph the function:

1. Press [F3], enter **xmin=0** and **xmax=1000**.
2. Select **f** as the dependent variable and **vel** as the independent variable.
3. Move the cursor to the bottom of the screen and press [ENTER] to select **Full Screen** mode.
4. Press [F3] to plot the function.
5. Once the graph has been made, press [F5]: Math, [4]: Maximum. A prompt will appear asking you to select a Lower Bound.
6. Move the cursor to the left side of the maximum and press [ENTER]. A prompt asks you to enter the upper bound. Move the cursor to the right side of the maximum point and press [ENTER].
7. A maximum value appears on the bottom of the screen as shown in the screen display above.

Given	**Solution**
Δv = 10 m/s	f = 0.02102
MWT = 32 g/mol	
T= 300 K	
vel = 400 m/s	

13.3 Real Gas Laws

Ideal gas laws assume the gas molecules are perfectly elastic spheres and that the average distance between molecules is large. This generally occurs at high temperatures or low pressure. However, under conditions of high pressure or low temperature and when the concentration of molecules increase, the forces between molecules become more significant and cause noticeable deviations from ideal gas behavior. This generally occurs for conditions of high pressure or low temperature. The van der Waals and Redlich-Kwong equations are two of several methods used to approximate the effect of inter-molecular interactions in gas calculations. The coefficients for each method are specific to a particular gas and can be approximated from information using the critical temperature and critical pressure for a gas. Either method is an approximation and is likely to differ in its accuracy over certain regions. The equation sets for both the van der Waal and Redlich-Kwong methods have been adapted in molar and specific volume (gravimetric) forms.

13.3.1 van der Waals: Specific Volume

The van der Waals equation is one of the oldest methods to approximate the behavior of real gases. It has a reasonable accuracy for a wide range of conditions. The **first equation** is a modified form of the ideal gas equation found in 13.1.1: (Equation 3). The first coefficient, **avws** ($Pa\cdot m^6/kg^2$), accounts for the attractive or repulsive forces between molecules, which reduce or increase pressure. The affect of these inter-molecular forces, according to the equation 1, is inversely proportional to the square of the gas volume **vs** (m^3/kg). The second coefficient, **bvws** (m^3/kg)**,** accounts for the reduction in specific volume, **vs**, due to the space occupied by the gas molecules themselves. The **second** and **third equations** approximate the coefficients **avws** and **bvws** from the critical temperature **Tcr** (K), the temperature, above which, a liquid phase no longer exists for a particular gas, and critical pressure **pcr** (Pa), the pressure, at the critical temperature, when the phase boundary between the liquid and gas phase is non-existent. The **last equation** relates the molar mass, **MWT** (kg/mol), to the number of moles, **N** (mol), and total mass of the gas **m** (kg).

$$\left(p+\frac{avws}{vs^2}\right)\cdot(vs-bvws)=\frac{Rm}{MWT}\cdot T \qquad \textbf{Eq. 1}$$

$$avws=\frac{27}{64}\cdot\left(\frac{Rm}{MWT}\right)^2\cdot\frac{Tcr^2}{pcr} \qquad \textbf{Eq. 2}$$

$$bvws=\frac{1}{8}\cdot\frac{Rm}{MWT}\cdot\frac{Tcr}{pcr} \qquad \textbf{Eq. 3}$$

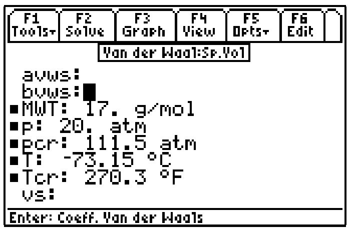
$$MWT=\frac{m}{N} \qquad \textbf{Eq. 4}$$

Variable	Description	Units
avws	Coefficient van der Waals	$Pa\cdot m^6/kg^2$
bvws	Coefficient van der Waals	m^3/kg
m	Mass	kg
MWT	Molecular weight	kg/mol
N	No. moles	mol

Variable	**Description**	**Units**
P	Pressure	Pa
pcr	Critical pressure	Pa
Rm	Molar gas constant	8.3145 J/(mol·K)
T	Temperature	K
Tcr	Critical temperature	K
vs	Specific volume	m^3/kg

Example 13.3.1:

What are approximated values for the van der Waals coefficients of ammonia (NH_3)? Use a specific volume for a pressure of 20 atm and a temperature of –73.15 ^oC. The molar mass of NH_3 is approximately 17 g/mol.



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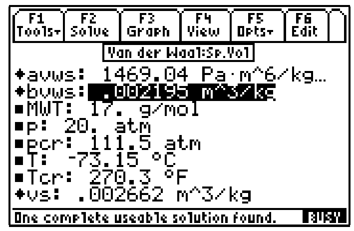


Computed Results

Solution – Select **equations one, two** and **three**. The critical temperature and pressure for several compounds, including ammonia are listed in the **Critical Data –Gases** topic of **Gases and Vapors** in the *Reference* section of ME•*Pro*. The critical temperature is listed as 270.3 ^oF and the critical pressure is 111.5 atm.

Given

MWT = 17 g/mol
p = 20 atm
pcr = 111.5 atm
T= -73.15 ^oC
Tcr = 270.3 ^oF

Solution

avws = 1469.04 Pa·m^6/kg^2
bvws = .002195 m^3/kg
vs = .002662 m^3/kg

13.3.2 van der Waals: Molar form

These equations are the *molar* versions of the *gravimetric* (mass) van der Waals equations listed in the previous section. In other words, the properties per quantity of gas are expressed in moles instead of mass, ex: **vs** (m^3/kg) are now **vm** (m^3/mol). The **first equation** is an adapted form of the ideal gas equation **p·V=n·Rm·T** which accounts for the effect of attractive and repulsive forces between molecules. The molar van der Waals coefficients, **avwm** (Pa·m^6/mol^2) and **bvwm** (m^3/mol), which approximate the effect of these forces for a particular gas, can be estimated from the critical temperature, **Tcr** (K), and critical pressure, **pcr** (Pa), data using **Eq. 2 and 3**.

$$\left(p+\frac{avwm}{vm^2}\right)\cdot\left(vm-bvwm\right)=Rm\cdot T \qquad \textbf{Eq. 1}$$

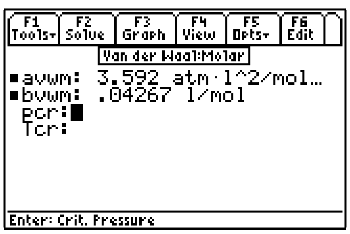
$$avwm=\frac{27}{64}\cdot Rm^2\cdot\frac{Tcr^2}{pcr} \qquad \textbf{Eq. 2}$$

$$bvwm = \frac{1}{8} \cdot Rm \cdot \frac{Tcr}{pcr}$$ **Eq. 3**

Variable	**Description**	**Units**
avwm	van der Waals coefficient - molar	$Pa \cdot m^6/mol^2$
bvwm	van der Waals coefficient-molar	m^3 /mol
p	Pressure	Pa
pcr	Critical pressure	Pa
Rm	Molar gas constant	8.3145 J/(mol·K)
T	Temperature	K
Tcr	Critical temperature	K
vm	Molar volume	m^3 /mol

Example 13.3.2:

K.A. Kobe and R.E.Lynn, Jr. (*Chem. Rev.*, 52:117-236 1953) reported critical pressure and temperature values for CO_2 as 7.39 MPa and 304.2 K. This data is also available in *Reference* under *Crit.Data-Gases* in *Gases and Vapors*. Determine whether the van der Waal coefficients for carbon dioxide listed in the *CRC 73rd ed.*, can be used to approximate these same values. The molar van der Waal coefficients for CO_2 listed in the *CRC 73rd Ed* (a, b) are 3.592 atm·liters^2/mol^2 and 0.04267 liters/mol. The molar mass of carbon dioxide is 44.01 g/mol.



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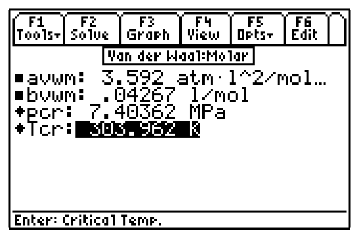


Computed Results

Solution – Select the **second** and **third equations** to solve this problem.

Given	**Solution**
avwm = 3.592 atm·l^2/mol^2	pcr = 7.40362 MPa
bvwm = 0.04267 l/mol	Tcr = 303.962 K

13.3.3 Redlich-Kwong: Sp.Vol

The Redlich-Kwong equations of state (*Chem. Rev.*, 44, 233, 1949) offer another method for accounting for inter-molecular forces of attraction and repulsion in calculating gas parameters. The **first equation** is a modified version of the ideal gas equation, **p·V=n·Rm·T**. The **second** and **third equations** approximate the gravimetric (mass) forms of the Redlich-Kwong coefficients, **arks** Pa·m^6/(kg^2·K$^{0.5}$) and **brks** (m^3/kg), from the critical pressure, **pcr** (Pa), and temperature, **Tcr** (K), of a particular gas. The **last equation** relates the molar mass, **MWT** (kg/mol), to the number of moles, **N** (mol) and total mass of the gas **m** (kg).

$$p = \frac{Rm \cdot T}{MWT \cdot (vs - brks)} - \frac{arks}{T^{.5} \cdot vs \cdot (vs + brks)}$$ **Eq. 1**

$$arks = .4275 \cdot \left(\frac{Rm}{MWT} \right)^2 \cdot \left(\frac{Tcr^{2.5}}{pcr} \right)$$ **Eq. 2**

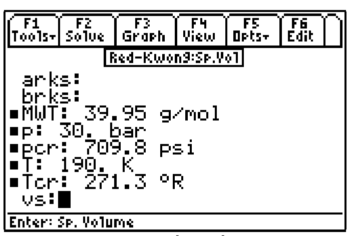
$$brks = .0867 \cdot \frac{Rm}{MWT} \cdot \frac{Tcr}{pcr}$$ **Eq. 3**

$$MWT = \frac{m}{N}$$ **Eq. 4**

Variable	Description	Units
arks	Redlich Kwong coefficient-mass	$Pa \cdot m^6/(kg^2 \cdot K^{.5})$
brks	Redlich Kwong coefficient-mass	m^3/kg
m	Mass	kg
MWT	Molar Mass	kg/mol
N	No. moles	mol
p	Pressure	Pa
pcr	Critical Pressure	Pa
Rm	Molar Gas constant	8.3145 J/(mol·K)
T	Temperature	K
Tcr	Critical Temperature	K
vs	Specific Volume	m^3/kg

Example 13.3.3:

Calculate the Redlich Kwong coefficients for Argon and the specific volume at a pressure of 30 bars and a temperature of 190 K. The molar mass of Argon is 39.95 g/mol.



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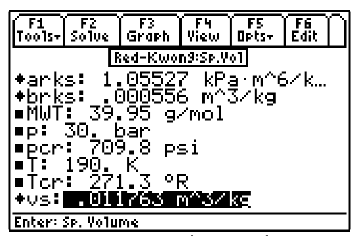


Computed Results

Solution – Select **equations 1, 2,** and **3** to solve this problem. The critical temperature and pressure for several compounds, including argon are listed in **Refrigerants-Cryogenic Properties** in the *Reference* section of ME•*Pro*. The critical temperature is listed as 271.3 °R and the critical pressure is 709.8 psi.

Given
MWT = 39.95 g/mol
p = 30 bar
pcr = 709.8 psi
T = 190 K
Tcr = 271.3 °R

Solution
arks = 1.05527 $kPa \cdot m^6/kg^2 \cdot \sqrt{K}$
brks = 0.000556 m^3/kg
vs = .0011763 m^3/kg

13.3.4 Redlich-Kwong: Molar

These equations are the molar form of the Redlich Kwong equations. The **first equation** is a modified version of the ideal gas equation, **p·V=n·Rm·T**. The **second** and **third equations** approximate the molar forms of the Redlich Kwong coefficients, **arkm** $Pa \cdot m^6/(mol^2 \cdot \sqrt{K})$ and **brkm** (m^3/mol), from the critical pressure, **pcr** (Pa), and temperature, **Tcr** (K), for a specific gas.

$$p = \frac{Rm \cdot T}{vm - brkm} - \frac{arkm}{\sqrt{T} \cdot vm \cdot (vm + brkm)} \qquad \textbf{Eq. 1}$$

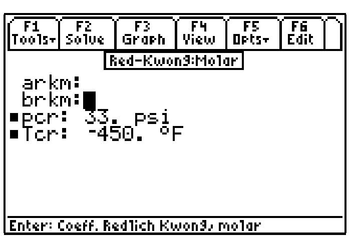
$$arkm = \frac{.4275 \cdot Rm^2 \cdot Tcr^{2.5}}{pcr} \qquad \textbf{Eq. 2}$$

$$brkm = \frac{.0867 \cdot Rm \cdot Tcr}{pcr} \qquad \textbf{Eq. 3}$$

Variable	**Description**	**Units**
arkm	Redlich Kwong coefficient-molar	$Pa \cdot m^6/(mol^2 \cdot K^{.5})$
brkm	Redlich Kwong coefficient, molar	m^3/mol
p	Pressure	Pa
pcr	Critical Pressure	Pa
Rm	Molar Gas constant	8.3145 J/(mol·K)
T	Temperature	K
Tcr	Critical Temperature	K
vm	Molar Volume	m^3/mol

Example 13.3.4:

The molar Redlich Kwong coefficients for Helium (He) are listed on page 143 of Reference 1 the listed values are 8.168 kPa $(m^3/kmol)^2$ and 0.01655 $(m^3/kmol)$ respectively. Verify that these values are consistent with calculations from critical pressure and temperature data for helium located in *Reference/Refrigerants/Selected Materials.*



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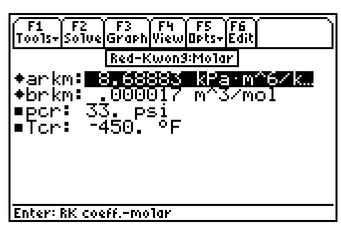


Computed Results

Solution – The values for critical temperature and pressure for Helium listed in *Reference* are –450°F and 33 psi, respectively. Select the **second** and **third equations** to solve this problem.

Given
pcr = 33 psi
Tcr = -450 °F

Solution
arkm =8.68883 $kPa \cdot m^6/kmol^2 \cdot \sqrt{K}$
brkm = .000017 m^3 /mol

13.4 Reverse Adiabatic

These equations compute internal energy change per unit mass, **us21** (J/kg), enthalpy change per unit mass, **hs21** (J/kg), and work performed **W12** (J), for an ideal gas undergoing a reversible adiabatic process. A reversible adiabatic process is also referred to as isentropic, i.e.: total entropy remains constant throughout the transition between initial and final states. **Equations 1, 2,** and **3** describe the relationships between pressure, temperature and volume for a reversible adiabatic process between initial state, **p1** (Pa), **V1** (m^3), **T1** (K), and final state, **p2** (Pa), **V2** (m^3), **T2** (K). **Equations 4, 5** and **6** compute the work, **W12**, performed for a reversible adiabatic transition between the initial and final states. **Equation 7** calculates the initial and final specific volumes, **vs1** (m^3/kg) and **vs2** (m^3/kg). **Equations 8** and **9** compute the change in specific energy, **us21** (J/kg), for an isentropic process. **Equation 10** calculates the change in specific enthalpy, **hs21** (J/kg), for a reversible adiabatic process. **Equation 11** computes **k,** the ratio of the specific heats at constant pressure **cp** J/(kg·K), and volume **cv (**J/(kg·K)). **Equation 12** relates molecular mass, **MWT** (kg/mol), to the number of moles, **N** (mol), and total mass of the gas, **m** (kg). **Equations 13** and **14** compute the specific volumes of the initial and final specific volumes, **vs1** and **vs2**. The **final three equations** are specific for an ideal gas. **Equations 15** and **16** express the ideal gas law of the initial and final states for specific volume, **vs1** and **vs2**. The **last equation** computes the specific heat at constant pressure, **cp,** for an ideal gas from the specific heat at constant volume, **cp**, and molar mass **MWT.**

$$\frac{p1}{p2}=\left(\frac{V2}{V1}\right)^{k}$$ **Eq. 1**

$$\frac{T2}{T1}=\left(\frac{V1}{V2}\right)^{k-1}$$ **Eq. 2**

$$\frac{T2}{T1}=\left(\frac{p2}{p1}\right)^{\frac{k-1}{k}}$$ **Eq. 3**

$$W12=m\cdot cv\cdot\left(T2-T1\right)$$ **Eq. 4**

$$W12=\frac{p2\cdot V2-p1\cdot V1}{k-1}$$ **Eq. 5**

$$W12=\frac{p1\cdot V1\cdot\left(\left(\frac{p2}{p1}\right)^{\frac{k-1}{k}}-1\right)}{k-1}$$ **Eq. 6**

$$vs2=vs1\cdot\left(\frac{T1}{T2}\right)^{\frac{1}{k-1}}$$ **Eq. 7**

$$us21=cv\cdot\left(T2-T1\right)$$ **Eq. 8**

$$us21 = \frac{p2 \cdot vs2 - p1 \cdot vs1}{k - 1}$$ **Eq. 9**

$$hs21 = \frac{k \cdot (p2 \cdot vs2 - p1 \cdot vs1)}{k - 1}$$ **Eq. 10**

$$k = \frac{cp}{cv}$$ **Eq. 11**

$$MWT = \frac{m}{N}$$ **Eq. 12**

$$vs1 = \frac{V1}{m}$$ **Eq. 13**

$$vs2 = \frac{V2}{m}$$ **Eq. 14**

$$p1 \cdot vs1 = \frac{Rm}{MWT} \cdot T1$$ **Eq. 15**

$$p2 \cdot vs2 = \frac{Rm}{MWT} \cdot T2$$ **Eq. 16**

$$cp = cv + \frac{Rm}{MWT}$$ **Eq. 17**

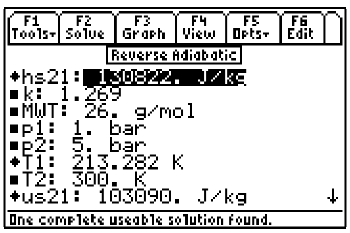
Variable	Description	Units
cp	Specific Heat-constant pressure	J/(mol·K)
cv	Specific heat -constant volume	J/(mol·K)
hs21	Enthalpy change	J/kg
k	Specific heat ratio	Unitless
m	Mass	kg
MWT	Molar mass	kg/mol
N	No. moles	mol
p1	Initial pressure	Pa
p2	Final pressure	Pa
Rm	Molar gas constant	8.3145 J/(mol·K)
T1	Initial temperature	K
T2	Final temperature	K
us21	Internal energy change	J/kg
V1	Initial volume	m^3
V2	Final volume	m^3
vs1	Initial specific volume	m^3 /kg
vs2	Final specific volume	m^3/kg

Variable	**Description**	**Units**
W12	Work performed	J

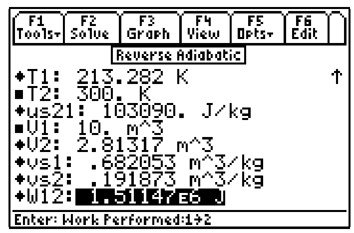
> **Caution:** Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 13.4:

Acetylene (C_2H_2, MWT=26 g/mol) undergoes adiabatic compression from an initial pressure of 1 bar at an initial volume of 10 m^3 to a final pressure of 5 bars. The temperature, following compression, is 300 K. What is the initial temperature, final volume, work performed, change in internal energy and enthalpy? Information regarding the specific heat of acetylene can be found in the **Reference** section under **Thermal Properties/Specific Heat/Cp Liquids and Gases** and **Cp/Cv Liq. and Gases at 1 atm**. Assume that the ratio of specific heats **cp/cv** remains constant over this pressure range and acetylene exhibits ideal gas behavior.



Upper Display



Lower Display

Solution – Select **Equations, one, two, six, seven, nine, ten** and **fifteen.** Use a listed value of cp/cv = 1.269 for acetylene from *Thermal Properties/Specific Heat/Cp/Cv Liq. and Gases at 1 atm* in the *Reference* section.

Given	**Solution**
k = 1.269	hs21 = 130822 J/kg
MWT = 26 g/mol	T1 = 213.282 K
p1 = 1 bar	us21 = 103090 J/kg
p2 = 5 bar	V2 = 2.81317 m^3
T2 = 300 K	vs1 = .682053 m^3/kg
V1 = 10 m^3	vs2 = .191873 m^3/kg
	W12 = 1.51147E6 J

13.5 Polytropic Process

A quasi-equilibrium compression or expansion of an ideal gas is often referred to as polytropic. A criteria for a polytropic process is that it occurs at a sufficiently slow rate, that the system boundary is *fully resisted* by an external force at every stage between the initial and final state that the system, and therefore can be said to be in equilibrium at any stage in the entire process. The magnitude of work performed between the initial and final states is equivalent to the sum (integral) of the incremental work performed at each equilibrium stage between the initial state, **p1** (Pa), **V1** (m^3), **vs1** (m^3/kg), and **T1** (K). and final state, **p2** (Pa), **V2** (m^3), **vs2** (m^3/kg), and **T2** (K). **Equation 1** relates the pressure **p1**, **p2** to volume **V1**, **V2** in a polytropic process. **Equation 2** computes the polytropic specific heat, **cn** (J/(kg·K)), from the polytropic exponent λ, the specific heat ratio **k** and the specific heat at constant volume **cv**. **Equations 3** and **4** relate temperature to volume and temperature to pressure in a polytropic process. **Equations 5** and **6** compute the work, **W12** (J), performed from an initial state (**p1**, **V1**, **vs1**, and **T1**) and final state (**p2**, **V2**, **vs2**, and **T2**). **Equation 7** computes the heat change, **Q12** (J), of the system from the initial and final temperatures **T1** and **T2**, and the specific heat for a polytropic process **cn**. The **last two**

equations describe the relationship between the specific heats at constant pressure **cp** (J/(kg·K)), and constant volume **cv** (J/(kg·K)), for an ideal gas, and compute the specific heat ratio **k**.

$$\frac{p1}{p2} = \left(\frac{V2}{V1}\right)^{\lambda} \quad \textbf{Eq. 1}$$

$$cn = \frac{cv \cdot (\lambda - k)}{\lambda - 1} \quad \textbf{Eq. 2}$$

$$\frac{T2}{T1} = \left(\frac{V1}{V2}\right)^{\lambda - 1} \quad \textbf{Eq. 3}$$

$$\frac{T2}{T1} = \left(\frac{p2}{p1}\right)^{\frac{\lambda - 1}{\lambda}} \quad \textbf{Eq. 4}$$

$$W12 = \frac{p2 \cdot V2 - p1 \cdot V1}{\lambda - 1} \quad \textbf{Eq. 5}$$

$$W12 = \frac{-p1 \cdot V1\left(\left(\frac{p2}{p1}\right)^{\frac{\lambda - 1}{\lambda}} - 1\right)}{\lambda - 1} \quad \textbf{Eq. 6}$$

$$Q12 = m \cdot cn \cdot (T2 - T1) \quad \textbf{Eq. 7}$$

$$cp = cv + \frac{Rm}{MWT} \quad \textbf{Eq. 8}$$

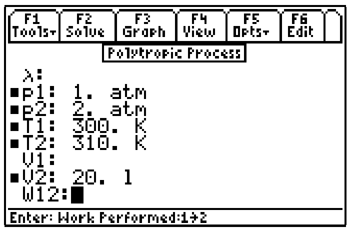
$$k = \frac{cp}{cv} \quad \textbf{Eq. 9}$$

Variable	**Description**	**Units**
λ	Polytropic coefficient, λ≠1	Unitless
cn	Specific heat-polytropic process	J/(mol·K)
cp	Specific heat – constant pressure	J/(kg·K)
cv	Specific heat-constant volume	J/(mol·K)
k	Specific heat ratio	Unitless
m	Mass	kg
MWT	Molar mass	kg/mol
p1	Initial pressure	Pa
p2	Final pressure	Pa
Q12	Heat transfer	J
Rm	Molar gas constant	8.3145 J/(mol·K)
T1	Initial temperature	K
T2	Final temperature	K

Variable	Description	Units
V1	Initial volume	m^3
V2	Final volume	m^3
W12	Work performed	J

Example 13.5:

Suppose that inflating a balloon is a quasi-equilibrium process. If the air pressure is 1 atm outside of the balloon and the pressure of the *same* air inside a balloon, having an internal volume of 20 liters, is 2 atm, compute the work performed to inflate the balloon. Assume air behaves as an ideal gas, the balloon does not transfer heat to its surroundings, and the air temperature increases (due to air compression only) from 300-310 K.



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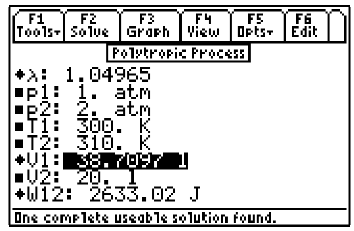


Computed Results

Solution – Select **equations, one, four** and **five** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
p1 = 1 atm	λ = 1.04965
p2 = 2 atm	V1 = 38.7097 l
T1 = 300 K	W12 = 2633.02 J
T2 = 310 K	
V2 = 20 l	

References:

1. Lynn D. Russell, and George A. Adebiyi, Classical Thermodynamics, Saunders College publishing, Harcourt Brace Jovanavich College Publishers, Fort Worth TX, 1993.
2. Michael R. Lindeburg: Mechanical Engineering Reference Manual, Eighth Ed.
3. Physical Chemistry, Peter Atkins, 5th Edition, Freeman Publishing Company, NY 1994.
4. CRC Handbook of Chemistry and Physics, 73rd ed. 1992-1993, CRC Press, Boca Raton Fl, 1992.
5. Redlich, O. and Kwong, J. N. S. 1949. “On the Thermodynamics of Solutions. V. An Equation of State: Fugacities of Gaseous Solutions.” *Chemical Reviews.* Vol. 44, No. 1. 233-244.
6. K.A. Kobe and R.E.Lynn, Jr, *Chemical Reviews*, Vol. 52: 117-236 1953.

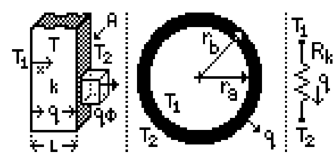
Chapter 14: Heat Transfer

This section covers heat transfer due to conduction, convection and radiation. The equations in this chapter are grouped into four main categories. A list of thermal properties and conductivities for many materials is available in the *Reference* section under **Thermal Properties/Thermal Conductivity**.

- **Basic Transfer Mechanics**
- **1D Heat Transfer**
- **Semi-infinite Solid**
- **Radiation**

14.1 Basic Transfer Mechanisms

14.1.1 Conduction



This section describes steady-state conduction of heat through objects with rectangular and circular cross-sections. The rate of heat transfer, **q** (W), is due to the thermal conductivity of a material, **k** (W/(m·K)), and the temperatures J, (K) and J_2 (K) at the surfaces 1 and 2 over a distance, **L** (m). The thermal conductivity, **k**, depends on thermo physical properties and thermodynamic properties of the substance at a given temperature. The shape factor for conduction, **S**, is determined from the geometric properties of the object, including length **L**, area **A** (m^2) of the rectangular surface, inner radius **ra** (m), and outer radius **rb** (m) for a circular conductor. **Equation 1** computes the rate of heat loss, **q**, for a rectangular flat plate with a cross-sectional area, **A**, and a temperature difference, (**T1-T2**), across length, **L**.   **Equation 2** computes the rate of heat transfer, **q**, through an object with a geometric shape factor, **S**. **Equation 3** computes the rate of heat transfer per rectangular surface area, **qϕ** (W/m^2), of the material. The **next three equations** compute the shape factor, **S**, for a rectangular thin plate (Eq.4), a cylinder with inner radius, **ra**, outer radius, **rb**, and length, **L** (Eq. 5), and a hollow sphere with inner radius, **ra** and outer radius, **rb** (Eq. 5). These equations assume steady-state transfer of heat, uniform conductivity, and the absence of thermal gradients perpendicular to the direction of main heat transfer.

$$q = k \cdot A \cdot \frac{(T1 - T2)}{L} \qquad \textbf{Eq. 1}$$

$$q = k \cdot S \cdot (T1 - T2) \qquad \textbf{Eq. 2}$$

$$q\phi = \frac{q}{A} \qquad \textbf{Eq. 3}$$

$$S = \frac{A}{L} \qquad \textbf{Eq. 4}$$

$$S = \frac{2 \cdot \pi \cdot L}{\ln\left(\frac{rb}{ra}\right)} \qquad \textbf{Eq. 5}$$

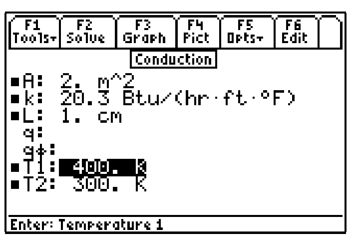
$S = \dfrac{4 \cdot \pi \cdot ra \cdot rb}{rb - ra}$	**Eq. 6**

Variable	Description	Units
A	Area	m^2
k	Thermal conductivity	W/(m·K)
L	Length	m
q	Heat transfer rate - conduction	W
qϕ	Heat flux	W/m^2
ra	Inner radius	m
rb	Outer radius	m
S	Shape factor	m
T1	Temperature at 1	K
T2	Temperature at 2	K

Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 14.1.1:

A lead slab, laid flat, has a cross-sectional area of 2 m^2 and a thickness of 1 cm. If the temperatures of the upper and lower sides are maintained at 400K and 300 K, what is the rate of heat transfer per area of the metal?



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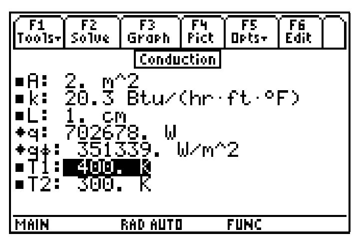


Computed results

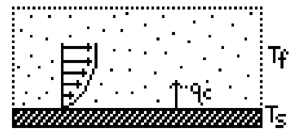
Solution –The thermal conductivity of lead is listed as k=20.3 Btu/(hr·ft·°F) in the temperature range 32-500°F (see **Reference/Thermal Properties/Thermal Conductivity/Elemental**). Use the **first** and **third equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
A = 2 m^2
k = 20.3 Btu/(hr·ft·°F)
L = 1 cm
T1=400 °C
T2=300 °C

Solution
q = 702678 W
qϕ = 351339 W/m^2

14.1.2 Convection

Convection is the transfer of heat from a surface to a moving fluid. It is a combination of heat transfer due to conduction, from the surface to the fluid immediately adjacent to the surface, and transport of the heat due to fluid motion. The mean coefficient of heat transfer, **h**, depends on several factors:



- The shape, size, roughness, and thermal conductivity of the surface.
- The thermal and physical properties of the fluid, including density, specific heat, and conductivity at a specific temperature.
- The flow properties of the fluid: velocity, viscosity and stability.

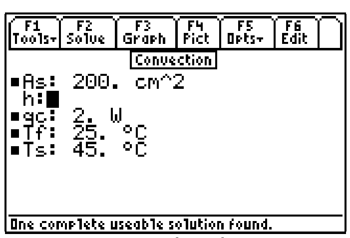
The single equation in this section describes a steady state process for heat transfer from a surface due to convection, **qc** (W), from a surface with temperature, **Ts** (K), and an area, **As** (m^2), to a flowing fluid with average temperature, **Tf** (K). **Equation 1** is commonly known as Newton's Law for cooling.

$$qc = h \cdot As \cdot (Ts - Tf) \qquad \textbf{Eq. 1}$$

Variable	**Description**	**Units**
As	Total surface area	m^2
h	Convection coefficient	$W/(m^2 \cdot K)$
qc	Heat transfer rate-convection	W
Tf	Temperature of fluid	K
Ts	Surface temperature of emitter	K

Example 14.1.2:

Convective transfer occurs from a surface, with an average temperature of 45°C and an area of 200 cm^2 to an air stream with an average temperature of 25°C. The rate of heat loss is 2 watts. What is the coefficient of transfer due to convection for this system?



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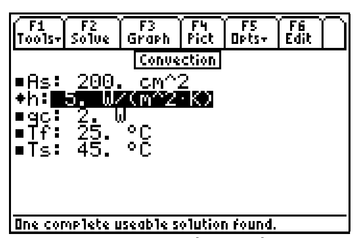


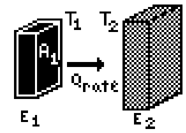
Computed results

Solution –Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
As = 200 cm^2
qc = 2 W
Tf = 25 °C
Ts = 45 °C

Solution
h = 5 $W/(m^2 \cdot K)$

14.1.3 Radiation



Radiation is the transmission of heat in the form of electromagnetic waves. An object absorbing all incoming radiation is defined as a blackbody. The following equations describe radiative transfer between two blackbodies. Often called Stefan-Boltzmann law, **equation 1** links emissive power of a black body, Eb (W/m^2) to the surface temperature of a radiating body Ts (K) and a constant σ. **Equations 2** and **3** compute the emissions, **E1** and **E2**, for objects with non-blackbody properties. The variables, **ε1** and **ε2**, account for the differences in emissivity, due to material type composition and temperature (ε=1 for a blackbody, ε=0 for a purely reflective body). **Equation 4** computes the net radiative transfer, **Qrate** (W), of heat between two bodies. **Fs2r** is the shape (or view) factor representing the fraction of radiation transmitted by one object, which is absorbed by the other. The shape factor, **Fs2r**, becomes 1 if the receiving surface area surrounds the emissive surface area, **A1** (m). **Ts** (K) and **Tr** (K) are the temperatures of the sending and receiving bodies, and **σ=**5.670 x 10^{-8} W/(m^2·K^4) is the Stefan-Boltzmann constant for radiation.

$$Eb = \sigma \cdot Ts^4 \quad \textbf{Eq. 1}$$

$$E1 = \varepsilon 1 \cdot \sigma \cdot T1^4 \quad \textbf{Eq. 2}$$

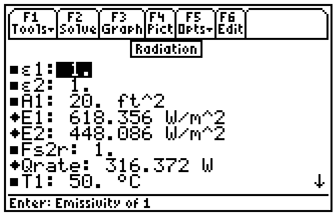
$$E2 = \varepsilon 2 \cdot \sigma \cdot T2^4 \quad \textbf{Eq. 3}$$

$$Qrate = A1 \cdot Fs2r \cdot \left(Eb1 - Eb2\right) \quad \textbf{Eq. 4}$$

Variable	**Description**	**Units**
σ	Stefan-Boltzmann constant	5.670x10^{-8} W/(m^2·K^4)
ε1	Emissivity of 1	unitless
ε2	Emissivity of 2	unitless
A1	Area of radiating surface	m^2
Eb	Black body emissive power	W/m^2
E1	Emissive power-radiating surface	W/m^2
E2	Emissive power-receiving surface	W/m^2
Fs2r	Radiation shape factor	unitless
Qrate	Heat transfer rate-radiation	W
T1	Surface temperature of emitter	K
T2	Temperature of receptor	K
Ts	Surface temperature of emitter	K

Example 14.1.3:

A machine case with a blackbody emissive surface area of 20 ft^2 is exposed to black body radiation with the surrounding walls and ceiling of a room. The temperature of the case and the walls is 50°C and 25°C, respectively. What is the heat transferred from the case to the ceiling and walls of the room?



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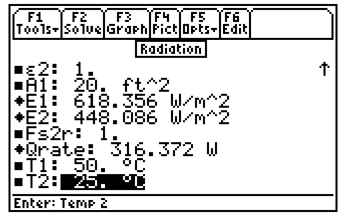


Lower Display

Solution – Select **equations 2, 3,** and **4**. The Stefan-Boltzmann constant, $\sigma = 5.670 \times 10^{-8}$ W/(m^2·K^4), is automatically inserted into the equation when solving.

Given
ε1 = 1
ε2 = 1
A1 = 20 ft^2
Fs2r = 1
T1= 50 ^{o}C
T2 = 25 ^{o}C

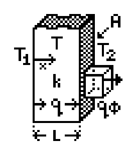
Solution
E1 = 618.356 W/m^2
E2 = 448.086 W/m^2
Qrate = 316.372 W

14.2 1 1D Heat Transfer

One-dimensional (1D) transfer equations describe heat exchange along a single axis (x-axis). No temperature gradients occur along the y and z-axes. Heat transfer is assumed to be time invariant.

14.2.1 Conduction

14.2.1.1 Plane Wall



The following equations describe heat transfer due to conduction along the horizontal axis of a plane wall. The wall, composed of a homogeneous material with thermal conductivity, **k** (W/(m·K)), separates temperatures **T1** (K) and **T2** (K). The wall has a length, **L** (m), along the axis of conduction and an area, **A** (m^2), normal to the direction of heat transfer. The **first equation** computes the temperature, **T** (K), at position **x** (m) from the side of the wall where the temperature is **T1**. The **second equation** computes the rate of heat transfer, **q** (W), between **T1** and **T2**. **Rk** (K/W) is the total thermal resistance to heat conduction of a rectangular plane wall having length **L**, area **A**, and conductivity, **k**.

$$T = T1 + \frac{T2 - T1}{L} \cdot x \qquad \textbf{Eq. 1}$$

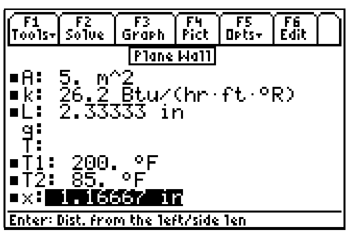
$$q = \frac{k \cdot A}{L} \cdot \left(T1 - T2\right) \qquad \textbf{Eq. 2}$$

$$Rk = \frac{L}{k \cdot A} \qquad \textbf{Eq. 3}$$

Variable	**Description**	**Units**
A	Area	m^2
k	Thermal conductivity	W/(m·K)
L	Length	m
q	Heat transfer rate - conduction	W
Rk	Thermal resistance - conduction	K/W
T	Temperature at x	K
T1	Temperature at 1	K
T2	Temperature at 2	K
x	Distance from the left/side length	m

Example 14.2.1.1:

Steel (1% carbon) is listed in the **Reference** section of the software as having a thermal conductivity of 26.2 Btu/(hr·ft·°F) for the temperature range 60-212 °F. Compute the steady state heat transfer across a steel plate with a cross-sectional area of 5 m^2 and a thickness of 2⅓". The temperatures on each side of the plate are maintained at 200°F and 85°F. Calculate the temperature at the midpoint inside the plate.



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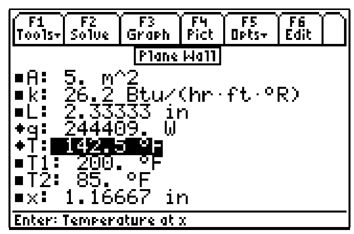
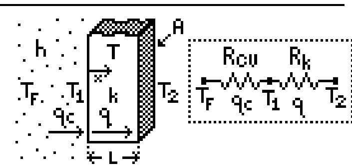


Computed results

Solution – Select the **first** and **second equations** to solve this problem. The thermal conductivity for Steel (1% C) is listed in **Reference/Thermal Properties/Thermal Conductivity/Alloys**. The entries and results are shown in the screen displays above.

Given	**Solution**
A=5 m^2	q = 244409 W
k = 26.2 Btu/(hr·ft·°F)	T = 142.5 °F
L = 2.33333 in	
T1=200 °F	
T2=85°F	
x = 1.16667 in	

14.2.1.2 Convective Source



These equations compute heat transfer for cases where convection and conduction occur in sequence. The diagram to the right portrays heat transfer between a fluid with temperature, **Tf** (K), to a surface with temperature, **T1** (K), and area, **A** (m^2). Heat is then conducted across the thickness of the plate, **L** (m), where the temperature is **T2**. **Equation 1** computes the temperature, **T** (K), at a distance, **x** (m) inside the plate from **T1**. The thermal conductivity, **k** (W/(m·K)), measures the plate material's ability to conduct heat. The convective coefficient, **h** W/(m^2·K), measures the fluid's ability to transport heat away from the surface and is dependent on the flow rate, viscosity, and specific heat of the fluid. The **second** and **third equations** compute the thermal resistances for conduction, **Rk** (K/W), and convection **Rcv** (K/W). Thermal resistances are often used in viewing the heat transfer process as analogous to an electrical circuit.† **Equations 4** and **5** calculate, **T1**, the temperature of the surface adjacent to the fluid. **Equation 6** computes **qc** (W), the rate of heat transfer due to convection. **Equations 7** and **9** compute **q** (W), the rate of heat transfer due to conduction. **Equation 8** assumes a steady state process of heat transfer for the convection and conduction series transfer mechanisms.

$$\frac{T-T2}{Tf-T2}=\frac{L-x}{L+\frac{k}{h}}$$ **Eq. 1**

$$Rk=\frac{L}{k\cdot A}$$ **Eq. 2**

$$Rcv=\frac{1}{h\cdot A}$$ **Eq. 3**

$$T1=\frac{\frac{Tf}{Rcv}+\frac{T2}{Rk}}{\frac{1}{Rcv}+\frac{1}{Rk}}$$ **Eq. 4**

$$T1=\frac{Tf\cdot h+\frac{T2\cdot k}{L}}{h+\frac{k}{L}}$$ **Eq. 5**

$$qc=\frac{Tf-T1}{Rcv}$$ **Eq. 6**

$$q=\frac{T2-T1}{Rk}$$ **Eq. 7**

$$q=qc$$ **Eq. 8**

$$q=\frac{Tf-T2}{Rcv+Rk}$$ **Eq. 9**

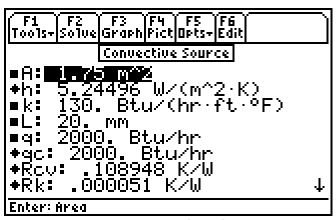
Variable	**Description**	**Units**
A	Area	m^2
h	Convection coefficient	$W/(m^2\cdot K)$

† Ohm's law (i=V/R) states that the rate of electron transfers, **I, which flows through a circuit,** is proportional to the voltage difference, **V**, over the resistance of the circuit, **R**. Heat transfer can be viewed as an analogy of Ohm's Law (q=ΔT/R) where **q** is the rate of heat transfer, **T** is the temperature difference, and **R** is the resistance to heat transfer. See section 14.2.2 for more explanation.

Variable	Description	Units
k	Thermal conductivity	W/(m·K)
L	Length	m
q	Heat transfer rate-conduction	W
qc	Heat transfer rate-convection	W
Rcv	Resistance to convection	K/W
Rk	Thermal resistance - conduction	K/W
T	Temperature at x	K
T1	Temperature at 1	K
T2	Temperature at 2	K
Tf	Temperature of fluid	K
x	Distance from the left/side length	m

Example 14.2.1.2:

Compute the average convective transfer coefficient and surface temperature for air flowing over an aluminum plate having a surface area of 1.75 m^2, a thickness of 20 mm, an average air temperature of 85^{o}F and a surface temperature on the opposite side of the plate of 200^o F. The rate of heat transfer is 2000 Btu/hr. Which mechanism of transfer (convection or conduction) limits transfer the most?



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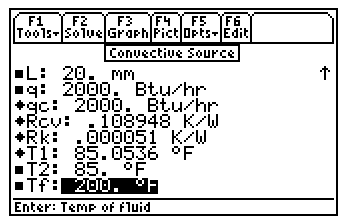
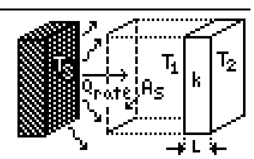


Lower Display

Solution – Aluminum has a thermal conductivity of 130 Btu/(hr·ft·^{o}F), see **Reference/Thermal Properties/Thermal Conductivity/Elements**. Select the **third** through the **seventh equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. The thermal resistance having the higher value limits heat transfer the most (convection in this example).

Given	Solution
A = 1.75 m^2	h = 5.24496 W/(m^2·K)
k = 130 Btu/(hr·ft·^{o}F)	qc = 2000 Btu/hr
L = 20 mm	Rcv = .108948 K/W
q = 2000 Btu/hr	Rk = .000051 K/W
T2 = 85 ^{o}F	T1 =85.0536 ^{o}F
Tf = 200 ^{o}F	

14.2.1.3 Radiative Source



These equations describe a sequential transfer of heat, first due to radiation, and secondly by conduction through a rectangular plane wall. The **first equation** calculates the rate of heat transferred, **Qrate**, (w) from an emitting object with emissivity, **εs**, and surface temperature, **Ts**, (K) to a blackbody plane wall with surface temperature, **T1** (K). The **second equation** computes the temperature, **T1**, (K) at the surface of the plane wall. **Fs2r**, is the shape factor, representing the fraction of radiation transmitted from the emitting surface area, **As,** (m^2) that is absorbed by the plane wall. The **last equation** computes heat conducted through a plane wall with thermal conductivity, **k** (W/m·K), length, **L** (m), surface temperature, **T1** (K) (facing the emitter), and a surface temperature, **T2**, (K) on the opposite side of the plane wall.

$$Qrate = \sigma \cdot Fs2r \cdot As \cdot \varepsilon \cdot \left(Ts^4 - T1^4\right)$$ **Eq. 1**

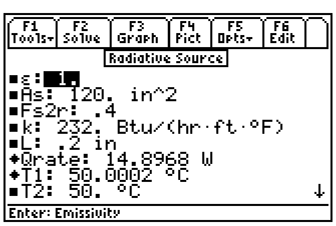
$$Qrate = \frac{k}{L} \cdot \left(T1 - T2\right)$$ **Eq. 2**

$$T1 = \frac{T2 + Fs2r \cdot \frac{\sigma \cdot L}{k} \cdot Ts^4}{1 + Fs2r \cdot \frac{\sigma \cdot L}{k} \cdot T1^3}$$ **Eq. 3**

Variable	**Description**	**Units**
σ	Stefan-Boltzmann constant	5.670×10^{-8} W/($m^2 \cdot K^4$)
ε	Emissivity	unitless
As	Total Surface area	m^2
Fs2r	Radiation shape factor	unitless
k	Thermal conductivity	w/(m·K)
L	Length	m
Qrate	Heat transfer rate-radiation	W
T1	Temperature at 1	K
T2	Temperature at 2	K
Ts	Temperature of radiating surface	K

Example 14.2.1.3:

The larger surface of a rectangular flat copper plate with dimensions 12" x 10" x 0.2" is exposed to a radiating blackbody (ε=1) with a surface temperature of 100° C. If the temperature of the opposite side of the copper plate is maintained at 50°C, what are the heat transfer and the boundary temperature of the side facing the blackbody emitter? Assume the shape factor for the radiating body and the copper surface is 0.4.



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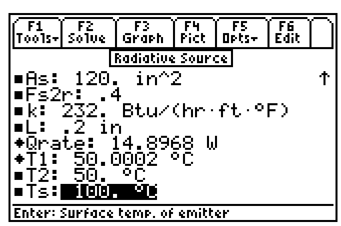

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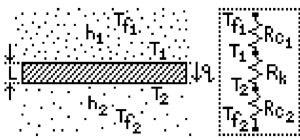
Solution – Copper has a thermal conductivity of 232 Btu/(hr·ft·°F), see **Reference/Thermal Properties/Thermal Conductivity/Elements**). The **first** and **second equations** are needed to solve the problem. The Stefan-Boltzmann constant, $\sigma = 5.670 \times 10^{-8}$ W/($m^2 \cdot K^4$), is automatically inserted into the equation when solving and does not appear in the list of variables.

Given
$\varepsilon = 1$
As = 120 in^2
Fs2r = .4
k= 232 Btu/(hr·ft·^{o}F)
L = .2 in
T2 = 50 ^{o}C
Ts = 100 ^{o}C

Solution
Qrate = 14.8968 W
T1 = 50.0002 ^{o}C

14.2.1.4 Plate and Two Fluids



These equations compute the rate of heat transfer between two fluids separated by a rectangular plate. **Equation 1** calculates the thermal resistance due to convection, **Rc1** (K/W), between a fluid with a convection coefficient of, **h1** (W/(m^2·K), and a plate surface with area, **A** (m). **Equation 2** computes the thermal resistance due to convection, **Rc2** (K/W), between a fluid with convection coefficient, **h2** W/(m^2·K), and the plate surface with area, **A** (m). The **third equation** calculates the thermal resistance, **Rk** (K/W), for a rectangular plate with a thermal conduction coefficient of, **k** (W/(m·K)), thickness **L** (m), and surface area **A** (m). **Equation 4** computes the rate of heat transfer, **q** (W), between two fluids, with temperatures, **Tf1** (K) and **Tf2** (K), separated by a plate having a thermal resistance, **Rk** (K/W). The **last two equations** estimate the surface temperatures on either side of the plate. **T1** (K), is the temperature of the plate on the surface adjacent to the fluid having a temperature, **Tf1**, (K), and thermal resistance to convection of **Rc1** (K/W). **T2** (K), is the temperature of the surface adjacent to the fluid having a temperature, **Tf2** (K), and a thermal resistance to convection of, **Rc2** (K).

$$Rc1 = \frac{1}{h1 \cdot A}$$ **Eq. 1**

$$Rc2 = \frac{1}{h2 \cdot A}$$ **Eq. 2**

$$Rk = \frac{L}{k \cdot A}$$ **Eq. 3**

$$q = \frac{Tf1 - Tf2}{Rc1 + Rc2 + Rk}$$ **Eq. 4**

$$T1 = Tf1 - Rc1 \cdot q$$ **Eq. 5**

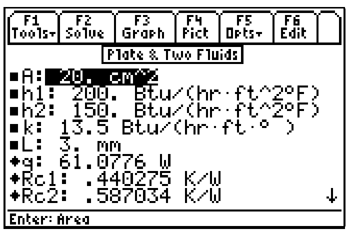
$$T2 = Tf2 + Rc2 \cdot q$$ **Eq. 6**

Variable	Description	Units
A	Area	m^2
h1	Convection coefficient	W/(m^2·K)
h2	Convection coefficient	W/(m^2·K)
k	Thermal conductivity	W/(m·K)
L	Length	m

Variable	**Description**	**Units**
q	Heat transfer rate-conduction	W
Rc1	Thermal resistance-convection	K/W
Rc2	Thermal resistance-convection	K/W
Rk	Thermal resistance-conduction	K/W
T1	Temperature at 1	K
T2	Temperature at 2	K
Tf1	Temperature of fluid 1	K
Tf2	Temperature of fluid 2	K

Example 14.2.1.4:

A stainless steel plate section in a heat exchanger has a thermal conductivity of 13.5 Btu/(h·ft·°F), a thickness of 3 mm and a surface area of 20 cm^2 in contact with the fluid. If the convection coefficients of two fluids flowing in opposite directions on each side of the plate are 200 Btu/(h·ft^2·°F) and 150 Btu/(h·ft^2·°F) and the temperatures of each fluid are 200 °F and 80 °F, respectively, what is the rate of heat transfer through the plate?



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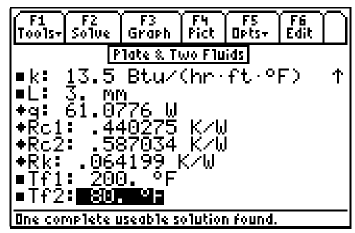

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Solution – Select equations, one, two, three and four to solve this problem.

Given
A = 20 cm^2
h1 = 200 Btu/(hr·ft^2·°F)
h2 = 150 Btu/(hr·ft^2·°F)
k = 13.5 Btu/(hr·ft·°F)
L = 3 mm
Tf1 = 200 °F
Tf2 = 80 °F

Solution
q = 61.0776 W
Rc1 = .440275 K/W
Rc2 = .587034 K/W
Rk = .064199 K/W

14.2.2 Electrical Analogy

The equation for the rate heat transfer, **q** (W), through a conducting medium between temperatures, **T1** (K) and **T2** (K), is similar to Ohm's Law for a current, **i** (A) moving through a resistive circuit between voltages, **V1** (V) and **V2** (V).

$$i = \frac{V1 - V2}{R}$$

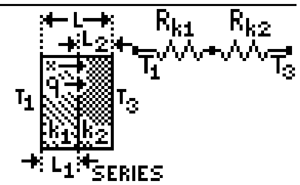
Ohm's Law

$$q = \frac{T1 - T2}{R}$$

Heat Transfer

The variable, **R**, in the equation for heat transfer, is the *thermal resistance to heat transfer* for a particular medium. **R** (K/W) depends on the physical dimensions of the conducting medium area, **A** (m^2), length **L** (m), or shape factor **Fs2r** and the convection, **h** W/(m^2·K), or conductivity, **k** W/(m·K), coefficient of the medium. Heat transfer through a plane wall composed of different materials in series or parallel arrangement to the direction of heat transfer can be viewed similarly to current movement through resistors in series or parallel arrangement. The following topics describe conduction of heat through composite materials, each having different conductivities, in parallel, series or combination arrangements.

14.2.2.1 Two Conductors in Series



These equations describe heat transfer and temperature profiles inside a plane wall composed of two materials. The materials have a series arrangement with respect to the direction of heat flow and perfect thermal contact exists between the two materials (i.e. there is no additional resistance at the contact point between the materials due to surface roughness). In addition, a steady state condition for heat transfer exists with no heat sources or sinks in the series arrangement. The two materials have lengths, in the direction of heat flow, **L1** (m) and **L2** (m), conductivities, **k1** W/(m·K) and **k2** W/(m·K), and cross-sectional areas, **A1** (m^2) and **A2** (m^2). **Equation 1** computes the thermal resistance for heat transfer in the first section of the plane wall, **Rk1** (K/W). **Equation 2** calculates the thermal resistance for the second material, **Rk2** (K/W). The **third equation** computes the rate of heat transfer, **q** (W), through the combined series arrangement between temperatures, **T1** (K) and **T3** (K). **Equation 4** computes, **q**, for the first material from temperatures, **T1** (K) and **T2** (K), and **Rk1**. **Equation 5** calculates, **q**, conducted through the second section of the wall with boundary temperatures, **T2** and **T3**. The **last two equations** compute the temperature, **T** (K), inside the wall at distance, **x** (m), from **T1**.

$$Rk1 = \frac{L1}{k1 \cdot A}$$ Eq. 1

$$Rk2 = \frac{L2}{k2 \cdot A}$$ **Eq. 2**

$$q = \frac{T1 - T3}{Rk1 + Rk2}$$ **Eq. 3**

$$q = \frac{T1 - T2}{Rk1}$$ **Eq. 4**

$$q = \frac{T2 - T3}{Rk2}$$ **Eq. 5**

When x < 0 and x < L1 the following equation is applicable

$$\frac{T - T1}{T2 - T1} = \frac{x}{L1}$$ **Eq. 6**

When L1 ≤ x and x < (L1+L2) the following equation is applicable

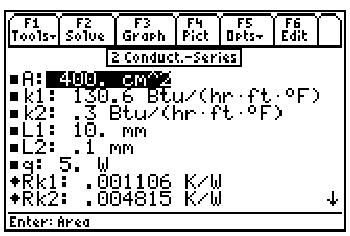
$$\frac{T - T2}{T3 - T2} = \frac{x - L1}{L2}$$ **Eq. 7**

Variable	Description	Units
A	Area	m^2
k1	Thermal conductivity	W/(m·K)
k2	Thermal conductivity	W/(m·K)
L1	Length	m
L2	Length	m
q	Heat Transfer rate - conduction	W
Rk1	Thermal resistance-Conduction	K/W
Rk2	Thermal resistance-Conduction	K/W
T	Temperature at x	K
T1	Temperature at 1	K
T2	Temperature at 2	K
T3	Temperature at 3	K
x	Distance from the left/side length	m

> **Caution:** Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 14.2.2.1:

A 0.1 mm oxide coating exists on an aluminum plate thick having a thickness of 10 mm and a surface area of 400 cm^2. If the rate of heat loss through the plate is 5 W and the surface temperature of the oxide coating is maintained 50 ^oC, what are the temperatures of the opposite side of the aluminum plate and the interface between the aluminum and the oxide? The thermal conductivity of aluminum is 130.6 Btu/(hr·ft·^{o}F) and the oxide coating is 0.3 Btu/(h·ft·^{o}F).



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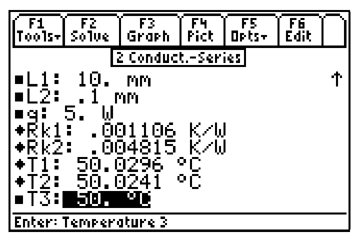

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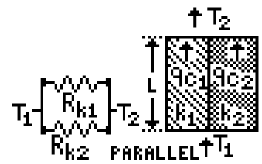
Solution – Select equations, one, two, three and four to solve this problem.

Given	Solution
A = 400 cm^2	Rk1 = .001106 K/W
k1 = 130.6 Btu/(hr·ft·^{o}F)	Rk2 = .004815 K/W
k2 = .3 Btu/(hr·ft·^{o}F)	T1 = 50.0296 ^{o}C
L1 = 10 mm	T2 = 50.0241 ^{o}C
L2 = .1 mm	
q = 5 W	
T3 = 50^o C	

14.2.2.2 Two Conductors in Parallel



These equations describe heat transfer in a composite material of two substances with the grain parallel to the direction of heat transfer. **Equations 1** and **2** compute the thermal resistances, **Rk1** (K/W) and **Rk2** (K/W), for the individual materials from lengths, **L1** (m) and **L2** (m), cross-sectional areas, **A1** (m^2) and **A2** (m^2), and thermal conductivities, **k1** W/(m·K) and **k2** W/(m·K). **Equations 3** and **4** calculate the rates of heat transfer, **qc1** (W) and **qc2** (W), through each material due to conduction. **Equation 5** computes the average temperature at distance, **x**, from **T1** (K). **Equation 6** calculates the total transfer of heat between temperatures, **T2** (K) and **T1**. The **last equation** calculates the equivalent resistance, **Rk** (K/W), for the parallel system from the individual resistances, **Rk1** and **Rk2**. These equations assume there is negligible transfer of heat between the two materials.

$$Rk1 = \frac{L}{k1 \cdot A1}$$ **Eq. 1**

$$Rk2 = \frac{L}{k2 \cdot A2}$$ **Eq. 2**

$$qc1 = \frac{T1 - T2}{Rk1}$$ **Eq. 3**

$$qc2 = \frac{T1 - T2}{Rk2}$$ **Eq. 4**

$$T - T1 = \frac{(T2 - T1) \cdot x}{L}$$ **Eq. 5**

$$qT = \frac{T1 - T2}{Rk}$$ **Eq. 6**

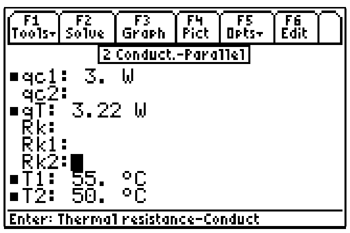
$$\frac{1}{Rk} = \frac{1}{Rk1} + \frac{1}{Rk2}$$ Eq. 7

Variable	**Description**	**Units**
A1	Area	m^2
A2	Area	m^2
k1	Thermal conductivity	W/(m·K)
k2	Thermal conductivity	W/(m·K)
L	Length	m
qc1	Heat transfer rate	W
qc2	Heat transfer rate	W
qT	Total heat transfer rate	W
Rk	Thermal Resistance-Conduction	K/W
Rk1	Thermal resistance-Conduction	K/W

Variable	**Description**	**Units**
Rk2	Thermal resistance-Conduction	K/W
T	Temperature at x or x, t	K
T1	Temperature at 1	K
T2	Temperature at 2	K
x	Distance from the left/side length	m

Example 14.2.2.2:

A CPU loses heat through a sink mounted with steel screws. The rate of heat transfer through the sink alone, with an internal temperature of 55 ^{o}C and surface temperature of 50 ^{o}C, is 3 W. The power loss for the sink, including the stainless steel screws, for the same surface temperature, is 3.22 W. Assume there is imperfect thermal contact between the screws and heat sink and negligible heat transfer occurs between the two. Compute the individual and total thermal resistances for the heat sink and the screws.



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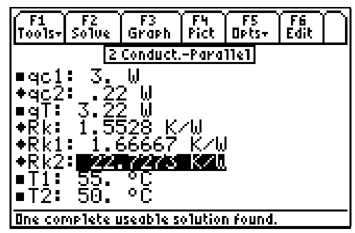
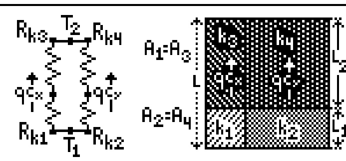


Computed results

Solution – Select **equations 3, 4, 6,** and **7**. The entries and results are shown in the screen displays above.

Given	**Solution**
qc1 = 3 W	qc2 = .22 W
qT = 3.22 W	Rk = 1.5528 K/W
T1 = 55 ^{o}C	Rk1 = 1.66667 K/W
T2 = 50 ^{o}C	Rk2 = 22.7273 K/W

14.2.2.3 Parallel-Series



This section describes the conduction of heat between temperatures, **T1** (K) and **T2** (K), through a parallel-series arrangement of four different materials. **Equation 1** computes the rate of heat transfer, **qc** (W), due to conduction through the composite system between **T1** and **T2**. **Req** (K/W), is the equivalent resistance of the combined parallel-series arrangement. **Equation 2** computes, **qc**, as the sum of the parallel heat transfer rates for the left, **qcx** (W), and right, **qcy** (W), sides. **Equation 3** calculates the rate of heat transfer, **qcx**, between temperatures, **T1** and **T2,** for the series arrangement of thermal resistances, **Rk1** (K/W) and **Rk3** (K/W). **Equation 4** computes the rate of heat transfer, **qcy**, in the second series arrangement, using thermal resistances, **Rk2** (K/W) and **Rk4** (K/W). **Equations 5**, **6**, **7**, and **8** compute the thermal resistances, **Rk1**, **Rk2**, **Rk3**, and **Rk4** from thermal conductivities, **k1**, **k2**, **k3**, and **k4**, W/(m·K) cross-sectional areas, **A1**and **A2** (m^2), and diffusion lengths, **L1** and, **L2** (m). The last equation calculates the equivalent resistance of the parallel-series arrangement, **Req** from **Rk1**, **Rk2**, **Rk3**, and **Rk4**.

$$qc = \frac{T1 - T2}{\mathrm{Re}\,q}$$ **Eq. 1**

$$qc = qcx + qcy$$ **Eq. 2**

$$qcx = \frac{T1 - T2}{Rk1 + Rk3}$$ **Eq. 3**

$$qcy = \frac{T1 - T2}{Rk2 + Rk4}$$ **Eq. 4**

$$Rk1 = \frac{L1}{k1 \cdot A1}$$ **Eq. 5**

$$Rk2 = \frac{L1}{k2 \cdot A2}$$ **Eq. 6**

$$Rk3 = \frac{L2}{k3 \cdot A1}$$ **Eq. 7**

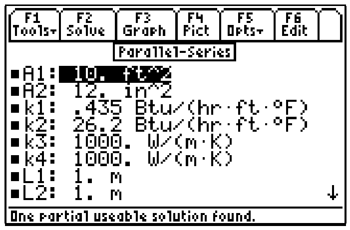
$$Rk4 = \frac{L2}{k4 \cdot A2}$$ **Eq. 8**

$$\frac{1}{Req} = \frac{1}{Rk1 + Rk3} + \frac{1}{Rk2 + Rk4}$$ **Eq. 9**

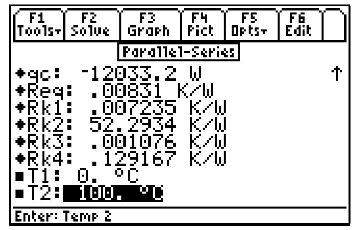
Variable	**Description**	**Units**
A1	Area	m^2
A2	Area	m^2
k1	Thermal conductivity	W/(m·K)
k2	Thermal conductivity	W/(m·K)
k3	Thermal conductivity	W/(m·K)
k4	Thermal conductivity	W/(m·K)
L1	Length	m
L2	Length	m
qc	Heat Transfer rate	W
qcx	Heat Transfer rate	W
qcy	Heat Transfer rate	W
Req	Equivalent thermal resistance	K/W
Rk1	Thermal resistance-Conduction	K/W
Rk2	Thermal resistance-Conduction	K/W
Rk3	Thermal resistance-Conduction	K/W
Rk4	Thermal resistance-Conduction	K/W
T1	Temperature at 1	K
T2	Temperature at 2	K

Example 14.2.2.3:

A 1 ft thick section of concrete wall, with four steel rods running parallel to the wall thickness, is exposed to a fluid having a temperature of 100 ^oC. The wall has a cross sectional area of 10 ft ^{2}. The rods, which run parallel to the direction of heat transfer, have a total cross sectional area of 12 in^2 and a length which is the same as the thickness of the wall. The temperature on the opposite side of the wall is 0 ^oC. If the thermal conductivity is 0.435 Btu/(hr·ft·^{o}F) for concrete, 26.2 Btu/(hr·ft·^{o}F) for steel (1% C), and the convection coefficient for the fluid are 1000 W/(m^2·K), what is the rate of heat transfer and thermal resistance in each material?



Upper Display



Lower Display

Solution – This problem can be solved using the conversion for the convection coefficient, h=k/L. Enter the value of h in the place k and enter L = 1. The units of L must be the same as the length units in k.

Example:

1. For h = 1000 W/(m2·K) k = 1000 W/(m·K)
   L = 1 m

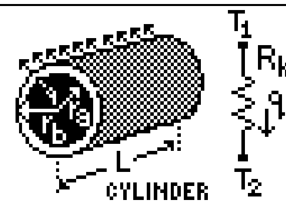
2. For h = 500 Btu/(hr·oF·ft2) k = 500 Btu/(hr·oF·ft)
   L = 1 ft

Select **equations, one, five, six, seven, eight and nine**. The entries and results are shown in the screen displays above. The value of qc is negative since heat travels in a direction opposite to the diagram above.

Given	**Solution**
A1 = 10 ft^2	qc = -12033.2 W
A2 = 12 in^2	Req = .00831 K/W
k1 = 26.2 Btu/(hr·ft·^{o}F)	Rk1 = .007235 K/W
k2 = .435 Btu/(hr·ft·^{o}F)	Rk2 = 52.2934 K/W
k3 = 1000 W/(m·K)	Rk3 = .001076 K/W
k4 = 1000 W/(m·K)	Rk4 = .129167 K/W
L1 = 1 ft	
L2 = 1 m	
T1 =0 ^{o}C	
T2 = 100 ^{o}C	

14.2.3 Radial Systems

14.2.3.1 Hollow Cylinder



These three equations compute heat conduction between the interior and exterior of a hollow cylinder made from a single material having a thermal conductivity, **k** (W/(m·K)). The length of the cylinder is, **L** (m), and the inner and outer radii of the cylinder are, **ra** (m) and **rb** (m). The surface temperatures at the inner and outer radii are, **T1** (K) and **T2** (K), respectively. The **first equation** computes the rate of heat transfer, **q** (W), due to conduction between the interior and the exterior of the cylinder. **Equation 2** calculates the thermal resistance, **Rk** (K/W), for conduction from **rb**, **ra, L** and **k**. The **last equation** estimates the temperature, **T** (K), at position, **r** (m), inside the cylinder wall, where **ra**<**r**<**rb**.

$$q = \frac{2 \cdot \pi \cdot L \cdot k}{\ln\left(\frac{rb}{ra}\right)} \cdot (T1 - T2)$$ **Eq. 1**

$$Rk = \frac{\ln\left(\frac{rb}{ra}\right)}{2 \cdot \pi \cdot L \cdot k}$$ **Eq. 2**

When r = ra and r ≤ rb the following equation is applicable

$$\frac{T - T1}{T2 - T1} = \frac{\ln\left(\frac{r}{ra}\right)}{\ln\left(\frac{rb}{ra}\right)}$$ **Eq. 3**

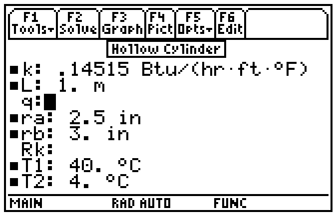
When ra < rb applies to all equations in this set.

Variable	**Description**	**Units**
k	Thermal conductivity	W/(m·K)
L	Length	m
q	Heat transfer rate - conduction	W
ra	Inner radius	m
rb	Outer radius	m
Rk	Thermal resistance-conduction	K/W
T	Temperature at x or x, t	K
T1	Temperature at 1	K
T2	Temperature at 2	K

Example 14.2.3.1:

A PVC pipe with an inner diameter 5” and an outer diameter of 6”has an interior surface temperature of 40°C and an outside surface temperature of 4°C. What is the rate of heat transfer per meter length of the pipe? Rigid poly-vinyl chloride (PVC) has a thermal conductivity of 0.14515 Btu/(hr·ft·°F).

Solution – Select **equations, one** and **two** to solve this problem.



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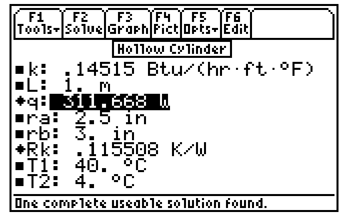
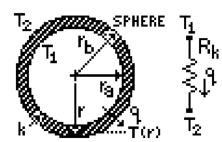


Computed results

Given
k = .14515 Btu/ hr·ft·F°
L = 1 m
ra = 2.5 in
rb = 3 in
T1 =40 °C
T2 = 4 °C

Solution
q = 311.668 W
Rk = .115508 K/W

14.2.3.2 Hollow Sphere



These equations compute the rate of heat transfer between the inner and outer surfaces of a hollow sphere due to conduction. The **first equation** calculates the rate of heat conducted, **q** (W), between the inner surface of the sphere, having temperature, **T1** (K), and the outer surface, having a temperature of, **T2** (K). **Equation 2** calculates the thermal resistance, **Rk** (K/W), between the inner and outer surfaces of the sphere from the inner radius, **ra** (m), outer radius, **rb** (m), and thermal conductivity, **k** W/(m·K). **Equation 3** computes the temperature **T** (K) at position **r** (m) where **ra**<**r**<**rb**.

$$q = \frac{T1 - T2}{Rk}$$ **Eq. 1**

$$Rk = \frac{rb - ra}{4 \cdot \pi \cdot rb \cdot ra \cdot k}$$ **Eq. 2**

When rb ≥ r ≥ ra the following equation is applicable.

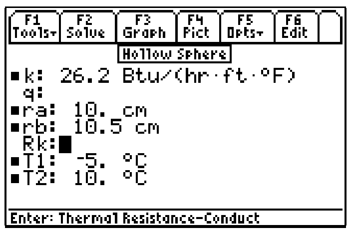
$$\frac{T - T1}{T2 - T1} = \frac{r - ra}{rb - ra} \cdot \frac{rb}{r}$$ **Eq. 3**

When ra < rb applies to all equations in this set.

Variable	Description	Units
k	Thermal conductivity	W/(m·K)
q	Heat transfer rate - conduction	W
r	Radius	m
ra	Inner radius	m
rb	Outer radius	m
Rk	Thermal resistance-conduction	K/W
T	Temperature at x	K
T1	Temperature at 1	K
T2	Temperature at 2	K

Example 14.2.3.2:

A hollow steel ball has an inner diameter of 20 cm and an outer diameter of 21 cm. The internal and external surface temperatures of the steel surface are –5°C and 10°C. What is the steady rate of heat transfer?



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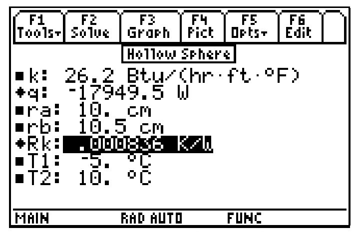
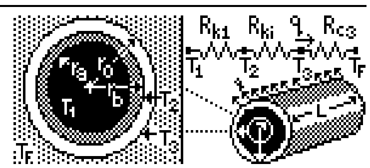


Computed results

Solution – **Equations one** and **two** are needed to solve the problem. Steel (1% carbon) has a thermal conductivity value of 26.2 Btu/(hr·ft·°F) in the range of 60-212°F (this value is listed in **Reference/Thermal Properties/Thermal Conductivity/Alloys**). Assume the listed value of thermal conductivity for steel has the same value at –4 °C and 10°C.

Given	Solution
k = 26.2 Btu/(hr·ft·°F)	q = -17949.5 W
ra = 10 cm	Rk = .000836 K/W
rb = 10.5 cm	
T1 = - 5 °C	
T2 = 10 °C	

14.2.3.3 Cylinder with Insulation Wrap



These equations describe a three-stage transfer of heat between an insulated pipe or cylinder, and a surrounding fluid. The first stage is conduction across the pipe material, having an inner surface radius, **ra** (m) and an outer radius, **rb** (m). The temperature on the interior surface of the pipe is **T1** (K), and the outer surface is **T2** (K). The second stage of heat transfer is conduction across a pipe insulation material having an inner radius, **rb**, and outer radius, **ro** (m). The temperature difference between the inner and outer surfaces of the insulation wrap is **T3-T2**. The final step is convection of heat between the outer surface of the insulation material and the surrounding fluid, having a temperature, **Tf** (K), and convective transfer coefficient, **h** (W/(m^2·K)). Since these steps occur in sequence, a series resistor analogy is used to describe the overall rate of heat exchange (see section 14.2.2). **Equation 1** computes the thermal resistance due to conduction, **Rk1** (K/W), between the inner and outer surfaces of a pipe having inner and outer radii, **ra** and **rb**, length **L,** and thermal conductivity, **k1** (W/(m·K)). **Equation 2** calculates the thermal resistance due to conduction for the insulation wrap, having an interior and exterior radii, **rb** and **ro**, length **L**, and thermal conductivity, **ki** W/(m·K). **Equation 3** computes the thermal resistance due to convection between the insulation wrap and surrounding fluid, having a convection coefficient, **h**. **Equation 4** calculates the equivalent resistance, **Req** (K/W), for heat transfer between the interior of the pipe and the fluid. The **next two equations 5 & 6** compute the rate of heat transfer, **q** (W), from the temperature differences and the thermal resistances to heat transfer for the pipe and insulation. **Equation 7** calculates the pipe/insulation boundary temperature, **T2**. The last equation computes the interfacial temperature, **T3**, for the surface of the insulation in contact with the fluid.

$$q = \frac{T1 - Tf}{Req}$$ **Eq. 1**

$$q = \frac{T1 - T2}{Rk1}$$ **Eq. 2**

$$T2 = T1 + \frac{T1 - Tf}{\mathrm{Re}q} \cdot Rk1$$ **Eq. 3**

$$T3 = Tf + \frac{T1 - Tf}{Req} \cdot Rc3$$ **Eq. 4**

$$Rk1 = \frac{\ln\left(\frac{rb}{ra}\right)}{2 \cdot \pi \cdot L \cdot k1}$$ **Eq. 5**

$$Rki = \frac{\ln\left(\frac{ro}{ra}\right)}{2 \cdot \pi \cdot L \cdot ki}$$ **Eq. 6**

$$Rc3 = \frac{1}{h \cdot 2 \cdot \pi \cdot ro \cdot L}$$ **Eq. 7**

$$\mathrm{Re}q = Rk1 + Rki + Rc3$$ **Eq. 8**

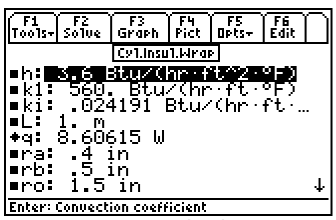
When ra < rb and bb < ro applies to all equations in this set.

Variable	**Description**	**Units**
h	Convection coefficient	W/(m^2·K)
ki	Thermal conduct. insulator	W/(m·K)
k1	Thermal conduct. cylinder	W/(m·K)
L	Length	m
q	Heat transfer rate - conduction	W
ra	Inner radius	m
rb	Outer radius	m

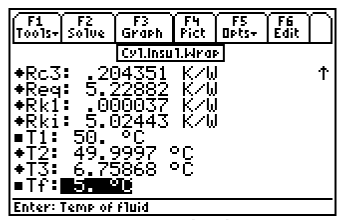
Rc3	Thermal resistance-convection	K/W
Req	Equivalent thermal resistance	K/W
Rk1	Thermal resistance-conduction	K/W
Rki	Thermal resistance-insulator	K/W
ro	Outer radius	m
T1	Temperature at 1	K
T2	Temperature at 2	K
T3	Temperature at 3	K
Tf	Temperature of fluid	K

Example 14.2.3.3:

A copper pipe having a conductivity of k=560 Btu/(hr·ft·^{o}F), an inner and outer radius of 0.4" and 0.5", is surrounded by a half-inch thick layer of polyurethane foam (PUF) k = 0.024191 Btu/(hr·ft·^{o}F). The temperature of the interior of the pipe is 50^{o}C. Cooling outside the pipe occurs due to natural convection with a fluid having a temperature of 5^{o}C and a convection coefficient for heat of h=3.6 Btu/(hr·ft^2·^{o}F). Assuming steady-state transfer of heat occurs, what is the rate of heat transfer to the surrounding fluid per meter length of the pipe? What is the temperature of the outside of the copper pipe and the outside of the insulation?



Upper Display

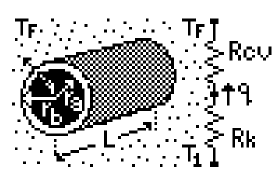


Lower Display

Solution – Select **equations, one, two, four, five, six, seven** and **eight**. The entries and results are shown in the screen displays above.

Given	**Solution**
h = 3.6 Btu/(hr·ft^2·^{o}F)	q = 8.60615 W
k1 = 560 Btu/(hr·ft·^{o}F)	Rc3 = .204351
ki = .024191 Btu/(hr·ft·^{o}F)	Req = 5.22882 K/W
L = 1 m	Rk1 = .000037 K/W
ra = .4 in	Rki = 5.02443 K/W
rb = .5 in	T2 = 49.9997 ^{o}C
ro = 1.5 in	T3 = 6.75868 ^{o}C
T1 = 50^{o}C	
Tf = 5^{o}C	

14.2.3.4 Cylinder - Critical radius



The equations in this section describe the transfer of heat from a pipe to a surrounding fluid. The **first equation** calculates the rate of heat transfer as a function of the temperature of the fluid **Tf** (K), inner surface temperature of the pipe **T1** (K), the resistance to conduction **Rk** (K/W), and the resistance to convection **Rcv** (K/W). **Equation 2** computes the resistance to conduction, **Rk** (K/W), of the pipe from the inner and outer pipe radii, **ra** (m), and, **rb** (m), the length of the pipe, **L** (m), and the thermal conductivity of the pipe material, **k** W/(m·K). **Equation 3** calculates the resistance due to convection, **Rcv**, for a fluid having a convection coefficient, **h** W/(m^2·K). **Equation 4** computes the critical radius, **rcrit** (m). If **rb** < **rcrit** the rate of transfer will increase as **rb** (pipe thickness) increases. This is due to a greater decrease of **Rcv** with increasing surface area, than increase in **Rk** with pipe thickness. If **rb**>**rcrit**, an increase in pipe thickness will cause a decrease in the rate of heat transfer.

$$q = \frac{T1 - Tf}{Rk + Rcv} \qquad \textbf{Eq. 1}$$

$$Rk = \frac{\ln\left(\frac{rb}{ra}\right)}{2 \cdot \pi \cdot L \cdot k} \qquad \textbf{Eq. 2}$$

$$Rcv = \frac{1}{h \cdot 2 \cdot \pi \cdot L \cdot rb} \qquad \textbf{Eq. 3}$$

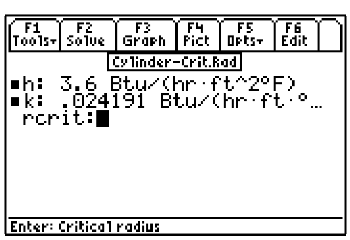
$$rcrit = \frac{k}{h} \qquad \textbf{Eq. 4}$$

When ra < rb applies to all equations in this set.

Variable	Description	Units
h	Convection coefficient	$W/(m^2 \cdot K)$
k	Thermal conductivity	$W/(m \cdot K)$
L	Length	m
q	Heat transfer rate - conduction	W
ra	Inner radius	m
rb	Outer radius	m
rcrit	Critical radius	m
Rcv	Thermal resistance - convection	K/W
Rk	Thermal resistance - conduction	K/W
T1	Temperature at 1	K
Tf	Temperature of fluid	K

Example 14.2.3.4:

Determine whether an increase in the thickness of the insulation wrap in the previous problem would increase or decrease the rate of heat transfer.



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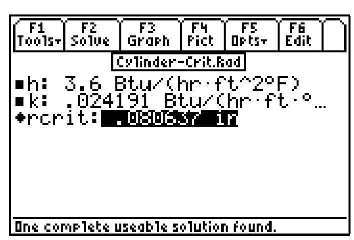
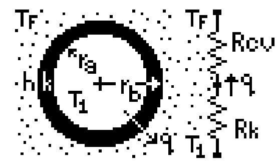


Computed results

Solution – Select the **last equation** to solve this problem. If the outer radius of the insulation (1.5 inches) is greater than the critical radius, **rcrit**, than an increase in thickness will result in a decrease in the rate of heat transfer. If rcrit < 1.5" than an increase in thickness will cause **q**, the rate of heat transfer, to increase.

Given	**Solution**
h = 3.6 Btu(hr·ft·°F)	rcrit = .080637 in
k = .024191 Btu/(hr·ft^2·°F)	

14.2.3.5 Sphere - Critical radius



These equations describe the transfer of heat from a hollow sphere to a surrounding fluid. The **first equation** calculates the rate of heat transfer, **q** (W), as a function of the temperature of the fluid, **Tf** (K), inner surface temperature of the sphere, **T1** (K), the resistance to conduction, **Rk** (K/W) between the inner and outer surfaces, and the resistance to convection of heat from the sphere surface to the fluid, **Rcv** (K/W). **Equation 2** computes the resistance to conduction, **Rk** (K/W) inside the sphere from the inner and outer radii, **ra** (m) and **rb** (m), and thermal conductivity of the material, **k** W/(m·K). **Equation 3** calculates the resistance due to convection from the heat conducting capacity of the fluid, **h** W/(m^2·K). **Equation 4** computes the critical radius, **rcrit** (m). If **rb** < **rcrit** the rate of transfer will increase as **rb** increases. This is due to a greater decrease of **Rcv** (increasing surface area), than increase in **Rk** (sphere thickness), with additional material. If **rb**>**rcrit**, an increase in sphere thickness will cause a decrease in heat transfer.

$$q = \frac{T1 - Tf}{Rk + Rcv} \quad \textbf{Eq. 1}$$

$$Rk = \frac{rb - ra}{4 \cdot \pi \cdot ra \cdot rb \cdot k} \quad \textbf{Eq. 2}$$

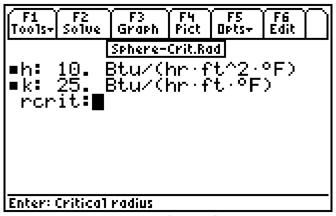
$$Rcv = \frac{1}{h \cdot 4 \cdot \pi \cdot rb^2} \quad \textbf{Eq. 3}$$

$$rcrit = \frac{2 \cdot k}{h} \quad \textbf{Eq. 4}$$

Variable	**Description**	**Units**
h	Convection coefficient	W/(m^2·K)
k	Thermal conductivity	W/(m·K)
q	Heat transfer rate - conduction	W
ra	Inner radius	m
rb	Outer radius	m
rcrit	Critical radius	m
Rcv	Thermal resistance	K/W
Rk	Thermal resistance-conduction	K/W
T1	Temperature at 1	K
Tf	Temperature of fluid	K

Example 14.2.3.5:

Compute the critical radius for a hollow sphere composed of a material having a thermal conductivity of 25 Btu/(hr·ft·°F) surrounding by a fluid having a convection coefficient of 10 Btu/(hr·ft·°F).



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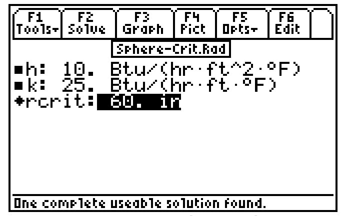


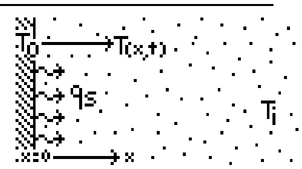
Computed results

Solution – Select the **last equation** to solve this problem. The entries and results are shown in the screen displays above.

Given	**Solution**
h = 10. Btu/(hr·ft²·°F).	rcrit = 60 in
k = 25. Btu/(hr·ft·°F).	

14.3 Semi-Infinite Solid

14.3.1 Step Change Surface Temperature



The equation set in this topic computes the unsteady case for one-dimensional heat transfer, in a semi-infinite solid, from a constant temperature source. **Equation 1** calculates the temperature, **T** (K), at position, **x** (m), from the constant temperature source after, **time** (s), has elapsed since the heat source was introduced. **To** (K), is the temperature of the constant heat source and **Ti** is the initial temperature of the solid at the time the heat source is introduced, i.e. **Ti** = T(x,0) and T(∞,time) **To**=T(0, time). **Equation 2** calculates the rate of heat transfer into the solid, **qs** (W), after **time**, has elapsed since the addition of the heat source. Area, **A** (m^2), is the area of the solid in direct contact with the heat source, **αd** (m^2/s), is thermal diffusion coefficient of heat in the solid, and **k** Btu/(ft·hr·°F) is the thermal conductivity of the solid. **Equation 3** computes the heat flux, **qf** (W/m^2), the rate of heat transfer per area. **Equation 1** incorporates the complimentary error function, **erfc**().

$$\frac{T-To}{Ti-To}=erf\left(\frac{x}{2\cdot\sqrt{\alpha d\cdot time}}\right) \qquad \textbf{Eq. 1}$$

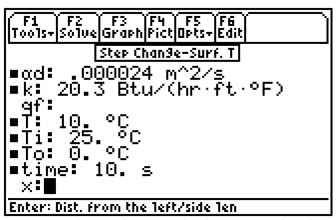
$$qf=\frac{k\cdot(To-Ti)}{\sqrt{\pi\cdot\alpha d\cdot time}} \qquad \textbf{Eq. 2}$$

$$qf=\frac{qs}{A} \qquad \textbf{Eq. 3}$$

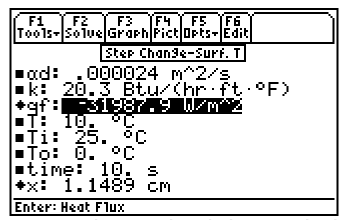
Variable	Description	Units
αd	Thermal diffusivity	m^2/s
A	Area	m^2
k	Thermal conductivity	W/(m·K)
qf	Heat flux	W/m^2
qs	Heat transfer rate	W
T	Temperature at x or x, t	K
Ti	Temperature (x, time = 0 or ∞, time = t)	K
To	Temperature (0, time = t)	K
time	Time	J
x	Distance from the left/side length	m

Example 14.3.1:

One end of a rectangular slab of lead, at 25°C, is brought into thermal contact with a mixture of ice and water. At what distance inside the slab, from the water contact point, has a temperature of 10°C after 10 seconds? After 20 seconds? After 30 seconds? What is the rate of heat transfer per square meter during these periods? Lead as a thermal conductivity of 20.3 Btu/(hr·ft·°F) and a thermal diffusion coefficient of $24.1x\ 10^{-6}\ m^2/s$ at room temperature.



Entered Values (First Solution)

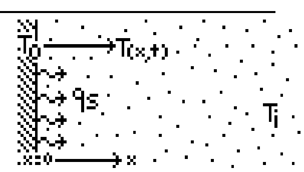


Computed Results (First Solution)

Solution – Select the **first** and **second equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. A notice will appear stating that **nsolve(…)**, numeric solve, will be used to compute the solution. Press [ENTER]. The entries and results are shown in the screen displays above.

Given	**Solution**
αd = 0.000024 m^2/s	x= 1.1489 cm (10 s)
k = 20.3 Btu/(hr·ft·°F)	x= 1.6248 cm (20 s)
T = 10°C	x= 1.98996 cm (30 s)
Ti = 25°C	qf = -31987.9 W/m^2 (10 s)
To = 0°C	qf = -22618.9 W/m^2 (20 s)
time = 10, 20, 30 s	qf = -18468 W/m^2 (30 s)

14.3.2 Constant Surface Heat Flux



These equations describe the unsteady case for heat diffusion through a conducting plane wall, with initial temperature, **Ti** (K), in contact with a heat source transferring a *constant rate of heat* with time, **qs** (W), i.e.: the temperature of the heat source varies with time to conduct the same amount of heat through the wall. **Equation 1** calculates the temperature, **T** (K), at position **x** (m), from the constant heat source inside the wall after, **time** (s), has elapsed since the heat source was introduced. **Equation 2** computes the temperature of the surface in contact with the constant heat source, **Ts** (K), at after **time** has elapsed. **Ti** (K), is the initial temperature of the plane wall at the time the heat source is introduced, i.e. **Ti** =T (x,0) and T(∞,time) and **Ts**=T(0, time). **Equation 3** computes the heat flux, **qf** (W/m^2), the rate of heat transfer over the area, **A** (m), of the conducting surface in contact with the emitting source. **αd** (m^2/s), is the thermal diffusion coefficient and, **k** Btu/(ft·hr·^{o}F), is the thermal conductivity for the wall material. **Equation 1** incorporates the Complimentary Error function, **erfc**(…) described in the previous section.

$$T = Ti + \frac{2 \cdot qf}{k} \cdot \sqrt{\frac{\alpha d \cdot time}{\pi}} \cdot e^{\frac{-x^2}{4 \cdot \alpha d \cdot time}} - \frac{qf \cdot x}{k} \cdot erfc\left(\frac{x}{2 \cdot \sqrt{\alpha d \cdot time}}\right) \qquad \textbf{Eq. 1}$$

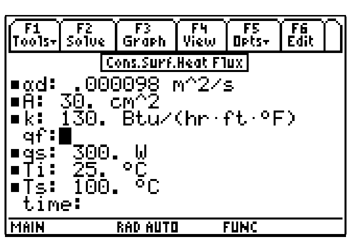
$$Ts = Ti + \frac{2 \cdot qf}{k} \cdot \sqrt{\frac{\alpha d \cdot time}{\pi}} \qquad \textbf{Eq. 2}$$

$$qf = \frac{qs}{A} \qquad \textbf{Eq. 3}$$

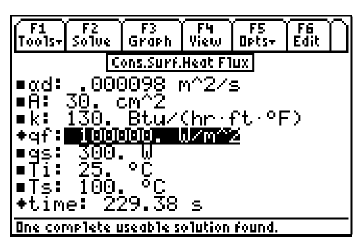
Variable	**Description**	**Units**
αd	Thermal diffusivity	m^2/s
A	Area	m^2
k	Thermal conductivity	W/(m·K)
qf	Heat flux	W/m^2
qs	Heat transfer rate	W
T	Temperature at x	K
Ti	Temperature	K
Ts	Surface temperature of emitter	K
time	Time	s
x	Distance from the left/side length	m

Example 14.3.2:

A heat element, with a surface area of 30 cm^2 and emitting a constant 300 W, is implanted in a semi-infinite block of aluminum. The aluminum has initial temperature of 25^{o}C, a thermal conductivity of 130 Btu/(hr·ft·^{o}F) (see **Reference/Thermal Properties/Thermal Conductivity/Elements**) and a thermal diffusion coefficient of 97.5 x 10^{-6} m^2/s. At what time following the activation of the heating element does the temperature of the aluminum adjacent to heat element reach 100^{o}C?



Entered Values



Computed results

Solution – Select the **second** and **third equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. A notice will appear stating that **nsolve()**

(numeric solve) will be used to compute the solution. Press [ENTER] to start the solver. The entries and results are shown in the screen displays above.

Given	**Solution**
αd = 97.5E-6 m^2/s	qf = 100000 W/m^2
A = 30 cm^2	time = 229.38 s
k = 130 Btu/(hr·ft·°F)	
qs = 300 W	
Ti = 25°C	
Ts = 100°C	

14.3.3 Surface Convection

The following equations describe the unsteady case of convective heat transfer from a fluid to a solid plane wall and diffusion of heat through the wall by means of conduction. The fluid has a convective transfer coefficient, **h** $W/(m^2 \cdot K)$, the solid has a thermal conductivity, **k** W/(m·K), and a thermal diffusion coefficient, **αd** (m^2/s). **Equation 1** computes temperature, **T** (K), at position **x** (m), and elapsed **time** (s), after the exposure of the solid to the fluid. The solid has an initial temperature, **Ti** (K), and the liquid maintains a constant temperature, **Tf** (K). The **second equation** computes the temperature of the solid surface at the solid/fluid interface at **time**. **Equation 3** computes the heat flux, **qf** (W/m^2), at after **time** has elapsed. The **last equation** computes the heat flux, **qf** (W/m^2), from the total flux, **qs**, over the fluid/solid contact surface area, **A** (m^2).

$$\frac{T-Ti}{Tf-Ti}=erfc\left(\frac{x}{2\cdot\sqrt{\alpha d\cdot time}}\right)-e^{\frac{h\cdot x}{k}+\frac{h^2\cdot\alpha d\cdot time}{k^2}}\cdot erfc\left(\frac{\frac{x}{2}}{\sqrt{\alpha d\cdot time}}+\frac{h\cdot\sqrt{\alpha d\cdot time}}{k}\right) \quad \textbf{Eq. 1}$$

$$\frac{Ts-Ti}{Tf-Ti}=1-e^{\frac{h^2\cdot\alpha d\cdot time}{k^2}}\cdot erfc\left(\frac{h\cdot\sqrt{\alpha d\cdot time}}{k}\right) \quad \textbf{Eq. 2}$$

$$qf=h\cdot(Tf-Ti)\cdot e^{\frac{h^2\cdot\alpha d\cdot time}{k^2}}\cdot erfc\left(\frac{h\sqrt{\alpha d\cdot time}}{k}\right) \quad \textbf{Eq. 3}$$

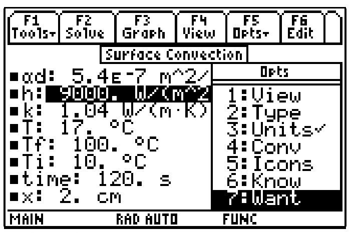
$$qf=\frac{qs}{A} \quad \textbf{Eq. 4}$$

Variable	**Description**	**Units**
αd	Thermal diffusivity	m^2/s
A	Area	m^2
h	Convection coefficient	$W/(m^2 \cdot K)$
k	Thermal conductivity	W/(m·K)
qf	Heat flux	W/m^2
qs	Heat transfer rate	W
T	Temperature at x or x,t	K
Tf	Temperature of fluid	K
Ti	Temperature (x, time = 0 or ∞, time = t)	K

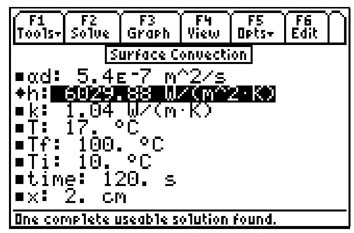
Variable	**Description**	**Units**
Ts	Surface temperature of emitter	K
time	Time	s
x	Distance from the left/side length	m

Example 14.3.3:

Boiling water is continuously sprayed onto a fireclay brick wall. A temperature probe, placed at the solid liquid interface at a distance of 2 cm inside the brick, records an initial temperature of 10°C and a temperature of 17°C, after 120 seconds have elapsed. The brick has a thermal conductivity of 1.04 W/(m·°K) and thermal diffusion coefficient of 5.4 x 10^{-7} m^2/s. What is the convection coefficient for the boiling water being sprayed on the brick surface?



Entered Values:Use F5:Opts, 7:Want to convert 'h' to an initial value for nsolve(..)



Computed results

Solution – Select the **first equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. **An initial guess value of h, is needed for nsolve (…) to converge to a solution**. The convection coefficient for boiling water typically ranges from 2000 to 50000 W/(m^2·K). Enter **h** = 9000 W/(m^2·K) and press [F5]:**Opts**, [7]**: Want**, to flag the entered value for **h** as a starting value for **nsolve (…)**. Press [F2] to compute the solution. The entries and results are shown in the screen displays above.

Given
αd = 5.4 E-7 m^2/s
k = 1.04 W/(m·K)
T = 17°C
Tf = 100°C
Ti = 10°C
time = 120 s
x = 2 cm

Solution
h = 6029.88 W/(m^2·K)

14.4 Radiation

14.4.1 Blackbody Radiation

Equation 1 is the Stefan-Boltzmann law for radiation emitted by a blackbody source. **Eb** (W/m^2), is the total emissive power for all wavelengths, **λ** (m), for an *ideal* blackbody and **Ts** (K), is the surface temperature of the blackbody emitter. The **second equation** computes **E** (W/m^2), the power emitted from a non-ideal blackbody source. The emissivity factor, **ε**, accounts for a particular material's radiative spectrum at a particular temperature (**ε =1** an ideal blackbody). The **third equation** is Planck's Law for monochromatic power, **Ebλ** (W/m^2), emitted by a blackbody surface at temperature, **Ts** and wavelength **λ** (m). The **last equation** computes the wavelength of maximum power emission from a blackbody, **λmax** (m), at blackbody surface temperature, **Ts**. The Stefan-Boltzmann constant for radiation, **σ**=5.670 x 10^{-8}

W/($m^2 \cdot K^4$), and the blackbody radiation constants, **brc1**=3.7417749 x 10^{-16} W·m^2, **brc2**=0.01438769 m·K, and the Wein displacement constant **brc3**=0.002897756 m·K, are values automatically inserted into the equation by ME•*Pro* during the solving process. The values for **σ**, **brc1, brc2** and **brc3** are listed in the **Reference/Engineering Constants** section of ME•*Pro.*

$$Eb = \sigma \cdot Ts^4 \quad \textbf{Eq. 1}$$

$$E = \varepsilon \cdot Eb \quad \textbf{Eq. 2}$$

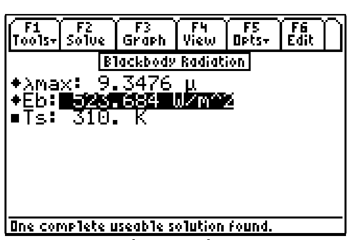
$$Eb\lambda = \frac{brc1}{\lambda^5 \cdot \left(e^{\frac{brc2}{\lambda \cdot Ts}} - 1 \right)} \quad \textbf{Eq. 3}$$

$$Ts \cdot \lambda\max = brc3 \quad \textbf{Eq. 4}$$

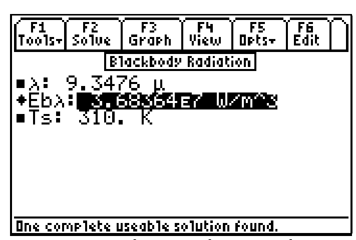
Variable	**Description**	**Units**
ε	Emissivity	unitless
σ	Stefan-Boltzmann constant for radiation	5.670 x 10^{-8} W/($m^2 \cdot K^4$)
λ	Wavelength	m
λmax	Peak wavelength	m
brc1	1st radiation constant	3.7417749 x 10^{-16} W·m^2
brc2	2nd radiation constant	0.01438769 m·K
brc3	Wein displacement constant	0.002897756 m·K
E	Emissive power	W/m^2
Eb	Black body emissive power	W/m^2
Ebλ	Monochromatic emissive power	W/m^3
Ts	Surface temperature of emitter	K

Example 14.4.1:

What is the total power emitted, per area, from an ideal blackbody at 310 K? What is the monochromatic power and wavelength at which the maximum emission occurs?



Computed Results: 1st step



Computed Results: 2nd step

Solution – Solve this problem in two steps. **Select equations one** and **four.** Enter the temperature and press [F2] to solve. **Deselect equations one** and **four,** select **equation 3**, press [F2] to solve. Enter the calculated value of **λmax** for **λ**. Press [F2] to solve.

Given	**Solution**
Ts = 310 K	λmax = 9.3476 µ

Given	**Solution**
	Eb = 523.684 W/m^2 Ebmax = 3.68364E7 W/m^3

14.4.2 Non-Blackbody radiation

Most surfaces are not blackbody surfaces but exhibit a combination of radiative and reflective properties (opaque). **Equation 1** is the equation for the rate of energy emitted per area, **Es** (W/m^2), by a non-blackbody surface at temperature **T** (K). **Equation 2** computes the radiosity, **Js** (W/m^2), per area from the surface, **As** (m^2), as the sum of the radiative, **Es**, and reflective, **Gs·ρs**, light components. **Gs** (W/m^2), is the thermal radiation per area incident on surface and, **ρs**, is the fraction of the incoming light which is reflected. **Equation 3** and 4 compute the radiative energy **qs** (W), leaving the opaque surface, where **αs** is the fraction of the light, incident on the opaque surface, which is absorbed and, **εs**, is the emissivity of the surface. The absorptivity, **αs**, is dependent on the wavelength of the incoming light. For objects exhibiting graybody characteristics, the light absorbed is independent of the wavelength of the incoming light and the assumption can be made that **αs ≈ εs ≈ 1-ρs**. **Equation 5** computes the thermal radiative-resistance-to-transfer, **Rs** (1/m^2), of the *second kind*‡. The **last equation** computes the relationship between the radiative, **αs**, and reflective, **ρs**, components of incident energy on the surface (**ρs=1, αs=0** for totally reflective, **ρs=0, αs=1** for blackbody).

$$Es = \varepsilon s \cdot \sigma \cdot T^4 \qquad \textbf{Eq. 1}$$

$$Js = Es + \rho s \cdot Gs \qquad \textbf{Eq. 2}$$

$$qs = As \cdot (Js - Gs) \qquad \textbf{Eq. 3}$$

$$qs = \frac{\frac{\varepsilon s}{\alpha s} \cdot Es - Js}{Rs} \qquad \textbf{Eq. 4}$$

$$Rs = \frac{1 - \alpha s}{\alpha s \cdot As} \qquad \textbf{Eq. 5}$$

$$\rho s = 1 - \alpha s \qquad \textbf{Eq. 6}$$

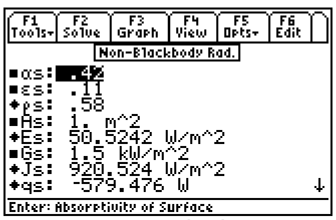
Variable	**Description**	**Units**
σ	Stefan-Boltzmann constant for radiation	5.670×10^{-8} W/($m^2 \cdot K^4$)
αs	Absorptivity	unitless
εs	Emissivity of shield	unitless

‡ Radiative transfer of the *first kind* views exchange of heat between two surfaces as powered by a thermal gradient, (**T1-T2**). The equation for this approach is, **q = (T1-T2)/Rf**, where **Rf** (K/W) is the resistance to transfer of the first kind and is a function of temperature, radiation area and shape factors. This approach works for heat exchange between two surfaces of having temperatures **T1** and **T2**. Radiative transfer of the *second kind* views heat exchange as powered by a difference in emission **(E1-E2)** between objects. The equation for this approach is, **q = (E1-E2)/Rs**, where **Rs** (1/m^2), is the resistance to transfer of the *second kind*. The second approach allows the advantage of modeling heat transfer between several surfaces (see Lindon Thomas, *Heat Transfer* p. 300).

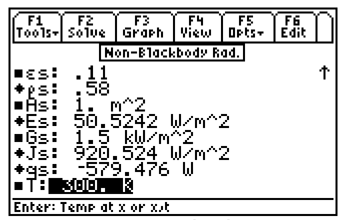
Variable	**Description**	**Units**
ρs	Reflectivity	unitless
As	Total surface area	m^2
Es	Emissive power-radiating surface	W/m^2
Gs	Irradiation	W/m^2
Js	Radiosity	W/m^2
qs	Heat transfer rate	W
Rs	Thermal resistance, 2nd kind	$1/m^2$
T	Temperature at x or x, t	K

Example 14.4.2:

Polished stainless steel has a solar absorptivity of 0.42, and an emissivity of 0.11 at room temperature (300 K). If sunlight, with an intensity of 1.5 kW/m^2 is irradiated on the steel surface, what is the total radiosity, and radiation emitted per square meter?



Upper Display



Lower Display

Solution – Select the **equations, one, two, three** and **six** to solve this problem. The entries and results are shown in the screen displays above.

Given	**Solution**
αs = .42	ρs = .58
ε s = .11	Es =50.5242 W/m^2
As = 1 m^2	Js = 920.524 W/m^2
Gs = 1.5 kW/m^2	qs = -579.476 W
T = 300 K	

14.4.3 Thermal Radiation Shield

Insulating walls are composed of layers of reflective materials separated by evacuated spaces. The reflective plates reduce heat transfer due to radiation and the spaces add extra insulation by reducing heat exchange due to thermal contact. In these equations, plates with surface areas **As** (m^2) and emissivity **εs** separate emissive graybodies with surface areas, **A1** (m^2) and **A2** (m^2), emissivities, **ε1** and **ε2**, and steady-state emissions, **E1** (W) and **E2** (W). These equations assume the following conditions:

- The shield surfaces are thin, and have a negligible resistance to conduction.
- The shape factors between shields and radiating surfaces are ≈1.
- Since the thermal shields are highly reflective, radiation received by the first gray-body, **E1** is assumed to originate primarily from itself.
- The non-isothermal gray-body approximation assumes **ε1/α1≈1**. Where **α1** is the absorptivity of surface **A1**.

Equations 1 and **2** compute the power emission for each surface. **Equation 3** computes the steady state heat transfer between, **A1**, and, **A2**, for a *series* arrangement of two reflective shields in the direction of heat transfer. Each shield has an emissivity, **εs**, and a reflective area, **As**. **Equation 4** calculates the case where **Ns** shields (**As**, **εs**) are placed in a parallel arrangement between **A1** and **A2**.

$$E1 = \varepsilon 1 \cdot \sigma \cdot T1^4 \qquad \textbf{Eq. 1}$$

$$E2 = \varepsilon 2 \cdot \sigma \cdot T2^4 \qquad \textbf{Eq. 2}$$

$$q12c = \frac{E1 - E2}{\frac{1-\varepsilon 1}{A1 \cdot \varepsilon 1} + \frac{1}{A1} + \frac{1-\varepsilon 2}{A2 \cdot \varepsilon 2} + \frac{2-\varepsilon s}{As \cdot \varepsilon s}} \qquad \textbf{Eq. 3}$$

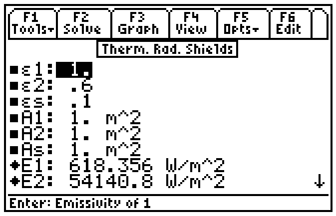
$$q12c = \frac{A1 \cdot (E1 - E2)}{\frac{1}{\varepsilon 1} + \frac{1}{\varepsilon 2} - 1 + \frac{(2-\varepsilon s) \cdot Ns}{\varepsilon s}} \qquad \textbf{Eq. 4}$$

Variable	**Description**	**Units**
σ	Stefan-Boltzmann constant for radiation	5.670×10^{-8} W/(m^2·K^4)
ε1	Emissivity of 1	unitless
ε2	Emissivity of 2	unitless
εs	Emissivity of shield	unitless
A1	Area	m^2
A2	Area	m^2
As	Total surface area	m^2
E1	Emissive power 1	W/m^2
E2	Emissive power 2	W/m^2
Ns	No. of shields	unitless
q12c	Heat flux	W/m^2
T1	Temperature 1	K
T2	Temperature 2	K

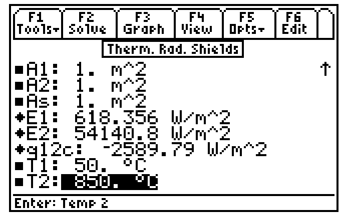
Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 14.4.3:

A blackbody plate with a temperature of 50°C exchanges radiation with a stainless steel plate having a temperature of 850°C and ε=0.6. If two thin, polished aluminum shields (ε=0.1), are placed in a series arrangement between the plates, what is net heat transfer per square meter between the two plates?



Upper Display



Lower Display

Solution – Select the **first three equations** to solve this problem. The entries and results are shown in the screen displays above.

Given
ε1 = 1
ε2 = .6
εs = .1
A1 = 1 m^2
A2 = 1 m^2
As = 1 m^2
T1 = 50°C
T2 = 850°C

Solution
E1 = 618.356 W/m^2
E2 = 54140.8 W/m^2
q12c = -2589.79 W/m^2

References:

1. Thomas, Lindon C, "Heat Transfer", Prentice Hall NJ, 1992

A useful source for heat transfer properties of many materials can be located on the web* at:
http://www.tak2000.com/
A glossary of websites for material related to this topic is located at:
http://www.uic.edu/~mansoori/Thermodynamic.Data.and.Property_html

*Note: These websites are not maintained or affiliated with da Vinci Technologies Group. da Vinci does not guarantee the availability and reliability of information located at these URL's.

Chapter 15: Thermodynamics

This section covers a wide range of topics under six subheadings.

- Fundamentals
- Vapor and Gas Mixture
- First Law
- System Properties
- Ideal Gas Properties
- Second Law

15.1 Fundamentals

These equations list various forms for expressing the quantity of a substance. **Equation 1** calculates the specific volume, **vs** (m^3/kg), which is the volume occupied per unit mass of a substance. The **second equation** computes density, **ρ** (kg/m^3). **Equation 3** calculates the specific weight, **γ** (N/m^3), from the total mass, **m** (kg), and volume, **V** (m^3) of the substance. The gravitational acceleration constant, **grav** (_g = 9.80665 m/s^2), is automatically inserted into the computation by ME•*Pro* and does not appear in the list of variables. The **last equation** computes the number of moles, **n** (mol), of a substance from its total mass, **m**, and molar mass, **MWT** (kg/mol).

$$vs = \frac{V}{m}$$ **Eq. 1**

$$\rho = \frac{m}{V}$$ **Eq. 2**

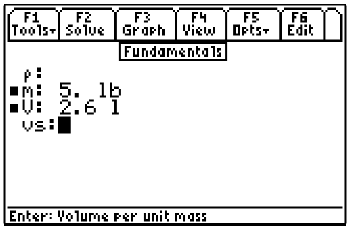
$$\gamma = \frac{m \cdot grav}{V}$$ **Eq. 3**

$$n = \frac{m}{MWT}$$ **Eq. 4**

Variable	Description	Units
γ	Specific weight	N/m^3
ρ	Density	kg/m^3
grav	Acceleration due to gravity	9.80665 m^2/s
m	Mass	kg
MWT	Molar mass	kg/mol
n	Number of moles	mol
V	Volume	m^3
vs	Volume per unit mass	m^3/kg

Example 15.1:

Compute the density and specific volume of a liquid having a volume of 2.6 liters and a mass of 5 pounds.



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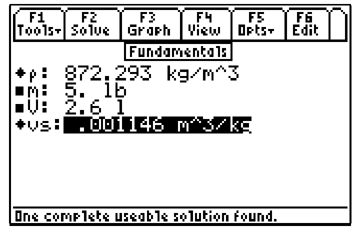


Computed results

Solution – Select the **first** and **second equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
m = 5 lb	ρ = 872.293 kg/m^3
V = 2.6 l	vs = .001146 m^3/kg

15.2 System Properties

15.2.1 Energy Equations

This section includes the general forms of the equations used to describe the state of a pure substance in thermodynamic equilibrium. **Equation 1** computes the enthalpy, **H** (J), as the sum of the internal energy, **UE** (J), of the material, and the work, **p·V** (J), performed by the system. The variables, **p** (Pa) and **V** (m^3), are the pressure and volume of the gas component of the system. The **second equation** computes the Helmoltz energy, **A** (J), the maximum work the system is able to perform. The quantity, **T·S** (J), is the energy in the system stored as random heat energy, and is unavailable for work. **T** (K), is the temperature and, **S** (J/K), is the total entropy of the system. **Equations 3** and **4** compute the Gibbs energy, **G** (J), the work that can be extracted from the system, after work, (**p·V)**, and the energy lost to increasing the entropy of the system, (**T·S**). The energies **H**, **U**, **A** and **G** are extensive properties; i.e., dependent on the quantity of the substance. **Equations 5-9** are differentiated forms of the first three equations and represent a thermodynamic transition of the system. **Δ** in **equations 10-17**, indicates a change in a property (**p, V, T, H, UE, G, A, S**) between state 1 and state 2.

$$H = UE + p \cdot V \qquad \textbf{Eq. 1}$$

$$A = UE - T \cdot S \qquad \textbf{Eq. 2}$$

$$G = H - T \cdot S \qquad \textbf{Eq. 3}$$

$$G = A + p \cdot V \qquad \textbf{Eq. 4}$$

$$\Delta UE = T \cdot \Delta S - p \cdot \Delta V \qquad \textbf{Eq. 5}$$

$$\Delta H = \Delta UE + p \cdot \Delta V + V \cdot \Delta p \qquad \textbf{Eq. 6}$$

$$\Delta A = \Delta UE - S \cdot \Delta T - T \cdot \Delta S \qquad \textbf{Eq. 7}$$

$$\Delta G = \Delta H - S \cdot \Delta T - T \cdot \Delta S \qquad \textbf{Eq. 8}$$

$\Delta G = \Delta A + p \cdot \Delta V + V \cdot \Delta p$	**Eq. 9**
$\Delta A = A2 - A1$	**Eq. 10**
$\Delta G = G2 - G1$	**Eq. 11**
$\Delta H = H2 - H1$	**Eq. 12**
$\Delta p = p2 - p1$	**Eq. 13**
$\Delta S = S2 - S1$	**Eq. 14**
$\Delta T = T2 - T1$	**Eq. 15**
$\Delta V = V2 - V1$	**Eq. 16**
$\Delta UE = UE2 - UE1$	**Eq. 17**

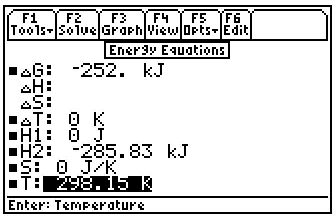
Variable	Description	Units
ΔA	Difference - Helmoltz function	J
ΔG	Difference - Gibbs free energy	J
ΔH	Difference - enthalpy	J
Δp	Difference - pressure	Pa
ΔS	Difference - entropy	J/K
ΔT	Difference - temperature	K
ΔUE	Difference - internal energy	J
ΔV	Difference - volume	m^3
A	Helmoltz function	J
A1	Helmoltz energy –1	J
A2	Helmoltz energy –2	J
G	Gibbs Free energy	J
G1	Gibbs energy –1	J
G1	Gibbs energy –2	J
H	Enthalpy	J
H1	Enthalpy –1	J
H2	Enthalpy –2	J
p	Pressure	Pa
p1	Partial pressure –1	Pa
p2	Partial pressure –2	Pa
S	Entropy	J/K
S1	Entropy –1	J/K
S2	Entropy –2	J/K
T	Temperature	K
T1	Temperature –1	K
T2	Temperature –2	K
UE	Internal energy	J
UE1	Internal energy – state –1	J
UE2	Internal energy – state –1	J
V	Volume	m^3
V1	Volume – state – 1	m^3
V2	Volume – state –2	m^3

Example 15.2.1:

One mole of hydrogen gas reacts with half a mole of molecular oxygen at room temperature (298.15 K) and standard atmospheric pressure (1 atm), to form 1 mol of liquid water.

$$H_2(g) + \frac{1}{2} \cdot O_2(g) \rightarrow 2 \cdot H_2O(l)$$

The enthalpy of formation for both hydrogen and oxygen under standard atmospheric and pressure conditions is 0 J/(mol·K). The enthalpy of formation for liquid water (298.15 K, 1 atm) is –285.83 kJ/mol. If the reaction creates –252 kJ of energy available for work, what is the change in entropy for the reaction?



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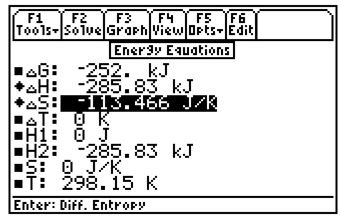


Computed results

Solution – Select the **eighth** and **twelfth equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters. A value for S is not needed since this is a constant temperature process. Press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
ΔG=-252 kJ	ΔH = -252 kJ
ΔT = 0 K	ΔS = -113.466 J/K
H1 = 0 J	
H2 = -285.83 kJ	
S = 0 J/K	
T=298.15 K	

15.2.2 Maxwell Relations

These four equations describe exact differential relationships used in thermodynamic calculations. An advantage of the Maxwell Relations is the ability to relate changes in observable parameters, such as pressure, volume and temperature (**p, V, T**), to changes in non-measurable quantities such as entropy (**S**). The subscripts to the bottom right of the parenthesis indicate a parameter being held constant while the other two parameters inside the parenthesis vary. **Δ** indicates the change in a property (**p, V, T,** and **S**).

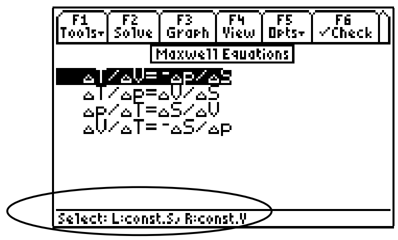
Note: Due to display limitations, the subscript variable, which remains constant during a transition, cannot be shown. The parameter being held constant on the left and right side of the equations appears in the status line at the bottom screen while the equation is being highlighted.

$$\left(\frac{\Delta T}{\Delta V}\right)_S = -\left(\frac{\Delta p}{\Delta S}\right)_V$$ **Eq. 1**

$$\left(\frac{\Delta T}{\Delta p}\right)_S = \left(\frac{\Delta V}{\Delta S}\right)_p \qquad \textbf{Eq. 2}$$

$$\left(\frac{\Delta p}{\Delta T}\right)_V = \left(\frac{\Delta S}{\Delta V}\right)_T \qquad \textbf{Eq. 3}$$

$$\left(\frac{\Delta V}{\Delta T}\right)_p = -\left(\frac{\Delta S}{\Delta p}\right)_T \qquad \textbf{Eq. 4}$$



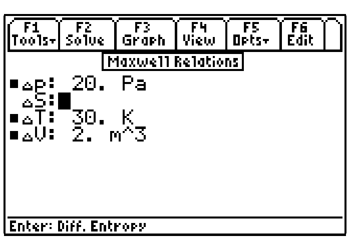
Status Line message for a highlighted Maxwell Relation
S (entropy) is constant for the left side of the equation
V (volume) is constant for the right side of the equation.

Variable	Description	Units
Δp	Difference - pressure	Pa
ΔS	Difference - entropy	J/K
ΔT	Difference - temperature	K
ΔV	Difference - volume	m^3

Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 15.2.2:

If the constant-volume heating of a gas produces an increase of 20 Pa with an increase in temperature of 30 K, what will be the change in entropy for the same gas if the volume expands by 2 m^3 during an isothermal heating?



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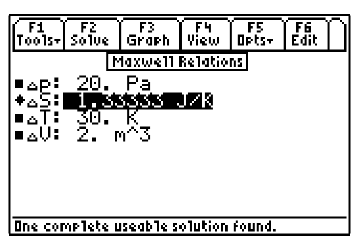


Computed results

Solution – Select the **third equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
Δp = 20 Pa
ΔT = 30 K
ΔV = 2 m^3

Solution
ΔS = 1.33333 J/K

15.3 Vapor and Gas Mixture

15.3.1 Saturated Liquid/Vapor

The following equations describe a pure liquid in equilibrium with its vapor at constant temperature. The chemical equation for conversion of a pure substance, **A**, from a liquid (l) to a vapor (v) is described below.

$$A(l) \longrightarrow A(v)$$
$$(1-x) \qquad (x)$$

Equation 1 computes **x**, the dryness fraction, or the degree of conversion of A from a liquid to a vapor, or, the mole fraction of A in vapor form (**x=0** when A is a liquid, **x=1** when **A** is a vapor). **Equation 2** calculates the total volume occupied by the chemical, **V** (m^3), as the sum of the total liquid volume, **Vtl** (m^3), and total vapor volume, **Vtv** (m^3), of **A.** **Equation 3** computes the total volume, **V**, from the mass of the liquid, **mlq** (kg), and vapor, **mv** (kg), components of A, in addition to the volumes occupied per mass of liquid, **vl** (m^3/kg), and gas, **vv** (m^3/kg). **Equation 4** calculates, **V**, from the total mass of the liquid and vapor, **m** (kg), and the volume occupied, per mass, by the liquid/vapor system, **vs** (m^3/kg). **Equation 5** computes the change in specific volume, **δv** (m^3), for the conversion of a pure substance from a liquid to a vapor. **Equation 6** computes, **vs**, from the mass of the vapor, **mv**, the total mass, **m**, and the specific volumes of the liquid, **vl** and vapor, **vv**. **Equation 7** calculates, **vs**, as the sum of the specific volume of the liquid, **vl**, and the additional volume contribution of the substance in vapor form **x·(vv-vl)**. **Equation 8** computes the change in internal energy per mass, **δu** (J/kg), for a liquid/vapor transition from the difference of the internal energy of the vapor, **ul** (J/kg), and the vapor, **uv** (J/kg). **Equation 9** calculates the internal energy per mass of the liquid/vapor **us** (J/kg) for the dryness fraction, **x**. **Equation 10** computes the specific enthalpy change, **δh** (J/kg), for converting a pure substance from a liquid to a vapor. **Equation 11** calculates the enthalpy per mass, **hs** (J/kg), of the liquid/vapor system for the dryness fraction, **x**. The last two equations, (**Eq. 12** and **13**), compute the entropy change per mass, **δs** J/(kg·K), of liquid converted to a vapor, and the entropy per mass, **ss** J/(kg·K), of the liquid/vapor system.

$$x = \frac{mv}{mlq + mv} \qquad \textbf{Eq. 1}$$

$$V = Vtl + Vtv \qquad \textbf{Eq. 2}$$

$$V = mlq \cdot vl + mv \cdot vv \qquad \textbf{Eq. 3}$$

$$V = m \cdot vs \qquad \textbf{Eq. 4}$$

$$\delta v = vv - vl \qquad \textbf{Eq. 5}$$

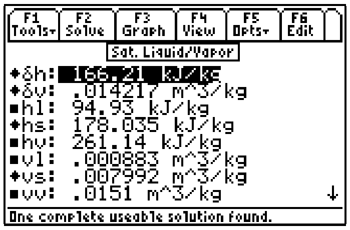
$vs = \frac{m - mv}{m} \cdot vl + \frac{mv}{m} \cdot vv$	**Eq. 6**
$vs = vl + x \cdot \delta v$	**Eq. 7**
$\delta u = uv - ul$	**Eq. 8**
$us = ul + x \cdot \delta u$	**Eq. 9**
$\delta h = hv - hl$	**Eq. 10**
$hs = hl + x \cdot \delta h$	**Eq. 11**
$\delta s = sv - sl$	**Eq. 12**
$ss = sl + x \cdot \delta s$	**Eq. 13**

Variable	Description	Units
δh	Difference in specific enthalpy	J/kg
δs	Difference in specific entropy	J/(kg·K)
δu	Difference in specific internal energy	J/kg
δv	Difference in specific volume	m^3/kg
hl	Specific enthalpy - liquid	J/kg
hs	Specific enthalpy	J/kg
hv	Specific enthalpy – vapor	J/kg
m	Mass	kg
mlq	Mass of liquid	kg
mv	Mass of vapor	kg
ss	Entropy per unit mass	J/(kg·K)
sl	Specific entropy – liquid	J/(kg·K)
sv	Specific entropy – vapor	J/(kg·K)
ul	Internal energy/mass (liquid)	J/kg
us	Internal energy/mass	J/kg
uv	Internal energy/mass (vapor)	J/kg
V	Volume	m^3
vl	Specific volume – liquid	m^3/kg
vs	Volume per unit mass	m^3/kg
Vtl	Total liquid volume	m^3
Vtv	Total vapor volume	m^3
vv	Specific vapor volume	m^3
x	Dryness fraction	unitless

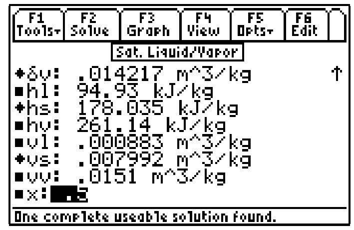
Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 15.3.1:

A table lists Refrigerant-22 as having a vapor quality (dryness fraction) of 0.5 at T=40°C. The saturated specific volumes of the refrigerant at this temperature are 0.000883 m^3/kg for the liquid and 0.0151 m^3/kg for the vapor. The specific enthalpies of the liquid and vapor are 94.93 kJ/kg and 261.14 kJ/kg. Find the specific volume and the specific enthalpy of the refrigerant.



Upper Display

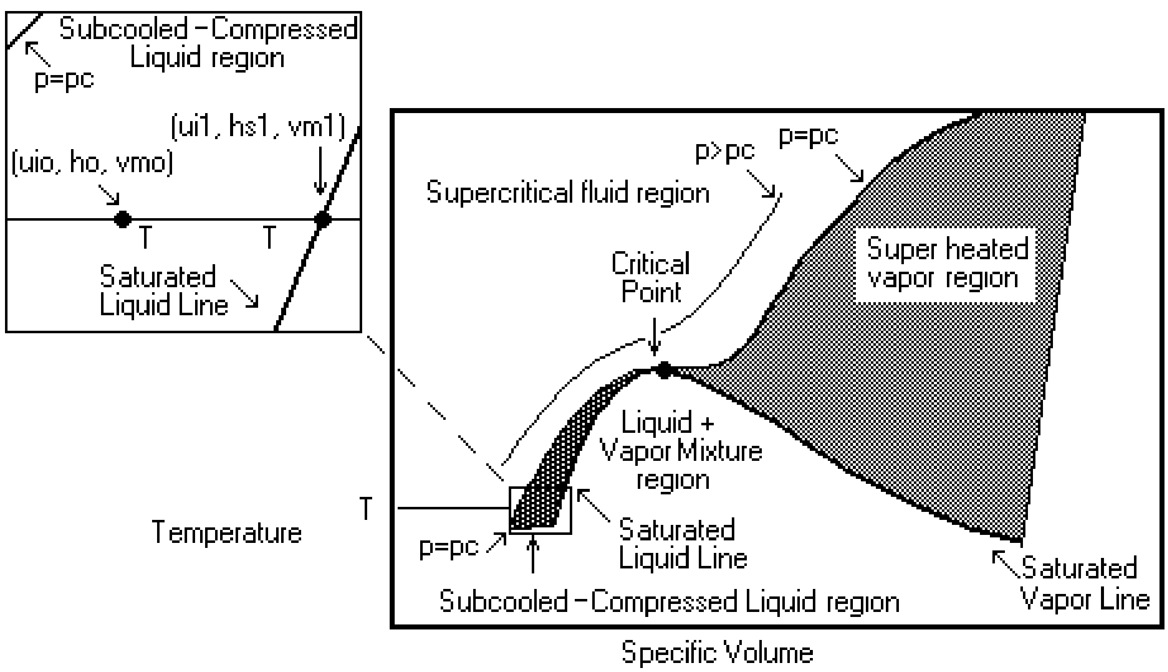


Lower Display

Solution – Select **equations 5**, **7**, **10**, and **11** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
hl = 94.93 kJ/kg	δh = 166.21 kJ/kg
hv = 261.14 kJ/kg	δv = .014217 m^3/kg
vl = .000883 m^3/kg	hs = 178.035 kJ/kg
vv = .0151 m^3/kg	vs = .007992 m^3/kg
x = .5	

15.3.2 Compressed Liquid-Sub cooled



Equation 1 describes the change in thermodynamic properties between initial state, 0 and final state, 1. Sub cooled liquid data for many substances is not available. **Equation 2** is an approximate version of the first equation for calculating of the thermodynamic state for a compressed or sub cooled liquid **ho** (J/kg) and **po** (Pa), from saturated liquid properties, **h1** (J/kg), **vm1** (m^3/mol), **p1** (Pa), at the same temperature. It is assumed that the internal energy and the specific volumes of the saturated liquid can be used for the sub-cooled state without a significant loss of accuracy. The assumptions are made are that **uio = ui1** (J/kg) and **vmo = vm1** (m^3/mol) at constant temperature, **T** (K).

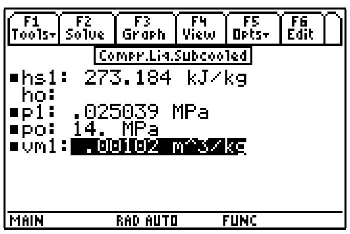
$$ho = hs1 + uio - ui1 + po \cdot vmo - p1 \cdot vm1 \quad \textbf{Eq. 1}$$

$$ho = hs1 + vm1 \cdot (po - p1) \quad \textbf{Eq. 2}$$

Variable	Description	Units
hs1	Specific enthalpy – state 1	J/(kg·K)
ho	Specific enthalpy – state o	J/(kg·K)
p1	Partial pressure – 1	Pa
po	Partial pressure – o	Pa
ui1	Internal energy/mass state 1	J/kg
uio	Internal energy/mass state o	J/kg
vm1	Volume per mass at state 1	m^3/kg
vmo	Volume per mass at state o	m^3/kg

Example 15.3.2:

Compute the sub cooled liquid properties for water at 65 °C and 14 MPa.



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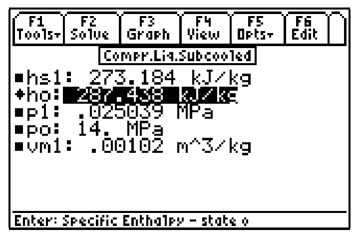


Computed results

Solution – Obtain the thermodynamic values for the saturated liquid state from **Analysis/Steam Tables/Saturated Steam** (see Chapter 3). Select **Temperature**, enter **Ts = 65°C** and press [F2] to solve. Use the values of **Ps**, **Vf**, and **Hf** for **p1**, **vm1** and **hs1 respectively,** in **equation 2** of this section. Use [F1]: **Tools**, [5]: **Copy**/[6]: **Paste**.

Given	Solution
T = 65°C	ho = 287.438 kJ/kg
hs1 (Hf) = 273.184 kJ/kg	
p1 (Ps) = .025039 MPa	
po = 14 MPa	
vm1 (Vf) = .00102 m^3/kg	

15.4 Ideal Gas Properties

15.4.1 Specific Heat

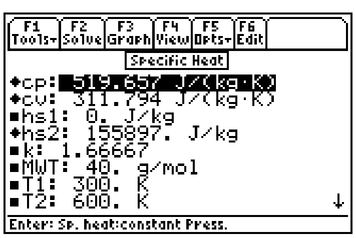
The following equations compute various thermodynamic properties of ideal gases. **Equation 1** is the ideal gas law. The **second equation** computes the difference in specific heats of constant pressure and constant volume for an ideal gas. **Equation 3** calculates the specific heat ratio, **k**. **Equations 4** and **5** compute the change in specific enthalpy, **hs2-hs1** (J/kg), and internal energy per mass, **ui2-ui1** (J/kg), with change in temperature, **T2-T1** (K). The specific heats at constant volume and pressure, **cv** J/(kg·K), and **cp** J/(kg·K), in equations 4 and 5, are assumed to remain constant with temperature.

$p \cdot vs = \frac{Rm}{MWT} \cdot T$	**Eq. 1**
$cp - cv = \frac{Rm}{MWT}$	**Eq. 2**
$\frac{cp}{cv} = k$	**Eq. 3**
$hs2 - hs1 = cp \cdot (T2 - T1)$	**Eq. 4**
$ui2 - ui1 = cv \cdot (T2 - T1)$	**Eq. 5**

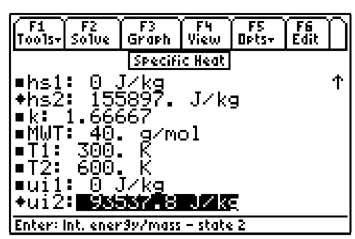
Variable	**Description**	**Units**
cp	Specific heat at constant pressure	J/(kg·K)
cv	Specific heat at constant volume	J/(kg·K)
vs	Specific volume	m^3/kg
hs1	Specific enthalpy – 1	J/kg
hs2	Specific enthalpy –2	J/kg
k	Specific heat ratio	unitless
MWT	Molar mass	kg/mol
p	Pressure	Pa
Rm	Molar gas constant	8.3145 J/(kg·K)
T	Temperature	K
T1	Temperature - 1	K
T2	Temperature – 2	K
ui1	Internal energy/mass state - 1	J/kg
ui2	Internal energy/mass state - 2	J/kg

Example 15.4.1:

An ideal monatomic gas, having a molar mass of 40 g/mol, is contained in a cylinder with a piston exerting a constant pressure. A heating element increases the temperature of the gas from 300 K to 600 K. What is the change in specific enthalpy, internal energy per kilogram, the specific heat at constant pressure and volume for the monatomic gas?



Upper Display



Lower Display

Solution – A specific heat ratio of k = 5/3 can be used for an ideal monatomic gas. Select **equations 2, 3, 4,** and **5** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
hs1 = 0 J/kg	cp = 519.657 J/(kg·°K)
k = 5/3	cv = 311.794 J/(kg·°K)
MWT = 40 g/mol	hs2 = 155897 J/kg
T1 = 300 K	ui2 = 93537.8 J/kg
T2 = 600 K	
ui1 = 0 J/kg	

15.4.2 Quasi-Equilibrium Compression

The following equations compute the work performed by a gas due to quasi-equilibrium expansion or compression. **Equation 1** computes the work performed, **Wp** (J), at a constant pressure, **p** (Pa), between an initial volume, **V1** (m^3), and final volume, **V2** (m^3). **Equation 2** computes the work performed at constant temperature, **Wt** (J), due to isothermal expansion/compression from an initial volume, **V1** to a final volume, **V2**. The **third equation** computes the work performed for a polytropic process, **Wpol** (J), (see Chapter 13.5 for more information on polytropic or quasi-equilibrium processes). The polytropic coefficient is restricted to values of $\boldsymbol{\lambda \neq 1}$. When $\boldsymbol{\lambda=0}$ (constant pressure), equation 3 simplifies to Equation 1. When $\boldsymbol{\lambda = \infty}$, the constant volume condition is fulfilled and **Wpol** becomes zero. The **fourth equation** computes **Wpol** for an ideal gas between an initial temperature **T1** (K) and final temperature **T2** (K). **Equation 5** shows the equivalence relationship between pressure and volume for the initial, **V1** (m^3) and **p1** (Pa), and final, **V2** (m^3) and **p2** (Pa), states of a polytropic process. **Equations 6** and **7** compute the heat flow, **Qpol** (J), into the system for a polytropic process.

$$Wp = p \cdot (V2 - V1) \qquad \textbf{Eq. 1}$$

$$Wt = p1 \cdot V1 \cdot \ln\left(\frac{V2}{V1}\right) \qquad \textbf{Eq. 2}$$

$$Wpol = \frac{p2 \cdot V2 - p1 \cdot V1}{1 - \lambda} \qquad \textbf{Eq. 3}$$

$$Wpol = \frac{\frac{m}{MWT} \cdot Rm \cdot (T2 - T1)}{1 - \lambda} \qquad \textbf{Eq. 4}$$

$$p1 \cdot V1^{\lambda} = p2 \cdot V2^{\lambda} \qquad \textbf{Eq. 5}$$

$$Qpol - Wpol = m \cdot (ui2 - ui1) \qquad \textbf{Eq. 6}$$

$$Qpol = m \cdot cv \cdot (T2 - T1) + Wpol \qquad \textbf{Eq. 7}$$

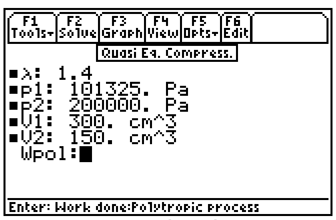
Variable	Description	Units
λ	Polytropic Index ($\lambda \neq 1$)	unitless
cv	Specific heat at constant volume	J/(K·kg)
m	Mass	kg
MWT	Molar mass	kg/mol
p	Pressure	Pa
p1	Pressure 1	Pa
p2	Pressure 2	Pa

Variable	Description	Units
Qpol	Heat Flow - polytropic	J
T1	Temperature 1	K
T2	Temperature 2	K
ui1	Internal energy/mass state 1	J/kg
ui2	Internal energy/mass state 2	J/kg
V1	Volume 1	m^3
V2	Volume 2	m^3
Wp	Work done: constant pressure	J
Wpol	Work done: polytropic process	J
Wt	Work: constant temperature	J

Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 15.4.2:

A piston compresses a gas reversibly from an initial pressure 101,325 Pa and an initial volume of 300 cm^3 to a final pressure of 200,000 Pa and a final volume of 150 cm^3. The polytropic index for the system is 1.4. Compute the work performed.



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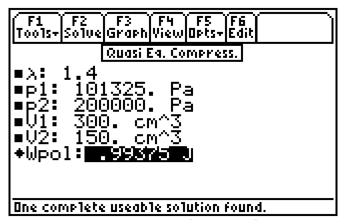


Computed results

Solution – To solve the problem select **Equation 3**. Highlight the equation by and press [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
$\lambda = 1.4$
p1 = 101325 Pa
p2 = 200000 Pa
V1 = 300 cm^3
V2 = 150 cm^3

Solution
Wpol = .99375 J

15.5 First Law

15.5.1 Total System Energy

The following equations describe the conservation of various forms of energy. The **first equation** computes the change in energy, **ΔE** (J), due to heat added to the system, **Q12** (J), less the work performed on the surroundings, **W12** (J). **Equation 2** computes the total change in energy as the sum of the changes of kinetic energy, **ΔKE** (J), potential energy, **ΔPE** (J), and internal energy, **ΔUE** (J). **Equation 3** calculates the difference in kinetic energy, **ΔKE**, between the initial, **KE1** (J), and final, **KE2** (J), states. **Equation 4** calculates the change in internal energy, **ΔPE**, from the initial, **PE1** (J), and final, **PE2** (J), energy states. **Equation 5** calculates the change in internal energy, **ΔUE**, from the initial, **UE1** (J), and final, **UE2** (J), energy states. **Equation 6** computes, **ΔKE**, due to difference in linear velocities, **vel1** (m/s) and **vel2** (m/s). **Equation 7** calculates the difference in potential energy, **ΔPE**, due to relative heights, **zh1** (m) and **zh2** (m), in the earth's gravitational field. The **last equation** computes **ΔUE** from the change in internal energy per mass, **ui1** (J/kg) and **ui2** (J/kg), and the total mass of the substance, **m** (kg).

$$\Delta E = Q12 - W12 \quad \textbf{Eq. 1}$$

$$\Delta E = \Delta KE + \Delta PE + \Delta UE \quad \textbf{Eq. 2}$$

$$\Delta KE = KE2 - KE1 \quad \textbf{Eq. 3}$$

$$\Delta PE = PE2 - PE1 \quad \textbf{Eq. 4}$$

$$\Delta UE = UE2 - UE1 \quad \textbf{Eq. 5}$$

$$\Delta KE = \frac{1}{2} \cdot m \cdot \left(vel2^2 - vel1^2\right) \quad \textbf{Eq. 6}$$

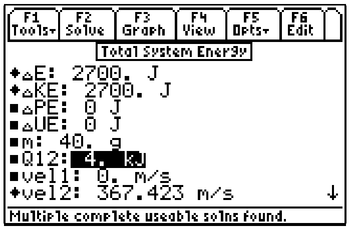
$$\Delta PE = m \cdot grav \cdot (zh2 - zh1) \quad \textbf{Eq. 7}$$

$$\Delta UE = m \cdot \left(ui2 - ui1\right) \quad \textbf{Eq. 8}$$

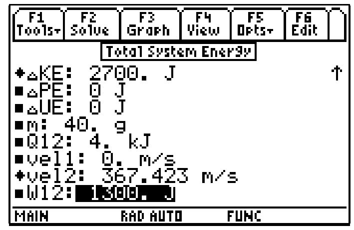
Variable	Description	Units
ΔE	Energy change	J
ΔKE	Difference in kinetic energy	J
ΔPE	Difference in potential energy	J
ΔUE	Difference in internal energy	J
grav	Gravitational constant	9.80665 m/s^2
KE1	Kinetic energy – state 1	J
KE2	Kinetic energy – state 2	J
m	Mass	kg
PE1	Potential energy – state 1	J
PE2	Potential energy – state 2	J
Q12	Heat Flow	J
ui1	Internal energy/mass state 1	J
ui2	Internal energy/mass state 2	J
UE1	Internal energy state 1	J
UE2	Internal energy state 2	J
vel1	Velocity – state 1	m/s
vel2	Velocity – state 2	m/s
W12	External work performed – 1 to 2	J
zh1	Height 1	m
zh2	Height 2	m

Example 15.5.1:

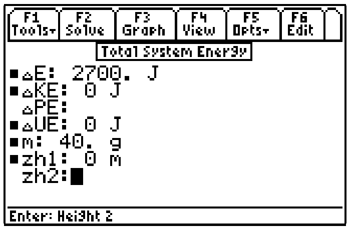
A 40 g bullet is fired vertically from a gun. The heat generated from gunpowder combustion inside the bullet is 4 kJ. The energy lost due to work and heating the gun is 1300 J. The remainder of the energy is transferred as kinetic energy to the bullet. If there is no wind friction, how high will the bullet travel?



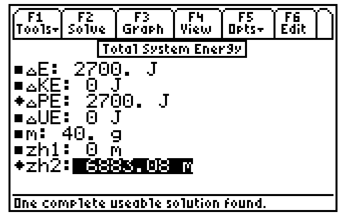
Step 1: Upper Display (Solution 2)



Step 1: Lower Display (Solution 2)



Step 2: Entered Values



Step 2: Computed Results

Solution – This problem needs to be solved in two stages. First, calculate the conversion of stored energy, in the gunpowder, to kinetic energy in the bullet using **Equations 1, 2,** and **6**. The first computation (**Step 1**) has two possible solutions, press [2] for the second solution for the results shown on this page. Then calculate the conversion of kinetic energy to potential energy using **Equations 2** and **7**. Enter the known variables for each step as shown in the screen displays above.

Step 1: Given
ΔPE = 0 J
ΔUE = 0 J
m = 40 g
Q12 = 4 kJ
vel1 = 0 m/s
W12 = 1300 J

Step 1: Solution
ΔE = 2700 J
ΔKE = 2700 J
vel2 = 367.423 m/s

Step 2: Given
ΔE = 2700 J
ΔKE = 0 J
ΔUE = 0 J
m = 40 g
zh1 = 0 m

Step 2: Solution
ΔPE = 2700 J
zh2 = 6883.08 m

15.5.2 Closed System: Ideal Gas

15.5.2.1 Constant Pressure

These equations describe the thermodynamic properties of a system undergoing transition under constant pressure. The **first equation** calculates the heat change of a system, **Q12** (J), as the sum of the change of the internal energy components, **ui2-ui1** (J/kg), and the work performed, **W12** (J). **Equation 2** computes **Q12** from the change in specific enthalpy,**hs2-hs1** J/(kg·K). **Equation 3** computes the work, **W12**, performed by the system during a expansion/compression between an initial volume, **V1** (m^3), and final volume, **V2** (m^3), by a constant pressure, **p** (Pa). **Equation 4** calculates the change in specific enthalpy, **hs2-hs1,** from the specific heat at constant pressure, **cp** J/(kg·K), and the change in temperature, **T2-T1** (K). **Equation 5** computes the change in internal energy, **ui2-ui1**, from the specific heat at constant volume, **cv** J/(kg·K). **Equation 6** calculates the change in specific enthalpy, **ss2-ss1** J/(kg·K), from **cp** and the initial and final temperatures, **T1** (K) and **T2** (K). **Equation 7** computes the total change in entropy of the system, **S21** (J/K). **Equation 8** calculates, **k**, the ratio of specific heats. **Equations 9 and 10** compute the ideal gas law at states 1 and 2.

$$Q12 = m \cdot (ui2 - ui1) + W12$$ **Eq. 1**

$$Q12 = m \cdot (hs2 - hs1)$$ **Eq. 2**

$$W12 = p \cdot (V2 - V1)$$ **Eq. 3**

$$hs2 - hs1 = cp \cdot (T2 - T1)$$ **Eq. 4**

$$ui2 - ui1 = cv \cdot (T2 - T1)$$ **Eq. 5**

$$ss2 - ss1 = cp \cdot \ln\left(\frac{T2}{T1}\right)$$ **Eq. 6**

$$S12 = m \cdot (ss2 - ss1)$$ **Eq. 7**

$$k = \frac{cp}{cv}$$ **Eq. 8**

$$p \cdot V1 = \frac{m}{MWT} \cdot Rm \cdot T1$$ **Eq. 9**

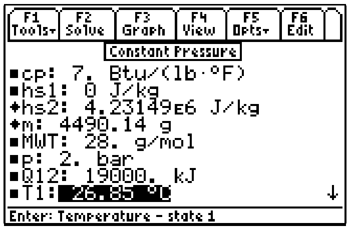
$$p \cdot V2 = \frac{m}{MWT} \cdot Rm \cdot T2$$ **Eq. 10**

Variable	Description	Units
cp	Specific heat at constant pressure	J/(K·kg)
cv	Specific heat at constant volume	J/(K·kg)
hs1	Specific enthalpy – state1	J/kg

Variable	Description	Units
hs2	Specific enthalpy – state 2	J/kg
m	Mass	kg
MWT	Molar mass	kg/mol
p	Pressure	Pa
Q12	Heat flow	J
Rm	Ideal gas constant	8.31451 J/(K·mol)
S12	Entropy	J/K
ss1	Entropy per mass – state 1	J/(K·kg)
ss2	Entropy per mass – state 2	J/(K·kg)
T1	Temperature – state 1	K
T2	Temperature – state 2	K
ui1	Internal energy/mass - state 1	J/kg
ui2	Internal energy/mass – state 2	J/kg
V1	Volume – 1	m^3
V2	Volume – 2	m^3
W12	External work done - 1 to 2	J

Example 15.5.2.1:

Nitrogen (28 g/mol), contained in a volume of 2 m^3, exerts a constant pressure of 2 bars against a movable, frictionless piston. The temperature of the nitrogen gas is 26.85^{o}C. A heating element exchanges 19,000 kJ of heat causing the volume to expand and the temperature to increase. The specific heat at constant pressure for nitrogen in this temperature range is approximately 7 Btu/(lb·F). What is the mass of nitrogen in the cylinder, the final volume and temperature the gas, and the work performed? Assume nitrogen has ideal gas behavior and the specific heat remains constant with temperature.



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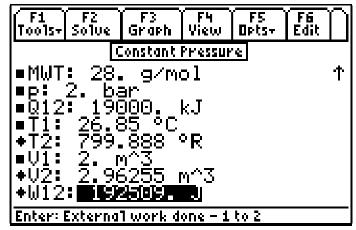

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Solution – Select the **second, third, fourth, ninth** and **tenth equations**. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
cp = 7 Btu/(lb·°F)	hs2 = 4.23149 E6 J/kg
hs1= 0 J/kg	m = 4490.14 g
MWT = 28 g/mol	T2 = 799.888 °R
p = 2 bar	V2 = 2.96255 m^3
Q12 = 19000 kJ	W12 = 192509 J
T1=26.85 °C	
V1=2. m^3	

15.5.2.2 Binary Mixture

The following equations compute the thermodynamic state for a mixing of two ideal gases. The gases are assumed to occupy the same volume before and after mixing. **Equations 1 and 2** compute the specific internal energy, **us** (J/kg), and the specific enthalpy, **hs** (J/kg), of the binary mixture at a temperature, **T** (K). The reference temperature, **Trf** (K), and the reference pressure, **pref** (Pa), must be identical for both gases. **Equation 3** calculates the specific entropy of the mixture, **ss** J/(kg·K), as the sum of the entropies of the individual gases at temperature, **T**, and entropy *of mixing* contribution (from the middle terms) due to the irreversible mixing of two gases. The two gases have partial pressures, **p1** (Pa) and **p2** (Pa), and occupy an initial and final volume, **V1** (m^3) and **V2** (m^3). **Equation 4** calculates the specific heat at constant pressure, **cp** J/(K·kg), of the gas mixture as the sum of the individual contributions of the specific heats of each gas, **cp1** J/(K·kg) and **cp2** J/(K·kg). **Equation 5** computes the mole ratio from the masses, **m1** (kg) and **m2** (kg), and the molar masses of each gas, **MWT1** (g/mol) and **MWT2** (g/mol). The **final equation** calculates the total mass, **m** (kg), of the gas mixture as the sum of the individual masses of each ideal gas, **m1** (kg) and **m2** (kg).

$$us = \frac{m1}{m}\cdot cv1\cdot(T - Trf) + \frac{m2}{m}\cdot cv2(T - Trf) + \frac{m1}{m}\cdot\left(h1rf - \frac{Rm}{MWT1}\cdot Trf\right) + \frac{m2}{m}\cdot\left(h2rf - \frac{Rm}{MWT2}\cdot Trf\right) \qquad \textbf{Eq. 1}$$

$$hs = \frac{m1}{m}\cdot cp1(T - Trf) + \frac{m2}{m}\cdot cp2\cdot(T - Trf) + \frac{m1}{m}\cdot h1rf + \frac{m2}{m}\cdot h2rf \qquad \textbf{Eq. 2}$$

$$ss = \frac{m1}{m}\cdot cp1\cdot\ln\left(\frac{T}{Trf}\right) + \frac{m2}{m}\cdot cp2\cdot\ln\left(\frac{T}{Trf}\right) - \left(\frac{m1\cdot Rm}{m\cdot MWT1}\cdot\ln\left(\frac{p1}{pref}\right) + \frac{m2\cdot Rm}{m\cdot MWT2}\cdot\ln\left(\frac{p2}{pref}\right)\right) + \frac{m1}{m}\cdot s1rf + \frac{m2}{m}\cdot s2rf \qquad \textbf{Eq. 3}$$

$$cp = \frac{m1}{m}\cdot cp1 + \frac{m2}{m}\cdot cp2 \qquad \textbf{Eq. 4}$$

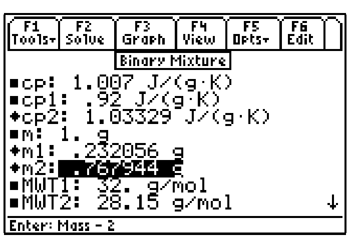
$$\frac{n2}{n1} = \frac{MWT1 \cdot m2}{MWT2 \cdot m1} \qquad \textbf{Eq. 5}$$

$$m = m1 + m2 \qquad \textbf{Eq. 6}$$

Variable	**Description**	**Units**
cp	Specific heat of mixture: Constant Pressure	J/(kg·K)
cp1	Specific heat: constant pressure – substance 1	J/(kg·K)
cp2	Specific heat: constant pressure – substance 2	J/(kg·K)
cv1	Specific heat: constant volume – substance 1	J/(kg·K)
cv2	Specific heat: constant volume – substance 2	J/(kg·K)
h1rf	Specific enthalpy – substance 1, at pref = Trf,	J/kg
h2rf	Specific enthalpy – substance 2, at pref = Trf	J/kg
hs	Specific enthalpy of mixture	J/kg
m	Mass of mixture	kg
m1	Mass 1	kg
m2	Mass 2	kg
MWT1	Molar mass: substance 1	kg/mol
MWT2	Molar mass: substance 2	kg/mol
n1	No. moles: substance 1	mol
n2	No. moles: substance 2	mol
p1	Partial pressure: substance 1	Pa
p2	Partial pressure: substance 2	Pa
pref	Reference partial pressure	Pa
ss	Specific entropy of mixture	J/(kg·K)
s1rf	Specific entropy – substance 1, at temp. = Trf, press. = pref	J/(kg·K)
s2rf	Specific entropy – substance 2, at temp. = Trf, press. = pref	J/(kg·K)
T	Temperature	K
Trf	Temperature – reference	K
us	Internal energy/mass	J/kg

Example 15.5.2.2:

Dry air has an approximate volume composition of 79 % Nitrogen (28.15 g/mol) and 21% oxygen (32 g/mol). The specific heat at constant pressure for dry air is listed as 1.007 J/g/K at 1 bar and 300 K. The specific heat at constant pressure for molecular oxygen, at the same temperature and pressure, is 0.92 J/(g·K). Approximate the specific heat at constant pressure for nitrogen.



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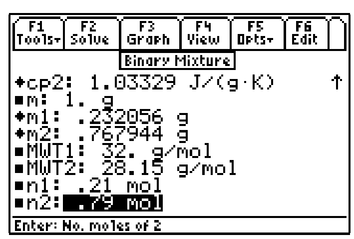

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Solution – Select the **last three equations** to solve this problem. The volumetric fractions of an ideal gas are the same as the mole fractions. Use a value of **m=1** so **m1** (oxygen) and **m2** (nitrogen) are computed as mass ratios. The actual value of the specific heat for nitrogen is 1.0302 J/(kg·K).

Given	**Solution**
cp = 1.007 J/(g·K)	cp2 = 1.03329 J/(kg·K)

Given
cp1=0.92 J/(g·K)
m = 1 g
MWT1 = 32 g/mol
MWT2 = 28.15 g/mol
n1 = .21 mol
n2 = .79 mol

Solution
m1 = . 232056 g
m2 = .767944 g

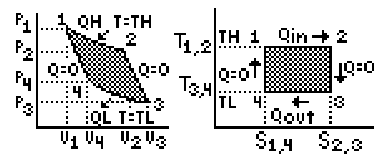
15.6 Second Law

15.6.1 Heat Engine Cycle

Heat engines exemplify the principles of the 2nd law of thermodynamics. By a series of reversible compression and expansion processes, work can be done.

15.6.1.1 Carnot Engine

The Carnot cycle, the most efficient heat engine, is composed of the following stages:



- 1→2: Reversible isothermal expansion (heat, **QH** (J), enters the system, temperature is constant).
- 2→ 3: Reversible adiabatic expansion (no heat is exchanged, **Q=0** J).
- 3→4: Reversible isothermal compression (heat, **QL** (J), is exchanged to the heat sink, temperature is constant).
- 4→1: Reversible adiabatic compression (no heat is exchanged).

The **first equation** computes the work performed, **Wnet** (J), as the difference of the heat extracted from the heat source, **QH**, and the heat deposited into a heat sink, **QL**. **Equation 2** calculates the heat removed from the heat source, **QH**, during isothermal expansion (step 1→2) from the mass of the ideal gas, **m** (kg), the molar mass of the gas, **MWT** (g/mol), the temperature of the heat source, **TH** (K), and the initial, **V1** (m^3), and final, **V2** (m^3), volumes during the isothermal expansion process. **Equation 3** computes the heat transferred to the heat sink, **QL**, during the isothermal compression (step 3→4). **TL** (K), is the temperature of the heat sink. **V3** (m^3) and **V4** (m^3) are the initial and final volumes of the reversible isothermal compression. **Equations 4** and **5** calculate the ratio of the initial and final volumes for the reversible adiabatic expansion (**V2→V3**) and compression (**V4→V1**) stages in the Carnot cycle from the ratio of the initial and final temperatures (**TL→TH** for stage 2, and **TH→TL** for stage 4). **Equations 6** and **7** compute the efficiency, **ζ**, of converting heat to work, for the Carnot cycle. **Equation 8** computes the specific heat ratio, **k**, from the specific heat of the gas at constant pressure, **cp** J/(kg·K), and constant volume, **cv** J/(kg·K). **Equation 9** and **10** calculate the relationship between the initial and final temperatures and pressures for an adiabatic process. The **last equation** computes the mass of the gas, **m** (kg), used in the Carnot cycle from the ideal gas relationship.

$$QH - QL = Wnet \qquad \textbf{Eq. 1}$$

$$QH = \frac{m \cdot Rm}{MWT} \cdot TH \cdot \ln\left(\frac{V2}{V1}\right) \qquad \textbf{Eq. 2}$$

$$QL = \frac{-m \cdot Rm}{MWT} \cdot TL \cdot \ln\left(\frac{V4}{V3}\right)$$ **Eq. 3**

$$\frac{TL}{TH} = \left(\frac{V2}{V3}\right)^{k-1}$$ **Eq. 4**

$$\frac{TL}{TH} = \left(\frac{V1}{V4}\right)^{k-1}$$ **Eq. 5**

$$\zeta = 1 - \frac{TL}{TH}$$ **Eq. 6**

$$\zeta = \frac{Wnet}{QH}$$ **Eq. 7**

$$k = \frac{cp}{cv}$$ **Eq. 8**

$$\frac{TH}{TL} = \left(\frac{p2}{p3}\right)^{\frac{k-1}{k}}$$ **Eq. 9**

$$\frac{TH}{TL} = \left(\frac{p1}{p4}\right)^{\frac{k-1}{k}}$$ **Eq. 10**

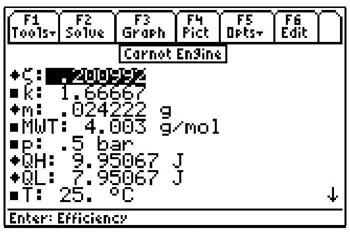
$$p \cdot V = \frac{m \cdot Rm}{MWT} \cdot T$$ **Eq. 11**

Variable	Description	Units
ζ	Efficiency	unitless
cp	Specific heat at constant pressure	J/(kg·K)
cv	Specific heat at constant volume	J/(kg·K)
k	Ratio of specific heats	unitless
m	Mass	kg
MWT	Molar mass	kg/mol
p	Pressure	Pa
p1	Pressure - 1	Pa
p2	Pressure – 2	Pa
p3	Pressure – 3	Pa
p4	Pressure – 4	Pa
QH	Heat flow at TH	J
QL	Heat flow at TL	J
Rm	Molar gas constant	8.3145 J/(kg·K)
T	Temperature	K
TH	Temperature	K

Variable	**Description**	**Units**
TL	Temperature	K
V	Volume	m^3
V1	Volume – 1	m^3
V2	Volume – 2	m^3
V3	Volume – 3	m^3
V4	Volume – 4	m^3
Wnet	Work done	J

Example 15.6.1.1:

A boiling pot of water (100 °C) and the ambient air (25°C) are the heat source and heat sink for a Carnot engine using helium. If the work generated per cycle is 2 J, the maximum volume for a compression cycle is 300 cm^3, and the measured pressure is at the maximum volume of 0.5 bars, what is the maximum efficiency of the engine, the heat absorbed and dispensed during a single cycle?



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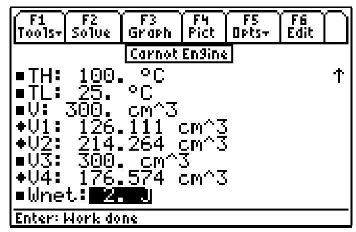

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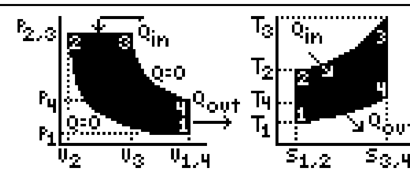
Solution – Select **equations 1, 3, 4, 5, 6, 7,** and **11**. The maximum volume in the cycle occurs at step 3 where the volume is **V3** and the temperature is **TL**. The molar mass (**MWT**) of helium is 4.003 g/mol. Use a value of **k**=5/3 for an ideal monatomic gas.

Given	**Solution**
k = 1.66667	ζ = .200992
MWT = 4.003 g/mol	m = 0.02422 g
p = .5 bar	QH = 9.95067 J
T = 25 °C	QL = 7.95067 J
TH = 100 °C	V1 = 126.111 cm^3
TL = 25°C	V2 = 214.264 cm^3
V = 300 cm^3	V4 = 176.564 cm^3
V3 = 300 cm^3	
Wnet = 2 J	

15.6.1.2 Diesel Cycle

The ideal diesel cycle consists of four reversible steps:



Stage 1: 1 → 2: Isentropic compression

Stage 2: 2 → 3: Reversible constant pressure heating

Stage 3: 3 → 4: Isentropic expansion

Stage 4: 4 → 1: Reversible constant volume cooling

The **first equation** calculates the rate heat intake **Qin** (W), into the ideal diesel cycle during the constant pressure-heating step (Stage 2). **Equation 2** computes the heat exhaust rate, **Qout** (W), during the constant volume-cooling step (Stage 4). **Equations 3** and **4** compute the net power, **Wr** (W), available for work. **Equations 5, 6,** and **7** compute the maximum efficiency, **ζ** (unitless), for the conversion of heat to work in the ideal diesel cycle. **Equation 8** calculates the specific heat ratio, **k**. **Equation 9** computes the work ratio, **Rw** (net work over of the work performed during isentropic expansion), from the compression ratio, **cr**, and the cutoff ratio, **cor**. **Equation 10** calculates the mean effective pressure, **mep** (Pa), or the constant pressure required to perform the network of the cycle (**Wr·time**) in a single compression stroke, **V1→V2** (m^3). The cutoff ratio, **cor**, is defined in **Equation 11** as the ratio of the volume of air at which constant pressure heating ends, **V3** (m^3), to the clearance volume, **V2** (m^3). **Equation 12** calculates **cor** in terms of **T1** (K) and **T2** (K). **Equation 13** computes the compression ratio, **cr**, the ratio of the initial volume, **V1** (m^3), and final volume, **V2** (m^3), in the first stage of the diesel cycle (the compression ratio must be greater than 14 and **T2** must be greater than the ignition temperature for the diesel to ignite). **Equations 14** and **15** relate the initial and final temperatures, **T1** and **T2,** and pressures, **p1** and **p2**, of the compression cycle (Stage 1), to the compression ratio, **V1/V2** and the specific heat ratio, **k**. **Equation 16** displays the pressure equivalence during the constant pressure expansion (Stage 2) of the ideal diesel cycle. **Equations 17** and **18** relate the initial and final temperatures, **T3** (K) and **T4** (K), of the isentropic expansion process (Stage 3) to the volume ratio (**V3/V4** or **cr/cor**) and the specific heat ratio, **k**. **Equations 19** and **20** compute the temperature, **T4** and **T1** (K), pressure, **p4** and **p1** (Pa), and volume, **V4** and **V1** (m^3), relationships for the constant volume cooling step of the ideal diesel cycle (stage 4). The **last two equations** compute the ideal gas law (**Eq. 21**) and the **time** (s) (**Eq. 22**) for a complete revolution of the diesel cycle from the mass flow rate, **mr** (kg/s), of fuel and air into the cycle and the mass of gas, **m** (kg), inside the cycle.

$$Qin = mr \cdot cp \cdot (T3 - T2)$$ **Eq. 1**

$$Qout = mr \cdot cv \cdot (T4 - T1)$$ **Eq. 2**

$$Wr = Qin - Qout$$ **Eq. 3**

$$Wr = mr \cdot cp \cdot T1 \cdot cr^{k-1} \cdot (cor - 1) - mr \cdot cv \cdot T1 \cdot (cor^k - 1)$$ **Eq. 4**

$$\zeta = 1 - \frac{Qout}{Qin}$$ **Eq. 5**

$$\zeta = 1 - \frac{T1}{k \cdot T2} \cdot \left(\frac{\frac{T4}{T1} - 1}{\frac{T3}{T2} - 1} \right)$$ **Eq. 6**

$$\zeta = 1 - \frac{\frac{1}{cr^{k-1}} \cdot (cor^k - 1)}{k \cdot (cor - 1)}$$ **Eq. 7**

$$k = \frac{cp}{cv}$$ **Eq. 8**

$$Rw = \frac{k(cor - 1) \cdot cr^{k-1} - (cor^k - 1)}{k(cor - 1) \cdot cr^{k-1} - (cor^k - cr^{k-1})}$$ **Eq. 9**

$$mep = \frac{Wr}{V1 \cdot \left(1 - \frac{1}{cr}\right)} \cdot time$$ **Eq. 10**

$$cor = \frac{V3}{V2}$$ **Eq. 11**

$$cor = \frac{T3}{T2}$$ **Eq. 12**

$$cr = \frac{V1}{V2}$$ **Eq. 13**

$$T2 = T1 \cdot cr^{k-1}$$ **Eq. 14**

$$p2 = p1 \cdot cr^{k-1}$$ **Eq. 15**

$$p2 = p3$$ **Eq. 16**

$$T4 = T3 \cdot \left(\frac{cor}{cr}\right)^{k-1}$$ **Eq. 17**

$$T4 = T3 \cdot \left(\frac{V3}{V4}\right)^{k-1}$$ **Eq. 18**

$$p4 = p1 \cdot \left(\frac{T4}{T1}\right)$$ **Eq. 19**

$$V1 = V4$$ **Eq. 20**

$$p \cdot V = \frac{m}{MWT} \cdot Rm \cdot T$$ **Eq. 21**

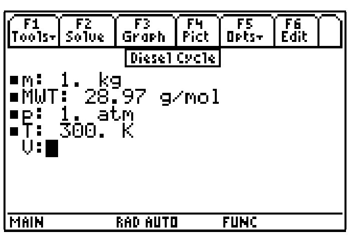
$$time = \frac{m}{mr}$$ **Eq. 22**

Variable	**Description**	**Units**
ζ	Efficiency	unitless
cor	Cutoff ratio	unitless
cp	Specific heat at constant pressure	J/(kg·K)
cr	Compression ratio	unitless
cv	Specific heat at constant volume	J/(kg·K)
k	Ratio of specific heats	unitless
m	Mass of gas in cycle	kg
mep	Mean effective pressure	Pa

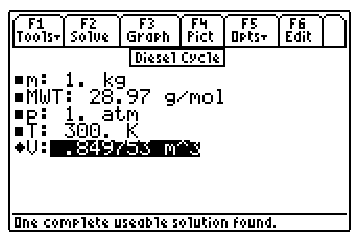
Variable	Description	Units
mr	Mass flow rate	kg/s
MWT	Molar mass	kg/mol
p	Pressure	Pa
p1	Pressure - 1	Pa
p2	Pressure – 2	Pa
p3	Pressure – 3	Pa
p4	Pressure – 4	Pa
Qin	Heat inflow rate	W
Qout	Heat outflow rate	W
Rm	Molar gas constant	8.3145 J/(kg·K)
Rw	Work ratio	unitless
T	Temperature	K
T1	Temperature - 1	K
T2	Temperature - 2	K
T3	Temperature - 3	K
T4	Temperature - 4	K
time	Time elapsed	s
V	Volume	m^3
V1	Volume - 1	m^3
V2	Volume – 2	m^3
V3	Volume – 3	m^3
V4	Volume – 4	m^3
Wr	Work rate	W

Example 15.6.1.2:

An air standard diesel engine has a compression ratio of 16 and operates at maximum and minimum temperatures, 1675 K and 300 K. The pressure at the beginning of compression is 1 atm, the specific heat ratio for air is assumed to be 1.4 and the specific heat of air at constant pressure is 1.005 kJ/(kg·K). Compute the pressure and temperature at each point in the cycle, the network performed per kilogram of gas in a single cycle, the maximum thermal efficiency, and the work ratio and mean effective pressure. The molar mass of air is 28.97 g/mol.

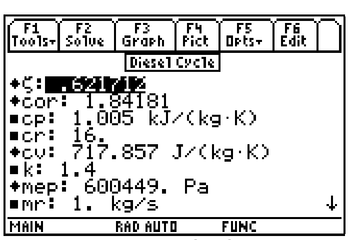


Step 1: Entered Values



Step 1: Computed results

Solution - Step 1–Solve this problem in two steps. Select **Equation 21** and compute **V** for State 1 (beginning of the isentropic compression). Record the computed value of **V**.



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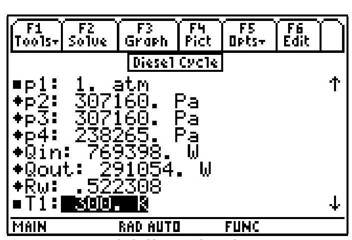

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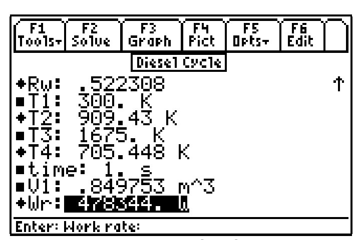

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Solution - Step 2 –Deselect **Equation 21** and select **Equations 1, 2, 3, 5, 8, 9, 10, 12, 14, 15, 16, 17**, and **19**. Enter the computed value of **V** = .849753 m^3 in the previous step for **V1**. Note, from the diagram above, that the high temperature occurs at **T3**. Since **Wr** will be computed as the network per kg of gas per cycle (J/kg), enter a mass flow rate of **mr** = 1kg/s and cycle time of **time**= 1 s so **Wr** displays the appropriate computed result for these units.

Step 1:

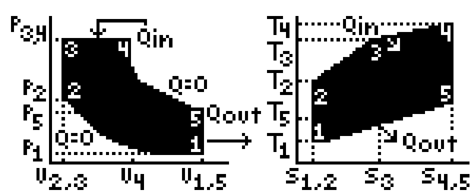
Given	Solution
MWT = 28.97 g/mol	V = .849753 m^3 (enter for V1 in Step 2)
p=1 atm	
T=300. K	
m= 1 kg	

Step 2:

Given	Solution
cp = 1.005 kJ/(kg·K)	ζ = .621712
cr = 16	cor = 1.84181
k = 1.4	cv = 717.857 J/(kg·K)
mr = 1kg/s	mep = 600449 Pa
p1 = 1 atm	p2 = 307160 Pa
T1 = 300 K	p3 = 307160 Pa
T3 = 1675 K	p4 = 238265 Pa
time = 1 s	Qin = 769398 W
V1 = .849753 m^3 (from step 1)	Qout = 291054 W
	Rw = .522308
	T2 = 909.43 K
	T4 = 705.448 K
	Wr = 478344 W

15.6.1.3 Dual Cycle

The ideal dual cycle better approximates the actual performance of a diesel engine. It is composed of 5 stages, including two heating steps:



Stage 1: 1→2 Adiabatic Compression.

Stage 2: 2→3 Constant Volume Heating.

Stage 3: 3→4 Constant pressure heating.

Stage 4: 4→5 Adiabatic Expansions.

Stage 5: 5→1 Constant Volume Cooling.

The **first equation** computes the total rate of heat intake, **Qin** (W), in Stages 2 and 3. **Equation 2** calculates the rate of heat exhaust, **Qout** (W), in Stage 5. **Equation 3** determines the net power available for work, **Wr** (W). **Equations 4, 5**, and **6** compute the maximum thermal efficiency, ζ,(unitless) of the heat cycle. **Equation 7** calculates the specific heat ratio, **k**. The pressure ratio, **rp**, is calculated in **equation 8**. The compression ratio, **cr**, is determined in **equation 9**. The cut-off ratio, **cor**, the ratio of volumes at the end of constant pressure heating, is computed in **equation 10**. The temperature/volume relationship during adiabatic compression (Stage 1) is computed in **equation 11**. **Equation 12** computes the temperature increase during the adiabatic compression step. **Equations 13** and **14** calculate the pressure/volume/temperature relationship for constant volume heating (Stage 2). **Equations 15** and **16** compute the gas properties for constant pressure heating (Stage 3). **Equation 17** and **18** computes the temperature/volume relationship for adiabatic expansion (Stage 4). **Equations 19** and **20** describe the constant volume cooling stage, the final step of the cycle (Stage 5). The **last two equations** compute the ideal gas law (**Eq. 21**) and the **time** (s) (**Eq. 22**) for a complete revolution of the dual cycle from the mass flow rate, **mr** (kg/s), of fuel and air into the cycle and the mass of gas, **m** (kg), inside the cycle.

Equation	
$Qin = mr \cdot cv \cdot (T3 - T2) + mr \cdot cp \cdot (T4 - T3)$	**Eq. 1**
$Qout = mr \cdot cv \cdot (T5 - T1)$	**Eq. 2**
$Wr = \zeta \cdot Qin$	**Eq. 3**
$\zeta = 1 - \frac{T5 - T1}{T3 - T2 + k \cdot (T4 - T3)}$	**Eq. 4**
$\zeta = 1 - \frac{Qout}{Qin}$	**Eq. 5**
$\zeta = 1 - \frac{\frac{1}{cr^{k-1}} \cdot (rp \cdot cor^{k} - 1)}{k \cdot rp \cdot (cor - 1) + rp - 1}$	**Eq. 6**
$\frac{cp}{cv} = k$	**Eq. 7**
$rp = \frac{p3}{p2}$	**Eq. 8**
$cr = \frac{V1}{V2}$	**Eq. 9**
$cor = \frac{V4}{V3}$	**Eq. 10**
$T2 = T1 \cdot cr^{k-1}$	**Eq. 11**
$T2 = T1 \cdot \left(\frac{p2}{p1}\right)^{\frac{k-1}{k}}$	**Eq. 12**
$T3 = T2 \cdot rp$	**Eq. 13**
$V2 = V3$	**Eq. 14**
$T4 = T3 \cdot cor$	**Eq. 15**

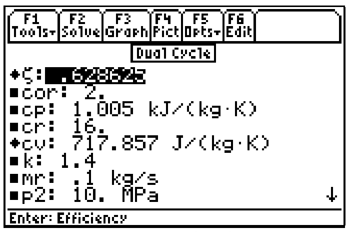
$p3 = p4$	**Eq. 16**
$T5 = T4 \cdot \left(\frac{V4}{V5}\right)^{k-1}$	**Eq. 17**
$T5 = T4 \cdot \left(\frac{p5}{p4}\right)^{\frac{k-1}{k}}$	**Eq. 18**
$T5 = \frac{T1 \cdot p5}{p1}$	**Eq. 19**
$V1 = V5$	**Eq. 20**
$p \cdot V = \frac{m}{MWT} \cdot Rm \cdot T$	**Eq. 21**
$time = \frac{m}{mr}$	**Eq. 22**

Variable	**Description**	**Units**
ζ	Efficiency	unitless
cor	Cutoff ratio	unitless
cp	Specific heat at constant pressure	J/(kg·K)
cr	Compression ratio	unitless
cv	Specific heat at constant volume	J/(kg·K)
k	Ratio of specific heats	unitless
m	Mass of gas in cycle	kg
mr	Mass Flow rate	kg/s
MWT	Molar mass	kg/mol
p	Pressure	Pa
p1	Pressure – 1	Pa
p2	Pressure – 2	Pa
p3	Pressure –3	Pa
p4	Pressure –4	Pa
p5	Pressure –5	Pa
Qin	Heat inflow rate	W
Qout	Heat outflow rate	W
Rm	Molar gas constant	8.3145 J/(kg·K)
rp	Pressure ratio	unitless
T	Temperature	K
T1	Temperature – 1	K
T2	Temperature – 2	K
T3	Temperature – 3	K
T4	Temperature – 4	K
T5	Temperature – 5	K
time	Cycle duration	s
V	Volume	m^3
V1	Volume - 1	m^3
V2	Volume – 2	m^3
V3	Volume – 3	m^3

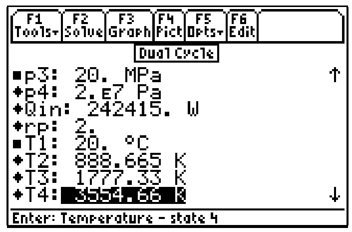
Variable	**Description**	**Units**
V4	Volume – 4	m^3
V5	Volume – 5	m^3
Wr	Work rate	W

Example 15.6.1.3:

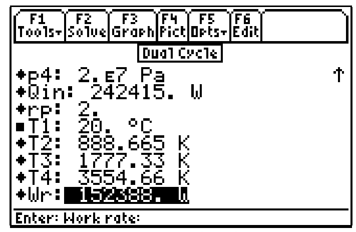
A dual cycle is used to estimate the performance of a piston engine. Atmospheric air, at 20°C, is drawn into the engine a rate of 0.1 kg/s. The air in the piston chamber is compressed to a pressure of 10 MPa. Following combustion, the pressure increases to 20 MPa. Assuming a cut-off ratio of 2 and a compression ratio of 16, compute the maximum thermal efficiency, the rate of heat intake and power available for work. Air has a specific heat ratio of 1.4 and a specific heat at constant pressure of 1.005 kJ/(kg·K).



Upper Display



Middle Display



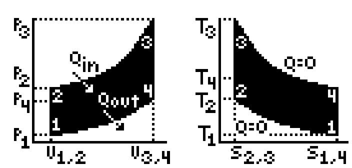
Lower Display

Solution – Solve this problem using **equations 1, 3, 6, 7, 8, 11, 13, 15,** and **16**. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
cor = 2	ζ = .628625
cp = 1.005 kJ/(kg·K)	cv = 717.857 J/(kg·K)
cr = 16	p4 = 2.E7 Pa
k = 1.4	Qin = 242415 W
mr = .1 kg/s	rp = 2
p2 = 10 MPa	T2 = 888.665 K
p3 = 20 MPa	T3 = 1777.33 K
T1 = 20°C	T4 = 3554.66 K
	Wr = 152388 W

15.6.1.4 Otto Cycle

The ideal Otto cycle is the approximate pattern of internal combustion (spark ignition) engines in automobiles.



Stage 1: 1→2 Isentropic compression
Stage 2: 2→3 Reversible constant volume heating
Stage 3: 3→4 Isentropic expansion
Stage 4: 4→1 Reversible constant volume cooling

The **first equation** computes the rate of heat intake, **Qin** (W), into the system. **Equation 2** calculates the rate of heat exhaust, **Qout** (W). **Equation 3** and **4** compute the power available for work, **Wr** (W). **Equations 5, 6**, and **7** calculate the maximum thermal efficiency of the ideal Otto cycle, **ζ**.(unitless) **Equation 8** calculates the mean effective pressure, **mep** (Pa), as the pressure needed to perform the net work per cycle (**Wr·time**) in a single compression stroke, **V1→V2**. **Equation 9** computes the work ratio, **Rw**, network over of the work performed during isentropic expansion, from the compression ratio, **cr**, and the low and high temperatures, **T1** (K) and **T3** (K). **Equation 10** calculates the specific heat ratio, **k**. **Equation 11** computes the compression ratio, **cr**, as the ratio of the initial and final volumes, **V1** (m^3) and **V2** (m^3), in the adiabatic compression (Stage 1). **Equation 12** computes the optimum compression ratio, **copt**, where **Wr** is maximized. **Equation 13** computes the temperature change during adiabatic compression (Stage 1). **Equation 14** states the constant volume condition during the heating process of Stage 2. **Equation 15** calculates the initial and final temperature of the isentropic expansion process (Stage 3). **Equation 16** states the constant volume condition for the cooling step (Stage 4). The **last two equations** compute the ideal gas law (**Eq. 17**) and the **time** (s) (**Eq. 18**) for a complete revolution of the dual cycle from the mass flow rate, **mr** (kg/s), of fuel and air into the cycle and the mass of gas, **m** (kg), inside the cycle at any time.

$$Qin = mr \cdot cv \cdot (T3 - T2)$$ **Eq. 1**

$$Qout = mr \cdot cv \cdot (T4 - T1)$$ **Eq. 2**

$$Wr = mr \cdot cv \cdot T3\left(1 - \frac{T1}{T3} \cdot cr^{k-1}\right) \cdot \left(1 - \frac{1}{cr^{k-1}}\right)$$ **Eq. 3**

$$Wr = \zeta \cdot Qin$$ **Eq. 4**

$$\zeta = 1 - \frac{T1}{T2} \cdot \frac{\left(\frac{T4}{T1} - 1\right)}{\left(\frac{T3}{T2} - 1\right)}$$ **Eq. 5**

$$\zeta = 1 - \frac{Qout}{Qin}$$ **Eq. 6**

$$\zeta = 1 - \frac{1}{cr^{k-1}}$$ **Eq. 7**

$$mep = \frac{Wr}{V1 \cdot \left(1 - \frac{1}{cr}\right)} \cdot time$$ **Eq. 8**

$$Rw = 1 - \frac{T1}{T3} \cdot cr^{k-1}$$ **Eq. 9**

$$k = \frac{cp}{cv}$$ **Eq. 10**

$$cr = \frac{V1}{V2}$$ **Eq. 11**

$$copt = \left(\frac{T3}{T1}\right)^{\frac{1}{2(k-1)}}$$ **Eq. 12**

$$\frac{T2}{T1} = cr^{k-1}$$ **Eq. 13**

$$V3 = V2$$ **Eq. 14**

$$\frac{T3}{T4} = \left(\frac{V4}{V3}\right)^{k-1}$$ **Eq. 15**

$$V1 = V4$$ **Eq. 16**

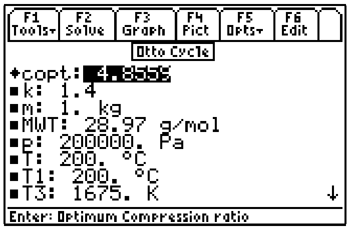
$$p \cdot V = \frac{m \cdot Rm}{MWT} \cdot T$$ **Eq. 17**

$$time = \frac{m}{mr}$$ **Eq. 18**

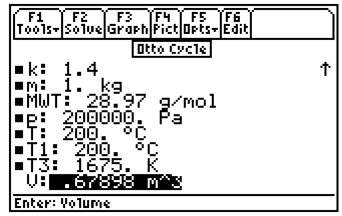
Variable	**Description**	**Units**
ζ	Efficiency	unitless
copt	Compression ratio	unitless
cp	Specific heat at constant pressure	J/(kg·K)
cr	Compression ratio	unitless
cv	Specific heat at constant volume	J/(kg·K)
k	Ratio of specific heats	unitless
m	Gas mass in cycle	kg
mep	Mean effective pressure	Pa
mr	Mass flow rate	kg/s
MWT	Molar mass	kg/mol
p	Pressure	Pa
Qin	Heat inflow rate	W
Qout	Heat outflow rate	W
Rm	Molar gas constant	8.3145 J/(kg·K)
Rw	Work ratio	unitless
T	Temperature	K
T1	Temperature - 1	K
T2	Temperature - 2	K
T3	Temperature - 3	K
T4	Temperature - 4	K
time	Time elapsed	s
V	Volume	m^3
V1	Volume – 1	m^3
V2	Volume – 2	m^3
V3	Volume – 3	m^3
V4	Volume – 4	m^3
Wr	Work rate	W

Example 15.6.1.4:

An air spark-ignition engine operates at a low pressure of 200 kPa, a low temperature of 200 °C and a high temperature of 1675 K. Compute the maximum work per mass of gas in one cycle revolution, the optimum compression ratio and the maximum thermal efficiency. Air has a molar mass of 28.97 g/mol, a specific heat ratio of 1.4 and a specific heat at constant pressure of 1.005 kJ/(kg·K).



Upper Display: Step 1

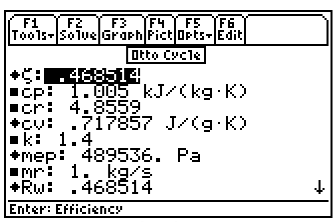


Lower Display: Step 1

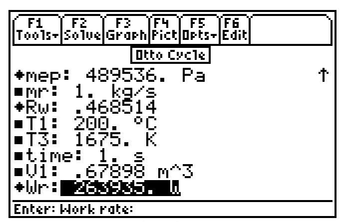
Solution – Step 1: Compute the solution in two steps. First compute the optimum compression ratio, **copt**, and the initial volume for a kilogram of air, **V**, using **equations 12 and 17**. Be sure to record the value of **copt** and **V** for **cr** and **V1**, respectively, in the next step.

Given (Step 1)
k = 1.4
m = 1 kg
MWT = 28.97 g/mol
p = 200000 Pa
T = 200 °C
T1 = 200 °C
T3 = 1675 K

Solution (Step 1)
copt = 4.8559 (enter for **cr** in Step 2)
V = .67898 m^3 (enter for **V1** in Step 2)



Upper Display: Step 2



Lower Display: Step 2

Solution – Step 2: After computing **V** and **copt, deselect the equations 12** and **17** by highlighting the equations and pressing [ENTER], and select **equations 3, 7, 8, 9, and 10**. Enter the previously calculated value of **copt** for **cr**, **V** for **V1**, a value of 1 s for cycle duration, **time**, and 1 kg/s for the mass flow rate, **mr**, so the computed value of **Wr** displays the work performed per mass of gas in a single cycle. Press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given (Step 2)
cp = 1.005 kJ/(kg·K)
cr = 4.8559 (computed in step 1)
k = 1.4
mr = 1 kg/s
T1 = 200 °C
T3 = 1675 K
time = 1 s
V1 = .67898 m^3 (computed in step 1)

Solution (Step 2)
ζ = .468514
cv = 717.857 J/(kg·K)
mep = 489536 Pa
Rw = .468514
Wr = 263935 W

15.6.1.5 Brayton Cycle

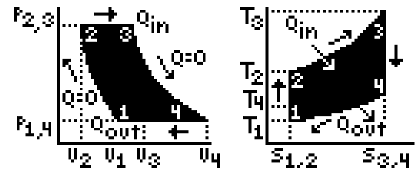
The Brayton Cycle is used to model power generation in gas turbine systems. It is composed of four stages:

Stage 1: 1 to 2: Isentropic compression.

Stage 2: 2 to 3: Constant pressure (Isobaric) heating.

Stage 3: 3 to 4: Isentropic expansion.

Stage 4: 4 to 1: Constant pressure (Isobaric) cooling.



The **first equation** computes the rate of heat intake, **Qin** (W), during the constant pressure-heating step (Stage 1). **Equation 2** computes the rate of heat exhaust, **Qout** (W). **Equation 3** calculates the power available for work, **Wr** (W). **Equation 4, 5, 6**, and **7** compute the maximum heat efficiency, **ζ**, (unitless) of the Brayton cycle. **Equation 8** computes the work ratio **Rw**, the network over of the work performed during isentropic expansion. **Equation 9** computes the optimum pressure ratio, **ropt**, required to perform the maximum rate of work for the Brayton Cycle temperature conditions. **Equation 10** calculates the specific heat ratio, **k**. **Equation 11** computes the pressure ratio, **rp**, of the isobaric heating process (Stage 2). **Equation 12** computes the temperature/pressure relationship for adiabatic compression (Stage 1). **Equation 13** states the constant pressure condition for isobaric heating (Stage 2). **Equation 14** computes the temperature/pressure relationship for adiabatic expansion (Stage 3). **Equation 15** states the constant pressure condition for isobaric cooling (Stage 4). The **last two equations** compute the ideal gas law (**Eq. 16**) and the **time** (s) (**Eq. 17**) for a complete revolution of the dual cycle, from the mass flow rate, **mr** (kg/s), of fuel and air into the cycle and the mass of gas, **m** (kg), inside the cycle.

$$Qin = mr \cdot cp \cdot (T3 - T2)$$ **Eq. 1**

$$Qout = mr \cdot cp \cdot (T4 - T1)$$ **Eq. 2**

$$Wr = \zeta \cdot Qin$$ **Eq. 3**

$$\zeta = 1 - \frac{\frac{T1}{T2} \cdot \left(\frac{T4}{T1} - 1\right)}{\left(\frac{T3}{T2} - 1\right)}$$ **Eq. 4**

$$\zeta = 1 - \frac{Qout}{Qin}$$ **Eq. 5**

$$\zeta = 1 - rp^{\frac{1-k}{k}}$$ **Eq. 6**

$$\zeta = 1 - \left(\frac{p1}{p2}\right)^{\frac{k-1}{k}}$$ **Eq. 7**

$$Rw = 1 - \frac{T1}{T3} \cdot rp^{\frac{k-1}{k}}$$ **Eq. 8**

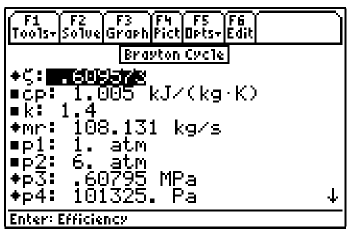
$ropt = \left(\frac{T3}{T1}\right)^{2 \cdot \frac{k}{k-1}}$	**Eq. 9**
$k = \frac{cp}{cv}$	**Eq. 10**
$rp = \frac{p2}{p1}$	**Eq. 11**
$rp = \left(\frac{T1}{T2}\right)^{\frac{k}{k-1}}$	**Eq. 12**
$p2 = p3$	**Eq. 13**
$\frac{p3}{p4} = \left(\frac{T3}{T4}\right)^{\frac{k}{k-1}}$	**Eq. 14**
$p1 = p4$	**Eq. 15**
$p \cdot V = \frac{m}{MWT} \cdot Rm \cdot T$	**Eq. 16**
$time = \frac{m}{mr}$	**Eq. 17**

Variable	**Description**	**Units**
ζ	Efficiency	unitless
cp	Specific heat at constant pressure	J/(kg·K)
cv	Specific heat at constant volume	J/(kg·K)
k	Ratio of specific heats	unitless
m	Mass of gas in cycle	kg
mr	Mass flow rate	kg/s
MWT	Molar mass	kg/mol
p	Pressure	Pa
p1	Pressure 1	Pa
p2	Pressure 2	Pa
p3	Pressure 3	Pa
p4	Pressure 4	Pa
Qin	Heat inflow rate	W
Qout	Heat outflow rate	W
Rm	Molar gas constant	8.3145 J/(kg·K)
ropt	Optimum pressure ratio	unitless
rp	Pressure ratio	unitless
Rw	Work ratio	unitless
T	Temperature	K
T1	Temperature – 1	K

Variable	Description	Units
T2	Temperature – 2	K
T3	Temperature – 3	K
T4	Temperature – 4	K
time	Time elapsed	s
V	Volume	m^3
Wr	Work rate	W

Example 15.6.1.5:

Air, having a pressure of 1 atm and temperature of 300 K, is drawn into a turbine where it is compressed to a pressure of 6 atm. The air is heated, at constant pressure, to a temperature of 1100 K. The rate of energy input during heating is 100 MW. Assume the turbine operates on the ideal Brayton cycle. Compute the temperature and pressure at each point in the cycle, the airflow rate (kg/s), the work ratio, and the thermal efficiency for the gas turbine. Air has a specific heat ratio of 1.4 and a specific heat at constant pressure of 1.005 kJ/(kg·K).



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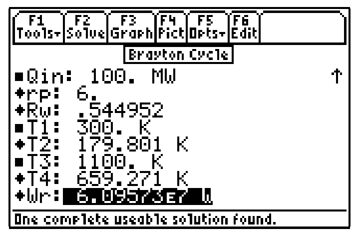


LowerDisplay

Solution – Select the following **Equations 1, 3, 4, 8, 11, 12, 13, 14, and 15** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
cp = 1.005 kJ/(kg·K)	ζ = .609573
k = 1.4	mr = 108.131 kg/s
p1 = 1 atm	p3 = .60795 MPa
p2 = 6 atm	p4 = 101325 Pa
Qin = 100 MW	rp = 6
T1 = 300 K	Rw = .544952
T3 = 1100 K	T2 = 179.801 K
	T4 = 659.271 K
	Wr = 6.09573 E7 W

15.6.2 Clapeyron Equation

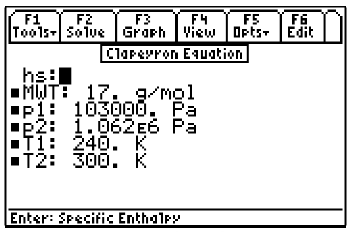
The Clausius Clapeyron equation describes the change in vapor pressure, of a pure substance, **p1-p2** (Pa), with change in temperature, **T1-T2** (K). It is assumed that the pure substance does not undergo a phase change between temperatures, **T1** and **T2**. The specific enthalpy, **hs** ($J\cdot kg\cdot K^{-1}$), determines the amount of energy-required convert a kilogram of pure substance from a solid to a vapor (sublimation) or liquid to a vapor (vaporization).

$$\ln\left(\frac{p2}{p1}\right) = \frac{MWT \cdot hs}{Rm} \cdot \left(\frac{1}{T1} - \frac{1}{T2}\right)$$ **Eq. 1**

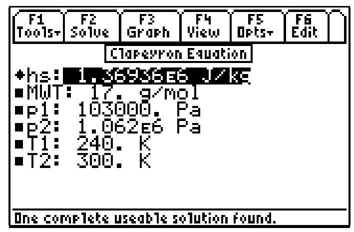
Variable	Description	Units
hs	Enthalpy per unit mass	J/(kg·K)
MWT	Molar mass	kg/mol
p1	Pressure 1	Pa
p2	Pressure 2	Pa
Rm	Molar gas constant	8.3145 J/(kg·K)
T1	Temperature – 1	K
T2	Temperature – 2	K

Example 15.6.2:

Liquid ammonia (molar mass: 17 g/mol) has a vapor pressure of 103,000 Pa at 240 K and 1,062,000 Pa at 300 K. What is the approximate enthalpy of vaporization per mass of ammonia?



Entered Values



Computed results

Solution – Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
MWT = 17 g/mol	hs=1.36936 E6 J/kg
p1 = 103,000 Pa	
p2 = 1,062,000 Pa	
T1 = 240 K	
T2 = 300 K	

References:

1. Lynn D. Russell, and George A. Adebiyi, Classical Thermodynamics, Saunders College publishing, Harcourt Brace Jovanavich College Publishers, Fort Worth TX, 1993
2. Michael R. Lindeburg, Mechanical Engineering reference manual, 8th Edition, Professional Publications, Belmont, CA 1990

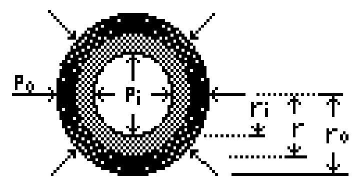
Chapter 16: Machine Design

This section of the software deals a wide range of calculations under the broad classification "Machine Design". Topics include designing springs, calculating induced stresses due to contact between cylinders or shafts, stresses in a variety of types of machine elements and bearings. The topics in this section are organized under four headings. They are:

- Stress - Machine elements
- Bearings
- Hertzian Stresses
- Spring Design

16.1 Stress: Machine Elements

16.1.1 Cylinders



Cylindrical pressure vessels, hydraulic cylinders, gun barrels, pipes carrying fluids at high pressures develop both radial and tangential stresses with values strongly dependent upon the radius of the elements under consideration. In determining the radial stress, **σr** (Pa), and tangential stress, **σt** (Pa), the assumption is made that the longitudinal elongation is constant. We designate the inner and outer radius of the cylinder as **ri** (m) and **ro** (m), while the internal and external pressures are designated by **pi** (Pa) and **po** (Pa). The **two equations** listed here determine the radial and tangential stresses respectively.

$$\sigma t = \frac{pi \cdot ri^2 - po \cdot ro^2 - \frac{ri^2 \cdot ro^2 \cdot (po - pi)}{r^2}}{ro^2 - ri^2}$$ **Eq. 1**

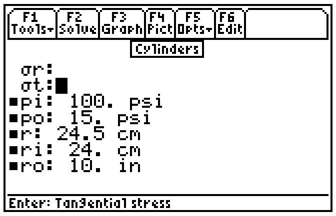
$$\sigma r = \frac{pi \cdot ri^2 - po \cdot ro^2 + \frac{ri^2 \cdot ro^2 \cdot (po - pi)}{r^2}}{ro^2 - ri^2}$$ **Eq. 2**

When r < ro and r > ri both equations give physically meaningful results.

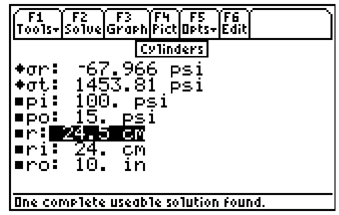
Variable	Description	Units
σr	Radial stress	Pa
σt	Tangential stress	Pa
pi	Pressure inside	Pa
po	Pressure outside	Pa
r	Radius	m
ri	Inside radius	m
Ro	Outside radius	m

Example 16.1.1:

A hollow cylinder, with an outer radius of 10 inches and an inside radius of 24 cm, carries a fluid at a pressure of 100 psi. The pressure outside is 15 psi. Find the radial and tangential stresses 0.5 cm away from the inner radius line.



Entered Values



Computed results

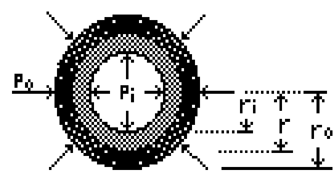
Solution – Select **both equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
pi = 100 psi	σr = -67.966 psi
po = 15 psi	σt = -1453.81 psi
r = 24.5 cm	
ri = 24 cm	
ro = 10 in	

The stress results shown here have been converted to psi units.

16.1.2 Rotating Rings

Many problems such as flywheels and blowers can be simplified to rotating ring problems to determine stresses. The two equations listed below describe the radial stress, **σr** (Pa), and tangential stress, **σt** (Pa), are linked to key parameters density, **ρ** (kg/m^3), angular velocity, **ω** (rad/s), Poisson's ratio, **ζ** (unitless), inside radius **ro** (m), outer radius **ro** (m) and radial distance, **r** (m), between **ri** and **ro**. The **first equation** computes the tangential stress, **σt** (Pa), while the **second equation** computes the radial, **σr**.



Eq. 1

$$\sigma t = \rho \cdot \omega^2 \cdot \frac{3+\zeta}{8} \cdot \left(ri^2 + ro^2 + \frac{ri^2 \cdot ro^2}{r^2} - \frac{1+3\cdot\zeta}{3+\zeta^2} \cdot r^2 \right)$$

Eq. 2

$$\sigma r = \rho \cdot \omega^2 \cdot \frac{3+\zeta}{8} \cdot \left(ri^2 + ro^2 - \frac{ri^2 \cdot ro^2}{r^2} - r^2 \right)$$

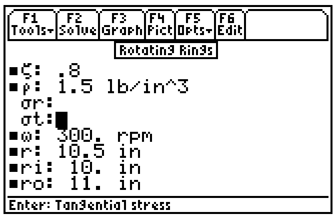
When r < ro and r > ri both equations give physically meaningful results.

Variable	Description	Units
ζ	Poisson's ratio	unitless
ρ	Density	kg/m^3
σr	Radial stress	Pa
σt	Tangential stress	Pa
ω	Angular velocity	rad/s
r	Radius	m

Variable	Description	Units
ri	Inside radius	m
ro	Outside radius	m

Example 16.1.2:

A material with a density of 1.5 lb/in^3 is a thin ring rotating at 300 rpm. The ring has an inner radius of 10 in and an outer radius of 11 in. Given the Poisson of 0.8, find the stress at mid-point of the ring thickness.



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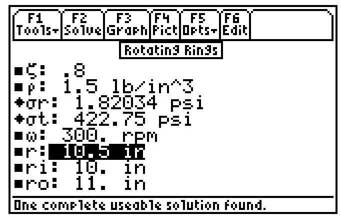
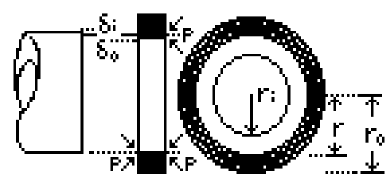


Computed results

Solution – Select **both equations** to solve the problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
ζ =.8	σr = 1.82034 psi
ρ = 1.5 lb/in^3	σt = 422.75 psi
ω = 300 rpm	
r = 10.5 in	
ri = 10 in	
ro = 11 in	

16.1.3 Pressure and Shrink Fits



Shrinking or pressing one part against the other, assembles two cylindrical parts, a contact pressure is created. The five equations presented in this equation set form the foundation for calculating stresses and dimensional changes in the two components involved in the assembly. The **first equation** computes the tangential stress, **σit** (Pa), at the transition radius, **r** (m), of the inner member, inner radius of the inner member, **ri** (m), and pressure, **p** (Pa). In a similar manner, the **second equation** calculates the tangential stress of the outer rim, **σot** (Pa), in terms of **p**, **r** and the inner surface radius of the outer member, **ro** (m). The **next two equations** determine the radial elongation/contraction, **δi** (m), and, **δo** (m), of the inner and outer members in terms of **r**, **ri**, **ro**, **p** and the respective modulus of elasticity, **Eo** (Pa), **Ei** (Pa) and Poisson's ratios, **ζi** (unitless) and **ζo** (unitless). The **last equation** calculates the radial interference, **δ** (m).

Eq. 1

$$\sigma it = \frac{-p \cdot \left(r^2 + ri^2\right)}{r^2 - ri^2}$$

Eq. 2

$$\sigma ot = \frac{p \cdot \left(ro^2 + r^2\right)}{ro^2 - r^2}$$

$$\delta o = \frac{p \cdot r}{Eo} \cdot \left(\frac{ro^2 + ri^2}{ro^2 - r^2} + \zeta o \right)$$ **Eq. 3**

$$\delta i = \frac{-p \cdot r}{Ei} \cdot \left(\frac{r^2 + ri^2}{r^2 - ri^2} - \zeta i \right)$$ **Eq. 4**

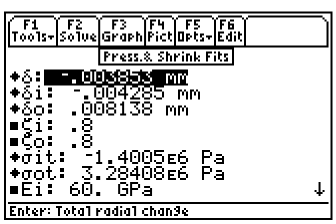
$$\delta = |\delta i| - |\delta o|$$ **Eq. 5**

When r < ro and r > ri all the equations give physically meaningful results.

Variable	Description	Units
δ	Total radial change	m
δi	Change in inner radius	m
δo	Change in outer radius	m
ζi	Poisson's ratio – inner ring	Unitless
ζo	Poisson's ratio – outer ring	Unitless
σit	Tangential stress – inner rim	Pa
σot	Tangential stress – outer rim	Pa
Ei	Young's modulus inner material	Pa
Eo	Young's modulus outer material	Pa
p	Pressure	Pa
r	Radius	m
ri	Inside radius	m
ro	Outside radius	m

Example 16.1.3:

Two cylinders of radius 7 inches and 10 inches are made of materials with modulus of elasticity 60 GPa and 100 GPa respectively and are press fitted with a pressure of 50 psi. The transition radius is 9 inches. Assume that the Poisson's ratios of the inner and outer members to be 0.8. Find the stresses in the inner and outer members at the transition radius. Compute the radial interference.



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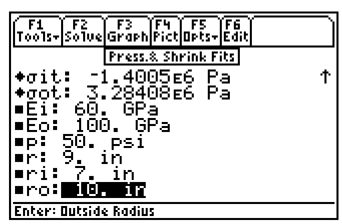

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Solution – Select **all of the equations** to solve this problem by pressing [ENTER]. Now press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

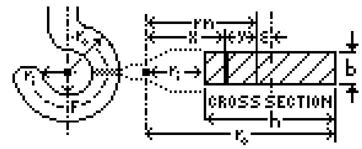
Given	Solution
ζi = 0.8	δ =-0.003853 mm
ζo = 0.8	δi =-0.004285 mm
Ei = 60 GPa	δo =0.008138 mm
Eo = 100 GPa	σit = -1.4005E6 Pa
p = 50 psi	σot = 3.28408E6 Pa
r = 9 in	

Given **Solution**

ri = 7 in

ro = 10 in

16.1.4 Crane Hook



The three equations in this section help solve the essential parameters involved in the use of a crane hook. The **first equation** calculates the cross sectional area, **A** (m^2), in terms of the width, **b** (m) and depth, **h** (m), of the crane hook. The **second equation** calculates the radius, **rn** (m), of the neutral axis using the inner and outer radius, **ri** (m) and **ro** (m). The **third equation** calculates the stress, **σ** (Pa), in terms of the force, **F** (N), area **A**, moment **MOM** (N·m), **rn**, eccentricity, **ε** (m). **Equation 4** relates **ri** and **ro** to the depth of the hook, **h**. The **last equation** computes the distances to an arbitrary position on the hook from the neutral axis **y** (m), and the central axis, **x** (m).

$$A = b \cdot h$$ **Eq. 1**

$$rn = \frac{h}{\ln\left(\frac{ro}{ri}\right)}$$ **Eq. 2**

$$\sigma = \frac{F}{A} + \frac{MOM \cdot y}{A \cdot \varepsilon \cdot (rn - y)}$$ **Eq. 3**

$$ro = ri + h$$ **Eq. 4**

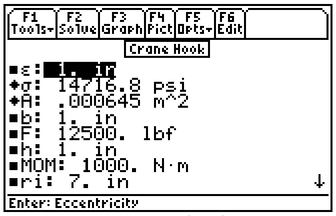
$$rn = x + y$$ **Eq. 5**

When rn < ro and rn > ri, these equations give physically meaningful results.

Variable	Description	Units
ε	Eccentricity	m
σ	Stress	Pa
A	Area	m^2
b	Width	m
F	Force	N
h	Height	m
MOM	Moment of inertia	N·m
ri	Inside radius	m
rn	Radius of neutral axis	m
ro	Outside radius	m
Y	Distance	m

Example 16.1.4:

A rectangular crane hook has to lift a load of 12500 lbf. The width of the hook is 1 inch and the thickness is 1 inch. The hook ring has an outer radius of 8 inches and an inner radius of 7 inches. The eccentric offset is 1 inch and the moment is applied 1.5 inches away from the center. A moment load of 1000 N·m is applied. Find the radius of neutral axis, and stress in the hook material.



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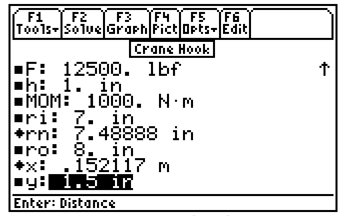

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Solution – Select **all the equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

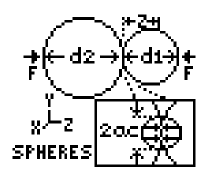
Given	**Solution**
ε = 1 in	σ = 14716.8 psi
b = 1 in	A = .000645 m^2
F = 12500 lbf	rn = 7.48888 in
h = 1 in	X = .152117 m
MOM = 1000 N·m	
ri = 7 in	
ro = 8 in	
y = 1.5 in	

16.2 Hertzian Stresses

When two spheres or cylinders with curved surfaces are pressed together to have a point or line contact stresses are developed in both bodies. The stresses developed in the two bodies are in general three dimensional in nature. The contact stresses arise in a number of practical cases such as wheel on a rail, in automotive cams and tappets, mating stresses in gear teeth and roller bearings.

The general case of contact stresses for two spheres or cylinders are called Hertzian Stresses.

16.2.1 Two Spheres



When two solid spheres of diameters, **d1** (m) and **d2** (m), are pressed together with a force, **F** (N), a circular contact area of radius, **ac** (m), is formed. The stress is maximum in the center of this circular contact area. By specifying the Young's modulus, **E1** (Pa) and **E2** (Pa), and the Poisson's ratio, **ζ1** and **ζ2** (unitless), for the two spheres, the contact radius, **ac** (m), is calculated from **equation 1**. The **second equation** computes the maximum pressure, **pmax** (Pa), at the center of the contact area. The principal stresses, **σxx** (Pa) and **σzz** (Pa), are computed from **equations 3 and 4** illustrating the variation of these stresses away from the principle point of contact with coefficient of friction **μf** (unitless).

$$ac = \left(\frac{\frac{3 \cdot F}{8}\left(\frac{1-\zeta 1^2}{E1} + \frac{1-\zeta 2^2}{E2} \right)}{\frac{1}{d1} + \frac{1}{d2}} \right)^{\frac{1}{3}}$$ **Eq. 1**

$$p\max = \frac{3 \cdot F}{2 \cdot \pi \cdot ac^2} \qquad \textbf{Eq. 2}$$

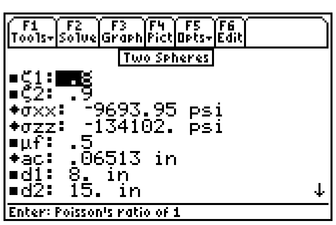
$$\sigma xx = -pmax \cdot \left(\left(1 - \frac{z}{ac} \cdot \tan^{-1}\left(\frac{ac}{z}\right)\right) \cdot (1 + \mu f) - \frac{1}{2 \cdot \left(1 + \frac{z^2}{ac^2}\right)} \right) \qquad \textbf{Eq. 3}$$

$$\sigma zz = \frac{-pmax}{1 + \frac{z^2}{ac^2}} \qquad \textbf{Eq. 4}$$

Variable	Description	Units
ζ1	Poisson's ratio of 1	unitless
ζ2	Poisson's ratio of 2	unitless
σxx	Stress along x axis	Pa
σzz	Stress along z axis	Pa
μf	Coefficient of friction	unitless
ac	Radius of contracted area	m
d1	Diameter of 1	m
d2	Diameter of 2	m
E1	Young's modulus –1	Pa
E2	Young's modulus -2	Pa
F	Force	N
pmax	Maximum pressure	Pa
z	z-axis distance from contact point	m

Example 16.2.1:

Two spheres, with diameters 8 inch and 15 inch, are pressing a load of 4000 lbf between them. The two spheres have elastic constants of 100 GPa and 125 GPa respectiv ely, and have Poisson's ratios of 0.8 each. If the friction coefficient is 0.5, find the stress 0.1 inches away from the point of impact.



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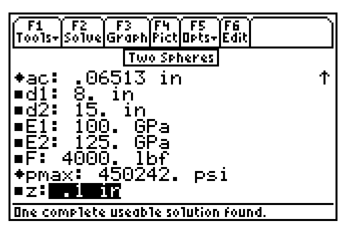


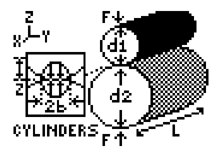
Lower Display

Solution – Select **all the equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

The problem is solved best in two stages. First solve the first two equations as a pair; then solve the last two equations as a new set knowing the values calculated from the first two equations. The composite results are shown below.

Given	**Solution**
ζ1 = .8	σxx = -9693.95 psi
ζ2 = .9	σzz = -134102 psi
μf = .5	ac = .06513 in
d1 = 8 in	pmax = 450242 psi
d2 = 15 in	
E1 = 100 GPa	
E2 = 125 GPa	
F = 4000 lbf	
z = .1 in	

16.2.2 Two Cylinders



When two solid cylinders with diameters **d1** (m), **d2** (m) and length **L** (m) are pressed together with a force **F** (N), a circular contact area of radius **ac** (m) is formed. The maximum stress is in the center of this circular contact area. By specifying the Young's modulus, **E1** and **E2** (Pa), and Poisson's ratio, **ζ1** and **ζ2** (unitless), for the two cylinders, the contact radius **ac** (m) is calculated from **equation 1**. The **second equation** computes the maximum pressure **pmax** (Pa) at the center of the contact area. The principal stresses **σxx** (Pa), **σyy** (Pa) and **σzz** (Pa) are computed from **equations 3, 4 and 5** illustrating the variation of these stresses away from the principle point of contact with coefficient of friction **μf** (unitless).

$$ac = \sqrt{\frac{\frac{2 \cdot F}{\pi \cdot L} \cdot \left(\frac{1-\zeta 1^2}{E1} + \frac{1-\zeta 2^2}{E2} \right)}{\frac{1}{d1} + \frac{1}{d2}}}$$ **Eq. 1**

$$pmax = \frac{2 \cdot F}{\pi \cdot ac \cdot L}$$ **Eq. 2**

$$\sigma xx = -2 \cdot \zeta \cdot pmax \cdot \left(\sqrt{1 + \frac{z^2}{ac^2}} - \frac{z}{ac} \right)$$ **Eq. 3**

$$\sigma yy = -p\max \cdot \left(\left(2 - \frac{1}{1 + \frac{z^2}{ac^2}} \right) \cdot \sqrt{1 + \frac{z^2}{ac^2}} - \frac{2 \cdot z}{ac} \right)$$ **Eq. 4**

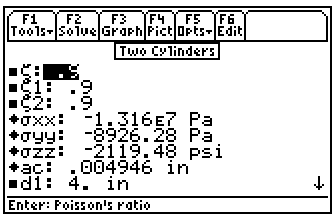
$$\sigma zz = \frac{-p\max}{\sqrt{1 + \frac{z^2}{ac^2}}}$$ **Eq. 5**

Variable	**Description**	**Units**
ζ1	Poisson’s ratio of 1	unitless
ζ2	Poisson’s ratio of 2	unitless
ζ	Poisson’s ratio	unitless
σxx	Stress along x axis	Pa

Variable	Description	Units
σyy	Stress along y axis	Pa
σzz	Stress along z axis	Pa
ac	Radius of contracted area	m
d1	Diameter of 1	m
d2	Diameter of 2	m
E1	Young's modulus –1	Pa
E2	Young's modulus –2	Pa
F	Force	N
L	Length	m
Pmax	Maximum pressure	Pa
Z	z-axis distance from contact point	m

Example 16.2.2:

Two cylinders 12 inches long, diameters 4 and 6 inches respectively, have Poisson's ratio of 0.9 and elasticity modulus of 50 and 60 GPa each. Find the stress at the line of impact when a 4000 -lbf load is applied.



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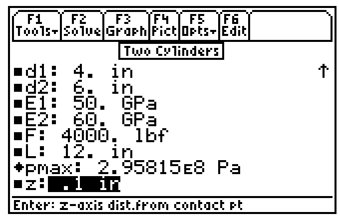


Lower Display

Solution – Select **all the equation**s to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. The problem takes about 20 seconds to solve.

Given	Solution
ζ = .9	σxx = -1.316 E7 Pa
ζ1 = .9	σyy = -8926.28 Pa
ζ 2= .9	σzz = -2116.48 psi
d1 = 4 in	ac = .004946 in
d2 = 6 in	pmax = 2.95815 E8 Pa
E1 = 50 GPa	
E2 = 60 GPa	
F = 4000 lbf	
L = 12 in	
z = 0.1 in	

16.3 Bearings

16.3.1 Bearing Life

Under the heading Bearing life, we have listed four equations that help design key bearing parameters in developing design equations. The main parameters of interest can be divided into manufacturer specified parameters are the rating load, **FR** (N), rated number of revolutions, **NR** (unitless), and the rated life, **LR** (s); in the design process, one specifies the corresponding design load, **FD** (N), design life, **LD** (s), and required design in the number of revolutions, **ND** (unitless). In addition, the rated speed, **NR1** (1/s), and designed speed, **ND1** (1/s), are also included in the calculations. The manufacturer supplies these ratings. The constant **β** is either 3 for ball bearings or 3.33 for collar bearings based on experience from the field.

$$ND = LD \cdot ND1 \quad \textbf{Eq. 1}$$

$$NR = LR \cdot NR1 \quad \textbf{Eq. 2}$$

$$\frac{ND}{NR} = \left(\frac{FR}{FD}\right)^{\beta} \quad \textbf{Eq. 3}$$

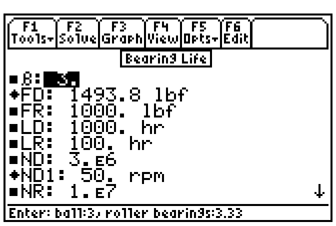
$$FR = FD \cdot \left(\frac{LD \cdot ND1}{LR \cdot NR1}\right)^{\frac{1}{\beta}} \quad \textbf{Eq. 4}$$

Variable	Description	Units
β	A Constant: enter 3 for ball bearings and 3.33 for roller bearings	unitless
FD	Radial design load	N
FR	Radial rating load	N
LD	Required design life	s
LR	Catalog rated life	s
ND	Required design: number of revolutions	unitless
ND1	Required design speed	1/s
NR	Rated number of revolutions	unitless
NR1	Rated speed	1/s

Example 16.3.1:

Design a bearing system from the following specifications:

Rating load is 1000 lbf; rated life is 100 hr; rated revolutions is 10000000; design goals are 3000000 revolutions for life, a design life of 1000 hrs. Assume a value of 3 for **β**.



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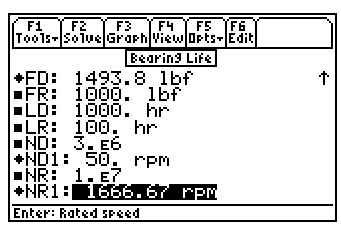
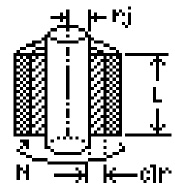


Lower Display

Solution – Select **all the equations** to solve this problem. Press [ENTER] to select all the equations and start the solving process by displaying all the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
β = 3	FD = 1493.8 lbf
FR = 1000 lbf	ND1 = 50 rpm
LD = 1000 hr	NR1 = 1666.67 rpm
LR = 100 hr	
ND = 3 E6	
NR = 1 E7	

16.3.2 Petroff's law



Petroff was one of the first to recognize clear connection between torque, speed, clearance, and coefficient of friction. In developing similar ideas, Sommerfeld was able to recognize a non-dimensional number named after him indicating a characteristic property of bearings. The four equations below form the core set of relationships. The **first equation** connects torque, **τ** (Nm), with the journal radius, **rj** (m), length **L** (m), clearance **clr** (m), absolute viscosity, **μ** (Pas), and significant speed, **N** (1/s). The **second equation** computes coefficient of friction, **cof** (unitless), in terms of **μ, N**, load pressure, **Pb** (Pa), and **clr**. The **third and fourth equations** are alternative ways to link the Sommerfeld number, **Sfld** (unitless), with **cof, rj, clr, μ, Pb.**

$$\tau = \frac{4 \cdot \pi^2 \cdot rj^3 \cdot L \cdot \mu \cdot N}{clr}$$ **Eq. 1**

$$cof = \frac{2 \cdot \pi^2 \cdot \mu \cdot N}{Pb} \cdot \frac{rj}{clr}$$ **Eq. 2**

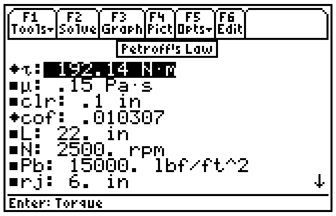
$$\frac{cof \cdot rj}{clr} = 2 \cdot \pi^2 \cdot Sfld$$ **Eq. 3**

$$Sfld = \left(\frac{rj}{clr}\right)^2 \cdot \left(\frac{\mu \cdot N}{Pb}\right)$$ **Eq. 4**

Variable	**Description**	**Units**
τ	Torque	N·m
μ	Absolute viscosity	Pa·s
clr	Clearance	m
cof	Coefficient of friction	Unitless
L	Length	m
N	Significant speed	1/s
Pb	Load per bearing area	Pa
rj	Journal radius	m
Sfld	Sommerfeld number	unitless

Example 16.3.2:

A journal bearing with a radius of 6 inches, 22 inches long, is working at a pressure of 15,000 lbf/ft^2. With a clearance of 0.1 in and an absolute viscosity of 0.15 Pa·s revolves at a speed of 2500 rpm. Find the torque, coefficient of friction and Sommerfeld number.



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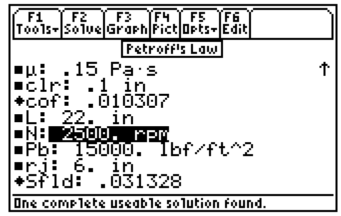


Lower Display

Solution – Select the **first three equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
μ = .15 Pa·s	τ = 192.14 N·m
clr =0.1 in	cof = .010307
L = 22 in	Sfld = .031328
N = 2500 rpm	
Pb = 15000 lbf/ft^2	
rj = 6 in	

16.3.3 Pressure Fed Bearings

The equations analyze the use of forced lubrication in a bearing system; enable the computation of heat dissipated and temperature rise in the fluid cooling the system.

$$Qs = \frac{\pi \cdot ps \cdot r \cdot clr^3}{3 \cdot \mu \cdot L} \cdot \left(1 + 1.5 \cdot ecc^2\right) \qquad \textbf{Eq. 1}$$

$$umax = \frac{ps \cdot clr^2}{8 \cdot \mu \cdot L} \qquad \textbf{Eq. 2}$$

$$\Delta H = \frac{2 \cdot \pi \cdot cof \cdot Pld \cdot r \cdot N}{J} \qquad \textbf{Eq. 3}$$

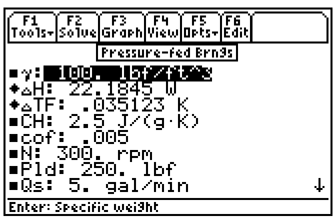
$$\Delta TF = \frac{2 \cdot \Delta H \cdot grav}{\gamma \cdot CH \cdot Qs} \qquad \textbf{Eq. 4}$$

Variable	Description	Units
γ	Specific weight	N/m^3
ΔH	Rate of heat loss	W
ΔTF	Temperature rise	K
μ	Absolute viscosity	Pa·s
CH	Specific heat	J/(kg·K)

Variable	**Description**	**Units**
clr	Clearance	m
Cof	Coefficient of friction	unitless
ecc	Eccentricity	unitless
J	Mechanical equivalent heat	unitless
L	Length	m
N	Significant speed	1/s
Pld	Load	N
ps	Supply pressure	Pa
Qs	Sideflow	m^3/s
R	Radius	m
umax	Maximum velocity	m/s

Example 16.3.3:

A cooling fluid with a specific weight of 100 lbf/ft^3 is forced into a system at the rate of 5 gal/min. The specific heat of the fluid is 2.5 J/(g·K). The shaft is rotating at a speed of 300 rpm. If the load is 250 lbf, find the amount of heat removed and the temperature rise of the fluid. Assume that the shaft radius is 5 inches.



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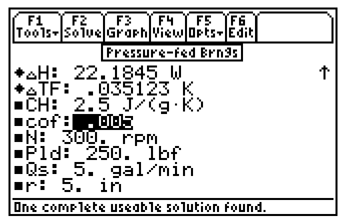


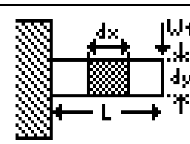
Lower Display

Solution – Select the **last two equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
$\gamma = 100\ lbf/ft^3$	$\Delta H = 22.1845$ W
CH = 2.5 J/(g·K)	$\Delta TF = .035123$ K
cof = 0.005	
N = 300 rpm	
Pld = 250 lbf	
Qs = 5 gal/min	
r = 5 in	

16.3.4 Lewis Formula

Two equations illustrate computing stress, **σ** (Pa), from different input data. The **first equation** computes **σ** from the length, **L** (m), along the z-axis, the load expressed as weight, **Wt** (N), and the x and y dimensions, **dx** (m) and **dy** (m), respectively. The **second equation** computes **σ** from **Wt, dx**, the diametrical pitch **pd** (1/m) and the Lewis form factor, **Lf** (unitless).



$$\sigma = \frac{6 \cdot Wt \cdot L}{dx \cdot dy^2} \qquad \textbf{Eq. 1}$$

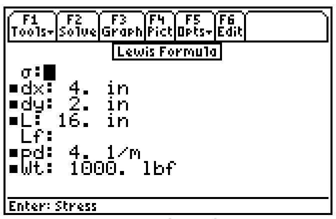
$$\sigma = \frac{Wt \cdot pd}{dx \cdot \pi \cdot Lf} \qquad \textbf{Eq. 2}$$

Variable	**Description**	**Units**
σ	Stress	Pa
Dx	X-section dimension	m
Dy	Y-section dimension	m
L	Length	m
Lf	Lewis form factor	unitless
Pd	Diametrical pitch	1/m
Wt	Load	N

Example 16.3.4:

Compute the Lewis form factor and stress in a mechanical system defined below:

The tooth is 4 inches wide, 2 inches high, and is 16 inches long. It is subjected to a weight of 1000 lbf. Assume diametrical pitch to be 4 1/m.



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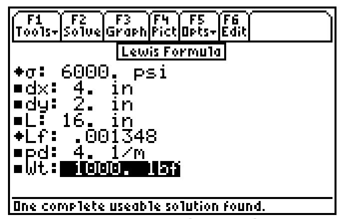


Computed results

Solution – Select **both equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
dx= 4 in	σ = 6000 psi
dy = 2 in	Lf = .001348
L = 16 in	
pd = 4 1/m	
Wt = 1000 lbf	

16.3.5 AGMA Stresses

American Gear Manufacturer's Association (AGMA) has established some industry wide standards for stress calculations. The two equations in this section reflect their standards. The **first equation** computes the bending stress, **σb** (Pa), of the gear in terms of load, **Wt** (N), face width, **fw** (m), pitch diameter, **dp** (m), a series of factors of the system such as geometry factor, **gf** (unitless), application factor, **Ka** (unitless), size factor, **Ks** (unitless), and dynamic factor, **Kv** (unitless), and load distribution factor, **Kld** (unitless). The **second equation** computes compressive stress, **σc** (Pa), in terms of the application parameter, **Ca** (unitless), surface condition factor, **Cf** (unitless), load distribution factor, **Cm** (unitless), elastic coefficient factor, **CP** (Pa), size factor **CS** (unitless), dynamic factor, **Cv** (unitless), **dp, gf,** and **fw**.

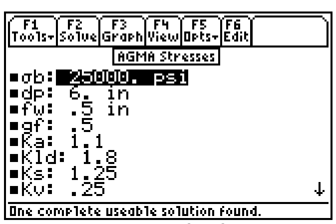
$$\sigma b = \frac{Wt \cdot Ka}{Kv} \cdot \frac{1}{fw \cdot dp} \cdot \frac{Ks \cdot Kld}{gf} \quad \textbf{Eq. 1}$$

$$\sigma c = \sqrt{\frac{CP \cdot Wt \cdot Ca \cdot CS \cdot Cm \cdot Cf}{Cv \cdot fw \cdot dp \cdot gf}} \quad \textbf{Eq. 2}$$

Variable	**Description**	**Units**
σb	Stress	Pa
σc	Compressive stress	Pa
Ca	Application factor	unitless
Cf	Surface condition factor	unitless
Cm	Load distribution factor	unitless
CP	Elastic coefficient	Pa
CS	Size Factor	unitless
Cv	Dynamic Factor	unitless
dp	Pitch diameter	m
fw	Face width	m
gf	Geometric factor	unitless
Ka	Application factor	unitless
Kld	Load distribution factor	unitless
Ks	Size factor	unitless
Kv	Dynamic Factor	unitless
Wt	Load	N

Example 16.3.5:

Compute the base stress for a system with a stress level of 25000 psi, application factor of 1.1, pitch diameter of 6 inches, dynamic factor of 0.25, face width of 0.5 inches, load distribution factor of 1.8, size factor of 1.25 and geometry factor of 0.5. Find the load the system can bear.



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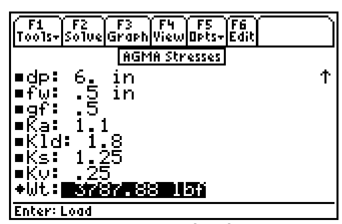


Lower Display

Solution – Select the **first equation** to solve this problem. Select it by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
σb = 25000 psi	Wt = 3787.88 lbf
dp = 6 in	
fw = .5 in	
gf = .5	
Ka = 1.1	
Kld = 1.8	
Ks = 1.25	
Kv = 0.25	

16.3.6 Shafts

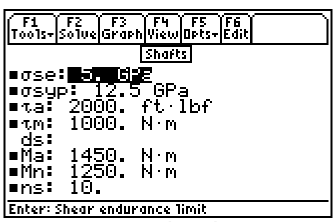
The equation below reflects a way to compute minimum shaft diameter, **ds** (m), given design criteria such as shear endurance limit, **σse** (Pa), and shear yield point, **σsyp** (Pa), torque amplitude, **τse** (N·m), mean torque, **τm** (N·m), moment amplitude, **Ma** (N·m), moment mean, **Mn** (m), and a factor of safety, **ns** (unitless).

$$ds = \left(\frac{32 \cdot ns}{\pi} \cdot \left(\left(\frac{\tau se}{\sigma se} + \frac{\tau m}{\sigma syp} \right)^2 + \left(\frac{Ma}{\sigma se} + \frac{Mn}{\sigma syp} \right)^2 \right)^{.5} \right)^{1/3} \qquad \textbf{Eq. 1}$$

Variable	Description	Units
σse	Shear Endurance limit	Pa
σsyp	Shear yield point	Pa
τa	Torque amplitude	N·m
τm	Mean torque	N·m
ds	Shaft diameter	m
Ma	Moment amplitude	N·m
Mn	Moment mean	N·m
ns	Factor of safety	unitless

Example 16.3.6:

Find the mean diameter of a shaft required to transmit a mean torque of 1000 N·m and torque amplitude of 2000 ft·lbf. The shaft has to withstand 5 GPa of an endurance limit, and a shear yield point of 12.5 GPa. The moment amplitude and mean moment of the shaft are 1450 N·m and 1250 N·m. Assume a factor of safety of 10.



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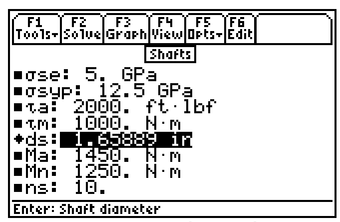


Computed results

Solution – Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
σse = 5 GPa	ds = 1.65889 in
σsyp = 12.5 GPa	
τa = 2000 ft·lbf	
τm = 1000 N·m	
Ma = 1450 N·m	
Mn = 1250 N·m	
ns = 10	

16.3.7 Clutches and Brakes

16.3.7.1 Clutches

16.3.7.1.1 Clutches

The six equations in this section are based on the following assumptions:

1. The pressure at any point is proportional to distance from the hinge.
2. The effect of centrifugal force neglected.
3. The shoe is assumed to be rigid.
4. Coefficient of friction does not vary with pressure.

For a clockwise rotation the pin reactions along x and y directions are shown by, **Rcx** (N) and **Rcy** (N), from **equations 1 and 2** in terms of maximum pressure, **pa** (Pa), width **b** (m), radius **r** (m), friction factor, **μf** (unitless), two computed unitless parameters, **conA** and **conB**, and loads, **Fx** (N) and **Fy** (N). The angle, **θa** (rad), represents the angle from the hinge. The **third and fourth equations** compute **Rax** (N) and **Ray** (N) representing the calculations for the counterclockwise case. The two angles, **θ1** (rad) and **θ2** (rad), represent the minimum and maximum angles of the brake shoe. The **last two equations** compute **conA** and **conB** in terms of **θ1** and **θ2**.

$$Rcx = \frac{pa \cdot b \cdot r}{\sin(\theta a)} \cdot (conA - \mu f \cdot conB) - Fx \qquad \textbf{Eq. 1}$$

$$Rcy = \frac{pa \cdot b \cdot r}{\sin(\theta a)} \cdot (conB + \mu f \cdot conA) - Fy \qquad \textbf{Eq. 2}$$

$$Rax = \frac{pa \cdot b \cdot r}{\sin(\theta a)} \cdot (conA + \mu f \cdot conB) - Fx \qquad \textbf{Eq. 3}$$

$$Ray = \frac{pa \cdot b \cdot r}{\sin(\theta a)} \cdot (conB - \mu f \cdot conA) - Fy \qquad \textbf{Eq. 4}$$

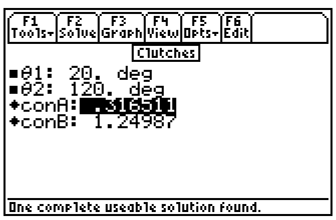
$$conA = \frac{(\sin(\theta 2))^2 - (\sin(\theta 1))^2}{2} \qquad \textbf{Eq. 5}$$

$$conB = \frac{\theta 2 - \theta 1}{2} - \frac{\sin(2 \cdot \theta 2) - \sin(2 \cdot \theta 1)}{4} \qquad \textbf{Eq. 6}$$

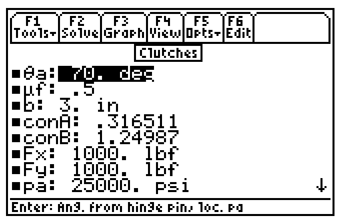
Variable	Description	Units
$\theta 1$	Minimum angle	rad
$\theta 2$	Maximum angle	rad
θa	Angle from hinge pin, location of pa	rad
μf	Coefficient of friction	unitless
b	Width	m
conA	Constant A	unitless
conB	Constant B	unitless
Fx	Force along x	N
Fy	Force along y	N
pa	Maximum pressure at θa	Pa
r	Radius	m
Rax	Reaction x-axis (anti-clockwise)	N
Ray	Reaction y-axis (anti-clockwise)	N
Rcx	Reaction x-axis (clockwise)	N
Rcy	Reaction y-axis (clockwise)	N

Example 16.3.7.1.1:

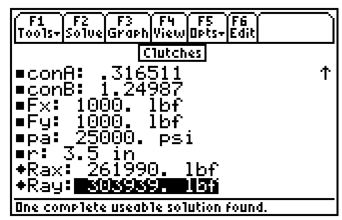
A clutch/brake system has a show that covers from 20 degrees to 120 degrees, with the angle from the hinge pin of 70 degrees. The coefficient of friction is 0.5. For a load of 1000 lbf along the x-axis and 1000 lbf along y-axis, and a maximum pressure of 25000 psi, find the counterclockwise reactions for a system 3 inches wide and 3 inches in radius.



Display: Step 1



Upper Display: Step 2



Lower Display: Step 2

Solution – Solve this problem in two steps. First select **equations 5 and 6** to solve for **conA** and **conB**. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. Following the calculations of **conA** and **conB**, deselect the last two equations and **select equations 3 and 4.** Designate the previously computed results for **conA** and **conB** as known values using the highlight bar and [F5]:**Opts**/[6]:**Know**. Press [F2] to solve for **Rax** and **Ray**.

Given (first step)
$\theta 1$ = 20 deg
$\theta 2$ = 120 deg

Solution (first step)
conA = .316511
conB = 1.24987

Given (second step)
θa = 70 deg
μf = .5
b = 3 in
Fx = 1000 lbf
Fy = 1000 lbf
pa = 25000 psi
r = 3 in

Solution (second step)
Rax = 261990 lbf
Ray = 303939 lbf

16.3.7.2 Uniform Wear - Cone Brake

The two equations in this section represent the force and torque required for a cone brake where uniform wear is assumed. The **first equation** represents the force **F** (N) necessary to actuate the brake system based on the maximum stress, **pmax** (Pa), outer diameter, **d1** (m), and inner diameter, **d2** (m). The **second equation** determines the torque, **τ** (N·m), needed in terms of the same parameters and cone angle, **α** (rad).

Eq. 1

$$F = \frac{\pi \cdot pmax \cdot d1}{2} \cdot \left(d2 - d1\right)$$

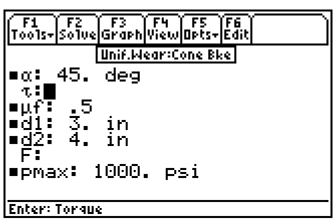
Eq. 2

$$\tau = \frac{\pi \cdot \mu f \cdot pmax \cdot d1}{8 \cdot \sin(\alpha)} \cdot \left(d2^2 - d1^2\right)$$

Variable	Description	Units
α	Half thread/cone angle	rad
τ	Torque	N·m
μf	Coefficient of friction	unitless
d1	Inner diameter	m
d2	Outer diameter	m
F	Force	N
Pmax	Maximum pressure	Pa

Example 16.3.7.2:

A cone brake with an angle of 45 degrees, an outer diameter of 4 inches, and an inner diameter of 3 inches, has coefficient of friction of 0.5. Find the force if the maximum pressure allowed is 1000 psi.



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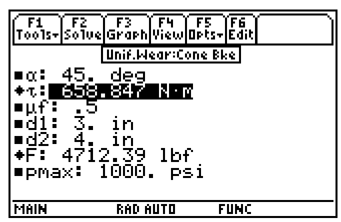


Computed results

Solution – Select **both equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
α = 45 deg	τ=658.647 N·m
μ = .5	F = 4712.39 lbf
d1 = 3 in	
d2 = 4 in	
pmax = 1000 psi	
uf = .5	

16.3.7.3 Uniform Pressure - Cone Brake

The two equations here reflect the force, **F** (N), and torque, **τ** (N·m), for the case where the pressure on the cone is assumed to be uniform. The **first equation** represents the force, **F** (N), necessary to actuate the brake system based on the maximum stress, **pmax** (Pa), outer diameter, **d1** (m), and inner diameter, **d2** (m). The **second equation** determines the torque, **τ** (N·m), needed in terms of the same parameters and cone angle, **α** (rad).

Eq. 1

$$F = \frac{\pi \cdot pmax \cdot \left(d2^2 - d1^2\right)}{4}$$

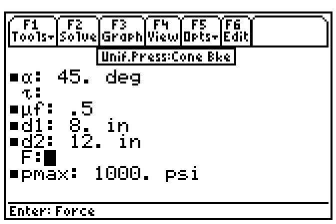
Eq. 2

$$\tau = \frac{\frac{F \cdot \mu f}{3 \cdot \sin(\alpha)} \cdot \left(d2^3 - d1^3\right)}{d2^2 - d1^2}$$

Variable	Description	Units
α	Half thread/cone angle	rad
τ	Torque	N·m
μf	Coefficient of friction	unitless
d1	Inner diameter	m
d2	Outer diameter	m
F	Force	N
pmax	Maximum pressure	Pa

Example 16.3.7.3:

A cone brake has an angle of 45 degrees, an outer diameter 12 inches and an inner diameter of 8 inches. Assuming a coefficient of friction of 0.5, find the force if the maximum pressure allowed is 1000 psi.



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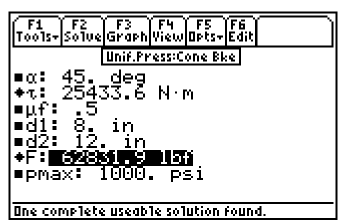


Computed results

Solution – Select **both equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
α = 45 deg	τ = 25433.6 N·m
μf= .5	F = 62831.9 lbf
d1 = 8 in	
d2 = 12 in	
pmax = 1000 psi	

16.4 Spring Design

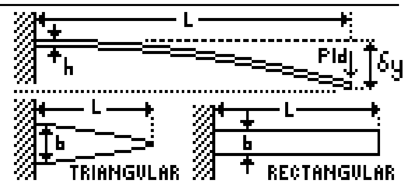
Design of springs has played a key role in a variety of situations where effort is made to find the proper combination of material, spring strength and volume of material. In this section we have treated a variety of spring designs:

1. Bending Springs – plate springs with a rectangular, triangular or semi-elliptical shapes.
2. Coiled Springs – Cylindrical helical springs with circular or rectangular cross section.
3. Torsional Springs – Circular or rectangular straight bar
4. Axially Loaded Springs – Conical circular section, or cylindrical helical spring with rectangular or circular cross section.

In the following subsections, the design equations for these springs are included and example problems are solved.

16.4.1 Bending

16.4.1.1 Rectangular Plate



The four equations in this section define the key design equations for a bending spring of rectangular cross section. The spring has a length, **L** (m), width **b** (m), and thickness, **h** (m). The spring is subjected to a load, **Pld** (N), resulting in a stress, **σs** (Pa), and a deflection, **δy** (m). **U** (J), represent the energy in the spring and the area moment is given by **I** (m^4). The **first equation** computes, **σs**, for a given load, **Pld**. The **second equation** computes the area moment, **I**, while the **third equation** estimates the energy, **U**. The **last equation** links up deflection, **δy**, in terms of **Pld**, **L**, Young's modulus, **E** (Pa), and **I**.

$$Pld = \frac{b \cdot h^2 \cdot \sigma s}{6 \cdot L}$$ **Eq. 1**

$$I = \frac{b \cdot h^3}{12}$$ **Eq. 2**

$$U = \frac{Pld \cdot \delta y}{2}$$ **Eq. 3**

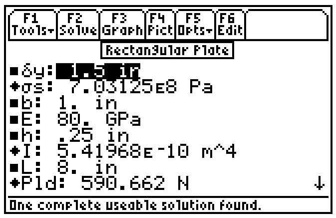
$$\delta y = \frac{Pld \cdot L^3}{3 \cdot E \cdot I}$$ **Eq. 4**

Variable	**Description**	**Units**
δy	Deflection	m
σs	Safe stress	Pa
b	Width	m
E	Young's modulus	Pa

Variable	Description	Units
h	Height	m
I	Area moment	m^4
L	Length	m
Pld	Load	N
U	Resilience	N·m

Example 16.4.1.1:

A rectangular plate spring is 8 inches long, 1 inch wide and 0.25 in thick. The material has a Young's modulus of 80 GPa, and flexes 1.5 inches. Find the maximum load the system can handle, the energy stored in the spring and the stress level for the maximum load.



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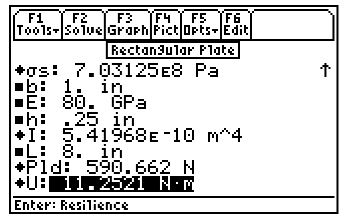
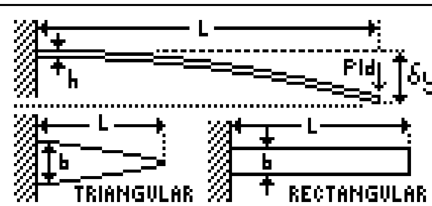


Lower Display

Solution – Select **all the equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
δy = 1.5 in	σs = 7.03125E8 Pa
b = 1 in	I = 5.41968E-10 m^4
E = 80 GPa	Pld = 590.662 N
h = .25 in	U = 11.2521 N·m
L = 8 in	

16.4.1.2 Triangular Plate



The four equations in this section define the key design equations for a bending spring of rectangular cross section. The spring has a length, **L** (m), and width, **b** (m), and thickness, **h** (m). The spring is subjected to a load, **Pld** (N), resulting in a stress, **σs** (Pa) and a deflection, **δy** (m). **U** (J), represent the energy in the spring and the area moment is given by **I** (m^4). The **first equatio**n computes **σs** for a given load, **Pld**. The **second equation** computes the area moment, **I**, while the **third equation** estimates the energy in the spring, **U**. The **last equation** links up deflection, **δy**, in terms of **Pld**, **L**, Young's modulus, **E** (Pa), and **I**.

$$Pld = \frac{b \cdot h^2 \cdot \sigma s}{6 \cdot L} \quad \textbf{Eq. 1}$$

$$I = \frac{b \cdot h^3}{12} \quad \textbf{Eq. 2}$$

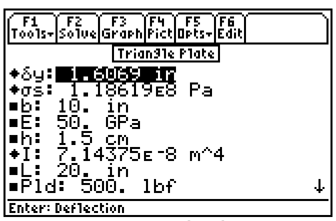
$$U = \frac{Pld \cdot \delta y}{2} \qquad \textbf{Eq. 3}$$

$$\delta y = \frac{Pld \cdot L^3}{2 \cdot E \cdot I} \qquad \textbf{Eq. 4}$$

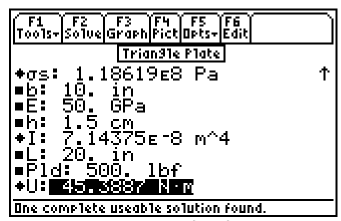
Variable	**Description**	**Units**
δy	Deflection	m
σs	Safe stress	Pa
b	Width	m
E	Young's modulus	Pa
h	Height	m
I	Area moment	m^4
L	Length	m
Pld	Load	N
U	Resilience	N·m

Example 16.4.1.2:

A triangular plate 20 inches long with a base width of 10 inches and a thickness of 1.5 cm is subject to a load of 500 lbf. Find the deflection due to this load, the energy of the spring, and the maximum stress introduced into the material. The spring has a modulus of elasticity of 50 GPa.



Upper Display

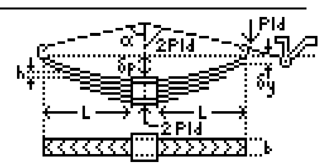


Lower Display

Solution – Select **all the equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
b = 10 in	δy = 1.6069 in
E = 50 GPa	σs = 1.18619 E8 Pa
h = 1.5 cm	I = 7.14375 E-8 m^4
L = 20 in	U = 45.3887 N·m
Pld = 500 lbf	

16.4.1.3 Semi-Elliptical



The two equations in this section specify key design properties for a semi-elliptical spring design. These equations are useful for designing springs for automobiles. The spring has a half-length, **L** (m), width, **b** (m), and a leaf thickness, **h** (m). The spring is subjected to a load 2***Pld** (N) resulting in a stress, **σs** (Pa), and a deflection, **δy** (m). The **first equation** computes **σs** for a load, **Pld,** given the number of leaves, **nf,** (unitless), and the no-load sag, **δp** (m). The **second equation** computes the deflection, **δy** (m), in terms of **L**, **nf**, **b**, **h**, **Pld**, **δp,** Young's modulus, **E** (Pa), and the half cone angle, **α** (rad).

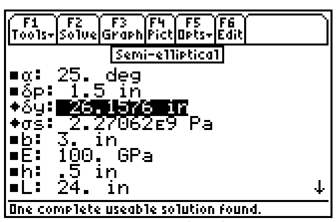
$$Pld = nf \cdot b \cdot h^2 \cdot \frac{\sigma s}{6 \cdot \left(L + \delta p \cdot \tan(\alpha)\right)}$$ **Eq. 1**

$$\delta y = \frac{6 \cdot L^2}{nf \cdot b \cdot h^3} \cdot Pld \cdot \frac{L + \delta p \cdot \tan(\alpha)}{E}$$ **Eq. 2**

Variable	**Description**	**Units**
α	Half thread/cone angle	rad
δp	Sag without load	m
δy	Deflection	m
σs	Safe stress	Pa
b	Width	m
E	Young's modulus	Pa
H	Height	m
L	Length	m
Nf	Number of coils/leaves	unitless
Pld	Load	N

Example 16.4.1.3:

A semi-elliptical spring has 6 leaves and is 24 inches long, 3 inches wide and .5 inches thick. The Young's modulus for the steel spring is 100 GPa. Find the deflection and stress for a 10000-lbf load. Assume the spring has initial sag of 1.5 inches. The half angle formed at the top of the spring is 25 degrees.



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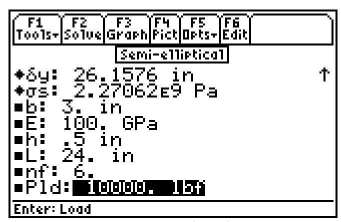


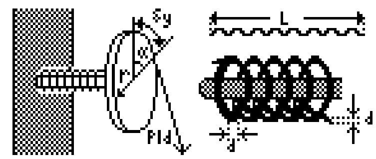
Lower Display

Solution – Select **both equations** to solve this problem. Select these by highlighting the equations and pressing ENTER. Press F2 to display the variables. Enter the values for the known parameters and press F2 to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
α = 25 deg	δy = 26.1576 in
δp = 1.5 in	σs = 2.27062E9 Pa
b = 3 in	
E = 100 GPa	
h = .5 in	
L = 24 in	
nf = 6	
Pld = 10000 lbf	

16.4.2 Coiled Springs

16.4.2.1 Cylindrical Helical - Circular wire



The seven equations in this section form the design equations for a cylindrical helical spring with a circular wire. The wire of the spring has a diameter, **d** (m), a coil radius, **r** (m), and length, **L** (m). The spring is subjected to a load, **Pld** (N), resulting in a stress, **σs** (Pa), and a deflection, **δy** (m). **U** (J) represents the energy in the spring and the area moment is given by **I** (m^4). The **first equation** computes **σs** for a given load, **Pld**. The **second equation** computes the area moment, **I**, while the **third equation** estimates the energy, **U**. The **fourth equation** links up deflection, **δy**, in terms of **Pld**, **L**, Young's modulus, **E** (Pa), and **I**. **Equations 6 and 7** define the geometry factor, **ksp** (unitless), and the geometry ratio, **kw** (unitless). The last equation computes the deflection, **δy**, from the coil radius, **r**, and the deflection angle, **α**.

$$Pld = \frac{\pi \cdot h^3 \cdot \sigma s}{32 \cdot r \cdot ksp}$$ **Eq. 1**

$$I = \frac{\pi \cdot d^4}{64}$$ **Eq. 2**

$$U = \frac{\sigma s^2 \cdot V}{8 \cdot E \cdot ksp^2}$$ **Eq. 3**

$$\delta y = \frac{2 \cdot r \cdot L \cdot \sigma s}{d \cdot E \cdot ksp}$$ **Eq. 4**

$$kw = \frac{2 \cdot r}{h}$$ **Eq. 5**

$$ksp = \frac{4 \cdot kw - 1}{4 \cdot kw - 4}$$ **Eq. 6**

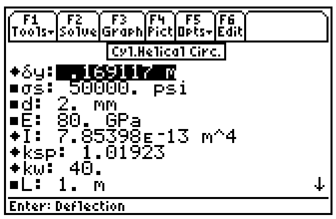
$$\delta y = r \cdot \alpha$$ **Eq. 7**

Variable	**Description**	**Units**
α	Half thread/cone angle	rad
δy	Deflection	m
σs	Safe stress	Pa
d	Diameter	m
E	Young's modulus	Pa
h	Height	m
I	Area moment	m^4
ksp	Geometry factor	unitless
kw	Geometry ratio	unitless

Variable	Description	Units
L	Length	m
nf	Number of coils/leaves	unitless
Pld	Load	N
r	Radius	m
U	Resilience	N·m
V	Volume of spring	m^3

Example 16.4.2.1:

A helical spring is to be designed with a circular wire. The wire used has a diameter 2 mm, and is wound as a spring of radius 4 cm. The total length of the wire is 1 m, and a safe stress is established at 50000 psi. Assume that the elastic modulus for the wire material is 80 GPa. Find the load the spring is designed to carry and the moment of inertia for the wire.



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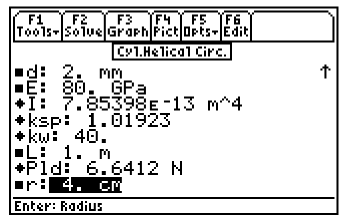

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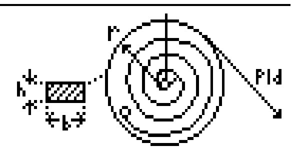
Solution – Select the **equations 1, 2, 4, 5** and **6** in this set to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
σs = 50000 psi	δy = .169117 m
d = 2 mm	I = 7.85398 E-13 m^4
E = 80 GPa	ksp =1.01923
L = 1 m	kw = 40
r = 4 cm	Pld = 6.6412 N

16.4.2.2 Rectangular Spiral



The five equations in this section define the key design equations for a bending spring having a rectangular cross section. The spring has a width, **b** (m), thickness, **h** (m), and a radius, **r** (m). The spring is subjected to a load, **Pld** (N) resulting in a stress, **σs** (Pa), and a deflection, **δy** (m). **U** (J) represents the energy in spring and the area moment is given by, **I** (m^4). The **first equation** computes **σs** for a given load, **Pld**. The **second equation** computes the area moment, **I**, while the **third equation** estimates the energy, **U**, in terms of the volume of the spring, **V** (m^3). The **last two equations** define the geometry factor **ksp** (unitless) and the geometry ratio **kw** (unitless).

$$Pld = \frac{b \cdot d^2 \cdot \sigma s}{6 \cdot r \cdot ksp} \quad \textbf{Eq. 1}$$

$$I = \frac{b \cdot h^3}{12} \quad \textbf{Eq. 2}$$

$$U = \frac{\sigma s^2 \cdot V}{8 \cdot E \cdot ksp^2} \qquad \textbf{Eq. 3}$$

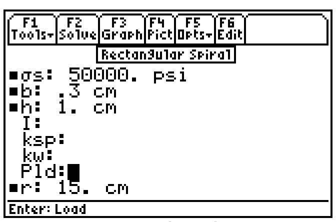
$$kw = \frac{2 \cdot r}{d} \qquad \textbf{Eq. 4}$$

$$ksp = \frac{3 \cdot kw - 1}{3 \cdot kw - 3} \qquad \textbf{Eq. 5}$$

Variable	Description	Units
σs	Safe stress	Pa
B	Width	m
E	Young's modulus	Pa
h	Height	m
I	Area moment	m^4
ksp	Geometry factor	unitless
kw	Geometry ratio	unitless
Pld	Load	N
r	Radius	m
U	Resilience	N·m
V	Volume of spring	m^3

Example 16.4.2.2:

A spring design calls for a 0.3 cm wide bar of height 1 cm, is wound to a radius of 15 cm. The safe stress is 50000 psi. Find the area moment of the spring and load it is designed to handle.



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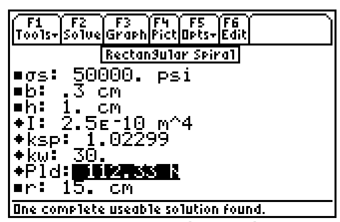


Computed results

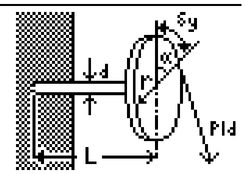
Solution – Select **equations 1, 2, 4,** and **5** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
σs = 50000 psi	I = 2.5 E-10 m^4
b = 0.3 cm	ksp = 1.02299
h = 1 cm	kw = 30
r= 15 cm	Pld = 112.33 N

16.4.3 Torsional Spring

16.4.3.1 Circular Straight Bar

The four equations in this section define the design equations for a spring designed as a circular straight bar. The spring bar has a diameter, **d** (m), length **L**, (m) and is subject to a load, **Pld** (N), resulting in a stress, **σs** (Pa), and a deflection, **δy** (m). **U** (J) represents the energy in the spring. The **first equation** computes **σs** for a given load **Pld**. The **second equation** computes the spring energy, **U**. The **last two equations** compute the deflection due to the load, **δy** (m).



$$Pld = \frac{\pi \cdot d^3 \cdot \sigma s}{16 \cdot r}$$ **Eq. 1**

$$U = \frac{\sigma s^2 \cdot V}{4 \cdot G}$$ **Eq. 2**

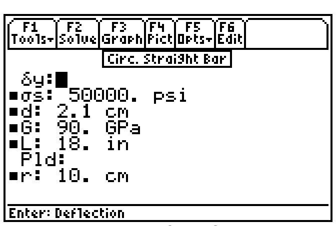
$$\delta y = \frac{2 \cdot r \cdot L \cdot \sigma s}{d \cdot G}$$ **Eq. 3**

$$\delta y = r \cdot \alpha$$ **Eq. 4**

Variable	**Description**	**Units**
α	Half thread/cone angle	rad
δy	Deflection	m
σs	Safe stress	Pa
d	Diameter	m
G	Modulus of rigidity	Pa
L	Length	m
Pld	Load	N
r	Radius	M
U	Resilience	N·m
V	Volume of spring	m^3

Example 16.4.3.1:

A circular straight bar, 18 inches long, has a diameter of 2.1 cm, and a safe stress level of 50000 psi. The material of the spring has a bulk elasticity of 90 GPa. The radius of the circular plate is 10 cm. Find the load and deflection for such a spring.



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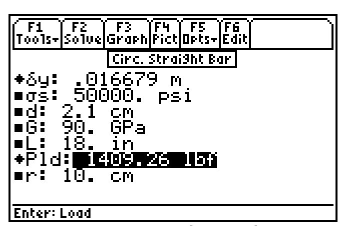
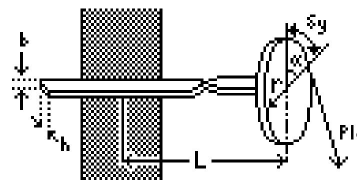


Computed results

Solution – Select equations **1** and **3** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
σs = 50000 psi	δy = .016679 m
d = 2.1 cm	Pld = 1409.26 lbf
G = 90 GPa	
L = 18 in	
r = 10 cm	

16.4.3.2 Rectangular Straight Bar



The five equations in this section define the key design equations for a rectangular straight bar with length, **L** (m), and width, **b** (m), and thickness, **h** (m). The spring is subjected to a load, **Pld** (N), resulting in a stress **σs,** (Pa) and a deflection, **δy** (m). **U** (J) represents the energy in the spring. The **first equation** computes **σs** for a given load, **Pld**. The **second equation** computes the spring energy, **U**. The **third equation** computes **C**, a geometry factor. The **last two equations** compute the deflection, **δy**, in alternate forms.

$$Pld = \frac{2 \cdot b^2 \cdot h \cdot \sigma s}{9 \cdot r}$$ **Eq. 1**

$$U = \frac{4 \cdot \sigma s^2 \cdot V\left(C^2 + 1\right)}{45 \cdot G}$$ **Eq. 2**

$$C = \frac{b}{h}$$ **Eq. 3**

$$\delta y = \frac{\frac{4}{5} \cdot r \cdot L \cdot \sigma s \cdot \left(b^2 + h^2\right)}{b \cdot h^2 \cdot G}$$ **Eq. 4**

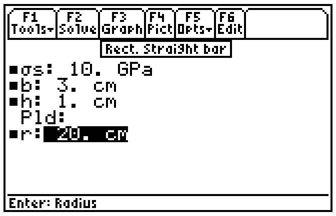
$$\delta y = r \cdot \alpha$$ **Eq. 5**

Variable	**Description**	**Units**
α	Half thread/cone angle	rad
δy	Deflection	m
σs	Safe stress	Pa
b	Width	m
C	Constant: max when C=1	unitless
G	Modulus of rigidity	Pa
h	Height	m
L	Length	m
Pld	Load	N
r	Radius	m
U	Resilience	N·m

Variable	**Description**	**Units**
V	Volume of spring	m^3

Example 16.4.3.2:

A rectangular straight bar is 3 cm wide and 1 cm high. If the radius of the load from the torque center is 20 cm and the allowed safe stress is 10 GPa, find the allowed load.



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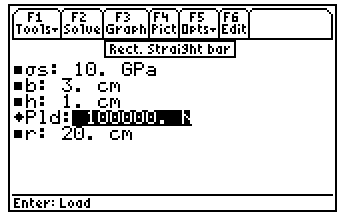


Computed results

Solution – Select the **first equation** to solve this problem. Select by this highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
σs = 10 GPa	Pld = 100000 N
b = 3 cm	
h = 1 cm	
r = 20 cm	

16.4.4 Axial Loaded

16.4.4.1 Conical Circular Section

The five equations in this section define the key design equations for an axial-loaded conical spring, having a circular cross section. The spring has a coil diameter, **d** (m), and is subject to a load, **Pld** (N), resulting in a stress, **σs** (Pa), and a deflection, **δy** (m). **U** (J) represents the energy in the spring. The **first equation** computes **σs** for a given load, **Pld**, spring radius, **r** (m), and wire diameter, **d**. The **second equation** computes the energy, **U**, in terms of bulk modulus, **G** (Pa), spring volume, **V** (m^3), and **σs**. The **next two equations** compute geometrical, unitless parameters, **ksw** and **kw**, that must be considered in computing load. The **last equation** calculates the deflection due to the load, **δy** (m).

$$Pld = \frac{\pi \cdot d^3 \cdot \sigma s}{16 \cdot r \cdot ksp} \qquad \textbf{Eq. 1}$$

$$U = \frac{\sigma s^2 \cdot V}{8 \cdot G \cdot ksp^2} \qquad \textbf{Eq. 2}$$

$$ksp = \frac{4 \cdot kw - 1}{4 \cdot kw - 4} + \frac{.615}{kw} \qquad \textbf{Eq. 3}$$

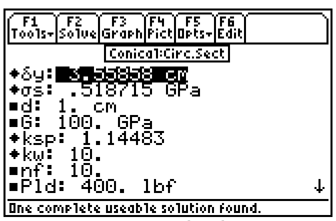
$$kw = \frac{2 \cdot r}{d}$$ **Eq. 4**

$$\delta y = \frac{\pi \cdot nf \cdot r^2 \cdot \sigma s}{d \cdot G \cdot ksp}$$ **Eq. 5**

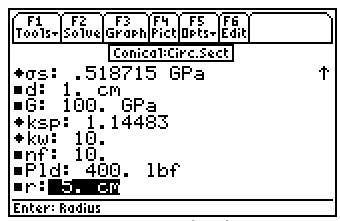
Variable	Description	Units
δy	Deflection	m
σs	Safe stress	Pa
d	Diameter	m
G	Modulus of rigidity	Pa
ksp	Geometry factor	unitless
kw	Geometry ratio	unitless
nf	Number of coils/leaves	unitless
Pld	Load	N
r	Radius	m
U	Resilience	N·m
V	Volume of spring	m^3

Example 16.4.4.1:

A 1 cm diameter wire is wound into a coil, 5 cm in radius, as an axially loaded spring. There are 10 rounds in the spring. If a load of 400 lbf is applied to this spring, what is the deflection and stress in the wire? Use a value of 100 GPa for the bulk modulus.



Upper Display



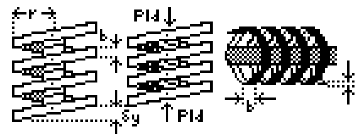
Lower Display

Solution – Select **equations 1, 3, 4, and 5** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
d = 1 cm	δy = 3.55858 cm
G = 100 GPa	σs = .518715 GPa
nf = 10	ksp = 1.14483
Pld = 400 lbf	kw = 10
r = 5 cm	

16.4.4.2 Cylindrical - Helical

16.4.4.2.1 Rectangular Cross Section



The six equations in this section define the key design equations for a bending spring of rectangular cross section. The spring ribbon has a width, **b** (m), and thickness, **h** (m). The spring is subject to a load, **Pld** (N), resulting in a stress, **σs** (Pa), and a deflection, **δy** (m). **U** (J) represents the energy in the spring. The **first equation** computes **σs** for a given load, **Pld**. The **second equation** computes a parameter, **C**, that expresses the aspect ratio of the ribbon material. The **third equation** estimates the energy of the spring, **U,** from the volume, **V** (m^3), and bulk modulus, **G** (Pa). The **fourth equation** computes **ksp**, a geometric factor in terms of **kw**, computed by **equation 6**. The deflection, **δy**, is computed by **equation 5**.

$$Pld = \frac{2 \cdot b^2 \cdot h \cdot \sigma s}{9 \cdot r \cdot ksp} \quad \textbf{Eq. 1}$$

$$C = \frac{b}{h} \quad \textbf{Eq. 2}$$

$$U = \frac{4 \cdot \sigma s^2 \cdot V \cdot \left(C^2 + 1\right)}{45 \cdot G \cdot ksp^2} \quad \textbf{Eq. 3}$$

$$ksp = \frac{4 \cdot kw - 1}{4 \cdot kw - 4} + \frac{.615}{kw} \quad \textbf{Eq. 4}$$

$$\delta y = \frac{7.2 \cdot \pi \cdot nf \cdot r^3 \cdot Pld \cdot \left(b^2 + h^2\right)}{b^3 \cdot h^3 \cdot G} \quad \textbf{Eq. 5}$$

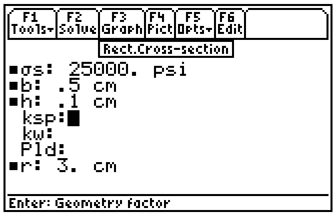
$$kw = \frac{2 \cdot r}{b} \quad \textbf{Eq. 6}$$

Variable	Description	Units
δy	Deflection	m
σs	Safe stress	Pa
b	Width	m
C	Constant: maximum when C=1	unitless
G	Modulus of rigidity	Pa
h	Height	m
ksp	Geometry factor	unitless
kw	Geometry ratio	unitless
nf	Number of coils/leaves	unitless
Pld	Load	N
r	Radius	m
U	Resilience	N·m

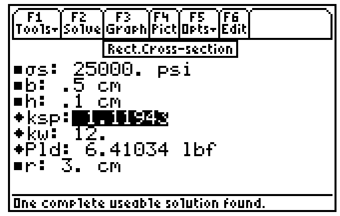
Variable	**Description**	**Units**
V	Volume of spring	m^3

Example 16.4.4.2.1:

Find the load for a helical ribbon 0.5 cm wide, 0.1 cm thick, wound as a spring 3 cm in radius. Assume that allowed stress is 25000 psi.



Entered Values

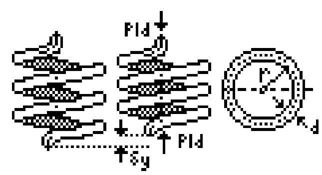


Computed results

Solution – Select **equations 1, 4** and **6** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
σs = 25000 psi	kw =12
b = .5 cm	ksp = 1.11943
h = .1 cm	Pld = 6.41034 lbf
r = 3 cm	

16.4.4.2.2 Circular Cross Section



The five equations in this section represent the key design equations for a bending spring of rectangular cross section. The spring has a wire diameter, **d** (m), and spring radius, **r** (m). The spring is subject to a load, **Pld** (N), resulting in a stress, **σs** (Pa), and a deflection, **δy** (m). **U** (J) represents the energy in the spring. The **first equation** computes **σs** for a given load, **Pld**. The unitless parameters, **kw** and **ksp**, represent the impact of the geometry of the wire. **Equation 2** computes the geometric ratio of the spring, **kw** (unitless). The **third equation** estimates the energy, **U**, using the spring volume, **V** (m^3), and bulk modulus, **G** (Pa). The deflection due to the load, **δy**, is computed by **equation 4**. The **last equation** computes, **ksp** a geometric factor, in terms of **kw**, computed by **equation 2**.

Eq. 1

$$Pld = \frac{\pi \cdot d^3 \cdot \sigma s}{16 \cdot r \cdot ksp}$$

Eq. 2

$$kw = \frac{2 \cdot r}{d}$$

Eq. 3

$$U = \frac{\sigma s^2 \cdot V}{4 \cdot G \cdot ksp^2}$$

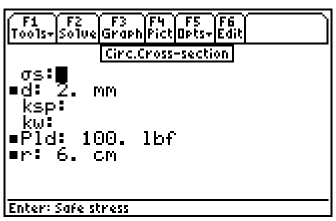
$$\delta y = \frac{4 \cdot \pi \cdot nf \cdot r^2 \cdot \sigma s}{d \cdot G \cdot ksp}$$ **Eq. 4**

$$ksp = \frac{4 \cdot kw - 1}{4 \cdot kw - 4} + \frac{.615}{kw}$$ **Eq. 5**

Variable	Description	Units
δy	Deflection	m
σs	Safe stress	Pa
d	Diameter	m
G	Modulus of rigidity	Pa
ksp	Geometry factor	unitless
kw	Geometry ratio	unitless
nf	Number of coils/leaves	unitless
Pld	Load	N
r	Radius	m
U	Resilience	N·m
V	Volume of spring	m^3

Example 16.4.4.2.2:

A spring coil 2 mm in diameter is wound as a coil 6 cm in radius. A load of 100 lbf is applied. What is the stress in the spring coil?



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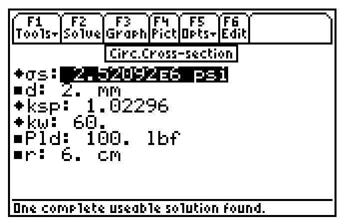


Computed results

Solution – Select **equations 1, 2** and **5** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
d = 2 mm	σs= 2.52092 E6 psi
Pld = 100 lbf	ksp = 1.02296
r = 6 cm	kw = 60

References:

1. Joseph E. Shigley and Charles R. Miseke, Mechanical Engineering Design, 5th Edition, McGraw-Hill Publishing company, New York, NY 1989
2. Eugene A. Avallone and Theodore Baumeister III, Standard Handbook for Mechanical Engineers, 9th Edition, McGraw-Hill Book Company, New York, 1984
3. Michael R Lindeburg, Mechanical Engineer's Reference Manual, Professional Publications, Belmont, CA 1990
4. Joseph E. Shigley and Charles R. Miseke, Mechanical Engineering Design, 3rd Edition, McGraw-Hill Publishing company, New York, NY 1977

Chapter 17: Pumps and Hydraulics

This portion of the software engages in solving problems encountered in dealing with pumps and hydraulic machines. The equations are presented under, three classifications listed below.

- Basic Definitions
- Centrifugal Pumps
- Pump Power

17.1 Basic Definitions

This area of mechanical engineering is a rather specialized area and commands a vocabulary of its own. The **first equation** describes friction head, **hf** (m), defined as the resistance to flow in the pipe, fittings valves, entrance and exits. The friction head, **hf**, results from friction coefficient **f** (unitless), a velocity, **V** (m/s), diameter, **d** (m), the effective length, **Le** (m), and the acceleration due to gravity, **grav** (m/s^2). The **second equation** defines the kinetic head, **hv** (m), from the velocity, **V** (m/s), and gravity, **grav**. The **third** and **fourth equations** are the so-called atmospheric head, **pa** (Pa), and pressure head, **hp** (m), due to a pressure, **p** (Pa). Fluid vapor pressure, **pvp** (Pa), converted to an equivalent head is defined as **hvp** (m) in **equation 5**. The **sixth** and **seventh equations** show the combined effects of the various heads defined here. For example, **hd** (m), represents total dynamic discharge head, **hd** (m), while, **nps** (m), represents the net positive suction head.

$$hf = \frac{f \cdot Le \cdot V^2}{2 \cdot d \cdot grav} \quad \textbf{Eq. 1}$$

$$hv = \frac{V^2}{2 \cdot grav} \quad \textbf{Eq. 2}$$

$$ha = \frac{pa}{\rho \cdot grav} \quad \textbf{Eq. 3}$$

$$hp = \frac{p}{\rho \cdot grav} \quad \textbf{Eq. 4}$$

$$hvp = \frac{pvp}{\rho \cdot grav} \quad \textbf{Eq. 5}$$

$$hd = hs + hv + hf \quad \textbf{Eq. 6}$$

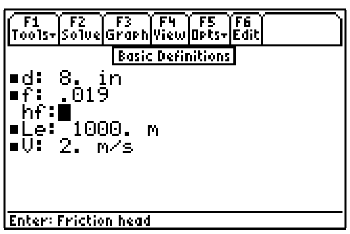
$$nps = ha + hs - hf - hvp \quad \textbf{Eq. 7}$$

Variable	Description	Units
ρ	Density	kg/m^3
d	Diameter	m
f	Friction coefficient	unitless

Variable	**Description**	**Units**
grav	Gravitational acceleration	9.80665 m/s^2
ha	Atmospheric head	m
hd	Discharge head	m
hf	Friction head	m
hp	Pressure head	m
hs	Static head	m
hv	Velocity head	m
hvp	Vapor pressure head	m
Le	Equivalent length	m
hps	Net positive suction head	m
p	Pressure	Pa
pa	Air pressure	Pa
pvp	Vapor pressure	Pa
V	Velocity	m/s

Example 17.1:

Water (T=20^{o}C) flows through an eight-inch diameter asphalted cast iron pipe with a velocity of 2 m/s. The friction coefficient is computed from a Moody diagram to be 0.019. What is the head loss per kilometer length of pipe?



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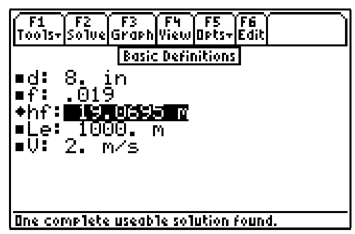


Computed results

Solution – Select the **first equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
d = 8 in
f = .019
Le = 1000 m
V = 2 m/s

Solution
hf = 19.0695 m

17.2 Pump Power

The following equations compute the energy (head) added by a pump, **hap** (m), to a flowing fluid. The **first equation** is a modified version of the Bernoulli equation. The variables, **po** (m) and **pi** (m) are the pressures at the outlet and inlet of the pump. **Vo** (m/s) and **Vi** (m/s) are the flow velocities at the outlet and inlet sides of the pump; **zh2** (m) is the height of the pump outlet and **zh1** (m) is the height of the inlet. The **second equation** computes the hydraulic horsepower, **whp** (W) from the mass flow rate, **Qm** (kg/s), and head added by the pump, **hap**. **Equation 3** computes the brake horsepower delivered to the pump shaft, **bhp** (W) from **whp** and the pump efficiency, **effp**. The friction horsepower, **fhp** (W), is calculated in **equation 4**. The electrical horsepower of the motor, **ehp** (W), is computed in **equation 5**. The **sixth equation** computes the overall efficiency of the pump, **effo**. The **last equation** computes the mass flow rate, **Qm**, from the volume flow rate, **Q** (m^3/s), and the fluid density, **ρ** (kg/m^3).

$$hap = \frac{po}{\rho} - \frac{pi}{\rho} + \frac{Vo^2}{2 \cdot grav} - \frac{Vi^2}{2 \cdot grav} + zh2 - zh1 \quad \textbf{Eq. 1}$$

$$whp = Qm \cdot grav \cdot hap \quad \textbf{Eq. 2}$$

$$bhp = \frac{whp}{effp} \quad \textbf{Eq. 3}$$

$$fhp = bhp - whp \quad \textbf{Eq. 4}$$

$$ehp = \frac{bhp}{effe} \quad \textbf{Eq. 5}$$

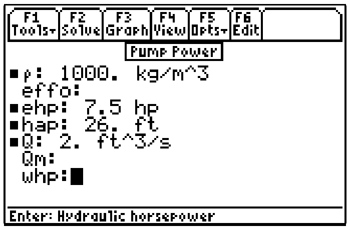
$$effo = \frac{whp}{ehp} \quad \textbf{Eq. 6}$$

$$Qm = Q \cdot \rho \quad \textbf{Eq. 7}$$

Variable	Description	Units
ρ	Density	kg/m^3
bhp	Brake horsepower	W
effe	Motor efficiency	unitless
effo	Overall efficiency	unitless
effp	Pump efficiency	unitless
ehp	Electric horsepower to the motor	W
fhp	Frictional horsepower	W
grav	Gravitational acceleration	9.80665 m/s^2
hap	Head added by pump	m
pi	Pressure at inlet	Pa
po	Pressure at outlet	Pa
Q	Discharge rate	m^3/s
Qm	Mass discharge rate	kg/s
Vi	Velocity at inlet	m/s
Vo	Velocity at outlet	m/s
whp	Hydraulic horsepower	W
zh1	Elevation at inlet	m
zh2	Elevation at outlet	m

Example 17.2:

An electric pump consuming 7.5 hp adds a 26 ft head to a flow rate of 2 ft^3/s. What is the overall efficiency of the pump? Assume the density of water is 1000 kg/m^3.



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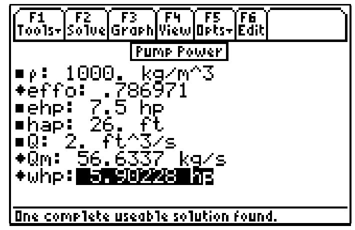


Computed results

Solution – Select **equations 2, 6** and **7** to solve this problem. Select these by highlighting the equations and pressing ENTER. Press F2 to display the variables. Enter the values for the known parameters and press F2 to solve for the unknown variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. The entries and results are shown in the screen displays above.

Given
ρ = 1000 kg/m^3
ehp = 7.5 hp
hap = 26 ft
Q = 2 ft^3/s

Solution
effo = .786971
Qm =56.6337 kg/s
whp = 5.90228 hp

17.3 Centrifugal Pumps

17.3.1 Affinity Law-Variable Speed

The following equations compute the change in pump performance (discharge rate, head and brake horsepower) with variation of pump speed, assuming the efficiency remains constant.

$$\frac{Qd2}{Qd1} = \frac{n2}{n1}$$ **Eq. 1**

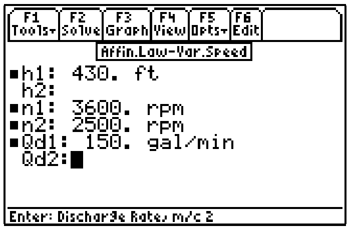
$$\frac{h2}{h1} = \left(\frac{n2}{n1}\right)^2$$ **Eq. 2**

$$\frac{bhp2}{bhp1} = \left(\frac{n2}{n1}\right)^3$$ **Eq. 3**

Variable	**Description**	**Units**
bhp1	Brake horsepower, machine 1	W
bhp2	Brake horsepower, machine 2	W
h1	Head for machine 1	m
h2	Head for machine 2	m
n1	Revolutions per second, machine 1	1/s
n2	Revolutions per second, machine 2	1/s
Qd1	Discharge rate, machine 1	m^3/s
Qd2	Discharge rate, machine 2	m^3/s

Example 17.3.1:

A 3600-rpm pump adds a head of 430 ft to a discharge rate of 150 gallons per minute. What will be the head and discharge rate for a similar pump at 2500 rpm?



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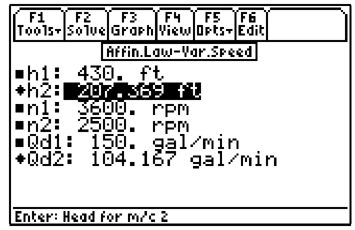


Computed results

Solution – Select the **first two equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
h1 = 430 ft
n1 = 3600 rpm
n2 = 2500 rpm
Qd1 = 150 gal/min

Solution
h2 = 207.369 ft
Qd2 = 104.167 gal/min

17.3.2 Affinity Law-Constant Speed

The following equations compute the variation of pump parameters (discharge rate, head and brake horsepower) with impeller diameter, for a homologous pump assuming the pump speed remains constant.

$$\frac{Qd2}{Qd1} = \frac{d2}{d1} \qquad \textbf{Eq. 1}$$

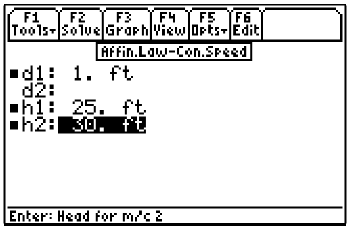
$$\frac{h2}{h1} = \left(\frac{d2}{d1}\right)^2 \qquad \textbf{Eq. 2}$$

$$\frac{bhp2}{bhp1} = \left(\frac{d2}{d1}\right)^3 \qquad \textbf{Eq. 3}$$

Variable	Description	Units
bhp1	Brake horsepower, machine 1	W
bhp2	Brake horsepower, machine 2	W
d1	Diameter, machine 1	m
d2	Diameter, machine 2	m
h1	Head for machine 1	m
h2	Head for machine 2	m
Qd1	Discharge rate, machine 1	m^3/s
Qd2	Discharge rate, machine 2	m^3/s

Example 17.3.2:

What ratio of impeller diameter increase is needed to raise the head from 25 to 30 ft?



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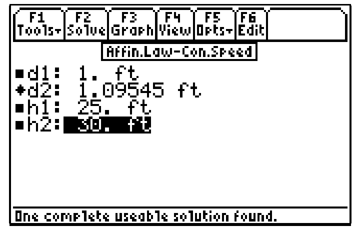


Computed results

Solution – Select the **second equation** to solve this problem. Since a ratio of diameter increase is being computed, enter an arbitrary value of 1 ft for **d1**. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
d1 = 1 ft
h1 = 25 ft
h2 = 30 ft

Solution
d2 = 1.09545 ft (~10% increase)

17.3.3 Pump Similarity

These equations are used to predict the performance of homologous pump (2) from the known parameters of another pump. These similarity laws assume that both pumps operate in the turbulent region, have the same operating efficiency, specific speed, cavitation number and the same percentage of wide-open flow.

$$\frac{n1 \cdot d1}{\sqrt{h1}} = \frac{n2 \cdot d2}{\sqrt{h2}} \qquad \textbf{Eq. 1}$$

$$Qd1 \cdot d2^2 \cdot \sqrt{h2} = Qd2 \cdot d1^2 \cdot \sqrt{h1} \qquad \textbf{Eq. 2}$$

$$\frac{bhp1}{bhp2} = \frac{\rho 1 \cdot d1^2 \cdot h1^{1.5}}{\rho 2 \cdot d2^2 \cdot h2^{1.5}} \qquad \textbf{Eq. 3}$$

$$\frac{Qd1}{Qd2} = \frac{n1 \cdot d1^3}{n2 \cdot d2^3} \qquad \textbf{Eq. 4}$$

$$\frac{bhp1}{bhp2} = \frac{\rho 1 \cdot n1^3 \cdot d1^5}{\rho 2 \cdot n2^3 \cdot d2^5} \qquad \textbf{Eq. 5}$$

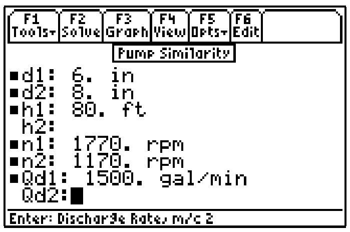
$$\frac{n1}{n2} = \sqrt{\frac{Qd2}{Qd1}} \cdot \left(\frac{h1}{h2}\right)^{\frac{3}{4}} \qquad \textbf{Eq. 6}$$

Variable	**Description**	**Units**
ρ1	Fluid density in machine 1	kg/m^3
ρ2	Fluid density in machine 2	kg/m^3
bhp1	Brake horsepower, machine 1	W
bhp2	Brake horsepower, machine 2	W

Variable	Description	Units
d1	Diameter, machine 1	m
d2	Diameter, machine 2	m
h1	Head for machine 1	m
h2	Head for machine 2	m
n1	Revolutions per second, machine 1	1/s
n2	Revolutions per second, machine 2	1/s
Qd1	Discharge rate, machine 1	m^3/s
Qd2	Discharge rate, machine 2	m^3/s

Example 17.3.3:

Compute the head and discharge capacity of a 8" pump operating at 1170 rpm from a similar pump having a diameter of 6", a discharge capacity of 1500 gallons per minute against a head pressure of 80 ft and operating at a speed of 1770 rpm.



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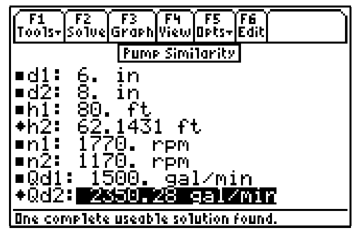


Computed results

Solution – Select the **first** and **fourth equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
d1 = 6 in
d2 = 8 in
h1 = 80 ft
n1 = 1770 rpm
n2 = 1170 rpm
Qd1 = 1500 gal/min

Solution
h2 = 62.1431 ft
Qd2 = 2350.28 gal/min

17.3.4 Centrifugal Compressor

Centrifugal compressors are used to increase pressures for compressible fluids. The theoretical power of a compressor, **Pth** (W), is computed for two cases in this section. The **first equation** assumes the compression process is adiabatic. The **second equation** computes **Pth** for an isothermal process in a water-cooled compressor. **Qd1** (m^3/s) is the volumetric flow entering the pump, **p1**and **p2** (Pa) are the pressures at the inlet and outlet of the pump, respectively.

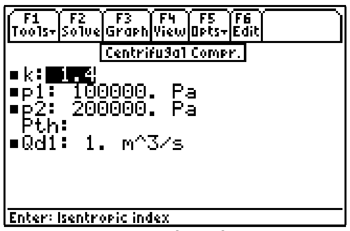
$$Pth = \frac{k}{k-1} \cdot Qd1 \cdot p1 \cdot \left(\left(\frac{p2}{p1} \right)^{\frac{k-1}{k}} - 1 \right) \quad \textbf{Eq. 1}$$

$$Pth = p1 \cdot Qd1 \cdot \ln\left(\frac{p2}{p1} \right) \quad \textbf{Eq. 2}$$

Variable	**Description**	**Units**
k	Isentropic index	unitless
p1	Pressure 1	Pa
p2	Pressure 2	Pa
Pth	Theoretical power	W
Qd1	Discharge rate, machine 1	m^3/s

Example 17.3.4:

What is the theoretical power for an air compressor having an intake rate of 1 m^3/s, increasing the pressure from 100 kPa, to 200 kPa. The specific heat ratio (isentropic index) for air is k=1.4.



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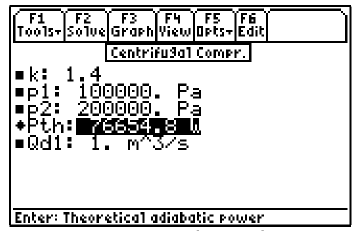


Computed results

Solution – Select the **first equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
k = 1.4	Pth = 76654.8 W
p1 = 100000 Pa	
p2 = 200000 Pa	
Qd1 = 1 m^3/s	

17.3.5 Specific Speed

The specific speed is a parameter used to rate the suitability of a pump or impeller type for a specific application. It is computed in the following equations from the pump parameters including the head coefficient, **CH**, discharge coefficient, **CQ**, rotational speed, **n** (1/s), head, **Δh** (m), and the rate of discharge, **Q** (m^3/s).

$$ns = \frac{CQ^{.5}}{CH^{.75}} \qquad \textbf{Eq. 1}$$

$$ns = \frac{n \cdot \sqrt{Q}}{(grav \cdot \Delta h)^{.75}} \qquad \textbf{Eq. 2}$$

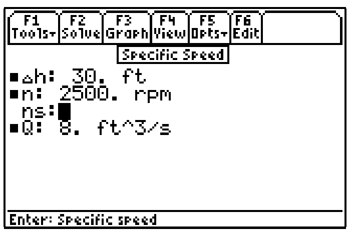
$$CH = \frac{\Delta h \cdot grav}{d^2 \cdot n^2} \qquad \textbf{Eq. 3}$$

$$CQ = \frac{Q}{n \cdot d^3} \qquad \textbf{Eq. 4}$$

Variable	**Description**	**Units**
Δh	Head	m
CH	Head coefficient	unitless
CQ	Discharge coefficient	unitless
d	Diameter	m
grav	Gravitational acceleration	9.80665 m/s^2
n	Revolutions per second	Hz
ns	Specific speed	unitless
Q	Discharge rate	m^3/s

Example 17.3.5:

Compute the specific speed for a pump having a rotational speed, 2500 rpm, a flow rate of 8 ft^3/s and a head increase of 30 ft.



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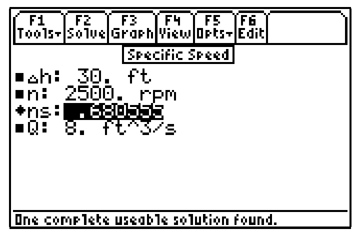


Computed results

Solution – Select the **second equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
Δh = 30 ft
n = 2500 rpm
Q = 8 ft^3/s

Solution
ns = .680555

References:

1. Michael R. Lindeburg, Mechanical Engineering Reference Manual, 8th Edition, Professional Publications, Belmont, CA 1990
2. John A. Roberson and Clayton T. Crowe, Engineering Fluid Mechanics, 5th Edition, Houghton Mifflin Company, Boston, MA, 1993

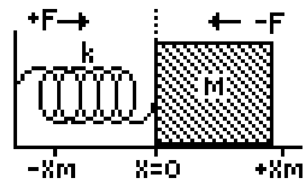
Chapter 18: Waves and Oscillation

This section contains equations describing oscillation or vibration of mechanical devices.The equations are presented under, three classifications listed below.

- Simple Harmonic Motion
- Pendulums
- Natural and Forced Vibrations

18.1 Simple Harmonic Motion

18.1.1 Linear Harmonic Oscillation



These equations describe the oscillation of a mass, **m** (kg), attached to the end of an ideal spring. The **first equation** is called Hooke's Law to compute the restoring force of the spring, **F** (N), in a direction opposite to the direction of displacement, **x** (m), from its equilibrium position. **Equation 2** calculates the natural frequency of vibration, **ω** (rad/s), of the system with a mass, **m**, and a spring stiffness, **k** (N/m). **Equation 3** computes the period of oscillation, **Tp** (s). The **last equation** calculates the oscillation frequency, **freq** (Hz).

$$F = -k \cdot x \quad \textbf{Eq. 1}$$

$$\omega = \sqrt{\frac{k}{m}} \quad \textbf{Eq. 2}$$

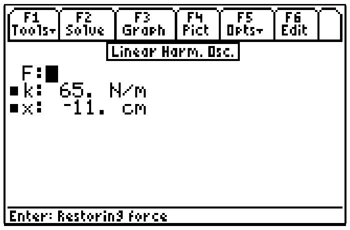
$$Tp = 2 \cdot \pi \cdot \sqrt{\frac{m}{k}} \quad \textbf{Eq. 3}$$

$$freq = \frac{1}{Tp} \quad \textbf{Eq. 4}$$

Variable	Description	Units
ω	Radian frequency	rad/s
F	Restoring force	N
freq	Frequency	Hz
k	Stiffness	N/m
m	Mass	kg
Tp	Period	s
x	Displacement from fest position	m

Example 18.1.1:

A block is attached to a spring having a stiffness of 65 N/m. The block is pushed, compressing the spring to a distance 11 cm from its equilibrium position. What is the restoring force exerted by the spring?



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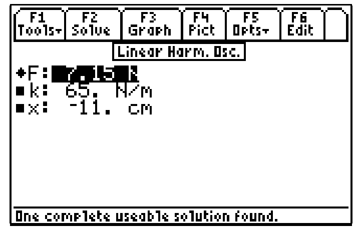
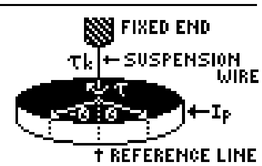


Computed results

Solution – Use the **first equation** to solve this problem. Select it by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter a negative value for displacement (x=-11 cm) since we want to compute the force in the opposite direction. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
k = 65 N/m	F = 7.15 N
x = -11 cm	

18.1.2 Angular Harmonic Oscillation



These equations describe the angular oscillation of a mass, with a rotational moment of inertia, **Ip** (kg·m^2), attached to the end of a torsion spring. If the mass is twisted (or displaced) by an angle, **θ** (rad), from its equilibrium position, the wire will exert a restoring torque, **τ** (N·m), in the opposite direction. **Equation 1** computes the period of oscillation, **Tp** (s), for the system. **Equation 2** computes the, restoring torque, **τ**, from the torsion constant of the suspension wire, **τk** (N·m/rad), and the displacement angle, **θ**. The **last equation** calculates the angular frequency of oscillation, **ω** (rad/s), from the period of oscillation, **Tp**.

$$Tp = 2 \cdot \pi \cdot \sqrt{\frac{Ip}{\tau k}} \qquad \textbf{Eq. 1}$$

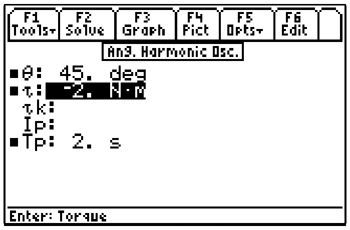
$$\tau = -\tau k \cdot \theta \qquad \textbf{Eq. 2}$$

$$\omega = \frac{2 \cdot \pi}{Tp} \qquad \textbf{Eq. 3}$$

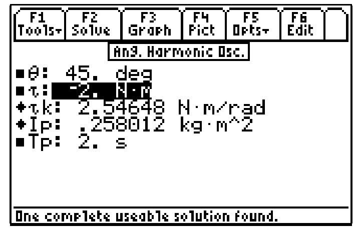
Variable	**Description**	**Units**
θ	Displacement angle	rad
τ	Torque	N·m
τk	Torsion constant	N·m/rad
ω	Radian frequency	rad/s
Ip	Rotational inertia	kg·m^2
Tp	Period of oscillation	s

Example 18.1.2:

A suspension wire has a restoring torque of -2 N·m at a displacement angle of 45^o. A mass is then attached to the end of a suspension and allowed to oscillate freely. The period for one oscillation is 2 s. Compute the torsion constant of the spring and the rotational moment of inertia for the mass.



Entered Values



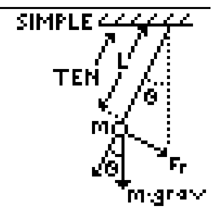
Computed results

Solution – Use the **first and second** equations to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
θ = 45 deg	τk = 2.54648 N·m/rad
τ = -2 N·m	Ip = .258012 kg·m^2
Tp = 2 s	

18.2 Pendulums

18.2.1 Simple Pendulum



The following equations describe the harmonic motion of a mass, **m** (kg), hanging from an ideal string (massless, non-stretchable) and having a length, **L** (m). The angle of displacement, **θ** (rad), is relative to a vertical line connecting the string to a mount. **Equation 1** computes the period of oscillation, **Tp** (s), for a simple pendulum with a string length, **L**. **Equation 2** calculates the tension, **Ten** (N), in the string at a displacement angle, **θ**, from the equilibrium position. **Equation 3** computes the restoring force, **Fr** (N), active in forcing the pendulum to its equilibrium position. The **last equation** calculates the oscillation frequency of the pendulum, **freq** (Hz).

$$Tp = 2 \cdot \pi \cdot \sqrt{\frac{L}{grav}} \qquad \textbf{Eq. 1}$$

$$Ten = -m \cdot grav \cdot \cos(\theta) \qquad \textbf{Eq. 2}$$

$$Fr = m \cdot grav \cdot \sin(\theta) \qquad \textbf{Eq. 3}$$

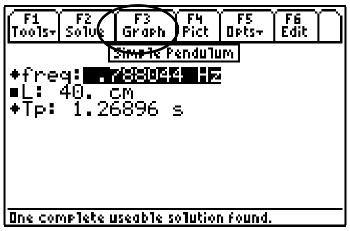
$$freq = \frac{1}{Tp} \qquad \textbf{Eq. 4}$$

Variable	Description	Units
θ	Displacement angle	rad
Fr	Restoring force	N
freq	Frequency	Hz
grav	Gravitation acceleration constant	9.08665 m/s^2
L	Length	m
m	Mass	kg

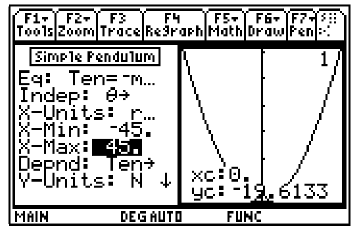
Variable	Description	Units
Ten	Tension	N
Tp	Period of oscillation	s

Example 18.2.1:

Compute the oscillation period and frequency for a simple pendulum having a cord length of 40 cm. If a mass, weighing 2 kg, is attached to the end of the pendulum, what is the maximum tension exerted by the gravitational field and angle of displacement for this tension?



Step 1: Computed results



Step 2: Graphing Ten vs. θ
(Tracing the minimum value of Ten using F3: Trace)

Solution – This solution can be computed in two steps. The TI graphing features will be used in the second part. **Step 1:** use the **first** and **fourth equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter a value for **L** = 40 cm and press [F2] to solve for **Tp** and **freq**. The entries and results are shown in the screen displays above. Note that the gravitational acceleration constant, **grav** = 9.08665 m/s^2, is automatically inserted into the calculation by the software and does not appear in the variable list.

Step 2: Check the [MODE] settings of your calculator to make sure the ANGLE mode is set to DEGREE. Access the mode screen by pressing [MODE], and move the highlight bar to ANGLE. If 'RADIAN' is listed, press the right arrow key ▶ and select DEGREE. Press [ENTER] twice to save the settings and return list of variables.

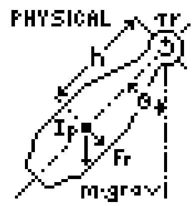
Continuing the solution for the example, press [ESC] to view the equations; select the **second equation**; press [F2] to display the list of variables; and enter **m** = 2 kg. Now press [F3] to graph the newly selected equation. Select the second equation, identify **θ** as the **Independent** variable and **ten** as the **Dependent** variable.

Enter **X-Min**: –45 and **X-Max**: 45.

Press [F3] to graph the selected equation. In this case, the graph appears in the right side of the screen. Press [2nd] and [APPS] to switch the active window from ME•*Pro* to the **Graph** interface. To switch back to ME•*Pro* press [2nd] and [APPS]. Press [F3] **Trace** and move the cursor to the minimum point (**θ**=0, **Ten**= **-**19.6 N). The value for **ten** is negative since the force is in a direction opposite to **ten** in the diagram. An alternative method for finding the minimum: Press [F5] **Math**, [3]: **Minimum**. Move the cursor to a location to the left of the minimum for the '**Lower Bound**' and press [ENTER], and then move the cursor to a location on the right of the minimum and press [ENTER] for the '**Upper Bound**'. To switch back to ME•*Pro* press [2nd] [APPS].

Given	Solution
L = 40 cm (Step 1)	freq = .788044 Hz
m = 2 kg (Step 2)	Tp = 1.26896 s
	Ten = **-**19.6133 (from Graph)

18.2.2 Physical Pendulum



A physical pendulum represents the behavior of real pendulums to a greater degree than the 'simple' case. The equations in this section describe a physical pendulum, having a mass, **m** (kg), and a rotational moment of inertia at the pivot point, **Ip** (kg·m^2), with its center of mass located at a distance, **h** (m), from the pivot point. When the pendulum is displaced from its equilibrium position at an angle, **θ** (rad), relative to the gravitational field, a restoring torque, **τr** (N·m) appears (**Eq.1**). The period of motion, **Tp** (s), for the physical pendulum can be calculated from **Ip**, **m**, and **h** using **equation 2**. The restoring force, **Fr** (N), acting at the center of mass is computed in the **last equation**.

$$\tau r = m \cdot grav \cdot h \cdot \sin(\theta) \qquad \textbf{Eq. 1}$$

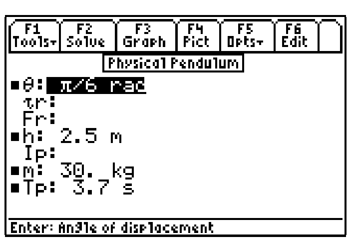
$$Tp = 2 \cdot \pi \cdot \sqrt{\frac{Ip}{m \cdot grav \cdot h}} \qquad \textbf{Eq. 2}$$

$$Fr = \frac{\tau r}{h} \qquad \textbf{Eq. 3}$$

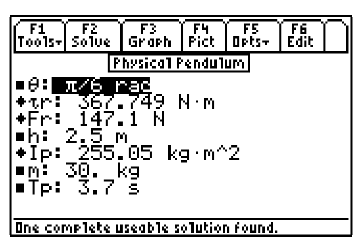
Variable	Description	Units
θ	Displacement angle	rad
τr	Restoring torque	N·m
Fr	Restoring force	N
grav	Gravitation acceleration constant	9.08665 m/s^2
h	Distance from pivot to center to	m
Ip	Rotational inertia	kg·m^2
m	Mass	kg
Tp	Period of oscillation	s

Example 18.2.2:

A steel cord weighing 30 kg and having a length of 5 m is suspended from a hook. What is the restoring torque and restoring force, at the center of mass, for a displacement angle of π/6 rad? Assume the center of mass is located at the midpoint of the cable. If the period of oscillation is 3.7 seconds, what is the moment of inertia of the cable at the pivot point?



Entered Values



Computed results

Solution – Select **all the equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
$\theta = \pi/6$ rad
h = 2.5 m
m = 30 kg
Tp = 3.7 s

Solution
τr = 367.749 N·m
Fr = 147.1 N
Ip = 255.05 kg·m^2

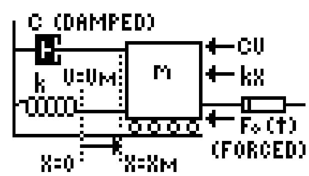
18.2.3 Torsional Pendulum

This menu heading reflects the contents of section 18.1.2 'Angular Harmonic Motion' and contains the same equation set.

18.3 Natural and Forced Vibrations

18.3.1 Natural Vibrations

18.3.1.1 Free Vibration



These equations compute the position and velocity of a free, linear, undamped oscillating system with respect to time, **t** (s). The **first equation** computes the displacement, **x** (m), of the oscillating system, relative to its equilibrium position (**x=0**) at time, **t**. The **second equation** computes the linear velocity of the system, **v** (m/s), at position, **x**, and time, **t**. **Equations 3** and **4** relate the angular offset, **ψ** (rad), used in **equations 1** and **2**, to the initial displacement, **xo** (m), the velocity at **t=0**, **vo** (m/s), and the natural oscillation frequency, **ωn** (rad/s). **Equation 5** computes the maximum displacement amplitude from the equilibrium position, **xm** (m). **Equation 6** computes the maximum velocity of the oscillating object, **vm** (m/s). **Equation 7** calculates the angular offset, **ψ**, from the initial position and velocity, **xo** and **vo,** and the natural frequency, **ωn**. **Equation 8** computes the natural vibration frequency, **ωn**, of the oscillating system from the mass of the object, **m** (kg), attached to a spring having a stiffness, **k** (N/m). The **last equation** calculates the period, **Tn** (s), of the natural vibration.

$$x = xm \cdot \sin(\omega n \cdot t + \psi)$$ **Eq. 1**

$$v = xm \cdot \omega n \cdot \cos(\omega n \cdot t + \psi)$$ **Eq. 2**

$$xo = xm \cdot \sin(\psi)$$ **Eq. 3**

$$vo = xm \cdot \omega n \cdot \cos(\psi)$$ **Eq. 4**

$$xm = \sqrt{xo^2 + \left(\frac{vo}{\omega n}\right)^2}$$ **Eq. 5**

$$vm = xm \cdot \omega n$$ **Eq. 6**

$$\psi = \tan^{-1}\left(\frac{xo \cdot \omega n}{vo}\right) \qquad \textbf{Eq. 7}$$

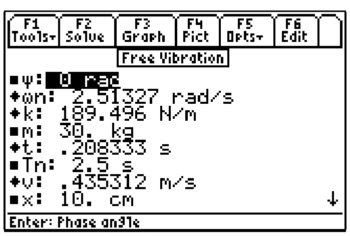
$$\omega n = \sqrt{\frac{k}{m}} \qquad \textbf{Eq. 8}$$

$$Tn = \frac{2 \cdot \pi}{\omega n} \qquad \textbf{Eq. 9}$$

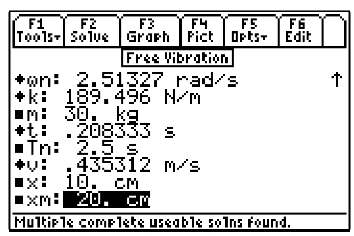
Variable	**Description**	**Units**
ψ	Phase angle	rad
ωn	Natural frequency	rad/s
k	Stiffness	N/m
m	Mass	kg
t	Time	s
Tn	Natural oscillation period	s
v	Velocity	m/s
vm	Maximum velocity	m/s
vo	Initial velocity	m/s
x	Displacement	m
xm	Maximum displacement from rest position	m
xo	Initial displacement	m

Example 18.3.1.1:

A mass (30 kg) attached to a spring oscillates once every 2.5 seconds and has a maximum displacement amplitude of 22 cm. If a stopwatch is set when the mass passes the equilibrium position of the spring, at what time will the mass reach a distance halfway between the equilibrium position and the position of maximum displacement? What will be the linear velocity at that time?



Upper Display: First Solution
Arbitrary Integer, 0 (Principal solution)

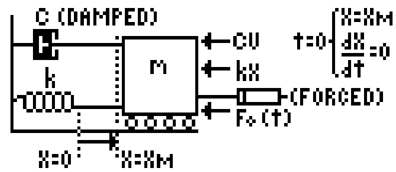


Lower Display: First Solution
Arbitrary Integer, 0 (Principal solution)

Solution – Select **Equations 1, 2, 8** and **9** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. **Select an arbitrary integer of 0** to compute the principal solution (the principal solution, P, in a periodic trigonometric function, trig (…), is P=trig (θ + n·π) and n is the arbitrary integer). **Select '1'** in the 'Multiple solutions dialog box to display the first solution. The entries and results are shown in the screen displays above.

Given	**Solution**
ψ = 0 rad	ωn = 2.51327 rad/s
m = 30 kg	k = 189.496 N/m
Tn = 2.5 s	t = .208333 s
x = 10 cm	v = .435312 m/s
xm = 20 cm	

18.3.1.2 Overdamped Case (ξ>1)



A damped oscillation system has a mechanism for removing mechanical energy from the vibrating system in the form of heat. **An over damped vibrating system is defined as the case where the damping ratio, ξ>1 (Eq.4), and the roots λ1 and λ2 (Equations 5 and 6) are distinct real negative numbers.** The dashpot, in the diagram to the right, has a viscous damping coefficient of **c** (N·s/m) and exerts a frictional force proportional to the velocity of the oscillating mass, **m** (kg). **Equation 1** computes the displacement of the mass from the rest position, **x** (m), at time, **t** (s), following release of the object. **Equations 2** and **3** calculate the constants, **A1** (m) and **A2** (m), for the case when the timer is started (**t=0**) when the mass is located at the amplitude of maximum displacement, **xm** (m), and the linear velocity ceases, (**dx/dt = 0**). **Equation 4** calculates the damping ratio, **ξ**, from the viscous damping coefficient, **c**, and the natural radian frequency, **ωn** (rad/s). **Equations 5** and **6** compute the roots, **λ1** (rad/s) and **λ2** (rad/s) from the damping ratio, **ξ**. The **last two equations** compute the natural frequency of the oscillating system, **ωn** (rad/s) and the natural period, **Tn** (s).

$$x = A1 \cdot e^{\lambda 1 \cdot t} + A2 \cdot e^{\lambda 2 \cdot t}$$ **Eq. 1**

$$A1 = xm \cdot \frac{\lambda 2}{\lambda 2 - \lambda 1}$$ **(When x=xm and dx/dt=0 at t=0)** **Eq. 2**

$$A2 = \frac{-xm \cdot \lambda 1}{\lambda 2 - \lambda 1}$$ **(When x=xm and dx/dt=0 at t=0)** **Eq. 3**

$$\xi = \frac{c}{2 \cdot \pi \cdot \omega n}$$ **Eq. 4**

$$\lambda 1 = \omega n \cdot \left(-\xi + \sqrt{\xi^2 - 1}\right)$$ **Eq. 5**

$$\lambda 2 = \omega n \cdot \left(-\xi - \sqrt{\xi^2 - 1}\right)$$ **Eq. 6**

$$\omega n = \sqrt{\frac{k}{m}}$$ **Eq. 7**

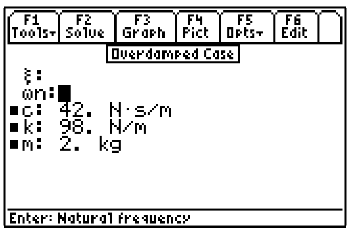
$$Tn = \frac{2 \cdot \pi}{\omega n}$$ **Eq. 8**

Variable	**Description**	**Units**
λ1	Natural radian frequency 1	rad/s

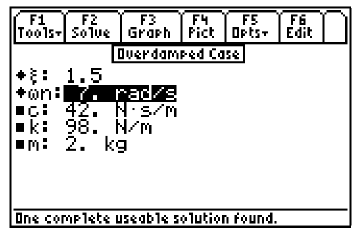
Variable	Description	Units
λ2	Natural radian frequency 2	rad/s
ξ	Damping ratio	unitless
ωn	Natural frequency	rad/s
A1	Constant, when t=0, x=xm	m
A2	Constant, when t=0, x=xm	m
c	Viscous damping coefficient	N·s/m
k	Stiffness	N/m
m	Mass	kg
t	Time	s
Tn	Natural oscillation period	s
x	Displacement from rest position	m
xm	Maximum displacement from rest position	m

Example 18.3.1.2:

What is the damping ratio for a damped oscillating system having a spring stiffness of 98 N/m, a viscous damping coefficient of 42 N·s/m, and a mass of 2 kg?



Entered Values

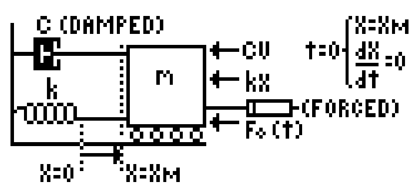


Computed results

Solution – Select the **fourth** and **seventh equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
c = 42 N·s/m	ξ = 1.5
k = 98 N/m	ωn = 7 rad/s
m = 2 kg	

18.3.1.3 Critical Damping (ξ=1)



Critical damping exists when the frequency of the damping mechanism is the same as the natural frequency, **ωn** (rad/s), or **ξ=1**. A critically damped system approaches equilibrium faster than the overdamped case. The following equations compute the unique case when the object is released at **t=0** (**x=xm** (m) and **dx/dt=0**). The coefficients for critically damped case (**λ1** and **λ2** in the overdamped case, section 18.3.1.2), are equivalent to the natural vibration frequency (i.e.**λ1 = λ2 = -ωn**). **Equation 1** computes the displacement from the equilibrium position, **x** (m), at time, **t** (s), following release. **Equation 2** computes the linear velocity, **v** (m/s), at time, **t**. **Equations 3** and **4** compute the coefficients, **A1** (m) and **A2c** (m·rad/s), from the initial (maximum) displacement, **xm = xo** (m) and the natural oscillation frequency, **ωn**. The **fifth equation** computes **ωn** from the spring constant, **k** (N/m), and the mass of the oscillating object, **m** (kg). The **last equation** calculates the period of natural vibration, **Tn** (s).

$$x = \left(A1 + A2c \cdot t\right) \cdot e^{-\omega n \cdot t}$$ **Eq. 1**

$$v = -\omega n \cdot \left(A1 + A2c \cdot t\right) \cdot e^{-\omega n \cdot t} + A2c \cdot e^{-\omega n \cdot t}$$ **Eq. 2**

$A1 = xm$ **(When x=xm and dx/dt=0 at t=0)** **Eq. 3**

$A2c = A1 \cdot \omega n$ **(When x=xm and dx/dt=0 at t=0)** **Eq. 4**

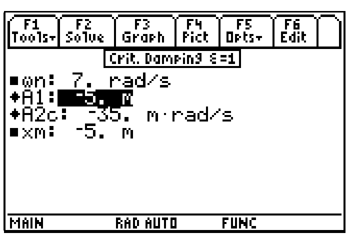
$$\omega n = \sqrt{\frac{k}{m}}$$ **Eq. 5**

$$Tn = \frac{2 \cdot \pi}{\omega n}$$ **Eq. 6**

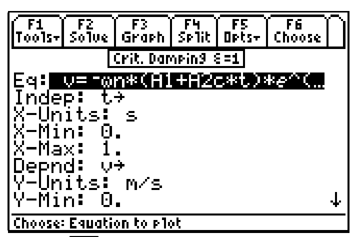
Variable	Description	Units
ωn	Natural frequency	rad/s
A1	Constant, when t=0, x=xm	m
A2c	Constant, when t=0, x=xm	m·rad/s
k	Stiffness	N/m
m	Mass	kg
t	Time	s
Tn	Natural oscillation period	s
v	Velocity	m/s
x	Displacement from rest position	m
xm	Maximum displacement from rest position	m

Example 18.3.1.3:

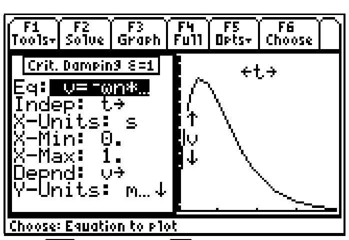
A critically damped system has a natural oscillation frequency of 7 rad/s and an initial displacement of 5 m. What is the maximum linear velocity between the time of release and the time the system has reached equilibrium?



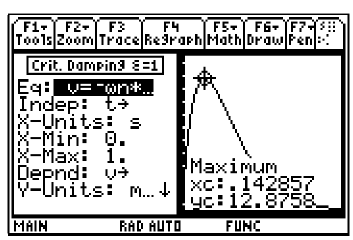
1: Entered values and computed results.



2: Press F3 for Graph Interface Screen.



3: F5: Math, 4: Maximum.
Select Lower and Upper bounds.

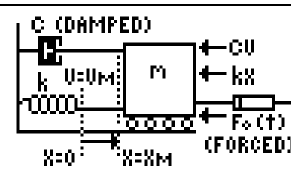


4: Result for maximum velocity.

Solution – This problem can be solved in **two steps**. **First**, compute the constants, **A1** and **A2c**, from **xm** and **ωn** using the **third** and **fourth equations**. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters. Enter a negative value for displacement since we want to compute a positive velocity in the other direction. Press [F2] to solve for the unknown variables: Frame 1. Next, press [F3]: **Graph** and use the right ▶ arrow key to display the equations in this set. Select the second equation by highlighting and pressing [ENTER]. Select **t** as the **independent** variable and **v** as the **dependent**. Enter **Xmin = 0** and **Xmax=1**, **Ymin=0** and **Ymax=15** (Frame 2). Deselect **Autoscale**. Press [F3]: **Graph** to plot the equation on the right side of the screen. **Press [2nd] [APPS] to switch from ME•*Pro* to the graphing window. The maximum velocity occurs at the maximum of the graph.** To locate the maximum, Press [F5]: **Math** and [4]: **Maximum**. Move the cursor to a point on the left side of the curve using the left arrow key ◀ and press [ENTER] (Frame 3). Next move to the right side of the curve using the right arrow key ▶ and press [ENTER]. The x and y coordinate of the maximum are displayed (Frame 4). **To switch back to ME•*Pro*, Press [2nd] [APPS].**

Given	**Solution**
ωn = 7 rad/s	A1 = -5 m
xm = -5 m	A2c = -35 rad·m/s
	v = 12.8758 m/s (maximum value)

18.3.1.4 Underdamped Case (ξ<1)



The following equations compute the vibration properties of an underdamped oscillation system. An underdamped system does not cease oscillating after a single period (ξ<1). The **first equation** computes the displacement, **x** (m), of the oscillating system relative to its equilibrium position, **x=0**, at time, **t** (s). The **second equation** computes the linear velocity of the system, **v** (m/s), at position, **x**, and time, **t**. **Equations 3** and **4** calculate the initial displacement, **xo** (m), and velocity, **vo** (m/s), at **t=0**. **Equation 5** computes the damping frequency, **ωd** (rad/s), for a system having a damping ratio, **ξ<1**. **Equation 6** computes the natural vibration frequency, **ωn**, of the oscillating system from the mass of the object, **m** (kg), attached to a spring having a stiffness, **k** (N/m). **Equation 7** computes the maximum displacement amplitude from the equilibrium position, **xm** (m). **Equation 8** computes the angular offset, **ψ** (rad), at **t=0**. **Equations 9** and **10** calculate the damping ratio, **ξ**, from either an observed value of the logarithmic damping factor, **δ** (m), or the viscous damping factor, **c** (N·s/m). **δ** represents the decrease in oscillation amplitude, **xm**, vs. time due to damping. The **last two equations** calculate the periods of the damping oscillation, **Td** (s) and the natural frequency of vibration, **Tn** (s).

$$x = xm \cdot e^{-\xi \cdot \omega n \cdot t} \cdot \sin(\omega d \cdot t + \psi) \quad \textbf{Eq. 1}$$

$$v = -\xi \cdot \omega n \cdot xm \cdot e^{-\xi \cdot \omega n \cdot t} \cdot \sin(\omega d \cdot t + \psi) + xm \cdot e^{-\xi \cdot \omega n \cdot t} \cdot \omega d \cdot \cos(\omega d \cdot t + \psi) \quad \textbf{Eq. 2}$$

$$xo = xm \cdot \sin(\psi) \quad \textbf{Eq. 3}$$

$$vo = -\xi \cdot \omega n \cdot xm \cdot \sin(\psi) + \omega d \cdot xm \cdot \cos(\psi) \quad \textbf{Eq. 4}$$

$$\omega d = \omega n \cdot \sqrt{1 - \xi^2} \quad \textbf{Eq. 5}$$

$$\omega n = \sqrt{\frac{k}{m}} \quad \textbf{Eq. 6}$$

$$xm = \sqrt{xo^2 + \left(\frac{vo}{\omega n}\right)^2} \qquad \textbf{Eq. 7}$$

$$\psi = \tan^{-1}\left(\frac{xo \cdot \omega n}{vo}\right) \qquad \textbf{Eq. 8}$$

$$\xi = \frac{\delta}{\sqrt{4 \cdot \pi^2 + \delta^2}} \qquad \textbf{Eq. 9}$$

$$\xi = \frac{c}{2 \cdot m \cdot \omega n} \qquad \textbf{Eq. 10}$$

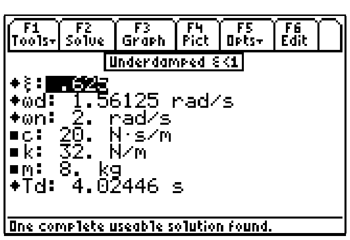
$$Td = \frac{2 \cdot \pi}{\omega d} \qquad \textbf{Eq. 11}$$

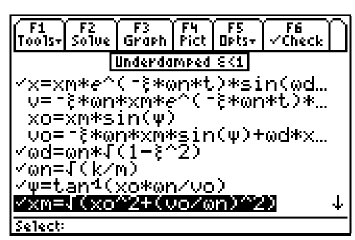
$$Tn = \frac{2 \cdot \pi}{\omega n} \qquad \textbf{Eq. 12}$$

Variable	**Description**	**Units**
δ	Logarithmic decrement	unitless
ξ	Damping ratio	unitless
ψ	Phase angle	rad
ωd	Damping frequency	rad/s
ωn	Natural frequency	rad/s
c	Viscous damping coefficient	N·s/m
k	Stiffness	N/m
m	Mass	kg
t	Time	s
Td	Underdamped oscillation period	s
Tn	Natural oscillation period	s
v	Velocity	m/s
vo	Initial velocity	m/s
x	Displacement	m
xm	Maximum displacement from rest position	m
xo	Initial displacement	m

Example 18.3.1.4:

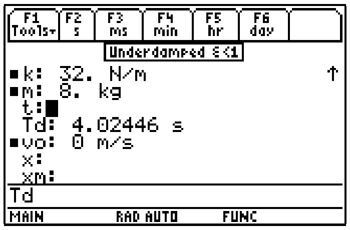
An 8 kg mass attached to a damped spring is released at a distance of 0.2 m from its equilibrium position. The spring stiffness is 32 N/m and the viscous damping coefficient is 20 N·s/m. Compute the amplitude after one complete oscillation.

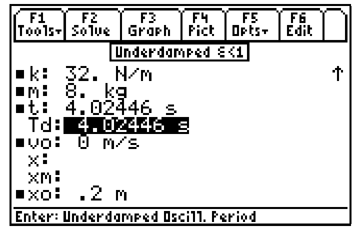




1: Entered Values and computed Results

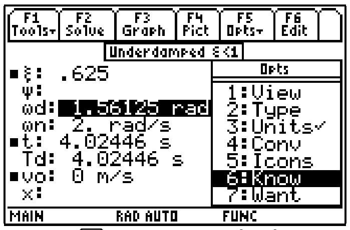
2: Equations Screen

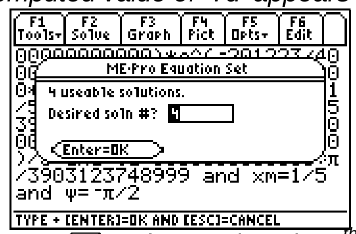




3: Enter 'Td' for 't'.

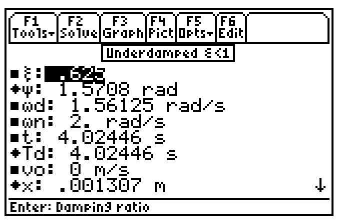
4: Computed value of 'Td' appears for 't'.



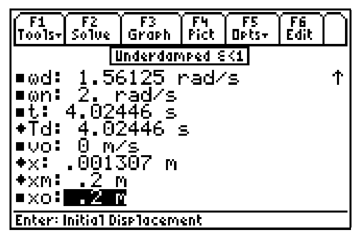


5: Use F5*:Opts,* 6*: Know to designate previously computed values as Known '▪'.*

6: Press F2*: Solve. Select the 4th of 4 possible solutions (ψ = π/2).*



7: Results: Upper Display



8: Results: Lower Display

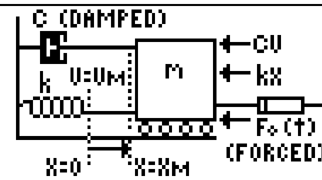
Solution – Compute this solution in two steps. The time, **t**, of a complete oscillation is equivalent to the damped period, **Td**. The computed value of **Td** will need to be entered for **t** in the first equation. To compute the period of the damped oscillation, **Td**, select the **fifth, sixth, tenth**, and **eleventh** equations. Select these by highlighting the equations and pressing ENTER. Press F2 to display the variables. Enter the values for the known parameters (**c, k,** and **m**) and press F2 to solve for the unknown variables (**ωd, ωn, ξ,** and **Td**). Next, press ESC to view the equation list. **Deselect Equations 5, 6** and **10** by highlighting the equations and pressing ENTER and **select equations 1, 7** and **8**; i.e.: **Equations. 1, 7, 8,** and **11** should now be selected. Press F2 to display the variables. Move the cursor to each of the variables, **ξ, ωn,** and **ωd**. Press F5:**Opts**, 6: **Know** to select each of the previously computed values to an entered value for computations. For **t**, enter the variable '**Td**' and press ENTER, the computed value of **Td** will appear for **t**. Enter values for **xo, vo** and press F2 to solve for the unknown variables (**ψ, x** and **xm**). When the ME•*Pro* dialogue box appears, select the fourth solution (**ψ** = π/2). The entries and results are shown in the screen displays above.

Given
c = 20 N·s/m
k = 32 N/m
m = 8 kg
vo = 0 m/s
xo = .2 m

Solution
ξ = .625
ψ = 1.5708 rad
ωd = 1.56125 rad/s
ωn = 2 rad/s
t = 4.02446 s (computed value of Td)
Td = 4.02446 s
x = .001307 m (less than 1% of xm)
xm = .2 m

18.3.2 Forced Vibrations

18.3.2.1 Undamped Forced Vibration



These equations describe the steady state oscillation conditions of an undamped system, experiencing an external periodic force, **Fo** (N). The undamped system has a natural vibration frequency, **ωn** (rad/s). The external vibration force has a frequency and magnitude of **ωf** (rad/s) and **Fo**. The **first equation** computes the particular solution (solution of the system in steady state) of the position, **xp** (m), vs. time, **t** (s). The equation assumes the velocity ceases, **(dx/dt) = 0**, when **xp = xm** at **t = 0 s**. The amplitude of the forced, undamped system, **xm** (m), is computed in **equation 2** from the amplitude of the forced oscillation, **xf** (m), the angular frequency of the natural motion, **ωn**, and the forced motion, **ωf**. The magnification factor, **Mag**, in **Equation 3**, computes the degree of resonance between the forced and natural resonance frequencies. As **ωf** approaches **ωn**, the limits of the attached spring are tested as **Mag** and **xm** approach infinity. This condition is to be avoided. If **ωf<ωn**, **Mag>0** and the vibration is in phase with the force, **Fo**. If **ωf>ωn**, the vibration is out of phase with the external force, **Fo** and **Mag<0**. **Equation 4** computes the natural frequency of the oscillating system, **ωn**, from the spring constant, **k** (N/m), and the mass attached to the spring, **m** (kg). **Equation 5** calculates the amplitude of the external force, **Fo**, on the oscillating system.

$$xp = xm \cdot \cos(\omega f \cdot t) \quad \textbf{Eq. 1}$$

$$xm = \frac{xf}{1 - \left(\frac{\omega f}{\omega n}\right)^2} \quad \textbf{Eq. 2}$$

$$Mag = \frac{1}{1 - \left(\frac{\omega f}{\omega n}\right)^2} \quad \textbf{Eq. 3}$$

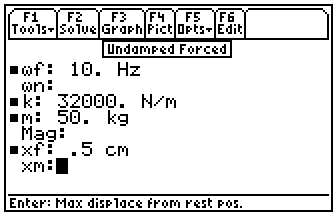
$$\omega n = \sqrt{\frac{k}{m}} \quad \textbf{Eq. 4}$$

$$Fo = k \cdot xf \quad \textbf{Eq. 5}$$

Variable	Description	Units
ωf	Forced radian frequency	rad/s
ωn	Natural frequency	rad/s
Fo	Magnitude of driving force	N
k	Stiffness	N/m
m	Mass	kg
Mag	Magnification	unitless
t	Time	s
xf	Forced displacement amplitude	m
xm	Maximum displacement of steady state	m
xp	Displacement: particular solution	m

Example 18.3.2.1:

A machine weighing 50 kg is mounted on four springs, each having a spring constant of 8000 N/m. The machine is mounted on a vibrating platform, having a vibration frequency of 10 Hz and a displacement of 0.5 cm. What is the maximum displacement of the forced system under steady-state conditions? Is the forced vibration in or out of phase with the natural vibration frequency? What is the maximum displacement of the forced system?



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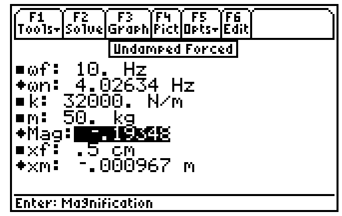
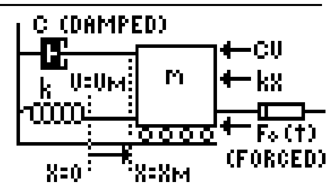


Computed results

Solution – Select the **second, third,** and **fourth** equations to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
ωf = 10 Hz	ωn = 4.02634 Hz
k = 32000 (4 x 8000) N/m	Mag = -1.9348 (out of phase)
m = 50 kg	xm = -.09674 cm (out of phase)
xf = .5 cm	

18.3.2.2 Damped Forced Vibration



These equations describe the features of a damped oscillating system experiencing a periodic, external vibration force having an angular frequency, **ωf** (rad/s) and amplitude, **xf** (m). The **first equation** computes the particular solution of the displacement amplitude, **xp** (m), at time, **t** (s), for the steady state condition of forced oscillation. The equation assumes **(dx/dt) = 0** and **xp = xm** at **t = ψ/ωf**. **Equation 2** calculates the maximum displacement of the damped, forced system, **xm** (m), from the natural frequency of the oscillating system, **ωn** (rad/s), the damping ratio, **ξ**, the forced amplitude, **xf**, and the frequency, **ωf**, of the external vibration. The phase offset, **ψ** (rad), or the degree the amplitude of response, **xp** (m), lags the amplitude of the applied force, **xf**, is computed in **equation 3**. The magnification factor, **Mag** (unitless), represents a degree of resonance between the forced, **ωf**, and natural, **ωn**, oscillation frequencies is computed in **Equation 4**. Resonance is achieved when **ψ = 90** deg for all values of **ξ**. **Equation 5** computes the damping ratio, **ξ**, from the viscous damping coefficient, **c** (N·s/m), of the damping system. The damping force (**c·v**) is proportional to the velocity of the spring, **v** (m/s). The maximum damping occurs when **ωf = ωn**. **Equation 6** computes the natural frequency of the oscillating system, **ωn**, from the spring stiffness, **k** (N/m), and the mass attached to the spring, **m** (kg). **Equation 7** calculates the maximum force, **Fo** (N), applied to the base of the system, as the sum of the spring and damping forces. The **last two equations** calculate the periods of the natural frequency of vibration, **Tn** (s), and the forced oscillation, **Tf** (s)

$$xp = xm \cdot \cos(\omega f \cdot t - \psi) \qquad \textbf{Eq. 1}$$

$$xm = \frac{xf}{\left(\left(1 - \left(\frac{\omega f}{\omega n}\right)^2\right)^2 + \left(2 \cdot \xi \cdot \frac{\omega f}{\omega n}\right)^2\right)^{.5}}$$ **Eq. 2**

$$\psi = \tan^{-1}\left(\frac{2 \cdot \xi \cdot \frac{\omega f}{\omega n}}{1 - \left(\frac{\omega f}{\omega n}\right)^2}\right)$$ **Eq. 3**

$$Mag = \frac{1}{\left(\left(1 - \left(\frac{\omega f}{\omega n}\right)^2\right)^2 + \left(2 \cdot \xi \cdot \frac{\omega f}{\omega n}\right)^2\right)^{0.5}}$$ **Eq. 4**

$$\xi = \frac{c}{2 \cdot \pi \cdot \omega n}$$ **Eq. 5**

$$\omega n = \sqrt{\frac{k}{m}}$$ **Eq. 6**

$$Fo = xm\sqrt{k^2 + c^2 \cdot \omega f^{\,2}}$$ **Eq. 7**

$$Tn = \frac{2 \cdot \pi}{\omega n}$$ **Eq. 8**

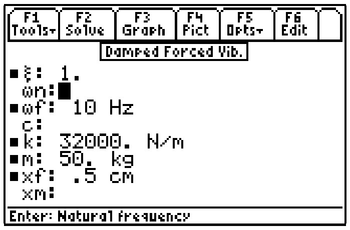
$$Tf = \frac{2 \cdot \pi}{\omega f}$$ **Eq. 9**

Variable	Description	Units
ξ	Damping ratio	unitless
ψ	Phase angle	rad
ωf	Forced radian frequency	rad/s
ωn	Natural frequency	rad/s
c	Viscous damping coefficient	N·s/m
Fo	Magnitude of driving force	N
k	Stiffness	N/m
m	Mass	kg
Mag	Magnification	unitless
t	Time	s
Tf	Forced oscillation period	s
Tn	Natural oscillation period	s
xf	Displacement due to Fo	m
xm	Maximum displacement of steady state	m

Variable	**Description**	**Units**
xp	Displacement: particular solution	m

Example 18.3.2.2:

The machine in Example 18.3.2.1 is equipped with four damp pads. What value for the viscous damping coefficient must each pad have to critically damp the system?



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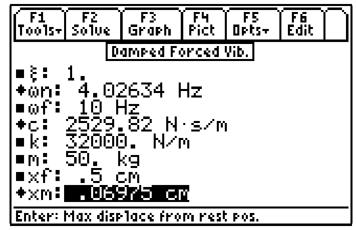


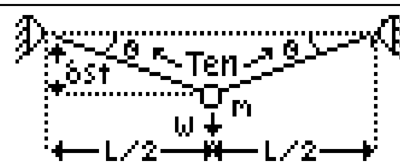
Computed results

Solution – A system is critically damped when **ξ=1**. **Equations 2**, **5**, and **6** are needed to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. The computed viscous damping coefficient value (2529.82 N·s/m), must be divided by four to compute the **c** for each pad (632.455 N·s/m).

Given	**Solution**
ξ = 1	ωn = 4.02634 Hz
ωf = 10 Hz	c = 2529.82 N·s/m (4 x 632.455 N·s/m)
k = 32000 N/m	xm = .06975 cm (86% reduction)
m = 50 kg	
xf = .5 m	

18.3.3 Natural Frequencies

18.3.3.1 Stretched String



The **first equation** calculates the natural frequency, **f1** (Hz), of longitudinal vibration for a hanging mass, **m** (kg), located at the center of a string. The string has two fixed ends located at distance, **L** (m), apart at the same height on an axis perpendicular to the gravitational field. **Ten** (N) is the tension in the string. The **second equation** computes the tension in the string, **Ten**, from the load and the deflection angle, **θ** (rad), from the horizontal axis. The load, **W** (N), is computed in the **third equation** from the mass of the hanging object, **m**, and the gravitational constant, **grav** (9.08665 m/s^2). The **last equation** computes the deflection, **δst** (m), the vertical distance of the object from the horizontal axis between the two mounts from the angle deflection at each end, **θ** (rad). The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables.

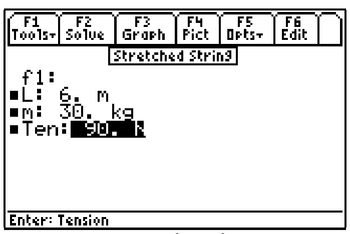
$$f1 = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{4 \cdot Ten}{m \cdot L}} \qquad \textbf{Eq. 1}$$

$Ten = \dfrac{W}{2 \cdot \sin\theta}$	**Eq. 2**
$W = m \cdot grav$	**Eq. 3**
$\tan\theta = \dfrac{2 \cdot \delta st}{L}$	**Eq. 4**

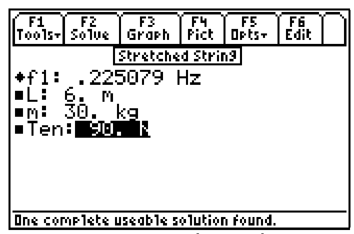
Variable	**Description**	**Units**
δst	Deflection	m
θ	Deflection angle	rad
f1	Natural frequency due to load (W)	Hz
grav	Gravitational acceleration	9.80665 m/s^2
L	Length	m
m	Mass	kg
Ten	Tension	N
W	Point Load	N

Example 18.3.3.1:

Compute the natural frequency for a mass of 3 kg hanging from a string at a horizontal distance of 3 m from each fixed point. The tension of the string is 90 N.



Entered Values

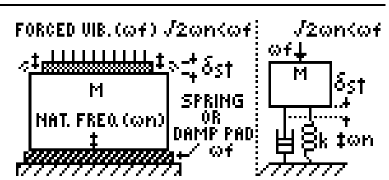


Computed results

Solution – Select the **first equation** to solve this problem. Highlight the equation using the cursor and press [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
L = 6 m	f1 = .225079 Hz
m = 30 kg	
Ten = 90 N	

18.3.3.2 Vibration Isolation



The isolation of equipment from a vibrating platform can be accomplished using pads or damped springs. The damped vibration system is similar to the damped forced vibration schematic on the right side of the diagram. The **first equation** computes the natural frequency of the damped vibration system, **ωn** (rad/s). **Equation 2** computes the amplitude of displacement, **δst** (m) for the vibrating system. **Equation 3** calculates the isolation efficiency, **eff**, from the forced vibration. **For a reduction in vibration to occur, the following condition must be met: √2·ωn<ωf**. The **last equation** relates the load, **W** (N), on the oscillating system to the mass of the machine, **m** (kg).

$$\omega n = \sqrt{\frac{grav}{\delta st}}$$ **Eq. 1**

$$\delta st = \frac{W}{k}$$ **Eq. 2**

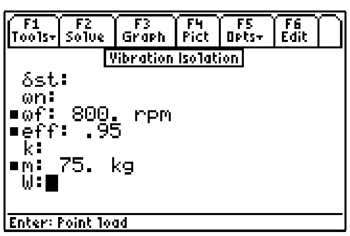
$$eff = 1 - \frac{1}{\left(\frac{\omega f}{\omega n}\right)^2 - 1}$$ Eq. 3

$$W = m \cdot grav$$ **Eq. 4**

Variable	**Description**	**Units**
δst	Deflection	m
ωn	Natural frequency due to load (W)	rad/s
ωf	Forcing frequency	rad/s
eff	Isolation efficiency	unitless
grav	Gravitation acceleration constant	9.08665 m/s^2
k	Stiffness	N/m
m	Mass	kg
W	Point load	N

Example 18.3.3.2:

A centrifugal fan (75 kg), rotating at 800 rpm, requires an isolation efficiency of 95%. What spring constant is required for the isolating material?



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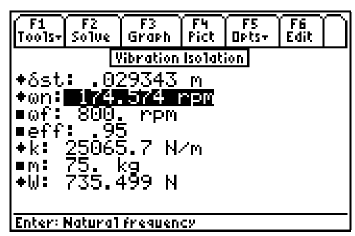


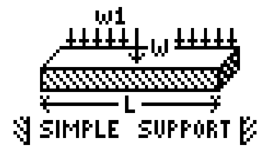
Computed results

Solution – **Select all of the equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
ωf = 800 rpm	δst = .029343 m
eff = .95	ωn = 174.574 rpm
m = 75 kg	k = 25065.7 N/m
	W = 735.499 N

18.3.3.3 Uniform Beams

18.3.3.3.1 Simply Supported



The following equations compute the natural frequency for a simply supported beam of length, **L** (m), with a point load, **W** (N), uniform load, **w1** (N/m), or both. The **first equation** computes the natural frequency of vibration, **f1** (Hz), for a beam having a length, **L**, an area moment of inertia, **I** (m^4), a modulus of elasticity (Young's modulus), **E** (Pa), and a point load, **W**, at the center of the beam. **Equation 2** calculates the frequency of vibration, **fn** (Hz), for a vibration mode (harmonic), **n**, for the same beam, having a uniform load, **w1**, instead of a point load, **W**. **Equation 3** computes the natural frequency of vibration, **f2** (Hz), for a beam with a combined point and uniform load (**W** and **w1**). Values for the vibration mode-based constant, **Kn**, in the second equation, are listed in **Table 18.1** for different vibration modes (harmonics).

Table 18.1: Kn for a simply supported, uniform beam, length L, vibration mode, n (Ref. #6)

Mode (n)	Kn	Nodal positions/L					
1	9.87	0.0	1.00				
2	39.5	0.0	0.50	1.00			
3	88.8	0.0	0.33	0.67	1.00		
4	158	0.0	0.25	0.50	0.75	1.00	
5	247	0.0	0.20	0.40	0.60	0.80	1.00

$$f1 = \frac{6.93}{2 \cdot \pi} \sqrt{\frac{E \cdot I \cdot grav}{W \cdot L^3}} \qquad \textbf{Eq. 1}$$

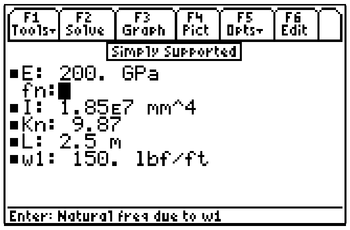
$$fn = \frac{Kn}{2 \cdot \pi} \cdot \sqrt{\frac{E \cdot I \cdot grav}{w1 \cdot L^4}} \qquad \textbf{Eq. 2}$$

$$f2 = \frac{6.93}{2 \cdot \pi} \sqrt{\frac{E \cdot I \cdot grav}{W \cdot L^3 + .486 \cdot w1 \cdot L^4}} \qquad \textbf{Eq. 3}$$

Variable	Description	Units
E	Young's modulus	Pa
f1	Natural frequency due to W	Hz
f2	Natural frequency due to W and w1	Hz
fn	Frequency for w1, mode 'n'	Hz
grav	Gravitation acceleration constant	9.08665 m/s^2
I	Area moment of inertia	m^4
Kn	Constant for vibration mode, 'n'	unitless
L	Beam length	m
W	Point load	N
w1	Load per unit length (Uniform)	N/m

Example 18.3.3.3.1:

Compute the natural frequency of a simply supported beam (**L** = 2.5 m, **E**=200 GPa, **I**=18.5 x 10^6 mm^4) having a uniform load of 150 lb/ft.



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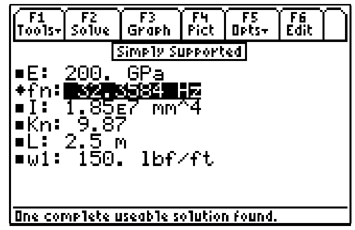


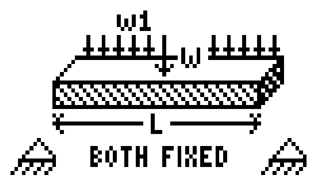
Computed results

Solution – Select the **second equation**. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Since we are calculating the natural frequency, the vibration mode-based constant is 1 and a value of **Kn = 9.87** is selected from **Table 18.1**. The entries and results are shown in the screen displays above.

Given
E = 200 GPa
I = 1.85 E 7 mm^4
Kn = 9.87
L = 2.5 m
w1 = 150 lb/ft

Solution
fn = 32.3584 Hz

18.3.3.3.2 Both Ends Fixed



The following equations compute the natural frequency for a uniform beam with both ends fixed. The **first equation** computes the natural frequency of vibration, **f1** (Hz), for a beam having a length, **L**, an area moment of inertia, **I** (m^4), a modulus of elasticity (Young's modulus), **E** (Pa), and a point load, **W** at the center of the beam. **Equation 2** calculates the frequency of vibration, **fn**, for a vibration mode (harmonic), **n**, for the same beam, having a uniform load, **w1**, instead of a point load, **W**. **Equation 3** computes the natural frequency of vibration, **f2** (Hz), for a beam with a combined point and uniform load (**W** and **w1**). Values for the vibration mode-based constant, **Kn**, in the second equation, are listed in **Table 18.2** for different vibration modes.

Table 18.2: Kn for a uniform beam, both ends fixed, length L, for vibration mode, n (Ref.#6).

n	Kn	Nodal positions/L					
1	22.4	0.0	1.00				
2	61.7	0.0	0.50	1.00			
3	121	0.0	0.36	0.64	1.00		
4	200	0.0	0.28	0.50	0.72	1.00	
5	299	0.0	0.23	0.41	0.59	0.77	1.00

$$f1 = \frac{13.86}{2 \cdot \pi} \cdot \sqrt{\frac{E \cdot I \cdot grav}{W \cdot L^3}}$$ **Eq. 1**

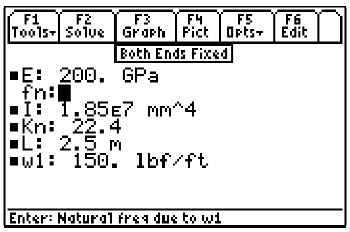
$$fn = \frac{Kn}{2 \cdot \pi} \cdot \sqrt{\frac{E \cdot I \cdot grav}{w1 \cdot L^4}}$$ **Eq. 2**

$$f2 = \frac{13.86}{2 \cdot \pi} \cdot \sqrt{\frac{E \cdot I \cdot grav}{W \cdot L^3 + .383 \cdot w1 \cdot L^4}}$$ **Eq. 3**

Variable	**Description**	**Units**
E	Young's modulus	Pa
f1	Natural frequency for W	Hz
f2	Natural frequency for W and w1	Hz
fn	Frequency for w1, mode 'n'	Hz
grav	Gravitation acceleration constant	9.08665 m/s^2
I	Area moment of inertia	m^4
Kn	Constant for vibration mode, 'n'	unitless
L	Beam length	m
W	Point load	N
w1	Load per unit length (Uniform)	N/m

Example 18.3.3.3.2:

Compute the natural frequency of a uniform beam with both ends fixed (**L** = 2.5 m, **E**=200 GPa, **I**=18.5 x 10^6 mm^4) having a uniform load of 150 lb/ft.



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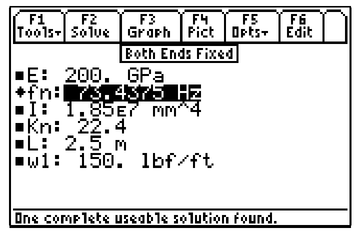
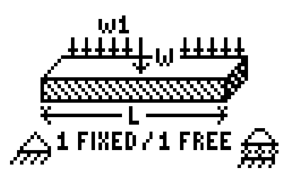


Computed results

Solution – Select the **second equation**. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Since we are calculating the natural frequency, the vibration mode-based constant is 1 and a value of **Kn** = 22.4 is selected from **Table 18.2.** The entries and results are shown in the screen displays above.

Given	**Solution**
E = 200 GPa	fn = 73.4375 Hz
I = 1.85 E7 mm^4	
Kn = 22.4	
L = 2.5 m	
w1 = 150 lb/ft	

18.3.3.3.3 1 Fixed End / 1 Free End



The following equations compute the natural frequency for a beam with one end fixed and one end free. The **first equation** computes the natural frequency of vibration, **f1** (Hz), for a beam having a length, **L**, an area moment of inertia, **I** (m^4), a modulus of elasticity (Young's modulus), **E** (Pa), and a point load, **W** at the center of the beam. **Equation 2** calculates the frequency of vibration, **fn**, for a vibration mode (harmonic), **n**, for the same beam, having a uniform load, **w1**, instead of a point load, **W**. **Equation 3** computes the natural frequency of vibration, **f2** (Hz), for a beam with a combined point and uniform load (**W** and **w1**). Values for the vibration mode-based constant, **Kn**, in the second equation, are listed in **Table 18.3** for different vibration modes.

Table 18.3:
Kn for a uniform beam, length L, One fixed/One free end, vibration mode, n (Ref.#6).

n	Kn	Nodal positions/L				
1	3.52	0.0				
2	22.0	0.0	0.783			
3	61.7	0.0	0.504	0.868		
4	121	0.0	0.358	0.644	0.905	
5	200	0.0	0.279	0.500	0.723	0.926

$$f1 = \frac{1.732}{2 \cdot \pi} \cdot \sqrt{\frac{E \cdot I \cdot grav}{W \cdot L^3}}$$ **Eq. 1**

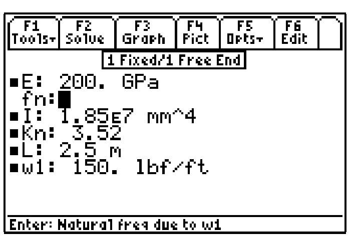
$$fn = \frac{Kn}{2 \cdot \pi} \cdot \sqrt{\frac{E \cdot I \cdot grav}{w1 \cdot L^4}}$$ **Eq. 2**

$$f2 = \frac{1.732}{2 \cdot \pi} \cdot \sqrt{\frac{E \cdot I \cdot grav}{W \cdot L^3 + .236 \cdot w1 \cdot L^4}}$$ **Eq. 3**

Variable	Description	Units
E	Young's Modulus	Pa
f1	Natural frequency due to W	Hz
f2	Natural frequency due to W and w1	Hz
fn	Frequency for w1, mode 'n'	Hz
grav	Gravitation acceleration constant	9.08665 m/s^2
I	Area moment of inertia	m^4
Kn	Constant for vibration mode, 'n'	unitless
L	Beam length	m
W	Point load	N
w1	Load per unit length (Uniform)	N/m

Example 18.3.3.3.3:

Compute the natural frequency of a uniform beam with both ends fixed (**L** = 2.5 m, **E**=200 GPa, **I**=18.5 x 10^6 mm^4) having a uniform load of 150 lb/ft.



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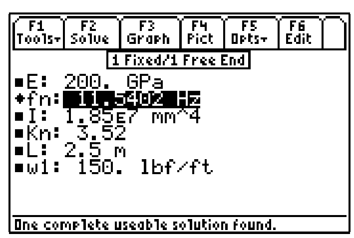


Computed results

Solution – Select the **second equation**. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Since the natural frequency is being calculated, the vibration mode-based constant is, **n=1**, and a value of **Kn** = 3.52 is selected from **Table 18.3**. The entries and results are shown in the screen displays above.

Given
E = 200 GPa
I = 1.85 E 7 mm^4
Kn = 3.52
L = 2.5 m
w1 = 150 lb/ft

Solution
fn = 11.5402 Hz

18.3.3.3.4 Both Ends Free

The following equation computes the frequency of vibration, **fn** (Hz), for a uniform beam with both ends free. The beam has a length **L** (m), an area moment of inertia, **I** (m^4), a modulus of elasticity (Young's modulus), **E** (Pa), and a uniform load, **w1** (N/m). Values for the vibration mode-based constant, **Kn**, are listed in **Table 18.4** for different vibration modes, **n**.

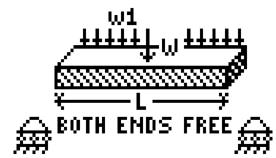


Table 18.4: Kn for a uniform beam, length L, both ends free, vibration mode, n (Ref.#6).

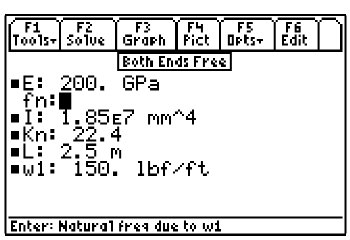
n	Kn	Nodal positions/L					
1	22.4	0.224	0.776				
2	61.7	0.132	0.500	0.868			
3	121	0.095	0.356	0.644	0.905		
4	200	0.074	0.277	0.500	0.723	0.926	
5	299	0.060	0.226	0.409	0.591	0.774	0.940

$$fn = \frac{Kn}{2 \cdot \pi} \cdot \sqrt{\frac{E \cdot I \cdot grav}{w1 \cdot L^4}} \qquad \textbf{Eq. 1}$$

Variable	Description	Units
E	Young's modulus	Pa
fn	Frequency for w1, mode 'n'	Hz
grav	Gravitation acceleration constant	9.08665 m/s^2
I	Area moment of inertia	m^4
Kn	Constant for vibration mode, 'n'	unitless
L	Beam length	m
w1	Load per unit length (Uniform)	N/m

Example 18.3.3.3.4:

Compute the natural frequency of a uniform beam with both ends free (**L** = 2.5 m, **E**=200 GPa, **I**=18.5 x 10^6 mm^4) having a uniform load of 150 lb/ft.



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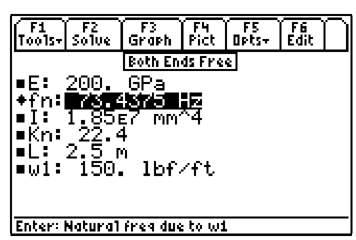


Computed results

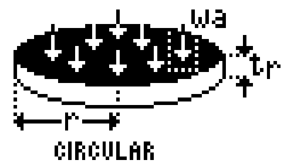
Solution – Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Since we are calculating the natural frequency, the vibration mode-based constant is 1 and a value of **Kn** = 22.4 is selected from **Table 18.4**. The entries and results are shown in the screen displays above.

Given
E = 200 GPa
I = 1.85 E 7 mm^4
Kn = 22.4
L = 2.5 m
w1 = 150 lb/ft

Solution
fn = 73.4375 Hz

18.3.3.4 Flat Plates

18.3.3.4.1 Circular Flat Plate



The following equations compute the natural vibration properties for a circular plate having a radius, **r** (m) and uniform thickness, **tr** (m) experiencing a uniform load per unit area, **wa** (N/m^2), which includes the weight of the plate. The **first equation** computes the frequency of vibration, **fn** (Hz). The **second equation** computes the flexural of rigidity for the plate, **D** (N·m), from the elastic properties of the material (Poisson's ratio, **ζ**, and Young's modulus, **E**) and the thickness of the plate, **tr**. Values for the vibration mode-based constant, **Kn**, in equation 1 are listed in **Table 18.5** for different mounting types and vibration modes (harmonics), **n**. The **third equation** computes the load per area, **wa**, from the total load, **W** (N), and the radius of the circle, **r** (m). The **last equation** adapts the previous three equations for an elliptical plate, having a major axis radius **ra** (m), and minor axis radius, **rb** (m), with its edges fixed.

Table 18.5: Kn for a Circular Plate, radius r, thickness, tr and an elliptical plate (Ref.#6).

Support Type (Circular)	n	Kn	Nodal positions/L
Edge fixed	**1**	10.2	Fundamental
	2	18.3	One nodal diameter
	3	34.9	Two nodal diameters
	4	39.8	One nodal circle
Edge Simply Supported	**1**	4.99	Fundamental
(ζ=0.3)	**2**	13.9	One nodal diameter
	3	25.7	Two nodal diameters
	4	29.8	One nodal circle
Edge Free	**1**	5.25	Two nodal diameters
(ζ=0.33)	**2**	9.08	One nodal circle
	3	12.2	Three nodal diameters
	4	20.5	One nodal diameter and One nodal circle
Elliptical Flat Plate (edge fixed)	**ra/rb**	**Kn (n=1)**	
Major Radius **ra,** Minor Radius **rb**	1	10.2	
	1.1	11.3	
	1.2	12.6	
	1.5	17.0	
	2.0	27.8	
	3.0	57.0	

$$fn = \frac{Kn}{2\cdot\pi}\cdot\sqrt{\frac{D\cdot grav}{wa\cdot r^4}}$$ **Eq. 1**

$$D = \frac{E \cdot tr^3}{12 \cdot \left(1 - \zeta^2\right)} \qquad \textbf{Eq. 2}$$

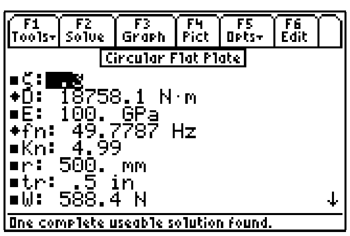
$$wa = \frac{W}{\pi \cdot r^2} \qquad \textbf{Eq. 3}$$

$$r = \frac{ra + rb}{2} \qquad \textbf{Eq. 4}$$

Variable	**Description**	**Units**
ζ	Poisson's ratio	unitless
D	Flexural of rigidity	N·m
E	Young's modulus	Pa
fn	Natural frequency for wa, mode 'n'	Hz
Kn	Constant for vibration mode, 'n'	unitless
r	Plate radius	m
ra	Major radius	m
rb	Minor radius	m
tr	Plate thickness	m
W	Total load	N
wa	Load per unit area	N/m^2

Example 18.3.3.4.1:

A simply supported, cast iron manhole cover weighs 60 kg (radius 500 mm, thickness ½"). Cast iron has a Young's Modulus of 100 GPa and a Poisson's ratio of 0.3. If the load is solely due to the plate's mass 588.4 N (60 kg * 9.80665 m/ s^2), compute the fundamental vibration frequency.



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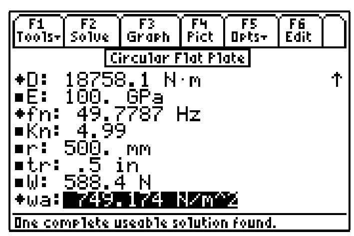
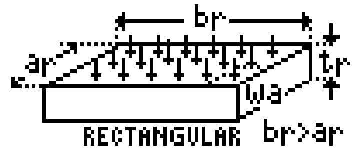


Lower Display

Solution –Select the **first three equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. A value of **Kn** = 4.99 is selected from **Table 18.5** for the fundamental vibration mode. The entries and results are shown in the screen displays above.

Given	**Solution**
ζ = 0.3	D = 18758.1 N·m
E = 100 GPa	fn = 49.7787 Hz
Kn = 4.99	wa = 749.174 N/m^2
r = 500 mm	
tr = .5 in	
W = 588.4 N	

18.3.3.4.2 Rectangular Flat Plate



The following equations compute the natural vibration properties for a rectangular plate having a short side length, **ar** (m), a longer side length, **br** (m) and uniform thickness, **tr** (m), experiencing a uniform load per unit area, **wa** (N/m^2), including the weight of the plate. The **first equation** computes the natural frequency of vibration, **fn** (Hz). The **second equation** computes the flexural of rigidity of the plate, **D** (N·m), from the elastic properties of the material (Poisson's ratio, **ζ**, and Young's modulus, **E**) and the thickness of the plate, **tr**. Values for the vibration mode-based constant, **Kn**, in equation 1 are listed in **Table 18.6** for some different mounting types and vibration modes (harmonics), **n**. The **third equation** computes the load per area, **wa**, from the total load, **W** (N), and the dimensions of the plate, **ar** and **br**. The **last equation** computes **Kn** for the special case where all edges are simply supported. The constants **na** and **nb** are positive integers and represent the number of vibration modes along the axis' of **ar** and **br**.

Table 18.6: Kn for a Rectangular Plate, thickness, tr (Ref.#6).

Rectangular Flat Plate	ar/br	Kn (n=1)
All edges fixed	1	36.0
ar – shorter edge length	0.9	32.7
br – longer edge length	0.8	29.9
	0.6	25.9
	0.4	23.6
	0.2	22.6
	0	22.4
All edges simply supported	Use Equation 4	Use Equation 4
2 'ar' fixed/1 'br' fixed/1 'br' free	3.0	213
	2.0	99
	1.6	67
	1.2	42.4
	1.0	33.1
	0.8	25.9
	0.6	20.8
	0.4	17.8
	0.2	16.2
	0	15.8

$$fn = \frac{Kn}{2 \cdot \pi} \cdot \sqrt{\frac{D \cdot grav}{wa \cdot ar^4}}$$ **Eq. 1**

$$D = \frac{E \cdot tr^3}{12 \cdot \left(1 - \zeta^2\right)}$$ **Eq. 2**

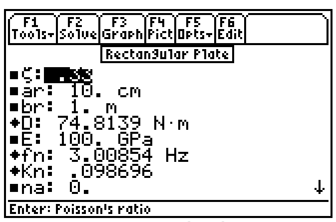
$$wa = \frac{W}{ar \cdot br}$$ **Eq. 3**

$$Kn = \pi^2 \cdot \left(na^2 + \left(\frac{ar}{br}\right)^2 \cdot nb^2 \right)$$ **(When all edges are supported)** **Eq. 4**

Variable	**Description**	**Units**
ζ	Poisson's ratio	unitless
ar	Shorter length of rectangular plate	m
br	Longer length of rectangular plate	m
D	Flexural rigidity	N·m
E	Young's modulus	Pa
fn	Frequency for wa, mode 'n'	Hz
Kn	Constant for vibration mode, 'n'	unitless
na	Mode of vibration: ar axis {1, 2, 3…}	unitless positive integer
nb	Mode of vibration: br axis {1, 2, 3…}	unitless positive integer
tr	Plate thickness	m
W	Total load	N
wa	Load per unit area	N/m^2

Example 18.3.3.4.2:

Compute the fundamental frequency for a titanium plate 2 mm thick with edge lengths 1m and 10 cm. The load, including the weight of the titanium, is 20 N. All of the edges are fixed. Pure titanium has a Young's modulus value of 110 GPa and a Poisson's ratio value of 0.33.



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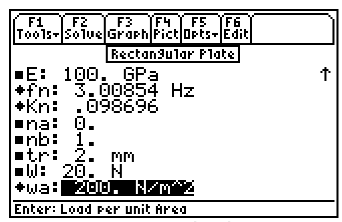

Lower Display

Solution – Select **all of the equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Enter values of **na** = 0 and **nb** = 1. The entries and results are shown in the screen displays above.

Given	**Solution**
ζ = 0.33	D = 74.8139 N·m
ar = 10 cm	fn = 3.00854 Hz
br = 1 m	Kn = .098696
E = 100 GPa	wa = 200 N/m^2
na = 0	
nb = 1	
tr = 2 mm	
W = 20 N	

References:

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2. Eugene A Avallone and Theodore Baumeister III, Mark's Handbook for Mechanical Engineers, 9th Edition, McGraw-Hill Book company, New York, NY 1986
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4. R. C. Hibbeler "Engineering Mechanics, Statics and Dynamics" 7th Edition Prentice Call, Engle Cliffs, NJ 1995
5. Meriam J.L., Kraige L.G. "Engineering Mechanics Vol. 2: Dynamics" 2nd Edition, John Wiley and Sons, NY, 1986
6. Young, W. "Roark's Formulas for Stress and Strain", 6th ed, McGraw Hill, NY, 1989

Chapter 19: Refrigeration and Air Conditioning

This portion of the software is designed to solve some of the common problems encountered in Refrigeration and Air conditioning. Two distinct areas form the basis for the calculations.

- **Heating Load**
- **Refrigeration**

19.1 Heating Load

Eleven equations listed in this category are used in the design of insulation and heating systems. The **first equation** computes the overall heat transfer coefficient, **U** W/(m^2·K), for a heat loss surface. The variables, **hin** and **hout** W/(m^2·K), are the film coefficients of the inside and outside surfaces. **Equation 2** computes the resistance due to convection in an air space, **Rh** (m^2·K/W); **ak** W/(m^2·K) is the thermal conductivity of the air space- typically 1.65 for inside (still) air and 6.0 for outside (~15 mph wind) air. **Equation 3** calculates the thermal resistance of conduction for three surfaces in series, **Rk** (m^2·K/W), each having thickness, **xd1, xd2,** and **xd3** (m), and conductivities **k1, k2,** and **k3** W/(m·K). **Equation 4** calculates the effective space emissivity, **E**, due to thermal conduction in air spaces. The variables, **e1** and **e2**, are the emissivities of side 1 and side 2 of the enclosure. **Equation 5** estimates the rate of heat transfer, **Qrate** (W), from the heat loss surface having an overall heat transfer coefficient, **U**, and area, **A** (m^2). **Equation 6** estimates **Qrate** through the edges of a floor surface. The perimeter, **peri** (m), is used when *slab edge* data is available. The heat loss coefficient, **F** W/(m·K), varies between 1.4 W/(m·K) for an exposed surface edge and 0.95 W/(m·K) for insulated edges. **Equations 7** and **8** compute the heat required to warm moisture in incoming air, **qw** (J), from the specific heat of water vapor, **cp** J/(kg·K), the inside and outside temperature, **Ti** (K) and **To** (K), the air density **ρ** (kg/m^3) and the humidity ratio, **ω** (unitless). **Equations 9** and **10** calculate the total heat required, to warm incoming moist air, **qt** (J). Values of **hi** J/(kg·K) and **ho** J/(kg·K), the enthalpies of moist air at temperatures,**Ti** and **To**, are available from psychometric charts (see Analysis Steam Tables). The **last equation** computes the volumetric flow rate of air, **Vf** (m^3/s) through a fixture using the *crack* method. If 1 wall is exposed, the total crack length is used for **L** (m), the greater crack length for 2 exposed walls, the crack length of two walls if 3 walls are exposed and one half the total crack length is used for 4 walls. **B** (m^2/hr) is the infiltration coefficient, which is specific to a type of fixture and the wind velocity.

$$U=\frac{1}{\frac{1}{hin}+\frac{1}{hout}+Rk+Rh}$$ **Eq. 1**

$$Rh=\frac{1}{ak}$$ **Eq. 2**

$$Rk=\frac{xd1}{k1}+\frac{xd2}{k2}+\frac{xd3}{k3}$$ **Eq. 3**

$$\frac{1}{E}=\frac{1}{e1}+\frac{1}{e2}-1$$ **Eq. 4**

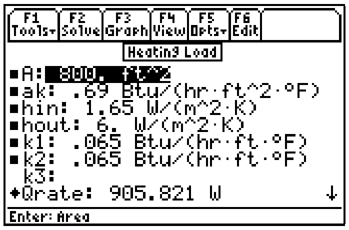
$$Qrate=U\cdot A\cdot(Ti-To)$$ **Eq. 5**

$Qrate = peri \cdot F \cdot (Ti - To)$	**Eq. 6**
$qw = \rho \cdot \omega \cdot Vf \cdot cp \cdot (Ti - To)$	**Eq. 7**
$qt = \rho \cdot Vf \cdot (hi - ho)$	**Eq. 8**
$qt = qa + qw$	**Eq. 9**
$Vf = B \cdot L$	**Eq. 10**

Variable	Description	Units
ρ	Density of incoming air	kg/m^3
ω	Humidity ratio	unitless
A	Area	m^2
ak	Thermal coefficient for air	W/(m·K)
B	Infiltration coefficient	m^2/hr
cp	Specific heat at constant pressure	J/(kg·K)
E	Effective space emissivity	unitless
e1	Emissivity	unitless
e2	Emissivity	unitless
F	Heat loss coefficient	$W/(m^2 \cdot K)$
hi	Enthalpy-inside Air	J/kg
hin	Film coefficient	$W/(m^2 \cdot K)$
ho	Enthalpy - outside Air	J/kg
hout	Film coefficient	$W/(m^2 \cdot K)$
k1	Thermal conductivity of 1	W/(m·K)
k2	Thermal conductivity of 2	W/(m·K)
k3	Thermal conductivity of 3	W/(m·K)
L	Length	m
ma	Air mass	kg
mw	Water vapor mass	kg
peri	Perimeter	m
qa	Heat transfer-dry air	J
Qrate	Rate of heat loss	W
qt	Rate of heat transfer-incoming air	W
qw	Rate of heat transfer-moisture	W
Rh	Resistance due to convection	$K \cdot m^2/W$
Rk	Resistance due to conduction	$K \cdot m^2/W$
Ti	Inside temperature	K
To	Outside temperature	K
U	Overall heat transfer coefficient	$m^2 \cdot K$
Vf	Volumetric flow rate	m^3/s
xd1	Thickness of material 1	m
xd2	Thickness of material 2	m
xd3	Thickness of material 3	m

Example 19.1:

A roof, having an area of 800 ft^2, is made of two layers of Oregon Pine (thickness of 1 in, thermal conductivity of 0.065 Btu/(ft·hr·°F) separated by an air space of ~ 4 inches having a convection coefficient of 0.69 Btu/(ft^2·hr·°F). Assuming no heat loss occurs due air seepage through cracks, what is the rate of heat loss if the inside temperature is 75°F and the outside temperature is 30°F. Assume the film coefficients for the inside and outside air are 1.65 and 6.0 W/(m^2·K).



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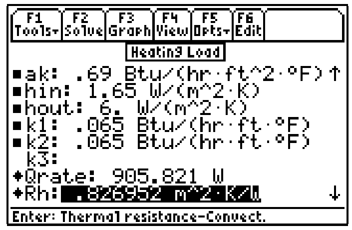
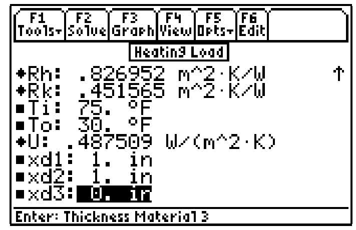


Middle Display



Lower Display

Solution – Select **equations 1, 2, 3,** and **5** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
A = 800 ft^2
ak = .69 Btu/(hr·ft^2·°F)
hin = 1.65 W/(m^2·K)
hout = 6 W/(m^2·K)
k1 = .065 Btu/(hr·ft·°F)
k2 = .065 Btu/(hr·ft·°F)
k3 = (blank)
Ti = 75 °F
To = 30 °F
xd1 = 1 in
xd2 = 1 in
xd3 = 0 in

Solution
Qrate = 905.821 W/(m^2·K)
Rh = .826952 m^2·K/W
Rk = .451565 m^2·K/W
U = .487509 W/(m^2·K)

19.2 Refrigeration

19.2.1 General Cycle

The following equations focus on the performance characteristics of refrigeration cycles. **Equation 1** computes the coefficient of performance for a refrigeration cycle, **COPr** (unitless). **Equation 2** calculates the coefficient of performance **COPh** (unitless) for a heat pump. The **third equation** shows the implicit relationship between **COPr** and **COPh**. The **last equation** calculates the work required to drive the refrigeration cycle, **Win** (J). **Qin** (J), is the heat input from the low temperature reservoir and **Qout** (J), is the exhaust of heat into the high temperature area.

$$COPr = \frac{Qin}{Qout - Qin}$$ **Eq. 1**

$$COPh = \frac{Qin + Win}{Qout - Qin}$$ **Eq. 2**

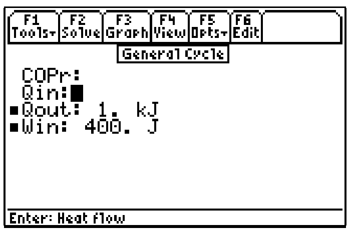
$$COPh = COPr + 1$$ **Eq. 3**

$Win = Qout - Qin$	**Eq. 4**

Variable	Description	Units
COPh	Performance coefficient -heat pump	unitless
COPr	Performance coefficient –refrigeration	unitless
Qin	Heat flow – from low temperature region	J
Qout	Heat flow – from high temperature region	J
Win	Work input	J

Example 19.2.1:

A refrigeration cycle requires 400 J of work and expels 1 kJ of heat. What is the heat drawn from the cool area and the coefficient of performance?



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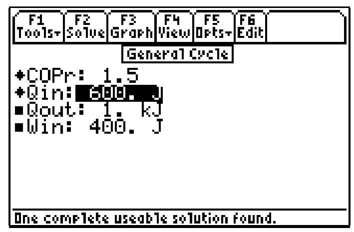


Computed results

Solution – Select the **first** and **last equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
Qout = 1 kJ	COPr = 1.5
Win = 400 J	Qin = 600 J

19.2.2 Reverse Carnot

The reverse Carnot cycle is composed of four steps:

1. Isentropic expansion
2. Isothermal heating (vaporization)
3. Isentropic compression
4. Isothermal cooling (condensation)

The coefficient of performance for the reverse Carnot cycle (refrigeration), **COPr** (unitless), is computed from the high and low temperatures, **Thi** (K) and **Tlo** (K). The coefficient of performance for the forward Carnot cycle (heat pump) **COPh** (unitless) is computed from **Thi** and **Tlo** in the second equation.

$$COPr = \frac{T1o}{Thi - T1o} \quad \textbf{Eq. 1}$$

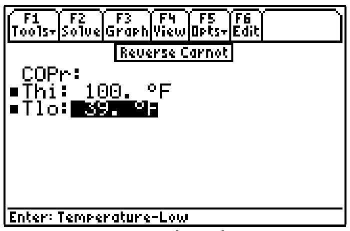
$$COPh = \frac{Thi}{Thi - T1o} \quad \textbf{Eq. 2}$$

$$COPh = COPr + 1 \quad \textbf{Eq. 3}$$

Variable	**Description**	**Units**
COPh	Performance coefficient -heat pump	unitless
COPr	Performance coefficient -refrigeration	unitless
Thi	Temperature – High	K
Tlo	Temperature – Low	K

Example 19.2.2:

What is the coefficient of performance for a reverse carnot cycle having a high temperature of 100 ^{o}F and a low temperature of 39 ^{o}F?



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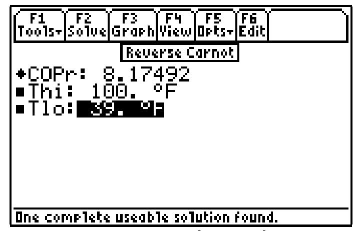


Computed results

Solution – Select the **first equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
Thi = 100 ^{o}F	COPr = 8.17492
Tlo = 39 ^{o}F	

19.2.3 Reverse Brayton

The reverse Brayton (air refrigeration) cycle is composed of four steps:

1. Isentropic expansion.
2. Constant pressure heating.
3. Isentropic compression.
4. Constant pressure cooling.

Equations 1 and **2** compute the coefficient of performance, **COP**, from the temperatures **T1** (K), **T2** (K), **T3** (K) and **T4** (K) at each stage in the cycle, the compression ratio, **pr** (unitless), and the specific heat ratio, **k** (unitless). The compression ratio, **pr**, is defined as the ratio of the high and low pressures in the third equation.

$$COP = \frac{T3 - T2}{T4 - T1 - (T3 - T2)}$$ **Eq. 1**

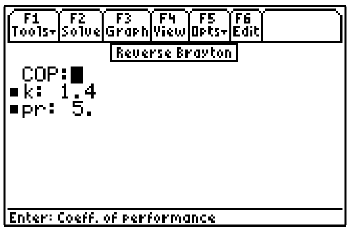
$$COP = \frac{1}{pr^{\frac{k-1}{k}} - 1}$$ **Eq. 2**

$$pr = \frac{phi}{plo}$$ **Eq. 3**

Variable	Description	Units
COP	Coefficient of performance	unitless
k	Adiabatic expansion coefficient	unitless
phi	Pressure – High	Pa
plo	Pressure – Low	Pa
pr	Pressure ratio	unitless
T1	Temperature	K
T2	Temperature	K
T3	Temperature	K
T4	Temperature	K

Example 19.2.3:

Compute the coefficient of performance for an air refrigeration cycle having a compression ratio of 5.



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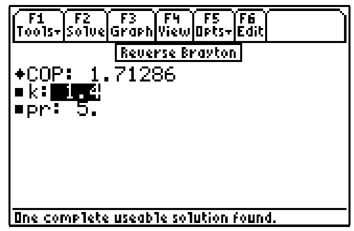


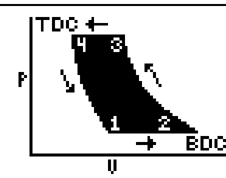
Computed results

Solution – Select the **second equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Use a value of k=1.4 for air (see *Reference/Thermal Properties/Specific Heat/ Cp/Cv Liquids and Gases at 1 atm*). Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
k = 1.4	COP = 1.71286
pr = 5	

19.2.4 Compression Cycle

The following equations describe the properties of a reciprocating single-stage compressor with clearance for an ideal gas. The four processes forming the foundation of the cycle are described below:



1. Intake valve opens and suction occurs at constant pressure.
2. Intake valve closes, polytropic compression occurs.
3. Discharge valve opens and discharge occurs at constant delivery pressure until piston.
4. System returns to its original state (residual gases from original compression exist

The pressure and volume of the gas at the beginning of the cycles are defined below:

1. Beginning of stroke 1 - Pressure **p1** (Pa) and volume **V1** (m^3)
2. Beginning of stroke 2 - Pressure **p2** (Pa) and volume **V2** (m^3)
3. Beginning of stroke 3 - Pressure **p3** (Pa) and volume **V3** (m^3)
4. Beginning of stroke 4 - Pressure **p4** (Pa) and volume **V4** (m^3)

The volume at the end of Stage 1, **V2**, is termed the bottom dead center (BDC). The volume at the end of Stage 3, **V4** is the top dead center (TDC). The **first and second equations** compute the compression ratio, **pr** (unitless), as the ratio of the high and low pressures in the cycle (**p4=p3**, high pressure; **p1=p2**, low pressure). **Equations 3** and **4** calculate the volumetric efficiency, **effv** (unitless), and the ratio of the mass of compressed gas to the mass of the swept volume. **Equations 5** and **6** relate the pressure/volume ratios for polytropic compression. The variable, **n** (unitless), is the polytropic exponent. If the compression is isentropic, **n**, has a value equivalent to the specific heat ratio of the gas (**cp/cv** where **cp** is the specific heat at constant pressure and **cv** is the specific heat at constant volume). The **last equation** computes the work performed by the system during polytropic compression, **W23** (J) (Stage 2).

$$pr = \frac{p4}{p1}$$ **Eq. 1**

$$pr = \frac{p3}{p2}$$ **Eq. 2**

$$effv = \frac{V2 - V1}{V2 - V4}$$ **Eq. 3**

$$effv = 1 - \frac{\left(pr^{\frac{1}{n}} - 1\right) \cdot V4}{V2 - V4}$$ **Eq. 4**

$$\frac{V1}{V4} = pr^{\frac{1}{n}}$$ **Eq. 5**

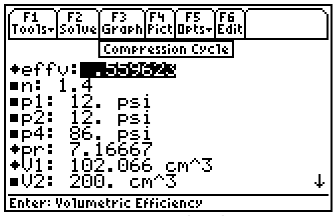
$$\frac{V2}{V3} = pr^{\frac{1}{n}}$$ **Eq. 6**

$$W23 = \frac{-n \cdot p2 \cdot V2}{1 - n} \cdot \left(pr^{\frac{n-1}{n}} - 1\right)$$ **Eq. 7**

Variable	Description	Units
effv	Volumetric efficiency	unitless
N	Polytropic exponent	unitless
p1	Pressure	Pa
p2	Pressure	Pa
p3	Pressure	Pa
p4	Pressure	Pa
Pr	Pressure ratio	unitless
V1	Volume	m^3
V2	Volume	m^3
V3	Volume	m^3
V4	Volume	m^3
W23	Work - compression/ stroke	J

Example 19.2.4:

What is the volumetric efficiency and work required for compression for a compressor which discharges air at constant pressure of 86 psi and draws in air at a constant pressure of 12 psi? The BDC volume is 200 cm^3 and the TDC volume is 25 cm^3. Assume the compression process is adiabatic (n=1.4).



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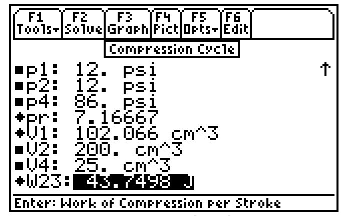

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Solution – Note: **V4**=TDC and **V2**=BDC, and **p1**=**p2**=low pressure, and **p3**=**p4**=high pressure. Select **equations 1, 3, 4, 5** and **7** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
n = 1.4	effv = .559623
p1 = 12 psi	pr = 7.16667
p2 = 12 psi	V1 = 102.066 cm^3
p4 = 86 psi	W23 = 43.7498 J
V2 = 200 cm^3	
V4 = 25 cm^3	

References: -

1. Michael R. Lindeburg, Mechanical Engineering Reference Manual, 8th Edition, Professional Publications, Belmont, CA 1990
2. John A. Roberson and Clayton T. Crowe, Engineering Fluid Mechanics, 5th Edition, Houghton Mifflin Company, Boston, MA, 1993
3. Eugene A Avallone and Theodore Baumeister III, Mark's Handbook for Mechanical Engineers, 9th Edition, McGraw-Hill Book company, New York, NY 1986

Chapter 20: Strength Materials

Strength of materials section is designed to cover a range of example problems encountered in characterizing materials under a variety of types of stresses. The topics are covered under these broad categories:

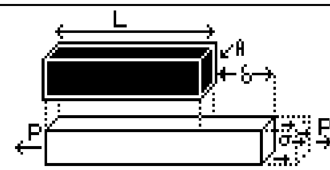
- Stress and Strain Basics
- Stress Analysis
- Torsion
- Load Problems
- Mohr's Circle

20.1 Stress and Strain Basics

Three basic definitions of stresses and strains are introduced in this section. They are Normal Stress and Strain, Volume dilation and Shear stress. Each of these sections is described here.

20.1.1 Normal Stress and Strain

Three equations describe the basic definitions of stress and strain. The **first equation** defines the relationship between stress, **σ** (Pa), load, **P** (N), and area, **A** (m^2). The **second equatio**n establishes the definition of strain, **ε** (no units), as the ratio of elongation, **δ** (m), due to the force, **P**, of a bar of length, **L** (m). The **third equation** represents Hooke's law relating stress, **σ**, to strain, **ε**, using the elasticity modulus, **E** (Pa).



$$\sigma = \frac{P}{A} \quad \textbf{Eq. 1}$$

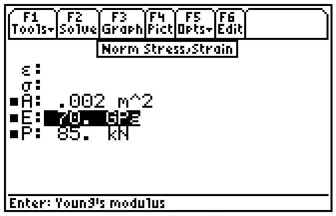
$$\varepsilon = \frac{\delta}{L} \quad \textbf{Eq. 2}$$

$$\sigma = \varepsilon \cdot E \quad \textbf{Eq. 3}$$

Variable	**Description**	**Units**
δ	Equivalent elongation	m
ε	Strain	unitless
σ	Stress	Pa
A	Area	m^2
E	Young’s modulus	Pa
L	Length	m
P	Load	N

Example 20.1.1:

Calculate the stress and strain of a bar with an Area of .002 m^2. The bar is made of an aluminum alloy (2014-T6) with a modulus of elasticity of 70 GPa. A load of 85 kN is placed upon this bar in tension mode.



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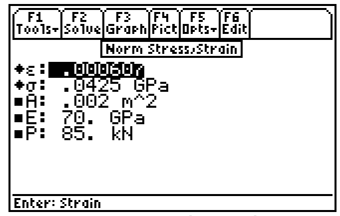
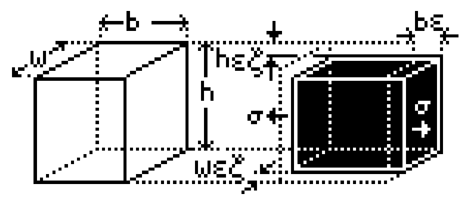


Computed results

Solution – Select the **first** and **third equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
A = .002 m^2	ε = .000607
E = 70 GPa	σ = .0425 GPa
P = 85 kN	

20.1.2 Volume Dilation



Volume dilation refers to volume changes in a solid body due to shear forces. A solid block defined by the dimensions base, **b** (m), height **h** (m), and width, **w** (m). The initial volume, **Vi** (m^3), of such a parallel-piped is defined by the **first equation**. A stress applied to this bar results in a strain, **ε** (no units), along the direction of the stress. However, the lateral dimensions shrink by the longitudinal strain modulated by the so-called Poisson's ratio, **ζ** (no units). This is illustrated by the **second equation**. The **last equation** defines the volumetric strain, **ev** (no units), in terms of **ε** and **ζ**.

$$Vi = b \cdot h \cdot w \quad \textbf{Eq. 1}$$

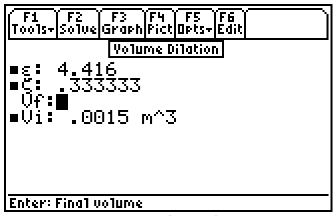
$$Vf = Vi \cdot (1+\varepsilon) \cdot (1-\zeta \cdot \varepsilon) \cdot (1-\varepsilon \cdot \zeta) \quad \textbf{Eq. 2}$$

$$ev = (1+\varepsilon) \cdot (1-\varepsilon \cdot \zeta)^2 - 1 \quad \textbf{Eq. 3}$$

Variable	**Description**	**Units**
ε	Strain	unitless
ζ	Poisson's ratio	unitless
B	Base	m
Ev	Volume dilation	unitless
H	Height	m
Vf	Final volume	m^3
Vi	Initial volume	m^3
W	Width	m

Example 20.1.2:

A copper bar, with an initial volume of .0015 m^3, is subject to a volumetric strain of 4.416. The Poisson's ratio for this copper bar is 1/3. Find the final volume of the copper bar after the stress has been applied.



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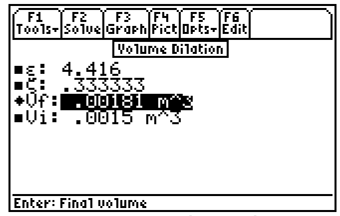


Computed results

Solution – Select the **second equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
$\varepsilon = 4.416$
$\zeta = 1/3$
$Vi = .0015\ m^3$

Solution
$Vf = .00181\ m^3$

20.1.3 Shear Stress and Modulus

The average shear stress, **τav** (Pa), on the cross section of an element such as a bolt is defined by the **first equation** as the shear force, **F** (N), acting over an area, **A** (m^2). The **second equation** expresses shear stress as a product of shear strain, **γ** (unitless), and bulk shear modulus, **G** (Pa), in a manner similar to Hooke's Law in shear. The **third equation** connects the modulus of elasticity in tension and shear (**E** (Pa) and **G** (Pa)) in terms of the Poisson's ratio, **ζ** (unitless).

$$\tau av = \frac{P}{A} \quad \textbf{Eq. 1}$$

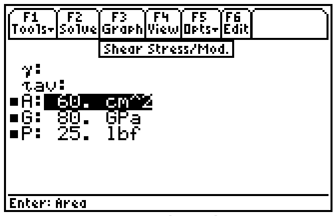
$$\tau av = G \cdot \gamma \quad \textbf{Eq. 2}$$

$$G = \frac{E}{2 \cdot (1+\zeta)} \quad \textbf{Eq. 3}$$

Variable	**Description**	**Units**
γ	Shear strain	Unitless
ζ	Poisson's ratio	unitless
τav	Shear stress	Pa
A	Area	m^2
E	Young's modulus	Pa
G	Shear modulus	Pa
P	Load	N

Example 20.1.3:

Find the shear stress and shear strain on a steel bolt that passes through one steel plate on a tractor and a U-shaped hitch on a trailer. The trailer is in a motionless position. As the tractor starts to move, it exerts a load on the bolt of 25 lbf. The area of the plate on the tractor is 60 cm^2 and the modulus of elasticity on the bolt is 80 GPa.



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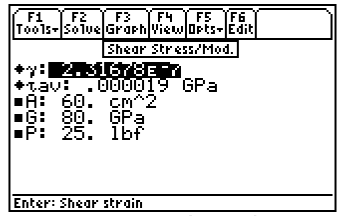


Computed results

Solution – Select the **first** and **second equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

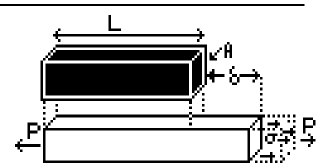
Given	**Solution**
A = 60 cm^2	γ = 2.31678E-7
G = 80 GPa	τav = .000019 GPa
P = 25 lbf	

20.2 Load Problems

This section is devoted to the behavior of a selected set of axially loaded members, which refers to structural elements having longitudinal axes and carrying only axial forces whether they are tensile or compressive. This family of problems is associated with bars in trusses, columns in buildings, spokes in bicycle wheels, and struts in aircraft engine mounts.

20.2.1 Axial Load

The elastic properties of a primitive bar of length, **L** (m), loaded in tension by axial forces, **P** (N), is characterized by the **first equation** defining the elongation of the bar. The material of the bar has a Young's modulus, **E** (Pa), and area, **A** (m^2), elongated by a distance, **δ** (m). For the axial bar, a convenient unit of measure is the stiffness, **k** (N/m) (i.e., the force needed to produce a unit elongation), computed in **Equation 2**.



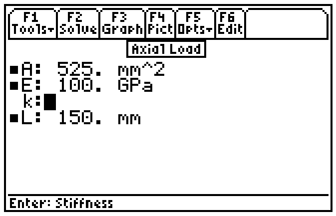
$$\delta = \frac{P \cdot L}{E \cdot A} \qquad \textbf{Eq. 1}$$

$$k = \frac{E \cdot A}{L} \qquad \textbf{Eq. 2}$$

Variable	**Description**	**Units**
δ	Equivalent elongation	m
A	Area	m^2
E	Young's modulus	Pa
k	Stiffness	N/m
L	Length	m
P	Load	N

Example 20.2.1:

Find the stiffness, the amount of force required for elongation, of a Titanium Alloy bar that is 150 mm long. This bar has an area of 525 mm^2, and the modulus of elasticity is 100 GPa.



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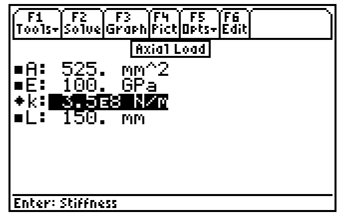


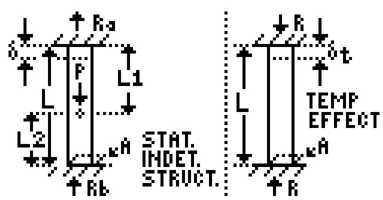
Computed results

Solution – Select the **second equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
A = 525 mm^2
E = 100 Gpa
L = 150 mm

Solution
K = 3.5E8 N/m

20.2.2 Temperature Effects



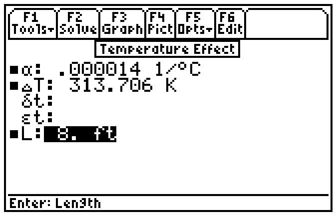
A change in the temperature of an object tends to produce a change in its dimensions. As shown in the schematic diagram shown here, a homogeneous rectangular block with length, **L** (m), area, **A** (m^2), is subject to a temperature change, **ΔT** (K), and a thermal expansion coefficient, **α** (1/K), results in a uniform thermal strain, **εt** (unitless) computed in **equation 1**. The **second equation** calculates the actual expansion, **δt** (m), due to thermal strain. The **third equation** computes the thermal strain force, **R** (N), induced in the member if the material is confined in a fixed space that does not allow the top to expand.

$\varepsilon t = \alpha \cdot \Delta T$	**Eq. 1**
$\delta t = \varepsilon t \cdot L$	**Eq. 2**
$R = A \cdot E \cdot \alpha \cdot \Delta T$	**Eq. 3**

Variable	**Description**	**Units**
α	Coefficient of thermal expansion	1/K
ΔT	Temperature difference	K
δt	Elongation	m
εt	Thermal strain	unitless
A	Area	m^2
E	Young's modulus	Pa
L	Length	m
R	Thermal strain force	N

Example 20.2.2:

If a porch had a roof that is held up by concrete pillars, how much will it move in an environment that has a 313.706 k temperature change from winter to summer? Also how much of a strain will this put on the pillars? The pillars are 8 ft tall, and the coefficient of thermal expansion for concrete is 14E-6 1/°C.



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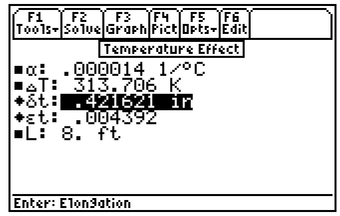
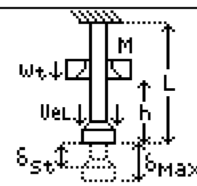


Computed results

Solution – Select the **first** and **second equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
α = 14E-6 1/°C	δt = .421621 in
ΔT = 313.706 k	εt = .004392
L = 8 ft	

20.2.3 Dynamic Load



Dynamic loads differ from static loads in that they are generally and loads can be applied suddenly and from an impact load. The eight equations in this section focus on impact of a dynamic load and helps compute its impact on properties of the system. We use the example of a collar and flange designed so that the collar sits on the flange and moves downwards with it. This situation is often referred to as *perfectly plastic impact*. Two key assumptions are made in this analysis -

- Disregard energy losses
- Assume all kinetic energy of the falling mass is converted to strain energy of the bar.

The **first equation** falls out of the conservation of energy principle whereby the potential energy lost by the mass, **M** (kg), with a weight, **Wt** (N), is transferred to the bar of length, **L** (m), area, **A** (m^2), height, **h** (m), elastic modulus, **E** (Pa), results in a maximum deflection, **δmax** (m). The **second equation** solves for the positive root of **δmax**. The **third equation** extracts the static elongation, **δst** (m), in terms of the static load parameters, **Wt**, **L**, **E** and **A**. The **next equation** is an alternative form of the first equation except **δmax** is expressed as a function of the static elongation, **δst**, and the effect of the dynamic load. The **next three equations** reflect a calculation of the maximum tensile stress, **σmax** (Pa), from parameters of the system. **Equation 7** describes maximum stress, **σmax**, in terms of velocity; weight, **Wt**, physical properties of the system such as area **A**, length **L**, velocity **vel** (m/s), and modulus of elasticity **E** (Pa). The **final equation** relates weight, **Wt**, and mass, **M**.

$$Wt \cdot (h + \delta max) = \frac{E \cdot A \cdot \delta max^2}{2 \cdot L} \qquad \textbf{Eq. 1}$$

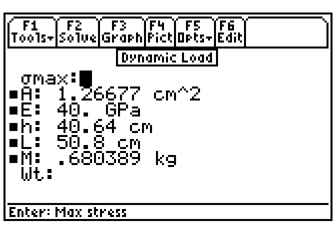
$$\delta max = \frac{Wt \cdot L}{E \cdot A} + \left(\left(\frac{Wt \cdot L}{E \cdot A} \right)^2 + \frac{2 \cdot Wt \cdot L \cdot h}{E \cdot A} \right)^{.5} \qquad \textbf{Eq. 2}$$

$\delta st = \dfrac{Wt \cdot L}{E \cdot A}$	**Eq. 3**
$\delta max = \delta st + \left(\delta st^2 + 2 \cdot h \cdot \delta st\right)^{.5}$	**Eq. 4**
$\sigma max = \dfrac{Wt}{A} + \left(\left(\dfrac{Wt}{A}\right)^2 + \dfrac{2 \cdot Wt \cdot h \cdot E}{A \cdot L}\right)^{.5}$	**Eq. 5**
$\sigma max = \sigma st + \left(\sigma st^2 + \dfrac{2 \cdot h \cdot E}{L} \cdot \sigma st\right)^{.5}$	**Eq. 6**
$\sigma max = \left(\dfrac{Wt \cdot vel^2 \cdot E}{A \cdot L \cdot grav}\right)^{.5}$	**Eq. 7**
$M = \dfrac{Wt}{grav}$	**Eq. 8**

Variable	**Description**	**Units**
δmax	Maximum elongation	m
δst	Static elongation	m
σmax	Maximum stress	Pa
σst	Static stress	Pa
A	Area	m^2
E	Young's modulus	Pa
h	Height	m
L	Length	m
M	Mass of load	kg
vel	Maximum velocity	m/s
Wt	Force	N

Example 20.2.3:

When a collar around a copper rod is dropped from a height of 40.64 cm, find the force of the collar, and the maximum stress experienced by the rod. The length of the rod is 50.8 cm and its modulus of elasticity is 40 GPa. The collar has a mass of .680389 kg and the rod has an area of 1.26677 cm^2.



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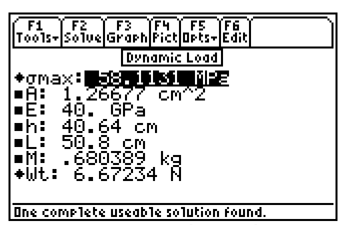


Computed results

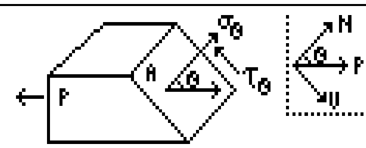
Solution – Select the **fifth** and **eighth equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
A = 1.26677 cm^2	σmax = 58.1147 MPa
E = 40 GPa	Wt = 6.67234 N
h = 40.64 cm	
L = 50.8 cm	
M = .680389 kg	

20.3 Stress Analysis

Five varieties of stress analysis problems are covered under this section.

20.3.1 Stress on an Inclined Section



By slicing a plane through a bar at an angle an inclined plane section is cut, **θ** (rad). The plane is subject to an axial force, **P** (N). The normal force, **N** (N), and shear force, **V** (N), result on the inclined plane and is defined by the **first two equations**. The normal force, **N**, represents the force perpendicular to the inclined plane, while shear force, **V**, represents the force parallel to the inclined plane. The **equations 3** and **4** formulate the normal stress, **σθ** (Pa) and shear stress, **τθ** (Pa), in terms of **P**, area, **A** (m^2), and **θ.**

$$N = P\cdot\cos(\theta) \qquad \textbf{Eq. 1}$$

$$V = P\cdot\sin(\theta) \qquad \textbf{Eq. 2}$$

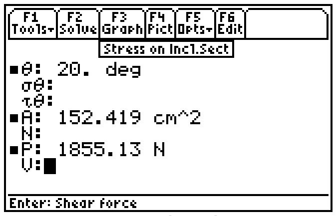
$$\sigma\theta = \frac{P}{A}\cdot(\cos(\theta))^2 \qquad \textbf{Eq. 3}$$

$$\tau\theta = \frac{-P}{A}\cdot\sin(\theta)\cdot\cos(\theta) \qquad \textbf{Eq. 4}$$

Variable	Description	Units
θ	Inclined plane angle	rad
σθ	Normal stress	Pa
τθ	Shear stress	Pa
A	Area	m^2
N	Normal force	N
P	Load	N
V	Shear force	N

Example 20.3.1:

Given the area of a bar, the load of an object, and the angle of the incline plane on the bar as 152.419 cm^2, 1855.125 N, and 20 deg respectively, find the shear force, the normal force, the normal stress, and the shear stress of the placed upon the bar by the object.



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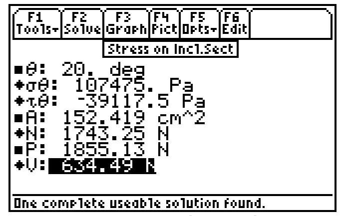


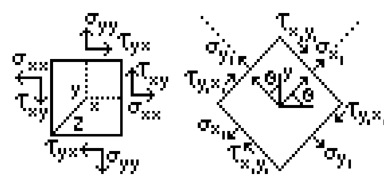
Computed results

Solution – Select **all** of the **equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
θ = 20 deg	σθ = 107475.0 Pa
A = 152.419 cm^2	τθ = -39117.5 Pa
P = 1855.125 N	N = 1743.25 N
	V = 634.49 N

20.3.2 Pure Shear

When a circular bar is subject to torsion, shear stresses act over the cross sections and longitudinal planes as shown in the thin cross-sectional element in the accompanying diagram. The **first equation** shows the shear stress along the principal stress, **σx1** (Pa), relates to the inclined plane angle, **θ** (rad), and shear stress, **τxy** (Pa). The **second equation** shows the shear stress, **τx1y1** (Pa), along the principal axis perpendicular to **σx1** in terms of **τxy** (Pa) and **θ.**



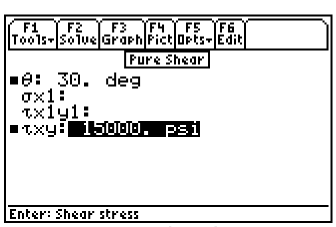
$$\sigma x1 = 2 \cdot \tau xy \cdot \cos(\theta) \cdot \sin(\theta) \qquad \textbf{Eq. 1}$$

$$\tau x1y1 = \tau xy \cdot \left((\cos(\theta))^2 - (\sin(\theta))^2 \right) \qquad \textbf{Eq. 2}$$

Variable	**Description**	**Units**
θ	Inclined plane angle	rad
σx1	Stress along principal axis	Pa
τx1y1	Shear stress along principal axis	Pa
τxy	Shear stress	Pa

Example 20.3.2:

Find the principal shear stress on a plane 30 deg from the longitudinal axis when a shear stress of 15000 psi.



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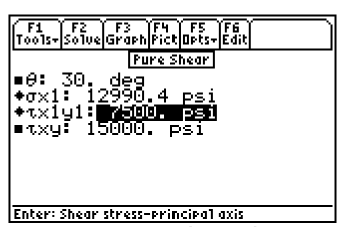
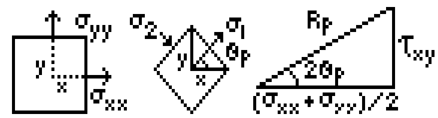


Computed results

Solution – Select **all** the **equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
θ = 30 deg	σx1 = 12990.4 psi
τxy = 15000 psi	τx1y1 = 7500 psi

20.3.3 Principal Stresses



The transformation equations for plane stresses to principal stresses form the backbone of these equations. From the normal stresses along the x and y-axis, **σxx** (Pa) and **σyy** (Pa), and shear stress, **τxy** (Pa), we can compute the angle of the plane, **θp** (rad), as shown by the **first equation**. The **second equation** computes a characteristic radius, **Rp** (Pa), often called the Mohr circle radius. The **third** and the **fourth equations** define the stresses, **σ1** (Pa) and **σ2** (Pa), along the principal axis. While the **fifth equation** shows a simple linear relationship amongst **σ1**, **σ2**, **σxx** and **σyy,** the **final two equations** show a method of computing **θp** using alternative methods.

$$\tan(2\cdot\theta p)=\frac{2\cdot\tau xy}{\sigma xx-\sigma yy}$$ **Eq. 1**

$$Rp=\sqrt{\left(\frac{\sigma xx-\sigma yy}{2}\right)^2+\tau xy^2}$$ **Eq. 2**

$$\sigma 1=\frac{\sigma xx+\sigma yy}{2}+\sqrt{\left(\frac{\sigma xx-\sigma yy}{2}\right)^2+\tau xy^2}$$ **Eq. 3**

$$\sigma 2=\frac{\sigma xx+\sigma yy}{2}-\sqrt{\left(\frac{\sigma xx-\sigma yy}{2}\right)^2+\tau xy^2}$$ **Eq. 4**

$$\sigma 1+\sigma 2=\sigma xx+\sigma yy$$ **Eq. 5**

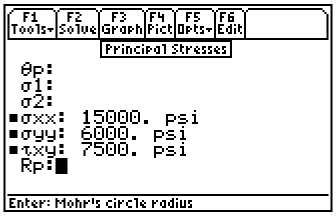
$$\cos(2\cdot\theta p)=\frac{\sigma xx-\sigma yy}{2\cdot Rp}$$ **Eq. 6**

$$\sin(2\cdot\theta p)=\frac{\tau xy}{Rp}$$ **Eq. 7**

Variable	**Description**	**Units**
θp	Principal angle	rad
σ1	Stress in 1	Pa
σ2	Stress in 2	Pa
σxx	Stress along x axis	Pa
σyy	Stress along y axis	Pa
τxy	Shear stress	Pa
Rp	Mohr's circle radius	Pa

Example 20.3.3:

Compute the Mohr radius given 15000 psi and 6000 psi for the normal stresses on the x and y axis', respectively. The shear stress is 7500 psi. Find the principal angle of the principal plane and values for the principal stresses.



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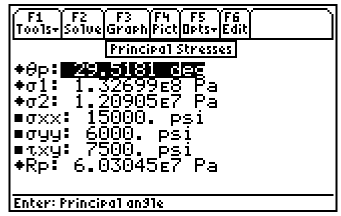


Computed results

Solution – Select the **first four equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. At the ME•*Pro* solver, **select an arbitrary integer of 0** to compute the principal solution (the principal solution, P, in a periodic trigonometric function, trig (…), is P=trig $(\theta + n\cdot\pi)$ and n is the arbitrary integer).

Given	**Solution**
σxx = 15000 psi	θp = 29.5181 deg
σyy = 6000 psi	σ1 = 1.32699 E8 Pa
τxy = 7500 psi	σ2 = 1.20905E7 Pa
	Rp = 6.03045E7 Pa

20.3.4 Maximum Shear Stress

The five equations below compute the maximum shear stress experienced in a member and the angle of the plane where this stress is experienced. The **first equation** defines **θs** (rad) as the angle of orientation of the plane of maximum shear stress in terms of the normal shear stresses, **σxx** (Pa) and **σyy** (Pa), along x and y axis respectively, and the shear stress, **τxy** (Pa). The **second equation** connects **θs** to **θp** (rad), the angle of orientation to the principal planes. The plane of algebraically maximum value of maximum shear, **τmax** (Pa), is shown by the **third equation**. The **last two equations** show alternate forms of computing **τmax** in terms of **σxx, σyy** and **τxy** or the corresponding principal stresses, **σ1 (Pa)** and **σ2 (Pa).**

$$\tan(2\cdot\theta s)=\frac{-(\sigma xx-\sigma yy)}{2\cdot\tau xy}$$ **Eq. 1**

$$\tan(2\cdot\theta s)=\frac{-1}{\tan(2\cdot\theta p)}$$ **Eq. 2**

$$\theta s1=\theta p-\frac{\pi}{4}$$ **Eq. 3**

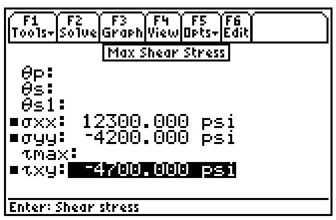
$$\tau max=\sqrt{\left(\frac{\sigma xx-\sigma yy}{2}\right)^2+\tau xy^2}$$ **Eq. 4**

$\tau max = \dfrac{\sigma 1 - \sigma 2}{2}$	**Eq. 5**

Variable	**Description**	**Units**
θp	Principal angle	rad
θs	Principal angle- maximum shear	rad
θs1	Principal angle- maximum shear 1	rad
σ1	Stress in 1	Pa
σ2	Stress in 2	Pa
σxx	Stress along x axis	Pa
σyy	Stress along y axis	Pa
τmax	Maximum shear stress	Pa
τxy	Shear stress	Pa

Example 20.3.4:

An element in plane stress is subject to a normal stress of 12300 psi along x-axis, and a stress of –4200 psi along y-axis, along with a shear stress of –4700 psi. Determine the principal stresses and the angle of maximum shear stress.



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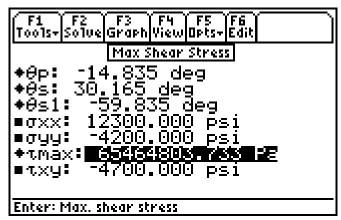
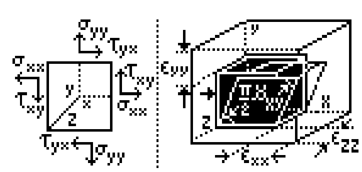


Computed results

Solution – Select the **first four equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. Because of the presence of trigonometric relationships, multiple solutions are possible and the principal solutions were chosen here. At ME•*Pro* solver and grapher, use “0” for the replacement integer of the arbitrary integer.

Given	**Solution**
σxx = 12300 psi	θp = -14.835 deg
σyy =-4200 psi	θs = 30.165 deg
τxy = -4700 psi	θs1 = -59.835 deg
	τmax = 6.54648E7 Pa

20.3.5 Plane Stress - Hooke's Law



Normal strains, **εxx** (unitless), **εyy** (unitless) and **εzz** (unitless), represent the changes in dimensions of an infinitesimally small cube having edges of unit length. The normal stress-strain relationships shown in the **first three equations** reflect Hooke’s law relationships, where stress and strain are related by the Young’s modulus of elasticity, **E** (Pa), and the Poisson’s ratio **ζ** (unitless). The **fourth equation** shows the Hooke’s law for shear stress, **τxy** (Pa), bulk modulus of elasticity, **G** (Pa) and shear strain, **γxy** (unitless). The **last two equations** complement the first two equations where stresses, **σxx** (Pa), **σyy** (Pa), and **σzz** (Pa), are expressed in terms of strains.

$$\varepsilon xx = \frac{1}{E} \cdot (\sigma xx - \zeta \cdot \sigma yy)$$ **Eq. 1**

$$\varepsilon yy = \frac{1}{E} \cdot (\sigma yy - \zeta \cdot \sigma xx)$$ **Eq. 2**

$$\varepsilon zz = \frac{-\zeta}{E} \cdot (\sigma xx + \sigma yy)$$ **Eq. 3**

$$\gamma xy = \frac{\tau xy}{G}$$ **Eq. 4**

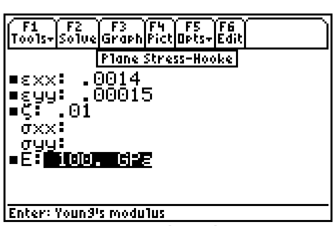
$$\sigma xx = \frac{E}{1-\zeta^2} \cdot (\varepsilon xx + \zeta \cdot \varepsilon yy)$$ **Eq. 5**

$$\sigma yy = \frac{E}{1-\zeta^2} \cdot (\varepsilon yy + \zeta \cdot \varepsilon xx)$$ **Eq. 6**

Variable	**Description**	**Units**
γxy	Shear strain along x, y axis	unitless
εxx	Strain-x axis	unitless
εyy	Strain-y axis	unitless
εzz	Strain-z axis	unitless
ζ	Poisson's ratio	unitless
σxx	Stress along x axis	Pa
σyy	Stress along y axis	Pa
τxy	Shear stress	Pa
E	Young's modulus	Pa
G	Shear modulus	Pa

Example 20.3.5:

A cubic structure is subject to linear strains of 1.4E-3 along the x-axis, and 1.5E-4 along the y-axis, and a Poisson's ratio of 0.01. If the Young's modulus of elasticity for this member is 100 GPa, find the stresses along x, and y-axis'.



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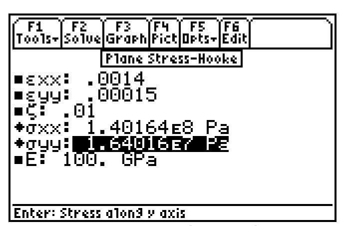


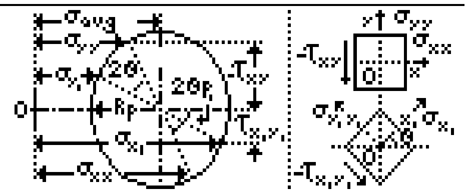
Computed results

Solution – Select the **last two equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above

Given	**Solution**
εxx =1.4E-3	σxx = 1.40164E8 Pa
εyy = 1.5E-4	σyy = 1.64016E7 Pa
ζ = 0.01	
E = 100 GPa	

20.4 Mohr's Circle Stress

20.4 Mohr's Circle Stress



A graphical presentation of transformation equations for plane stress is called the Mohr's circle. Six equations listed here form the core relationships that reflect various aspects of the components constituting the Mohr's circle. The **first equation** relates principal axis stress, **σx1** (Pa), to stresses along x and y-axis, **σxx** (Pa) and **σyy** (Pa), and the inclined plane angle, **θ** (rad). The **second equation** shows the relationship of these variables to the principal shear stress, **τx1yt1** (Pa). The **third equation** computes the average stress, **σav** (Pa). The Mohr's circle radius, **Rp** (Pa), is calculated from, **σx1**, **σav** and **τxy**. The **last two equations** show alternate ways to calculate **θp1** (rad), the angle of the principal axis.

$$\sigma x1-\frac{\sigma xx+\sigma yy}{2}=\frac{\sigma xx-\sigma yy}{2}\cdot\cos(2\cdot\theta)+\tau xy\cdot\sin(2\cdot\theta) \qquad \textbf{Eq. 1}$$

$$\tau x1y1=\frac{-(\sigma xx-\sigma yy)}{2}\cdot\sin(2\cdot\theta)+\tau xy\cdot\cos(2\cdot\theta) \qquad \textbf{Eq. 2}$$

$$\sigma av=\frac{\sigma xx+\sigma yy}{2} \qquad \textbf{Eq.3}$$

$$Rp^2=(\sigma x1-\sigma av)^2+\tau x1y1^2 \qquad \textbf{Eq. 4}$$

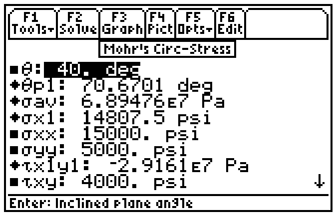
$$\cos(2\cdot\theta p1)=\frac{\sigma xx-\sigma yy}{2\cdot Rp} \qquad \textbf{Eq. 5}$$

$$\sin(2\cdot\theta p1)=\frac{\tau xy}{Rp} \qquad \textbf{Eq. 6}$$

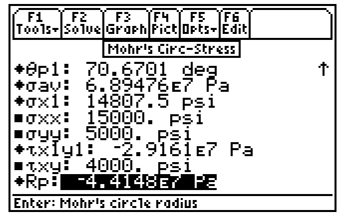
Variable	Description	Units
θ	Inclined plane angle	rad
θp1	Principal angle 1	rad
σav	Average stress	Pa
σx1	Stress-principal axis 1	Pa
σxx	Stress along x axis	Pa
σyy	Stress along y axis	Pa
τx1y1	Shear stress – principal axis	Pa
τxy	Shear stress	Pa
Rp	Mohr's circle radius	Pa

Example 20.4:

An element in plane stress is subject to a stress of 15000 psi along the x-axis, 5000 psi along the y-axis and a shear force of 4000 psi. The plane is at an angle of 40 degrees to the axis. Find the principal stress values and angle of the planes for maximum stress.



Upper Display



Lower Display

Solution – Select the **first five equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. At ME•*Pro* solver and grapher, use "0" for the replacement integer of the arbitrary integer. The first usable solution is shown in this example.

Given	**Solution**
θ = 40 deg	θp1 =70.6701 deg
σxx = 15000 psi	σav = 6.89476E7 Pa
σyy = 5000 psi	σx1 = 14807 .5 psi
τxy = 4000 psi	τx1y1 = -2.9161E7 Pa
	Rp = 4.4146E7 Pa

20.5 Torsion

Torsion refers to the twisting of a structural member, when it is loaded by a *couple* which produces rotation about the longitudinal axis.

20.5.1 Pure Torsion

Eight equations are listed here to illustrate various aspects of torsion. The **first two equations** focus on the definition of torsional shear strain, **γ** (unitless), in terms of the shaft radius, **rad** (m), and length **L** (m), angular twists per unit length, **θ1** (rad/m), and twist, **ϕ** (rad). The **third equation** represents Hooke's law for shear. The **fourth equation** yields the torque, **T** (N·m), for the angular twist modulated by the bulk modulus, **G** (Pa), and polar moment of inertia, **Ip** (m^4). The **fifth equation** computes **Ip** in terms of **rad**. The **sixth equation** shows the computation of the twist, **ϕ**, in terms of **T**, **L**, **G** and **Ip**. The **last two equations** show alternate forms of **τmax** (Pa), the maximum shear stress.

$$\gamma = rad \cdot \theta 1$$ **Eq. 1**

$$\gamma = \frac{rad \cdot \phi}{L}$$ **Eq. 2**

$$\tau = G \cdot \gamma$$ **Eq. 3**

$$T = G \cdot Ip \cdot \theta 1$$ **Eq. 4**

$$Ip = \frac{\pi \cdot rad^4}{2}$$ **Eq. 5**

$$\phi = \frac{T \cdot L}{G \cdot Ip}$$ **Eq. 6**

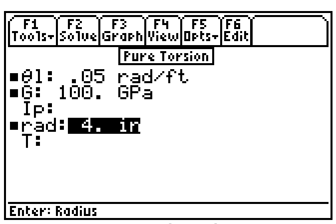
$$\tau max = \frac{T \cdot rs}{Ip}$$ **Eq. 7**

$$\tau max = \frac{16 \cdot T}{\pi \cdot d^3}$$ **Eq. 8**

Variable	**Description**	**Units**
γ	Shear strain	unitless
θl	Angle of twist/unit length	rad/m
τ	Shear stress	Pa
τmax	Maximum shear stress	Pa
ϕ	Angle of twist	rad
d	Diameter	m
G	Shear modulus	Pa
Ip	Polar moment of inertia	m^4
L	Length	m
rad	Radius	m
rs	Shaft radius	m
T	Torque	N·m

Example 20.5.1:

Compute the torque needed for a system with a radius of 4 inches, G=100 GPa, and a twist of 0.05 rad/ft.



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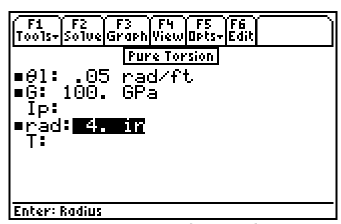


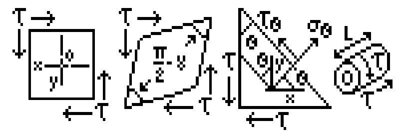
Computed results

Solution – Select the **fourth** and **fifth equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
$\theta 1$ = 0.05 rad/ft
G = 100 GPa
rad = 4 in

Solution
Ip = 0.000167 m^4
T = 2.74568E6 N·m

20.5.2 Pure Shear



When a circular bar either solid or hollow is subject to torsion, shear stress, **τ** (Pa), acts over the cross section of the inclined face of a plane inclined at an angle of **θ** (rad). The **first equation** in this set calculates, **σθ** (Pa), the normal stress perpendicular to the plane of interest in terms of shear stress, **τ** (Pa) and **θ**. The **second equation** gives the shear stress value, **τθ** (Pa), for the system on the plane. The **next two equations** complement the first two by expressing **τθ** and **σθ** in alternate forms using trigonometric identities. The **fifth equation** represents Hooke's law for shear linking shear strain, **γ** (unitless), with bulk modulus, **G** (Pa), and shear stress, **τ**. The **remaining equations** compute Young's modulus, **E** (Pa), strain, **εmax** (unitless), and elastic strain energy, **U** (J), with **G**, Poisson's ratio **ζ** (unitless), system torque, **T** (N·m), polar moment **Ip** (m^4).

$$\sigma\theta = 2 \cdot \tau \cdot \sin(\theta) \cdot \cos(\theta)$$ **Eq. 1**

$$\tau\theta = \tau \cdot \left((\cos(\theta))^2 - (\sin(\theta))^2 \right)$$ **Eq. 2**

$$\sigma\theta = \tau \cdot \sin(2 \cdot \theta)$$ **Eq. 3**

$$\tau\theta = \tau \cdot \cos(2 \cdot \theta)$$ **Eq. 4**

$$\gamma = \frac{\tau}{G}$$ **Eq. 5**

$$E = 2 \cdot G \cdot (1 + \varsigma)$$ **Eq. 6**

$$\varepsilon max = \frac{\tau}{E} \cdot (1 + \varsigma)$$ **Eq. 7**

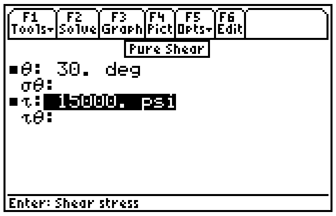
$$U = \frac{T^2 \cdot L}{2 \cdot G \cdot Ip}$$ **Eq. 8**

Variable	Description	Units
γ	Shear strain	unitless
εmax	Maximum strain	unitless
ζ	Poisson's ratio	unitless
θ	Inclined plane angle	rad
σθ	Normal stress	Pa
τ	Shear stress	Pa
τθ	Shear stress	Pa
E	Young's modulus	Pa
G	Shear modulus	Pa

Variable	Description	Units
Ip	Polar moment of inertia	m^4
L	Length	m
T	Torque	N·m
U	Elastic strain energy	J

Example 20.5.2:

Find the normal stress **σθ** and shear stress **τθ** for system when a shear stress of 15000 psi is applied to a plane at an angle of 30 degrees.



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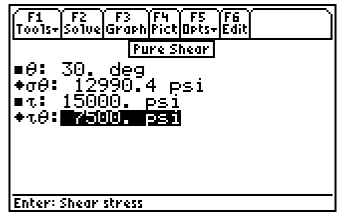
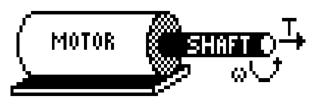


Computed results

Solution – Select the **first** and **second equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
θ = 30 degrees	σθ = 12990.4 psi
τ = 15000 psi	τθ = 7500 psi

20.5.3 Circular Shafts



Circular shafts are used invariably in transferring mechanical power from one system to another. The three equations below represent the basic properties of such a rotating system. The **first equation** connects the power, **Pwr** (W), with the torque, **T** (N·m), and angular frequency, **ω** (rad/s). The **second equation** shows the connection between **ω** and frequency, **freq** (Hz). The **last equation** expresses the relation between the rotational speed, **rpm** (rpm) and **freq**.

$$Pwr = T \cdot \omega \quad \textbf{Eq. 1}$$

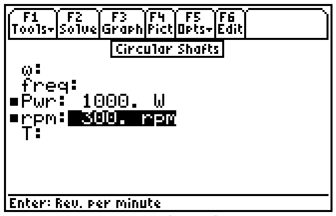
$$\omega = 2 \cdot \pi \cdot freq \quad \textbf{Eq. 2}$$

$$rpm = freq \quad \textbf{Eq. 3}$$

Variable	Description	Units
ω	Radian frequency	rad/s
freq	Frequency	Hz
Pwr	Power	W
rpm	Revolutions per minute	1/s
T	Torque	N·m

Example 20.5.3:

Find the torque needed to transfer 1000W of power at 300 revolutions per minute.



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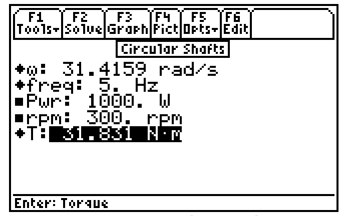
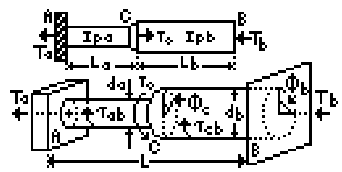


Computed results

Solution – Select **all** the **equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
Pwr = 1000 W	ω = 31.4159 rad/s
rpm = 300 rpm	freq = 5 Hz
	T = 31.831 N·m

20.5.4 Torsional Member



The ten equations in this section represent an analysis of the flexibility method applied to bars in torsion, as shown in the picture. We examine the case of two bars of diameters, **da** (m) and **db** (m), loaded by a torque, **To** (N·m), at the location C shown in the picture above. The object is to compute **Ta** (N·m) and **Tb** (N·m), the reactive torque at each end of the system. The angle of rotation, **ϕc** (rad), from C to A, and **ϕb** (rad), represents the angle of rotation from B to C. The two segments of the element have lengths, **La** (m) and **Lb** (m), giving a total length of **L** (m). **Ipa** (m^4) and **Ipb** (m^4) represent the polar moments of the regions of the member, while **τac** (Pa) and **τbc** (Pa), represent the maximum shear from A to C and maximum shear from B to C respectively. **G** (Pa), is defined as the bulk modulus. The equations listed below represent a complex set of relations between all the variables described here.

$$To = Ta + Tb \quad \textbf{Eq. 1}$$

$$\phi b = \frac{(To - Tb)\cdot La}{G\cdot Ipa} - \frac{Tb\cdot Lb}{G\cdot Ipb} \quad \textbf{Eq. 2}$$

$$Ta = \frac{To\cdot Lb}{L} \quad \textbf{Eq. 3}$$

$$Tb = \frac{To\cdot La}{L} \quad \textbf{Eq. 4}$$

$$L = La + Lb \quad \textbf{Eq. 5}$$

$$\tau ac = \frac{To\cdot Lb\cdot da}{2\cdot(La\cdot Ipb + Lb\cdot Ipa)} \quad \textbf{Eq. 6}$$

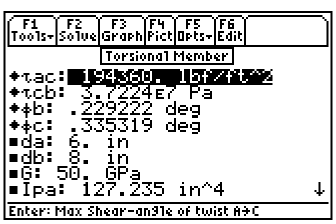
$$\tau cb = \frac{To\cdot La\cdot db}{2\cdot(La\cdot Ipb + Lb\cdot Ipa)} \quad \textbf{Eq. 7}$$

$\phi c = \dfrac{Ta \cdot La}{G \cdot Ipa}$	**Eq. 8**
$\phi c = \dfrac{Tb \cdot Lb}{G \cdot Ipb}$	**Eq. 9**
$\phi c = \dfrac{La \cdot Lb \cdot To}{G \cdot (La \cdot Ipb + Lb \cdot Ipa)}$	**Eq. 10**

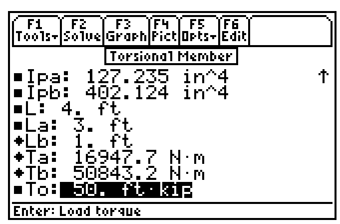
Variable	Description	Units
τac	Maximum shear at angle of twist A→C	Pa
τcb	Maximum shear at angle of twist C→B	Pa
φb	Angle of rotation at B	rad
φc	Angle of rotation at C	rad
da	Diameter at end A	m
db	Diameter at end B	m
G	Shear modulus	Pa
Ipa	Polar moment of inertia of A	m^4
Ipb	Polar moment of inertia of B	m^4
L	Length	m
La	Shaft length-diameter da	m
Lb	Shaft length-diameter db	m
Ta	Reactive torque at A	N·m
Tb	Reactive torque at B	N·m
To	Load torque	N·m

Example 20.5.4:

A shaft 4 feet long has a diameter of 6 inches for the first 3 feet and 8 inches for the remaining length. The bulk modulus has a value of 100 GPa. If a torque of 50 ft·kips is applied at the transition point, find the angles of twist, and shear stresses in the system. Assume that the polar moments are 127.235 in^4 and 402.124 in^4 respectively.



Upper Display



Lower Display

Solution – Select the **first eight equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
da = 6 in	τac = 194360 lbf/ft^2
db = 8 in	τcb = 3.7224E7 Pa
G = 50 GPa	φb = .229222 deg
Ipa = 127.235 in^4	φc = .335319 deg
Ipb = 402.124 in^4	Lb = 1 ft

Given
L = 4 ft
La = 3 ft
To = 50 ft·kip

Solution
Ta = 16947.7 N·m
Tb = 50843.2 N·m

References:

Gere and Timoshenko “Mechanics of Materials” 3rd Ed. PWS-Kent. 1990

Chapter 21: Fluid Mechanics

The fluid mechanics section of the equation library offers a collection of equations organized in convenient topics identified below. These topics have several subtopics to assist quick access to the subject matter of choice.

- **Fluid properties**
- **Fluid Dynamics**
- **Flow in Conduits**
- **Fluid Statics**
- **Surface Resistance**
- **Impulse/Momentum**

21.1 Fluid Properties

21.1.1 Elasticity

Elasticity is a measure of the deformation (expansion or contraction) of a fluid due to a pressure change. The **first equation** computes the bulk modulus of elasticity, **Ev** (Pa), for an ideal compressible fluid undergoing an **isothermal** process. **Equation 2** computes **Ev** for an **adiabatic** process. **Equation 3** computes the specific heat ratio, **k** (unitless), from the specific heat at constant pressure, **cp** (J/(kg·K)), and the specific heat at constant volume, **cv** (J/(kg·K)). **Equation 4** links **cp** and **cv** using the molecular mass, **MWT** (kg/mol) and **Rm** (J/(mol. K), the universal Gas Constant. **Equation 5** computes the density of the fluid, **ρ** (kg/m^3), from the mass, **m** (kg), occupied per volume, **vol** (m^3). The **last equation** is the ideal gas law.

$$Ev = \frac{\rho \cdot Rm \cdot T}{MWT} \quad \textbf{Eq. 1}$$

$$Ev = k \cdot p \quad \textbf{Eq. 2}$$

$$k = \frac{cp}{cv} \quad \textbf{Eq. 3}$$

$$cp = cv + \frac{Rm}{MWT} \quad \textbf{Eq. 4}$$

$$\rho = \frac{m}{vol} \quad \textbf{Eq. 5}$$

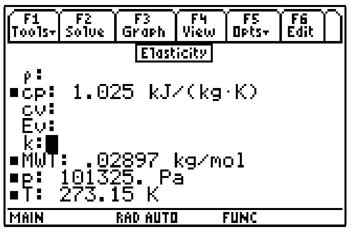
$$p = \frac{\rho \cdot Rm \cdot T}{MWT} \quad \textbf{Eq. 6}$$

Variable	Description	Units
ρ	Density	kg/m^3
cp	Specific heat at constant pressure	J/(kg·K)
cv	Specific heat at constant volume	J/(kg·K)

Variable	Description	Units
Ev	Bulk modulus for an ideal gas	Pa
k	Specific heat ratio	Unitless
m	Mass	Kg
MWT	Molar mass	kg/mol
p	Pressure	Pa
Rm	Ideal Gas constant	8.31451 J/(mol·K)
T	Temperature	K
vol	Volume	m^3

Example 21.1.1:

Compute the adiabatic elasticity for air using values for molar mass, standard temperature and pressure.



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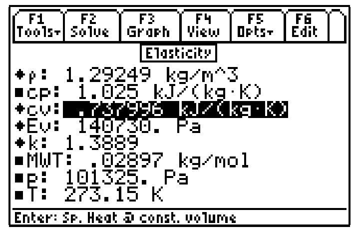
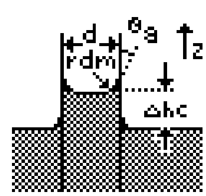


Computed results

Solution – The molar mass of dry air (ma = 0.02897 kg/mol), the standard temperature (ST = 273.15 K) and the standard pressure, (SP = 101325 Pa) are listed in ME•*Pro* under the **Reference** section in **Engineering Constants** (see Chapter 27). The specific heat for dry air, **cp** = 1.025 kJ/(kg·K), at 275 K is listed under **Reference/Thermal Properties/Specific Heat/Cp Liquids and Gases** (see Chapter 34). Select **Equations 2, 3, 4 and 6** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Note that **Rm**, is automatically inserted into the calculation by the software. The entries and results are shown in the screen displays above.

Given	Solution
cp = 1.025 kJ/(kg·K)	ρ = 1.29249 kg/m^3
MWT = .02897 kg/mol	cv = .737996 kJ/(kg·K)
p = 101325 Pa	Ev = 140730 Pa
T = 273.15 K	k = 1.3889

21.1.2 Capillary Rise



These equations compute the rise of a fluid inside a capillary tube due to surface tension. The **first equation** computes the surface tension, **σ** (N/m), of a fluid inside a capillary due to the vertical displacement of the fluid inside of the tube, **Δhc** (m), the inner diameter of the capillary, **d** (m), and the contact angle of the fluid with sides of the capillary tube, **θa** (rad). **Equation 2** computes the radius of the meniscus, **rdm** (m). The **last equation** computes the specific weight of the fluid, **γ** (N/m^3), from the density, **ρ** (kg/m^3) and the gravitational acceleration.

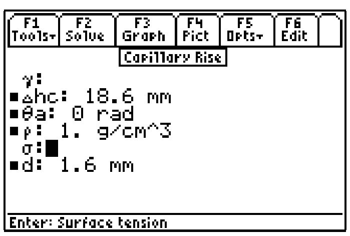
$$\Delta hc = \frac{4 \cdot \sigma \cdot \cos(\theta a)}{\gamma \cdot d}$$ **Eq. 1**

$rdm = \dfrac{d}{2 \cdot \cos(\theta a)}$	**Eq. 2**
$\rho = \dfrac{\gamma}{grav}$	**Eq. 3**

Variable	Description	Units
γ	Specific weight	N/m^3
θa	Contact angle	rad
Δhc	Capillary rise	m
ρ	Density	kg/m^3
σ	Surface tension	N/m
d	Diameter	m
grav	Gravitational acceleration	$9.80665\ m/s^2$
rdm	Meniscus radius	m

Example 21.1.2:

A capillary having an interior diameter of 1.6 mm is inserted into water (**ρ** = 1.0 g/cm^3). If a height displacement of 18.6 mm is measured and the contact angle of the meniscus is assumed to be zero, what is the surface tension?



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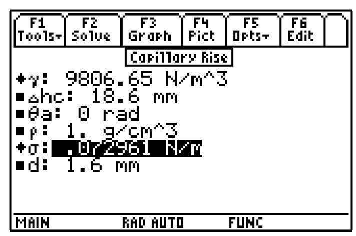


Computed results

Solution – Select the **first** and **third equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Note that **grav**, is automatically inserted into the calculation. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given

Δhc = 18.6 mm
θa = 0 rad
ρ = 1 g/cm^3
d = 1.6 mm

Solution

γ = 9806.65 N/m^3
σ = .072961 N/m

21.2 Fluid Statics

21.2 1 Pressure Variation

21.2.1.1 Uniform Fluid

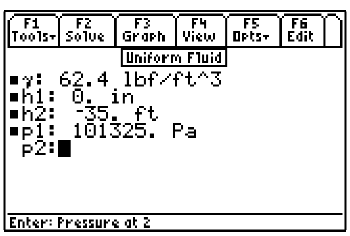
This equation computes the pressure variation, **p1** (Pa) and **p2** (Pa), between heights, **h1** (m) and **h2** (m), for a uniform density fluid, having a specific gravity, **γ** (N/m^3).

$$\frac{p1}{\gamma} + h1 = \frac{p2}{\gamma} + h2 \qquad \textbf{Eq. 1}$$

Variable	Description	Units
γ	Specific weight	N/m^3
h1	Height at 1	m
h2	Height at 2	m
p1	Pressure at 1	Pa
p2	Pressure at 2	Pa

Example 21.2.1.1:

Water has a specific weight of 62.4 lbf/ft^3 at 25 ^{o}C. If a pressure gauge is dropped 35 ft below the water surface (at sea level), and the temperature of the water is uniform, what is the pressure reading?



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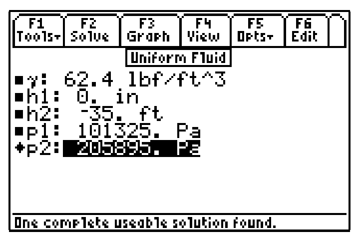


Computed results

Solution – Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknowns. The entries and results are shown in the screen displays above.

Hint: the value of specific weight, **γ** (N/m^3) can be calculated from the density, **ρ** (kg/m^3), of a fluid in the next section. The value of standard pressure for **p1**, (SP = 101325 Pa) is listed in ME•*Pro* under the **Reference** section in **Engineering Constants** (see Chapter 27).

Given
γ = 62.4 lbf/ft^3
h1 = 0
h2 = -35 ft
p1 = 101325 Pa

Solution
p2 = 205895 Pa

21.2.1.2 Compressible Fluid

These equations compute the specific weight, **γ** (N/m^3) and density, **ρ** (kg/m^3) for an ideal gas. The **first equation** calculates **γ**, from pressure, **p** (Pa), temperature, **T** (K), molar mass, **MWT** (kg/mol), and the gravitational acceleration, **grav** (9.80665 m/s^2). The **second equation** computes **γ** from its density, **ρ.**

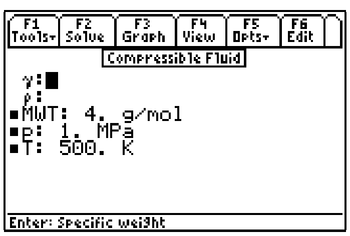
$$\gamma = \frac{p \cdot grav \cdot MWT}{Rm \cdot T} \quad \textbf{Eq. 1}$$

$$\rho = \frac{\gamma}{grav} \quad \textbf{Eq. 2}$$

Variable	Description	Units
γ	Specific weight	N/m^3
ρ	Density	kg/m^3
Grav	Gravitational acceleration	9.80665 m/s^2
MWT	Molar mass	kg/mol
Rm	Ideal gas constant	8.31451 J/(mol·K)
P	Pressure	Pa
T	Temperature	K

Example 21.2.1.2:

What is the specific weight and density for helium (molar mass = 4 g/mol) at 1 MPa and 500 K?



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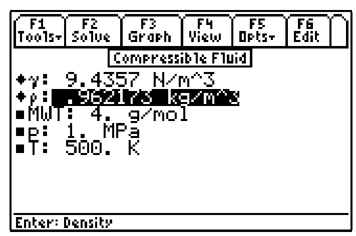


Computed results

Solution – Select **both equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. Note that **grav** and **Rm** are automatically inserted into the calculation by the software.

Given	Solution
MWT = 4 g/mol	γ = 9.4357 N/m^3
p = 1 MPa	ρ = .962173 kg/m^3
T = 500 K	

21.2.1 Pressure Variation

21.2.1.3 Troposphere

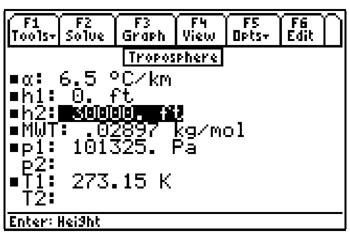
This section computes the temperature and pressure in the troposphere region of the atmosphere (up to 10,800 m above sea level). The atmospheric lapse rate in the troposphere is **α** = 6.50 K/km. These equations can also compute the variation of pressure and temperature with height for any compressible fluid.

$T2 = T1 - a \cdot (h2 - h1)$	**Eq. 1**
$p2 = p1 \cdot \left(\dfrac{T1 - a \cdot (h2 - h1)}{T1} \right)^{\frac{grav \cdot MWT}{\alpha \cdot Rm}}$	**Eq. 2**

Variable	Description	Units
α	Lapse rate	^{o}C/m
grav	Gravitational acceleration	9.80665 m/s^2
h1	Height at 1	m
h2	Height at 2	m
MWT	Molar mass	kg/mol
p1	Pressure at 1	Pa
p2	Pressure at 2	Pa
Rm	Ideal gas constant	8.31451 J/(mol·K)
T1	Temperature at 1	K
T2	Temperature at 2	K

Example 21.2.1.3:

Given the adiabatic lapse rate **α** = 6.5 °C/km, and the standard temperature and pressure conditions at sea level, calculate the atmospheric temperature and pressure at a height of 30000 ft above sea level?



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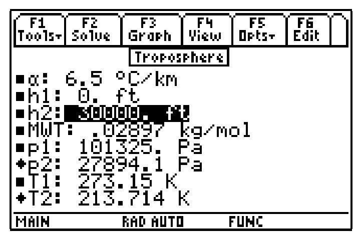


Computed results

Solution – The values for the standard temperature (ST = 273.15 K) and the standard pressure, (SP = 101325 Pa) and the molar mass of dry air (ma = 0.02897 kg/mol), are listed in ME•*Pro* under the **Reference** section of **Engineering Constants** (see Chapter 24). Select **both equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Note that **grav** and **Rm** are automatically inserted into the calculations. The entries and results are shown in the screen displays above.

Given
α = 6.5 K/km
h1 = 0 ft
h2 = 30000 ft
MWT = .02897 kg/mol
p1 = 101325 Pa
T1 = 273.15 K

Solution
p2 = 27894.1 (~28% of SP)
T2 = 213.714 K (~78% of ST)

21.2.1.4 Stratosphere

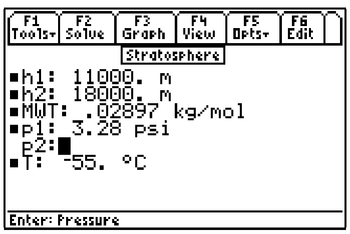
The following equation computes the variation of pressure with height in the stratosphere. The stratosphere occurs at heights greater than 10,769 m and extends to an elevation of 32.3 km. The usual assumption in stratosphere calculations is that the temperature is constant in the entire region.

$$p2 = p1 \cdot e^{\frac{-(h2-h1)\cdot MWT\cdot grav}{Rm\cdot T}} \qquad \textbf{Eq. 1}$$

Variable	**Description**	**Units**
grav	Gravitational acceleration	9.80665 m/s^2
h1	Height at 1	m
h2	Height at 2	m
MWT	Molar mass	kg/mol
p1	Pressure at 1	Pa
p2	Pressure at 2	Pa
Rm	Ideal gas constant	8.31451 J/(mol·K)
T	Temperature	K

Example 21.2.1.4:

If the pressure and temperature of the atmosphere are 3.28 psi and –55 ^{o}C at an elevation of 11,000 m, what is the pressure at 18,000 m, assuming the temperature remains constant with height in this range?



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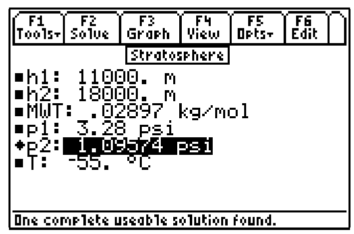
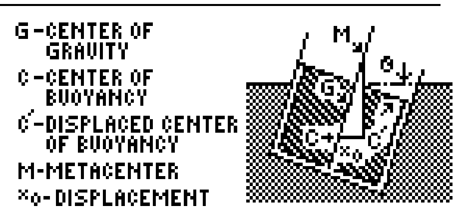


Computed results

Solution – The molar mass of dry air (ma = 0.02897 kg/mol), is listed in ME•*Pro* under the **Reference** section in **Engineering Constants** (see Chapter 24). Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. Note that **grav** and **Rm** are automatically inserted into the calculation by the software.

Given	**Solution**
h1 = 11000 m	p2 = 1.09574 psi
h2 = 18000 m	
MWT = .02897 kg/mol	
p1 = 3.28 psi	
T = -55 ^{o}C	

21.2.2.1 Floating Bodies



These equations compute the stability of a floating object due to buoyancy. A floating object has a center of gravity located at **G** and a center of buoyancy **C** along the axis of symmetry as shown in the schematic diagram shown here. The object has a second moment of **I00** (m^4) with respect to the waterline. When the object is tilted (heeled) at an angle **θ** (rad), the center of buoyancy is shifted by a distance, **xo** (m) to a new location, **C'**, due to the displacement of liquid, **Vd** (m^3), on one side of the moment axis. The intersection of the vertical line connecting **C'** to the moment axis is defined as the metacenter **M**. If the **M** is located above the **G**, the floating object is stable (ie. **CM** is positive). If **CM** is negative, **M** is below **G**, than the object is unstable (likely to tip over). The **first equation** calculates **CM** (m) from **I00** (m^4) and **Vd** (m^3). **Equation 2** computes the distance **GM** (m) between the metacenter and the center of gravity. The **last**

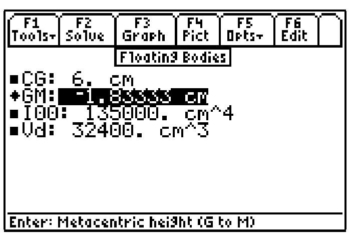
equation computes **Vd** (m^3), from the second moment of inertia, **I00**, the shift xo, between the **C and C'.**

Equation	
$$CM = \frac{I00}{Vd}$$	**Eq. 1**
$$GM = \frac{I00}{Vd} - CG$$	**Eq. 2**
$$Vd = \frac{I00 \cdot \tan(\theta)}{xo}$$	**Eq. 3**

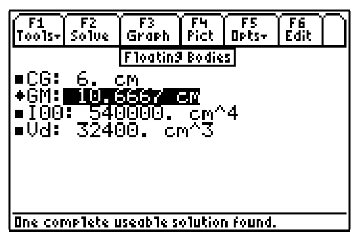
Variable	Description	Units
θ	Angle	rad
CG	Dist. from center of buoyancy to center of gravity	m
CM	Dist. from center of mass to metacenter	m
GM	Metacentric height (G to M)	m
I00	2^{nd} Moment of inertia	m^4
Vd	Displaced volume	m^3
xo	Displacement of buoyancy center	m

Example 21.2.2:

A block of wood (30 x 30x 60 cm) floats at an immersed depth of 18 cm with its longitudinal axis parallel to the water surface. The block has a longitudinal second moment of 135,000 cm^4 and a transverse second moment of 540,000 cm^4. Determine whether the object is stable on each axis.



Longitudinal Stability



Transverse Stability

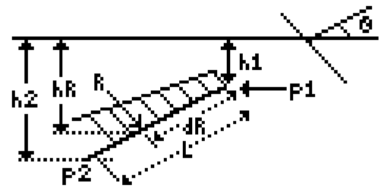
Solution – Select the **second equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. The center of gravity, **G**, is located at a distance of 15 cm (1/2 x 30 cm) from the bottom of the block and the center of buoyancy, **C**, is located at a distance of 9 cm (1/2 x 18 cm) from the bottom. The difference, **CG**, is 6 cm. The displaced volume, **Vd**, is 32,400 cm^3 (18 x 30 x 30 cm). Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
CG = 6 cm
I00 = 135,000 cm^4 (longitudinal moment)
I00 = 540,000 cm^4 (transverse moment)
Vd = 32400 cm^3

Solution
GM = -1.83333 cm (longitudinal) unstable
GM = 10.6667 cm (transverse) stable

21.2.2.2 Inclined Plane/Surface

These equations compute the pressure on a flat rectangular surface immersed in a liquid with one of its edges inclined at an angle, **θ** (rad), relative to the surface of the liquid. The **first equation** computes the density of the liquid, **ρ** (kg/m^3) from the specific weight, **γ** (N/m^3) and the gravitational acceleration, **grav** (9.80655 m/s^2). **Equation 2** computes the pressure on the inclined edge surface, **p1** (Pa), immersed at a vertical distance of **h1** (m) from the liquid's surface (see figure).



Equation 3 computes the pressure at the edge furthest from the liquid surface, **p2** (Pa), at a vertical distance **h2** (m) from the surface. **Equation 4** computes the average pressure, **pavg** (Pa), over the inclined surface. **Equation 5** calculates the vertical distance from the liquid surface to the center of pressure, **hR** (m), on the inclined surface. **Equation 6** computes the relationship between the length of the inclined side, **L** (m) and the heights, **h1** and **h2**. The **last equation** relates **hR**, to **dR** (m), the distance from the upward edge located at **h1** to the center of pressure.

$$\rho = \frac{\gamma}{grav} \quad \textbf{Eq. 1}$$

$$p1 = r \cdot grav \cdot h1 \quad \textbf{Eq. 2}$$

$$p2 = r \cdot grav \cdot h2 \quad \textbf{Eq. 3}$$

$$pavg = \frac{1}{2} \cdot \rho \cdot grav \cdot (h1 + h2) \quad \textbf{Eq. 4}$$

$$hR = \frac{2}{3} \cdot \left(h1 + h2 - \frac{h1 \cdot h2}{h1 + h2} \right) \quad \textbf{Eq. 5}$$

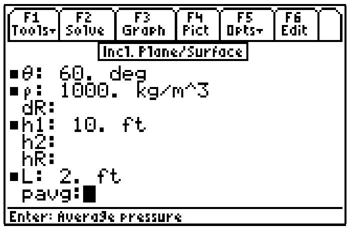
$$L = \frac{h2 - h1}{\sin(\theta)} \quad \textbf{Eq. 6}$$

$$dR = \frac{hr - h1}{\sin(\theta)} \quad \textbf{Eq. 7}$$

Variable	Description	Units
γ	Specific weight	N/m^3
θ	Angle	rad
ρ	Density	kg/m^3
dR	Distance to R from edge 1	m
grav	Gravitational acceleration	9.80665 m/s^2
h1	Height at 1	m
h2	Height at 2	m
hR	Distance to center of pressure	m
L	Length	m
p1	Pressure at 1	Pa
p2	Pressure at 2	Pa
pavg	Average pressure	Pa

Example 21.2.3:

A swimming pool has an inclined portion of its wall at an angle of 60 ^{o} and a vertical depth of 10 ft from the water's surface. The length of the incline is 2 ft. Compute the average pressure on the inclined surface and the location of the pressure center from the leading edge. Water has a density of 1000 kg/m^3.



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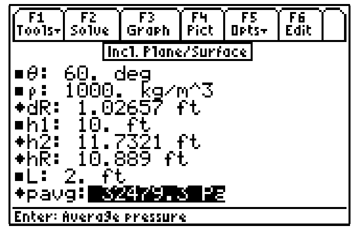


Computed results

Solution – Select the **last four equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Note that **grav** is automatically inserted into the calculations. The entries and results are shown in the screen displays above.

Given	**Solution**
θ = 60 deg	dR = 1.02657 ft
ρ = 1000 kg/m^3	h2 = 11.7321 ft
h1 = 10 ft	hR = 10.889 ft
L = 2 ft	pavg = 32479.3 Pa

21.3 Fluid Dynamics

21.3.1 Bernoulli Equation

The following equations expound the conservation of energy for fluid flow between an inlet 1 and an outlet 2. The fluid is assumed to be incompressible (i.e., the pressure does not change more than 10 % between 1 and 2) and the flow is frictionless and adiabatic. The **first equation** is the so called the Bernoulli equation. The first term on each side of the equation represents the energy associated with pressure **p1 (**Pa) and specific weight **γ** (N/m^3) at the inlet and the corresponding pressure **p2** (Pa) and **γ** (N/m^3). The second term on each side represents kinetic energy associated with the fluid flow due to velocities **v1** and **v2**.

Equation 2 is a duplicate of equation 1 with the total pressure head, **ht** (m), and equivalent to the left-hand side of the equation. **Equation 3** calculates the total pressure, **pt** (Pa), at the inlet (1) from the specific weight, **γ** (N/m^3) of the fluid and the total head, **ht** (m). **Equation 4** calculates the total head, **ht**, at the inlet (1) from the pressure, **p1** (Pa), velocity, **v1** (m/s), and head due to height, **h1** (m). **Equation 5** computes the impact head, **hs** (m). **Equation 6** calculates pressure head, **hp** (m). **Equation 7** estimates the velocity head, **hv** (m). **Equation 8** computes the total energy, **Et** (J), for a mass of fluid, **m** (kg), at the inlet. **Equation 9** computes the relationship between the specific weight, **γ** (N/m^3) and density, **ρ** (kg/m^3), of the fluid.

$$\frac{p1}{\gamma}+\frac{v1^2}{2\cdot grav}+h1=\frac{p2}{\gamma}+\frac{v2^2}{2\cdot grav}+h2 \qquad \textbf{Eq. 1}$$

$$ht = \frac{p2}{\gamma} + \frac{v2^2}{2 \cdot grav} + h2 \quad \textbf{Eq. 2}$$

$$pt = \gamma \cdot ht \quad \textbf{Eq. 3}$$

$$ht = hs + h1 \quad \textbf{Eq. 4}$$

$$hs = hp + hv \quad \textbf{Eq. 5}$$

$$hp = \frac{p1}{\gamma} \quad \textbf{Eq. 6}$$

$$hv = \frac{v1^2}{2 \cdot grav} \quad \textbf{Eq. 7}$$

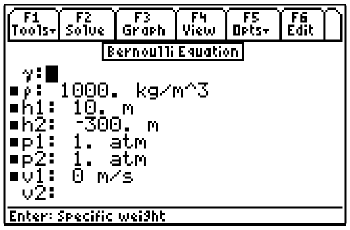
$$Et = m \cdot grav \cdot ht \quad \textbf{Eq. 8}$$

$$\rho = \frac{\gamma}{grav} \quad \textbf{Eq. 9}$$

Variable	Description	Units
γ	Specific weight	N/m^3
ρ	Density	kg/m^3
Et	Total energy of fluid	J
grav	Gravitational acceleration	$9.80665\ m/s^2$
h1	Height at 1	m
h2	Height at 2	m
hp	Pressure head	m
hs	Impact head	m
ht	Total head	m
hv	Velocity head	m
m	Mass	kg
p1	Pressure at 1	Pa
p2	Pressure at 2	Pa
pt	Total pressure	Pa
v1	Velocity at 1	m/s
v2	Velocity at 2	m/s

Example 21.3.1:

An intake pipe in a water reservoir is located 10 m below the water surface. The discharge pipe is located 300 m below the intake pipe. What is the velocity of the water leaving the discharge pipe? Assume no significant change in atmospheric pressure exists between the two heights and water has a density of 1000 kg/m^3.



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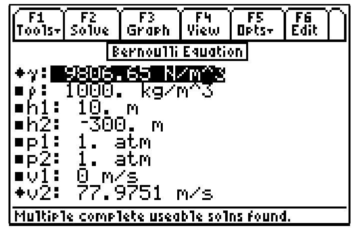


Computed results

Solution – Select the **first** and **last equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. The entries and results are shown in the screen displays above. Choose the second solution (positive velocity).

Given
ρ = 1000 kg/m^3
h1 = 10 m
h2 = -300 m
p1 = 1 atm
p2 = 1 atm
v1 = 0 m/s

Solution
γ = 9806.65 N/m^3
v2 = 77.9751 m/s

21.3.2 Reynolds Number

The following equations compute the Reynolds number, **Nre**. If **Nre** ≤ 2000, the flow is assumed to be laminar. If **Nre** >2000, the fluid flow is assumed to be turbulent[3]. **Equation 1** computes **Nre** from the equivalent diameter of the conduit, **de** (m) (determined in the next section), the average flow velocity, **v** (m/s), the fluid density, **ρ** (kg/m^3), and the absolute viscosity, **μ** ($N\cdot s/m^2$). **Equation 2** computes **Nre** from the kinematic viscosity of the fluid, **μk** (m^2/s). The **next three equations** calculate **Nre** from the mass flow rate per area, **Qf** ($kg/s/m^2$), mass flow rate, **Qm** (kg/s), and volume flow rate, **Qv** (m^3/s). The last equation relates the kinematic viscosity, **μk** (m^2/s) to the absolute viscosity, **μ** ($N\cdot s/m^2$).

$$Nre = \frac{de \cdot v \cdot \rho}{\mu} \quad \textbf{Eq. 1}$$

$$Nre = \frac{de \cdot v}{\mu k} \quad \textbf{Eq. 2}$$

$$Nre = \frac{de \cdot Qf}{\mu} \quad \textbf{Eq. 3}$$

$$Nre = \frac{4 \cdot Qm}{\pi \cdot \mu \cdot de} \quad \textbf{Eq. 4}$$

[3] A transition region between laminar and turbulent flow exists between **Nre**=2000 to 4000. However, for most cases for turbulent flow, values of **Nre** are well above 4000.

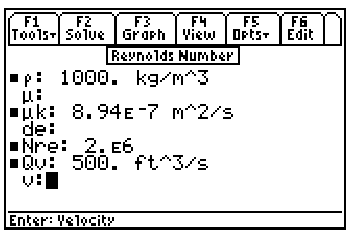
$$Nre = \frac{4 \cdot Qv \cdot \rho}{\pi \cdot \mu \cdot de}$$ **Eq. 5**

$$\mu k = \frac{\mu}{\rho}$$ **Eq. 6**

Variable	**Description**	**Units**
ρ	Density	kg/m^3
μ	Absolute viscosity	$N \cdot s/m^2$
μk	Kinematic viscosity	m^2/s
de	Equivalent diameter	m
Nre	Reynolds number	unitless
Qf	Mass flow rate per area	$kg/(s \cdot m^2)$
Qm	Mass flow rate	kg/s
Qv	Rate of volume discharge	m^3/s
v	Velocity	m/s

Example 21.3.2:

Water (25°C) has a kinematic viscosity of 8.94×10^{-7} m^2/s and a density of 1000 kg/m^3. Compute the equivalent diameter needed to support a volume flow rate of 500 cubic feet per second and maintain an approximate Reynolds value of 2×10^6. What is the linear velocity of flow?



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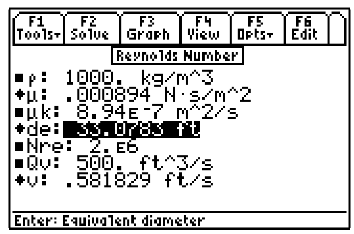


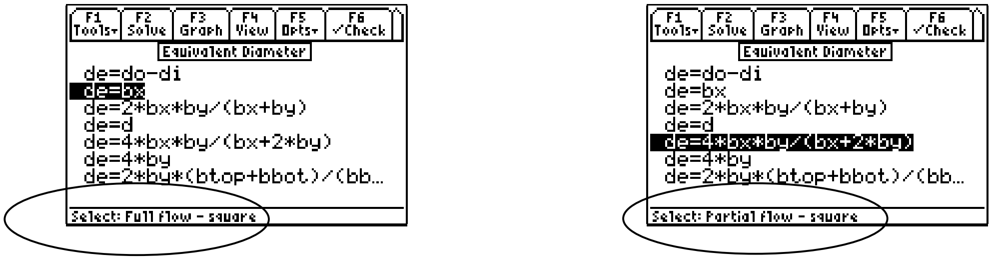
Computed results

Solution – Select the **second** and **last two equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
ρ = 1000 kg/m^3	μ = .000894 $N \cdot s/m^2$
μk = 8.94 E-7 m^2/s	de = 33.0783 ft
Nre = 2 E6	v = .581829 ft/s
Qv = 500 ft^3/s	

21.3.3 Equivalent Diameter

The following equations compute the circular pipe equivalent diameter, **de** (m), for different shaped conduit cross sections (annulus, square, rectangular and trapezoid). The equivalent diameter, **de**, is four times the ratio of the area in flow over the wetted perimeter, i.e.: **de = 4·(area in flow/wetted perimeter)**. The description of each equation appears at the bottom of the screen when the equation is highlighted with the cursor (see below). The description includes the name of the cross section and whether flow completely fills the cross-section.



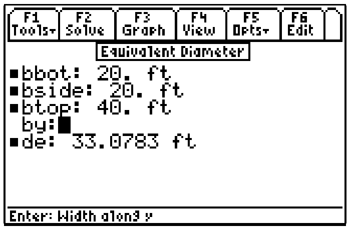
Equation description appears at the bottom of the screen

$de = do - di$	(Full flow-annulus cross-section, *do*-longer radius, *di*-shorter radius)	**Eq. 1**
$de = bx$	(Full flow-square, *bx*-side length)	**Eq. 2**
$de = \dfrac{2 \cdot bx \cdot by}{bx + by}$	(Full flow-rectangle, *bx*-width, *by*-depth)	**Eq. 3**
$de = d$	(Half-filled circle)	**Eq. 4**
$de = \dfrac{4 \cdot bx \cdot by}{bx + 2 \cdot by}$	(Partially full rectangle, *bx*-wide, *by*-depth of flow)	**Eq. 5**
$de = 4 \cdot by$	(Wide shallow stream, *by*-depth of flow)	**Eq. 6**
$de = \dfrac{2 \cdot by \cdot (btop + bbot)}{bbot + 2 \cdot bside}$	(Partial flow-trapezoid, *by*- flow depth, *btop*- top width, *bbot*- bottom width, *bs*- side length)	**Eq. 7**

Variable	**Description**	**Units**
bbot	Width along bottom	m
bside	Side of trapezoid	m
btop	Width along top	m
bx	Flow width -x	m
by	Flow depth – y	m
d	Diameter	m
de	Equivalent diameter	m
di	Inner diameter	m
do	Outer diameter	m

Example 21.3.3:

An equivalent diameter of 33.0783 ft was needed to achieve turbulent flow in the previous example (example 21.3.2). If the flow occurs in a trapezoid canal having a bottom width of 20 ft, a top width of 40 ft and a side length of 20 ft, what is the depth of flow?



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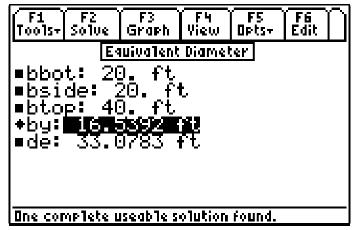


Computed results

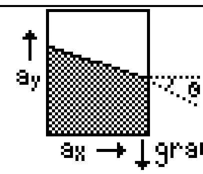
Solution – Select the **last equation** to solve this problem. Select this by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
bbot = 20 ft
bside = 20 ft
btop = 40 ft
de = 33.0783 ft

Solution
by = 16.5392 ft

21.3.4 Fluid Mass Acceleration

21.3.4.1 Linear Acceleration



These equations compute the change in surface angle of a contained liquid when subject to a linear acceleration from an applied force. The **first equation** computes the angle of incline for the liquid surface, **θ** (rad), due to the vertical, **ay** (m/s^2), horizontal, **ax** (m/s^2), and gravitational, **grav** (9.80665 m/s^2) components of acceleration. The vertical acceleration, **ay**, is opposite in direction to **grav**. The **second equation** calculates the head pressure of the liquid, **ph** (Pa), due to the height change, **Δh** (m), gravitation, **grav**, and vertical acceleration, **ay**.

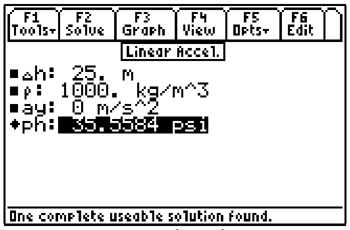
$$\tan(\theta) = \frac{ax}{ay + grav} \qquad \textbf{Eq. 1}$$

$$ph = \rho \cdot grav \cdot \Delta h \cdot \left(1 + \frac{ay}{grav}\right) \qquad \textbf{Eq. 2}$$

Variable	**Description**	**Units**
Δh	Height difference	m
θ	Angle	rad
ρ	Density	kg/m^3
ax	Acceleration along x	m/s^2
ay	Acceleration along y	m/s^2
grav	Gravitational acceleration	9.80665 m/s^2
ph	Pressure head	Pa

Example 21.3.4.1:

A pipe joint in a rocket is equipped with a seal that can withstand a fluid pressure of 100 psi. What is the increase in pressure for the pipe seal system, having a head of 25 m, in the earth's gravitational field, and then subjected to an additional vertical acceleration of 3 g's (gravitational fields) due to launching? Will the seal be able to withstand the additional pressure? Use water as a fluid for the pipe system and ignore the change in atmospheric pressure with height.



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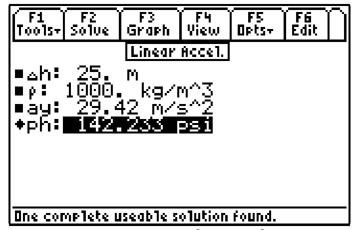


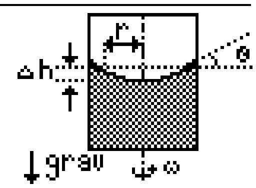
Computed results

Solution – Select the **second equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. The density of water is approximately 1000 kg/m^3. First compute the pressure, **ph**, where **ay=0**, then calculate the pressure when **ay = 3*grav**, where **grav** is a defined constant in ME•*Pro* for the gravitational acceleration constant (9.80665 m/s^2). The gravitational constant, **grav**, is automatically inserted into the calculation and does not appear in the list of variables. The entries and results are shown in the screen displays above.

Given
Δh = 25 m
ρ = 1000 kg/m^3
ay = 0 m/s^2
ay = 3* grav (29.42 m/s^2)

Solution
ph = 35.5584 psi (ay = 0 m/s^2)
ph = 142.233 psi (ay = 29.42 m/s^2)
(The seal does not have sufficient capacity)

21.3.4.2 Rotational Acceleration



When a liquid mass is rotated about its central axis at angular velocity, **ωr** (rad/s), the liquid surface forms into a parabolic shape (see diagram). The **first equation** computes the average angle of incline, **θ** (rad), of a liquid surface between the axis of rotation and the edge of the vessel containing the liquid. The **second equation** computes the change in height of the center of the meniscus due to rotation. The radius of the meniscus during rotation, **r** (m), is measured from the axis of rotation to the edge of the liquid at the height of the liquid surface prior to rotation. **Equation 3** computes the tangential velocity, **v** (m/s), of the rotating liquid at distance, **r**, from the axis of rotation.

$$\tan(\theta) = \frac{\omega^2 \cdot r}{grav} \qquad \textbf{Eq. 1}$$

$$\Delta h = \frac{(\omega \cdot r)^2}{2 \cdot grav} \qquad \textbf{Eq. 2}$$

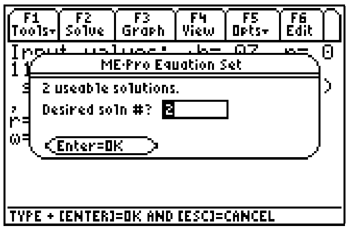
$$v = \omega \cdot r \qquad \textbf{Eq. 3}$$

Variable	Description	Units
Δh	Height difference	m
θ	Angle	rad

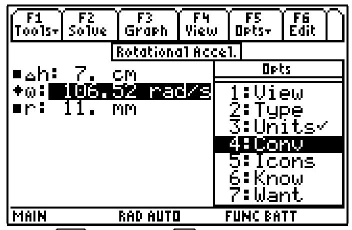
Variable	Description	Units
ω	Angular velocity	rad/s
grav	Gravitational acceleration	9.80665 m/s^2
r	Radius	m
v	Velocity	m/s

Example 21.3.4.2:

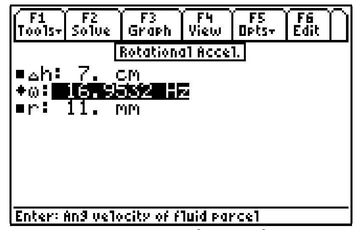
A 22 cm length test tube, having a diameter of 22 mm is filled with a liquid to a height of 15 cm. If the test tube is placed on an agitator, which spins the fluid, what is the maximum rotational velocity setting (Hz) that can be used to prevent the liquid from spilling due to rotation? Use the radius of the test tube as a rough estimate for the meniscus radius.



Dialogue displaying number of possible solutions (select 2)



*Use [F5]:**Opts**, [4] **Conv.** to convert ω to units of Hz*



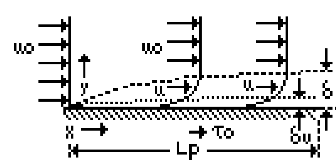
Computed results

Solution – Select the **second equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. Select the second of two possible solutions (positive angular velocity). To convert the value of **ω** (rad/s) to units of Hertz, highlight **ω**, and press [F5]: Opts, [4] Conv. The unit menu for **ω** will appear in the tool bar at the top. Press [F6] Hz. The entries and results are shown in the screen displays above.

Given	**Solution**
Δh = 7 cm	ω = 16.9532 Hz (2nd solution)
r = 11 mm	

21.4 Surface Resistance

21.4.1 Laminar Flow – Flat Plate



These equations describe the dimensions and velocity profile of boundary layer formation over a plate for laminar flow. When a fluid, having a mean velocity, **uo** (m/s), encounters a plate parallel to its stream, a velocity gradient forms over the plate. The velocity, **u** (m/s), at the surface of the plate is zero and increases with height until **u=uo**. The region where the velocity gradient occurs is the laminar boundary layer. The height at which **u=0.99·uo** is the boundary layer depth, **δ** (m). The thickness of the boundary layer, **δ** (m), increases with distance from the edge of the plate, **x** (m). The **first equation** computes the thickness of the boundary layer, **δ**, at distance **x** (m) from the edge the plate. **Equation 2** computes the shear stress, **τo** (N/m^2), on the plate at a given distance from the plate edge, **x**. **Equation 3** computes the shear force, **Fs** (N), exerted over an upwind portion of the plate having an area, **b·x** (m^2), where, **b** (m) is the width of the plate and **x** is the traveled distance of the fluid over the plate. **Equation 4** computes the Reynolds value, **Re**, from the mean velocity of the fluid, **uo**, the distance traveled by the fluid, **x**, and the kinematic viscosity, **μk** (m^2/s). **Equation 5** calculates the velocity, **u**, inside the laminar boundary layer at distance, **x**, from the plate edge and vertical distance, **y** (m) from the plate surface. **Equation 6** calculates the local shear stress coefficient, **cf**, from the shear, **τo**, and mean fluid velocity, **uo**. **Equation 7** computes the average shear stress coefficient, **Cof**, for the length of the plate, **Lp** (m). The **last equation** relates the absolute viscosity, **μ** ($N\cdot s/m^2$), to the kinematic viscosity, **μk** (m^2/s).

$$\frac{\delta}{x}\cdot\sqrt{\mathrm{Re}} = 5 \qquad \textbf{Eq. 1}$$

$$\tau o = \frac{\mu\cdot uo\cdot\sqrt{\mathrm{Re}}}{3\cdot x} \qquad \textbf{Eq. 2}$$

$$Fs = .664\cdot b\cdot\mu\cdot uo\cdot\sqrt{\mathrm{Re}} \qquad \textbf{Eq. 3}$$

$$\mathrm{Re} = \frac{uo\cdot x}{\mu k} \qquad \textbf{Eq. 4}$$

$$\frac{u}{uo} = \frac{.332\cdot y\cdot\sqrt{\mathrm{Re}}}{x} \qquad \textbf{Eq. 5}$$

$$cf = \frac{2\cdot\tau o}{\rho\cdot uo^2} \qquad \textbf{Eq. 6}$$

$$Cof = \frac{1.33}{\left(\frac{uo\cdot Lp}{\mu k}\right)^{.5}} \qquad \textbf{Eq. 7}$$

$$\mu = \mu k\cdot\rho \qquad \textbf{Eq. 8}$$

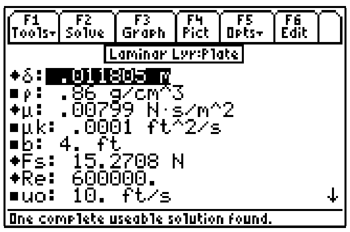
Variable	**Description**	**Units**
δ	Boundary layer thickness	m
ρ	Density	kg/m^3
τo	Shear stress	N/m^2
μ	Absolute viscosity	$N\cdot s/m^2$
μk	Kinematic viscosity	m^2/s
b	Breadth	m
cf	Local shear stress coefficient	unitless
Cof	Average shear stress coefficient	unitless
Fs	Shear force	N
Lp	Length of plate	m
Re	Reynolds value at x	unitless
u	Velocity	m/s
uo	Free atream velocity	m/s
x	Distance from edge	m
y	Depth: distance along y	m

Caution: *Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing* [F2] *will not select all the equations and start the solver*.

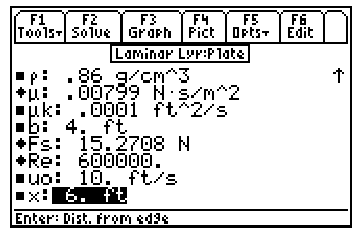
Example 21.4.1:

Caution: *Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing* [F2] *will not select all the equations and start the solver*.

Crude oil, at 70 °F ($\mu k = 10^{-4}$ft/s, $\rho = 0.86$ g/cm^3), flows over plate with a free stream velocity of 10 ft/s. The plate has a width of 4 ft and a length of 6 ft. Compute the thickness of the boundary layer at the end of the plate and the total drag force exerted on the plate by the fluid.



Upper Display

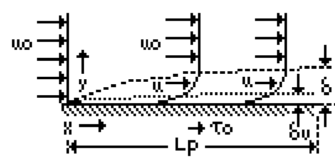


Lower Display

Solution – Select **Equations 1, 3, 4,** and **8** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
ρ = .86 g/cm^3	δ = .011805 m
μk = .0001 ft^2/s	μ = .00799 N·s/m^2
b = 4 ft	Fs = 15.2708 N
uo = 10 ft/s	Re = 600000
x = 6 ft	

21.4.2 Turbulent Flow – Flat Plate



The following equations describe turbulent flow over a smooth plate. Turbulent flow is more complex than the laminar case. Different velocity profiles occur in different regions above the surface and different equations are required for each. The boundary layer depth, δ, is defined as the region where **u ≤ 0.99·uo**. This section contains equations for three regions in the boundary layer region, δ (m), above a plate. The viscous sub layer, immediately above the surface, is characterized laminar flow, shear is constant with height, and momentum transport occurs due to viscous forces rather than turbulence. The viscous sub layer underneath a turbulent layer has a depth, **δv** (m), computed in **Equation 10** and a velocity profile described in **Equation 11**. A second region above, **δv**, is the turbulent layer where logarithmic velocity profile law applies (**Eq. 12**). The logarithmic profile is valid for heights above the surface, **y** (m), where **δv<y<0.15·δ.** The final region inside the boundary layer is approximated by the power law (**Eq. 13**). This equation overlaps region where the logarithmic profile equation and spans from **0.1·δ<y<δ** inside the boundary layer. The **first equation** relates absolute viscosity, μ (N·s/m^2) to the kinematic viscosity of the fluid, **μk** (m^2/s). **Equation 2** computes the local shear stress coefficient, **cf**, for the surface. **Equation 3** calculates the average shear stress coefficient, **Cof**, over the length of the plate, **Lp** (m). **Equation 4** calculates the shear force per area, **τo**, (N/m^2) on the plate. **Equation 5** computes the shear force, **Fs** (N), over the area of the plate having a width, **b** (m), from the fluid density, **ρ** (kg/m^3), the free stream velocity, **uo** (m/s), and boundary layer depth, **δ** (m). **Equation 6** computes the friction velocity, **uf** (m/s). The Reynolds value, **Re**, is computed in the **seventh equation**. The determinant, **Rf**, in **Equation 8**, is used to determine whether height, **y**, occurs inside the viscous sub layer (**Rf<11.84**), or in the logarithmic profile region (**30<Rf<500,** or **y/δ<0.15**). The boundary layer depth, **δ** (m), is computed in **Equation 9**. The depth of the viscous sub layer, **δv**, is determined in **equation 10**. **Equation 11** estimates the velocity, **u** (m/s), at height **y** inside the viscous sub layer. **Equation 12** computes the velocity, **u**, at height **y** in the turbulent region, **y>δv**. **Equation 11** is valid for Reynolds values, **Re<10^7**. **Equation 12**, is known as the power law, and approximates, to a good degree, the velocity profile over about 90% of the boundary layer **0.1<y/δ<1**. The **last two equations** compute the velocity, **u**, at a height, **y**, in an equilibriated turbulent boundary layer above a flat roughened surface having a roughness length, **kd** (m), height displacement, **hd** (m), friction velocity, **uf**, and the von Kármán constant, **vk** (~0.4). **ψ** is the stability factor (**ψ=0** neutral, **ψ>0** stable, **ψ<0** unstable) and is calculated in the last equation from the Monin-Obukhov path length, **Lm** (m) where **Lm= +5 m** for extremely stable conditions, **Lm= -5 m** for extremely unstable conditions, and **Lm= ∞** for neutral conditions.

$$\mu = \rho \cdot \mu k \qquad \textbf{Eq. 1}$$

$$cf = \frac{.058}{\mathrm{Re}^{1/5}} \qquad \textbf{Eq. 2}$$

$$Cof = \frac{.074}{\left(\dfrac{uo \cdot Lp}{\mu k}\right)^{1/5}} \qquad \textbf{Eq. 3}$$

$$\tau o = \frac{cf \cdot \rho \cdot uo^2}{2} \qquad \textbf{Eq. 4}$$

$$Fs = b \cdot \rho \cdot uo^2 \cdot \delta \qquad \textbf{Eq. 5}$$

$$uf = \sqrt{\frac{\tau o}{\rho}} \qquad \textbf{Eq. 6}$$

$$\mathrm{Re} = \frac{uo \cdot x}{\mu k} \qquad \textbf{Eq. 7}$$

$$Rf = \frac{uf \cdot y}{\mu k} \qquad \textbf{Eq. 8}$$

$$\delta = \frac{0.37 \cdot x}{\mathrm{Re}^{.2}} \qquad \textbf{Eq. 9}$$

$$\delta v = \frac{5 \cdot \mu k}{uf}$$ **Eq. 10**

when y ≤ δv the following equation is applicable

$$u = \frac{\tau o \cdot y}{\mu}$$ **Eq. 11**

when δv < y ≤ .15·δ the following equation is applicable

$$\frac{u}{uf} = 5.75 \cdot \log\left(\frac{y \cdot uf}{\mu k}\right) + 5.56$$ **Eq. 12**

when 0.1·δv < y < δ the following equation is applicable

$$u = uo \cdot \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$ **Eq. 13**

when kf>0 (turbulent flow over a rough surface) the following equations are applicable

$$\frac{u}{uf} = \frac{1}{vk} \cdot \left(\ln\left(\frac{y}{kf}\right) + \psi\right)$$ **Eq. 14**

$$\psi = \frac{4.7 \cdot y}{Lm}$$ **Eq. 15**

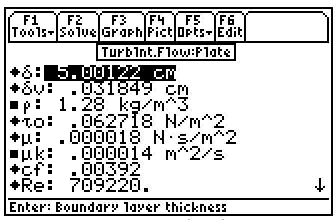
Variable	**Description**	**Units**
δ	Boundary layer thickness	m
δv	Viscous sub layer depth	m
ρ	Density	kg/m^3
τo	Shear stress	N/m^2
ψ	Stability (ψ>0 stable, ψ<0 unstable, ψ=0 neutral)	unitless
μ	Absolute viscosity	$N \cdot s/m^2$
μk	Kinematic viscosity	m^2/s
b	Breadth	m
cf	Local shear stress coefficient	unitless
Cof	Average shear stress coefficient	unitless
Fs	Shear force	N
kf	Roughness Length	m
Lm	Monin-Obukhov path length (L= +5m stable, L= -5m unstable, L=∞ neutral)	m
Lp	Length of plate	m
Re	Reynolds value at x	unitless
Rf	Reynolds determinant	unitless
u	Velocity	m/s
uf	Friction velocity	m/s

Variable	Description	Units
uo	Free stream velocity	m/s
vk	von Kármán constant (~0.4)	unitless
x	Distance from edge	m
y	Depth: distance along y	m

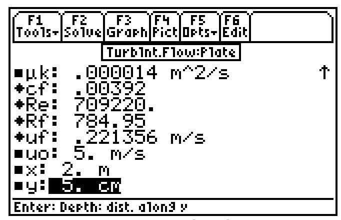
Caution: *Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing* F2 *will not select all the equations and start the solver*.

Example 21.4.2:

Air flows with a velocity of 5 m/s parallel with a smooth plate. What is the nominal thickness of the viscous layer, the shear force at a horizontal distance of 2 m from edge of the plate and a height of 5 cm above the surface? The kinematic viscosity and density of air at (10 ^oC, 1 atm) is 1.41 x 10^{-5} m^2/s and 1.28 kg/m^3 (the density of air can be calculated using the **last equation** in the first section of this chapter, 21.1.1:Elasticity using **MWT** = 28.97 g/mol).



Upper Display



Lower Display

Solution – Solve this problem in two steps. First determine whether y=5 cm is inside the boundary layer, and, if it is, which equation of velocity should be used. Select **Equations 1, 2, 4, 6, 7, 8, 9,** and **10.** Select these by highlighting the equations and pressing ENTER. Press F2 to display the variables. Enter the values for the known parameters and press F2 to solve for the unknown variables. Since **y** is close to the height of **δ** and **Rf>500**, **equation 13** (power law) should be used to compute the velocity. Press ESC, select the **last equation**, and press F2 to return to the variable list, and F2 to solve. The entries and results are shown in the screen displays above.

Given	**Solution**
ρ = 1.28 kg/m^3	δ = 5.00122 cm
μk = 1.41E^{-5}	δv = .031849 cm
uo = 5 m/s	τo = .062718 N/m^2
x = 2 m	μ = .000018 $N \cdot s/m^2$
y = 5 cm	cf = .00392
	Re = 709220
	Rf = 784.95
	u = 4.99983 m/s
	uf = .221356 m/s

21.4.3 Laminar Flow on an Inclined Plane

These equations describe laminar flow of a fluid down a smooth plate inclined at an angle **θ** (rad). The flow is uniform and occurs as a result of gravitational acceleration. These equations assume a constant flow depth, **d** (m), and velocity, **u** (m/s), with distance down the inclined plane. In addition, it is assumed that the shear, **τo** (N/m^2), decreases linearly with height from the plate surface (bottom), where shear is at it's maximum, to the height of the liquid surface, **d**, where shear with the atmosphere is assumed to be negligible. The criteria of whether laminar or turbulent flow exists is determined by the Reynolds number, **Nre**. In this case, if **Nre≤500**, the flow is considered laminar. **Equations 1** and **2** compute the average velocity of the fluid, **uavg** (m/s) from the specific weight, **γ** (N/m^3), the absolute viscosity, **μ** (N·s/m^2), kinematic viscosity, **μk** (m^2/s), the angle of incline, **θ**, and the depth of fluid flow, **d** (m). **Equation 3** computes the slope of the incline, **So**. **Equation 4** calculates the flow velocity, **u** (m/s), at height, **y** (m), from the plane. **Equation 5** computes the maximum velocity, **umax** (m/s), when **y=d**. The **sixth equation** computes the Reynolds number, **Nre**. **Equation 7** calculates the shear, **τo**, at height, **y**, from the plate surface. **Equation 8** relates the absolute viscosity, **μ** (N·s/m^2) to the kinematic viscosity, **μk** (m^2/s), of the fluid. The **last equation** relates the specific weight, **γ**, to the density **ρ** (kg/m^3) of the fluid.

$$uavg = \frac{\gamma \cdot d^2}{3 \cdot \mu \cdot \sin(\theta)}$$ **Eq. 1**

$$uavg = \frac{grav \cdot d^2}{3 \cdot \mu k} \cdot \sin(\theta)$$ **Eq. 2**

$$So = \tan(\theta)$$ **Eq. 3**

$$u = \frac{grav \cdot So}{2 \cdot \mu k} \cdot y \cdot (2 \cdot d - y)$$ **Eq. 4**

$$umax = \frac{grav \cdot So}{2 \cdot \mu k} \cdot d^2$$ **Eq. 5**

$$Nre = \frac{uavg \cdot d}{\mu k}$$ **Eq. 6**

$$\tau o = \gamma \cdot \sin\theta (d - y)$$ **Eq. 7**

$$\mu = \mu k \cdot \rho$$ **Eq. 8**

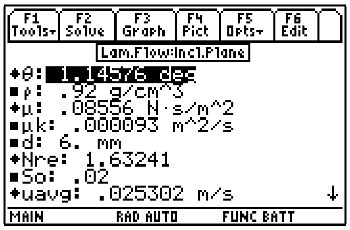
$$\gamma = \rho \cdot grav$$ **Eq. 9**

Variable	**Description**	**Units**
γ	Specific weight	N/m^3
θ	Angle	rad
ρ	Density	kg/m^3
τo	Shear stress	N/m^2
μ	Absolute viscosity	N·s/m^2
μk	Kinematic viscosity	m^2/s
d	Depth	m
grav	Gravitational acceleration	9.80665 m/s^2
Nre	Reynolds number	unitless
So	Slope	unitless
u	Velocity	m/s
uavg	Average velocity	m/s
umax	Maximum velocity	m/s

Variable	**Description**	**Units**
y	Depth: distance along y	m

Example 21.4.3:

Crude oil flows down a plate having a slope incline of 0.02. Crude oil has a density of $0.92 g/cm^3$ and a kinematic viscosity of 9.3×10^{-5} m^2/s. If the depth of flow is 6 mm, what are the maximum and average flow velocities? Is the flow laminar?



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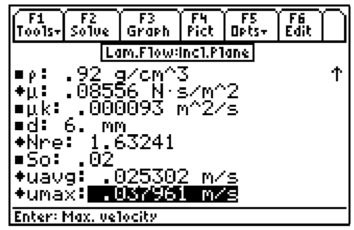


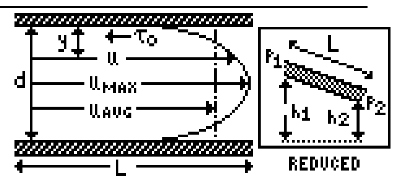
Lower Display

Solution – Select **Equations 2, 3, 5, 6,** and **8,** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. **Select an arbitrary integer of 0** to compute the principal solution. The entries and results are shown in the screen displays above.

Given	**Solution**
$\rho = .92\ g/cm^3$	$\theta = 1.14576$ deg
$\mu k = .000093\ m^2/s$	$\mu = .08556\ N\cdot s/m^2$
d = 6 mm	Nre = 1.63241 (laminar)
So = .02	uavg = .025302 m/s
	umax = .037961 m/s

21.5 Flow in Conduits

21.5.1 Laminar Flow: Smooth Pipe



These equations describe laminar flow of an incompressible fluid in a smooth pipe, having a circular cross-section and length, **L** (m). The **first equation** the computes the pressures, **p1** and **p2** (Pa), on two ends of a pipe due to height (or head) difference, **h1-h2** (m), and head loss due to friction, **hf** (m). **Equation 2** calculates the velocity, **u** (m/s), at a distance, **y** (m) from the wall towards the center of the pipe. **Equation 3** computes the shear, **τo** (N/m^2), on the surface of the pipe opposite to the direction of fluid flow. **Equation 4** estimates the head loss due to friction, **hf**, over the length of the pipe section, **L** (m), from the mean velocity, **uavg** (m/s), and the diameter of the pipe, **d** (m). **Equation 5** computes the Reynolds number, **Nre**, for the flow in the pipe. **If Nre<2000, flow is considered to be laminar**. **Equation 6** calculates the volume flow rate, **Q** (m^3/s), from the mean velocity of the fluid, **uavg**, and the area of the pipe. **Equation 7** calculates the resistance coefficient, **fr**, for laminar flow in a pipe having a Reynolds number, **Nre**. **Equation 8** relates the absolute viscosity, **μ** ($N\cdot s/m^2$), to the kinematic viscosity, **μk** (m^2/s), of the fluid. The **last equation** relates the specific weight, **γ** (N/m^3), to the density, **ρ** (kg/m^3), of the fluid.

$$\frac{p1}{\gamma} + h1 = \frac{p2}{\gamma} + h2 + hf \qquad \textbf{Eq. 1}$$

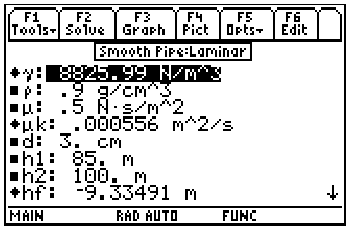
$u = \frac{d \cdot y - y^2}{4 \cdot \mu} \cdot \frac{hf \cdot \gamma}{L}$	**Eq. 2**
$\tau o = \frac{d}{4 \cdot L} \cdot hf \cdot \gamma$	**Eq. 3**
$hf = \frac{32 \cdot \mu \cdot L \cdot uavg}{\gamma \cdot d^2}$	**Eq. 4**
$Nre = \frac{\lvert uavg \rvert \cdot d}{\mu k}$	**Eq. 5**
$Q = \frac{uavg \cdot \pi}{4} \cdot d^2$	**Eq. 6**
$fr = \frac{64}{Nre}$	**Eq. 7**
$\gamma = \rho \cdot grav$	**Eq. 8**
$\mu = \mu k \cdot \rho$	**Eq. 9**

Variable	**Description**	**Units**
γ	Specific weight	N/m^3
ρ	Density	kg/m^3
τo	Shear stress	N/m^2
μ	Absolute viscosity	$N \cdot s/m^2$
μk	Kinematic viscosity	m^2/s
d	Diameter	m
fr	Friction coefficient	unitless
grav	Gravitational acceleration	$9.80665\ m/s^2$
h1	Height at 1	m
h2	Height at 2	m
hf	Head loss due to friction	m
L	Length	m
Nre	Reynolds number	unitless
p1	Pressure at 1	Pa
p2	Pressure at 2	Pa
Q	Rate of volume discharge	m^3/s
u	Velocity	m/s
uavg	Average velocity	m/s
y	Depth: distance along y	m

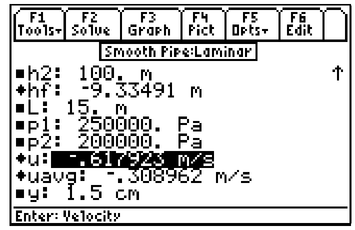
Caution: *Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing* [F2] *will not select all the equations and start the solver*.

Example 21.5.1:

Oil, having a density of 0.9 g/cm^3 and an absolute viscosity of 5 x 10^{-1} N·s/m^2, flows down a vertical 3 cm pipe. The pipe pressure at a height of 85 m is 250,000 Pa and the pressure at 100 m is 200,000 Pa. What is the direction of flow and the fluid velocity at the center of the pipe?



Upper Display



Lower Display

Solution – Select **Equations 1, 2, 4, 8** and **9** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. Note: the equations compute a positive velocity and head loss if flow occurs from 1$\rightarrow$2 and a negative velocity for flow in the direction of 2$\rightarrow$1. In this case, the lower height and pressure were entered for **h1** and **p1**, and the height and pressure at the higher elevation were entered as **h2** and **p2**. The negative velocity for **u** indicates a movement of fluid downwards due to gravity. The entries and results are shown in the screen displays above.

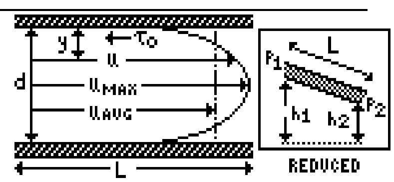
Given

ρ = .9 g/cm^3
μ = .5 N·s/m^2
d = 3 cm
h1 = 85 m
h2 = 100 m
L = 15 m
P1 = 250000Pa
P2 = 200000 Pa
Y= 1.5 cm

Solution

γ = 8825.99 N/m^3
μk = .000556 m^2/s
hf = -9.33491 m
u = -.617923 m/s
uavg = -.308962 m/s

21.5.2 Turbulent Flow: Smooth Pipe



Turbulent flow occurs in pipes when the Reynolds number **Nre>3000**. These equations describe turbulent flow of an incompressible fluid in through a smooth circular pipe of length, **L** (m) and constant diameter, **d** (m). The **first equation** the computes the pressures at each end of a pipe due to height (or head) difference, **h1-h2** (m), and head loss due to friction, **hf** (m). **Equation 2** (Darcy-Weisbach equation) estimates the head loss due to friction, **hf**, over the length of the pipe section, **L**, from the mean velocity, **uavg** (m/s), and diameter of the pipe, **d**. **Equation 3** is an analytical expression which relates the resistance coefficient, **fr**, for turbulent flow in a pipe, to the Reynolds number, **Nre**. **Equation 4** computes the friction velocity, **uf** (m/s). **Equation 5** calculates, **Rf**, a determinant used for the velocity profile equations, **Equation. 11** and **12**. The computed value of **Rf** determines whether a distance, **y** (m) from the side of the pipe, lies inside the viscous sub layer (non turbulent) region of pipe flow, or the turbulent region. **Equation 6** computes the Reynolds number, **Nre**, for the flow in the pipe. **Equation 7** computes the shear, τ**o** (N/m^2), on the surface of the pipe in the direction opposite to the direction of fluid flow. **Equation 8** calculates the volume flow rate, **Q** (m^3/s), from the mean velocity of the fluid, **uavg**, and the cross-sectional flow area of the pipe. **Equation 9** calculates the specific weight, **γ** (N/m^3), from the density of the fluid, **ρ** (kg/m^3), and the gravitational acceleration, **grav** (9.80655 m/s^2). **Equation 10** relates the absolute viscosity, **μ** (N·s/m^2) to the kinematic viscosity, **μk** (m^2/s) of the fluid. **Equations 11, 12** and **13** compute velocity profiles inside the pipe at vertical distance, **y**, from the edge. If **Rf<5** then **equation 11** is used to compute the velocity at height **y**. If **20<Rf<10^5**, than the logarithmic profile, **equation 12**, is should be used. The appropriate ranges of for the determinant, **Rf**, or Reynolds number, **Nre**, for each equation are listed in a **when (…)** clause preceding the equation. The coefficient, **n**, in the **last equation** varies with Reynolds number, **Nre**. The appropriate values of **n** for **Nre** are listed in **Table 21.1**.

Table 21.1 Empirical values of n vs. Nre in Equation 13[1]

Nre	n
4 x 10^3	6.0
2.3 x 10^4	6.6
1.1 x 10^5	7.0
1.1 x 10^6	8.8
3.2 x 10^6	10.0

$$\frac{p1}{\gamma}+h1=\frac{p2}{\gamma}+h2+hf \quad \textbf{Eq. 1}$$

$$|hf|=\frac{fr\cdot L\cdot uavg^2}{2\cdot grav\cdot d} \quad \textbf{Eq. 2}$$

$$\frac{1}{\sqrt{fr}}=2\cdot\log\left(Nre\cdot\sqrt{fr}\right)-.8 \quad \textbf{Eq. 3}$$

$$uf=uavg\cdot\sqrt{\frac{fr}{8}} \quad \textbf{Eq. 4}$$

$$Rf=\frac{|uf|\cdot y}{\mu k} \quad \textbf{Eq. 5}$$

$$Nre=\frac{|uavg|\cdot d}{\mu k} \quad \textbf{Eq. 6}$$

$$\tau o=uf^2\cdot\rho \quad \textbf{Eq. 7}$$

$$Q = \frac{uavg \cdot \pi}{4} \cdot d^2$$ **Eq. 8**

$$\gamma = \rho \cdot grav$$ **Eq. 9**

$$\mu = \mu k \cdot \rho$$ **Eq. 10**

when 0< Rf < 5, the following equation is applicable

$$|u| = \frac{uf^2 \cdot y}{\mu k}$$ **Eq. 11**

when 20< Rf < 1 E5, the following equation is applicable

$$\frac{u}{uf} = 5.75 \cdot \log\left(\frac{y \cdot |uf|}{\mu k}\right) + 5.5$$ **Eq. 12**

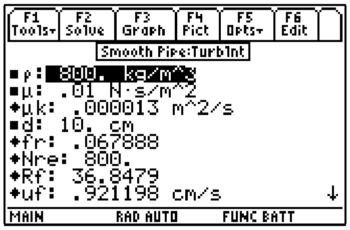
when 4 E3< Nre < 3.2 E6, the following equation is applicable

$$u = u\max \cdot \left(\frac{2 \cdot y}{d}\right)^{\frac{1}{n}}$$ **Eq. 13**

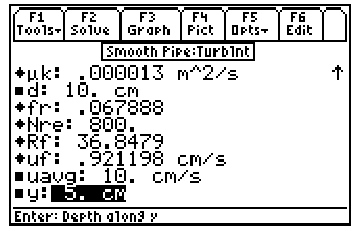
Variable	**Description**	**Units**
γ	Specific weight	N/m^3
ρ	Density	kg/m^3
τo	Shear stress	N/m^2
μ	Absolute viscosity	$N \cdot s/m^2$
μk	Kinematic viscosity	m^2/s
d	Diameter	m
fr	Friction coefficient	unitless
grav	Gravitational acceleration	$9.80665\ m/s^2$
h1	Height at 1	m
h2	Height at 2	m
hf	Head loss due to friction	m
L	Length	m
n	Constant: turbulent flow (see Table 21.1)	unitless
Nre	Reynolds number	unitless
p1	Pressure at 1	Pa
p2	Pressure at 2	Pa
Q	Volume discharge	m^3/s
Rf	Determinant	unitless
u	Velocity	m/s
uf	Friction velocity	m/s
uavg	Average velocity	m/s
umax	Maximum velocity	m/s
y	Depth: distance along y	m

Example 21.5.2:

A fluid having a density of 800 kg/m^3 and an absolute viscosity of 10^{-2} N·s/m^2, flows at a speed of 10 cm/s through a 10 cm diameter circular pipe. What is the maximum velocity?



Upper Display

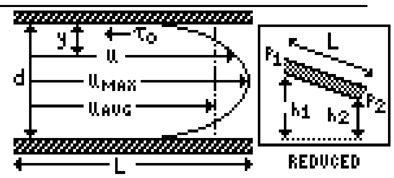


Lower Display

Solution – The maximum velocity occurs at the center of the pipe, **y=d/2**. To determine which equation for velocity profile should be used (**Eq. 11, 12, 13**), compute **Rf** and **Nre** in **equations 4** and **5**. Use **Equations 3, 4, 5, 6 and 10**. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for **Rf** and **Nre**. Since **20<Rf<1 E5**, select **equation 12** (logarithmic profile equation), to solve for the maximum velocity. The entries and results are shown in the screen displays above.

Given	**Solution**
ρ = 800 kg/m^3	μk =.000013 m^2/s
μ = .01 N·s/m^2	fr = .067888
d = 10 cm	Nre = 800
uavg = 10 cm/s	Rf = 36.8479
y = 5 cm	uf = .921198 cm/s

21.5.3 Turbulent Flow: Rough Pipe



The following equations compute turbulent flow in a circular pipe having a roughened surface. For high values of **Nre**, the resistance coefficient, **fr**, is solely a function of the mean roughness of the surface elements inside the pipe, **kf** (m), and the pipe diameter, **d** (m). The **first equation** computes the difference in pressures, **p1** (Pa) and **p2** (Pa), at each end of the pipe due to height (or head) difference, **h1-h2** (m), and head loss due to friction, **hf** (m). **Equation 2** calculates the volume flow rate, **Q** (m^3/s), from the mean velocity of the fluid, **uavg** (m/s), and the cross-sectional area of the pipe. **Equations, 3** and **4,** compute the Reynolds number, **Nre**, for flow in a pipe. **If Nre>3000, the flow in the pipe is considered turbulent.** **Equation 4** is an empirical equation, which calculates the resistance coefficient, **fr**, for turbulent flow in a pipe having a Reynolds number, **Nre**, length, **L**, and diameter, **d**. **Equation 5** calculates the velocity, **u** (m/s), at a distance, **y** (m) from the pipe wall towards the center of the pipe. **Equation 6** (Darcy-Weisbach equation) estimates the head loss due to friction, **hf** (m), over the length of the pipe section, **L**, from the mean velocity, **uavg** (m/s), and diameter of the pipe, **d**. **Equation 7** computes the friction velocity, **uf** (m/s). **Equation 8** calculates the shear, **τo** (N/m^2), on the surface of the pipe in the direction opposite to the fluid flow. **Equation 9** calculates the specific weight, **γ** (N/m^3), from the density **ρ** (kg/m^3) of the fluid and the gravitational acceleration, **grav** (9.80655 m/s^2). **Equation 10** relates the absolute viscosity, **μ** (N·s/m^2) to the kinematic viscosity, **μk** (m^2/s) of the fluid. **Equations 11, 12** and **13** are explicit formulas for relating **fr, hf, Q** and **d**. These formulas have been reported to have an accuracy of 3% for the following ranges of kf/d and Nre [1]: 10^{-5} **< kf/d < 2 x** 10^{-2} and **4 x** 10^3 **< Nre <** 10^8**.** **Since Eq. 11, 12** and **13** compute approximate relationships for **fr, hf, Q** and **d,** it is recommended, at the most, only one be used in a computation. Some roughness values, **kf** (m), for some common pipe surfaces are listed in **Table 21.2**.

Table 21.2: Relative roughness for various pipe surfaces[1,5]

Pipe Surface	kf
Riveted Steel	0.9 mm
Concrete	0.3 mm
Cast Iron	0.26 mm
Galvanized Iron	0.15 mm
Asphalt Cast Iron	0.12 mm
Commercial steel or wrought iron	0.046 mm
Brass and Copper	0.0015 mm
Drawn Tubing	0.0015 mm

$$\frac{p1}{\gamma}+h1=\frac{p2}{\gamma}+h2+hf$$ **Eq. 1**

$$Q=\frac{uavg\cdot\pi}{4}\cdot d^{2}$$ **Eq. 2**

$$Nre=\frac{|uavg|\cdot d}{\mu k}$$ **Eq. 3**

$$Nre=\frac{d^{3/2}}{\sqrt{fr}\cdot\mu k}\cdot\sqrt{\frac{2\cdot grav\cdot|hf|}{L}}$$ **Eq. 4**

$$\frac{u}{uf}=5.75\cdot\log\left(\frac{y}{kf}\right)+Kc$$ **Eq. 5**

$$|hf|=\frac{fr\cdot L\cdot uavg^{2}}{2\cdot grav\cdot d}$$ **Eq. 6**

$$uf=uavg\cdot\sqrt{\frac{fr}{8}}$$ **Eq. 7**

$$\tau o=uf^{2}\cdot\rho$$ **Eq. 8**

$$\gamma=\rho\cdot grav$$ **Eq. 9**

$$\mu=\mu k\cdot\rho$$ **Eq. 10**

when 4 E3 < Nre < 1E8 and when 1 E-5 < kf/d < 2 E-2, the following equations are applicable

$$fr = \frac{.25}{\left(\log\left(\frac{kf}{3.7 \cdot d} + \frac{5.74}{Nre^{.9}}\right)\right)^2}$$ **Eq. 11**

$$Q = -2.22 \cdot d^{5/2} \cdot \sqrt{\frac{grav \cdot |hf|}{L}} \cdot \log\left(\frac{kf}{3.7 \cdot d} + \frac{1.78 \cdot \mu k}{d^{3/2} \cdot \sqrt{\frac{grav \cdot |hf|}{L}}}\right)$$ **Eq. 12**

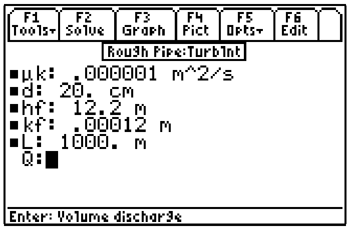
$$d = .66 \cdot \left(kf^{1.25} \cdot \left(\frac{L \cdot Q^2}{grav \cdot |hf|}\right)^{4.75} + \mu k \cdot Q^{9.4} \cdot \left(\frac{L}{grav \cdot |hf|}\right)^{5.2}\right)^{.04}$$ **Eq. 13**

Variable	Description	Units
γ	Specific weight	N/m^3
ρ	Density	kg/m^3
τo	Shear	N/m^2
μ	Absolute viscosity	$N \cdot s/m^2$
μk	Kinematic viscosity	m^2/s
d	Diameter	m
fr	Friction coefficient	unitless
grav	Gravitational acceleration	9.80665 m/s^2
h1	Height at 1	m
h2	Height at 2	m
hf	Head loss due to friction	m
kf	Roughness length (see Table 21.2)	m
Kc	Coefficient	unitless
L	Length	m
Nre	Reynolds number	unitless
p1	Pressure at 1	Pa
p2	Pressure at 2	Pa
Q	Rate of volume discharge	m^3/s
u	Velocity	m/s
uf	Friction velocity	m/s
uavg	Average velocity	m/s
y	Depth: distance along y	m

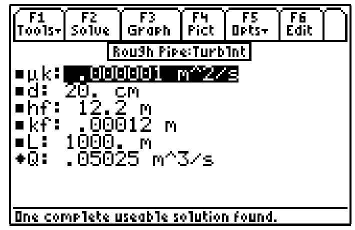
Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 21.5.3:

The head loss for water (20°C) in an asphalted cast-iron pipe, having a diameter of 20 cm and a length of 1 km, is 12.2 m. What is the volumetric flow rate in this pipe? Water has a kinematic viscosity of 1 x 10^{-6} m^2/s at T = 20°C. The roughness value, **kf**, for a cast iron pipe is listed as 0.12 mm in **Table 21.2**.



Entered Values



Computed results

Solution – Equation 12 can be used to solve this problem. Select it by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
µk = .000001 m^2/s
d = 20 cm
hf = 12.2 m
kf = .00012 m
L = 1000 m

Solution
Q = .05025 m^3/s

21.5.4 Flow pipe Inlet

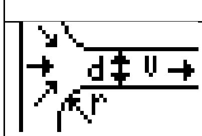
The following equation computes the head loss caused by flow separation or turbulence in pipe bends or transitions to different pipe sizes. **Table 21.3** lists the coefficient, **Kc**, for different inlets, outlets, or fittings for turbulent flow.

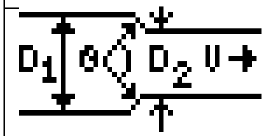
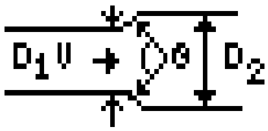
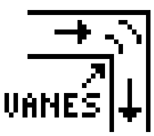
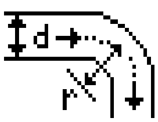
Eq. 1

$$hL = \frac{Kc \cdot v^2}{2 \cdot grav}$$

Variable	**Description**	**Units**
Grav	Gravitational acceleration	9.80665 m/s^2
HL	Head loss	m
Kc	Coefficient (see Table 21.3)	unitless
V	Velocity	m/s

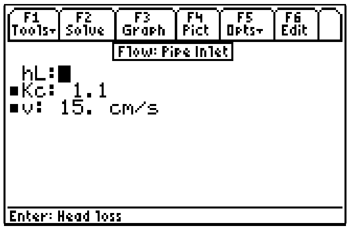
Table 21.3 Loss Coefficients for various Transitions and Fittings1

Pipe Entrance	**r/d**	*Kc*
	0.0 0.1 >0.2	0.50 0.12 0.03
Contraction	**D2/D1**	**Kc (θ=60°/θ=180°C)**

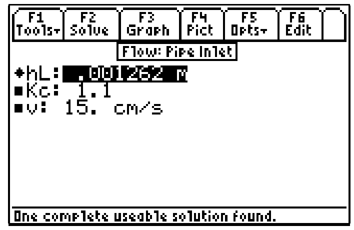
Pipe Entrance	**r/d**	*Kc*
	0.0	0.08/0.50
	0.20	0.08/0.49
	0.40	0.07/0.42
	0.60	0.06/0.32
	0.80	0.05/0.18
	0.90	0.04/0.10
Expansion	**D2/D1**	**Kc (θ=10°/θ=180°C)**
	0.0	--/1.00
	0.20	0.13/0.92
	0.40	0.11/0.72
	0.60	0.06/0.42
	0.80	0.03/0.16
90° Miter Bend	**Without Vanes**	**With Vanes**
	1.1	0.2
90° Smooth Bend	**r/d**	**Kc**
	1	0.35
	2	0.19
	4	0.16
	6	0.21
	8	0.28
	10	0.32
Threaded Pipe Fittings	**Globe valve-Wide Open**	10.0
	Angle valve-Wide Open	5.0
	Gate valve-Wide Open	0.2
	Gate valve-Half Open	5.6
	Return Bend	2.2
	Tee-straight-through flow	0.4
	Tee-side-outlet flow	1.8
	90° Elbow	0.9
	45° Elbow	0.4

Example 21.5.4:

Compute the head loss for a 90° miter pipe bend (w/o vanes) having a flow velocity of 15 cm/s.



Entered Values

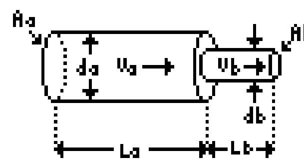


Computed results

Solution – From **Table 21.3**, select **Kc= 1.1**. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
Kc = 1.1 (from Table 21.3)	hL = .001262 m
v = 15 cm/s	

21.5.5 Series Pipe System



These equations compute the head loss of two pipes (a and b) connected in series. The head loss due to friction, **hf** (m), is computed as the sum of individual losses in each section. These equations can be used in conjunction with the pipe flow equations in the preceding sections, 21.5.1, 21.5.2, 21.5.3, and the equation for computing loss due to fittings or transitions (*Flow Pipe Inlet*, *21.5.4*). The **first equation** computes the conservation of volume flow in a series connection of two pipes each having velocities, **va** (m/s) and **vb** (m/s), and circular diameters, **da** (m) and **db** (m). **Equation 2** computes the head loss, **hf**, in both pipes from the individual losses due to friction in each pipe. The friction coefficients, **fa** and **fb**, for each individual pipe section, can be computed for specific pipes or flow features using equations which compute, **fr**, in sections, *Laminar Flow in Smooth Pipes, 21.5.1: Turbulent Flow in Smooth Pipes, 21.5.2:* and *Turbulent Flow in Rough Pipes 21.5.3:*).

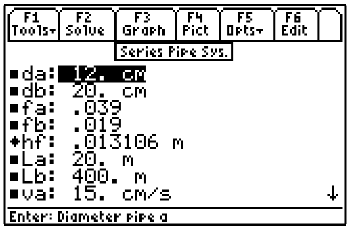
$$vb = \left(\frac{da}{db}\right)^2 \cdot va \qquad \textbf{Eq. 1}$$

$$hf = \frac{va^2}{2 \cdot grav} \cdot \left(\frac{fa \cdot La}{da} + \frac{fb \cdot Lb}{db} \cdot \left(\frac{da}{db}\right)^4\right) \qquad \textbf{Eq. 2}$$

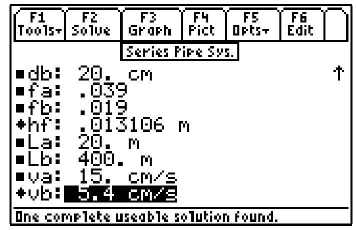
Variable	**Description**	**Units**
da	Diameter of pipe a	m
db	Diameter of pipe b	m
fa	Friction coefficient of pipe a	unitless
fb	Friction coefficient of pipe b	unitless
grav	Gravitational acceleration	9.80665 m/s^2
hf	Head loss due to friction	m
La	Length of pipe a	m
Lb	Length of pipe b	m
va	Flow velocity in pipe a	m/s
vb	Flow velocity in pipe b	m/s

Example 21.5.5:

A 12 cm diameter riveted steel pipe (**fr** = 0.039), having a flow velocity of 15 cm/s, is connected to an asphalt cast-iron pipe having a friction coefficient of 0.019 and a diameter of 20 cm. The lengths of each pipe are 20 m and 400 m, respectively. What is the flow velocity in the second pipe and what is the total head loss due to friction in both pipes?



Upper Display

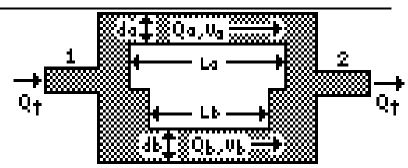


Lower Display

Solution – Select **all of the equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. The entries and results are shown in the screen displays above.

Given	**Solution**
da = 12 cm	hf = 0.013106 m
db = 20 cm	vb = 5.4 cm
fa = .039	
fb = .019	
La = 20 m	
Lb = 400 m	
va = 15 cm/s	

21.5.6 Parallel Pipe System



When a second pipe, **b**, is added to an existing line, **a**, the flow will divide itself so the head loss due to friction is the same in each pipe. The **first equation** describes the equivalence relationship of head loss in each pipe, **hfa** (m) and **hfb** (m). **Equations 2** and **3** compute the head loss in each pipe, **hfa** and **hfb** (m), due to friction. The friction coefficients, **fa** and **fb**, can either be obtained from Moody diagrams for different pipe surfaces or computed from the pipe roughness factor, **kf** (m), and the Reynolds number, **Nre**, using the formula for **fr** in *Turbulent Flow: Rough pipes*. The variables, **va** (m/s) and **vb** (m/s) are the velocities each pipe. **Equation 4** computes the total flow rate, **Qt** (m^3/s) as the sum of the flows in each pipe, **Qa** (m^3/s) and **Qb** (m^3/s). The **last two equations** compute the flow rates in each pipe, **Qa** and **Qb**, from the respective flow velocities, **va** and **vb**, and pipe diameters, **da** (m) and **db** (m).

$$hfa = hfb \qquad \textbf{Eq. 1}$$

$$va = \sqrt{\frac{2 \cdot da \cdot grav}{fa \cdot La} \cdot hfa} \qquad \textbf{Eq. 2}$$

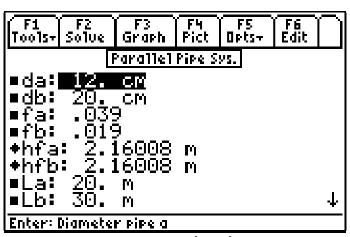
$$vb = \sqrt{\frac{2 \cdot db \cdot grav}{fb \cdot Lb} \cdot hfb} \qquad \textbf{Eq. 3}$$

$Qt = Qa + Qb$	**Eq. 4**
$Qa = \dfrac{va \cdot \pi \cdot da^2}{4}$	**Eq. 5**
$Qb = \dfrac{vb \cdot \pi \cdot db^2}{4}$	**Eq. 6**

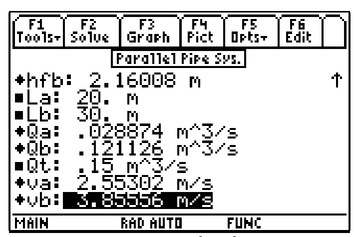
Variable	**Description**	**Units**
da	Diameter of ppe - a	m
db	Diameter of pipe - b	m
fa	Friction coefficient of pipe - a	unitless
fb	Friction coefficient of pipe - b	unitless
grav	Gravitational acceleration	9.80665 m/s^2
hfa	Head loss due to friction - a	m
hfb	Head loss due to friction - b	m
La	Length of pipe - a	m
Lb	Length of pipe - b	m
Qa	Rate of volume discharge - a	m^3/s
Qb	Rate of volume discharge - b	m^3/s
Qt	Rate of total volume flow	m^3/s
va	Flow velocity in pipe a	m/s
vb	Flow velocity in pipe b	m/s

Example 21.5.6:

A 12 cm diameter, 20 m length riveted steel pipe (**fr** = 0.039) is connected in parallel with a 20 cm diameter, 30 m length, and asphalt cast-iron pipe (**fr** = 0.019). If the total flow rate is 0.15 m^3, what are the flow rates through each pipe?



Upper Display



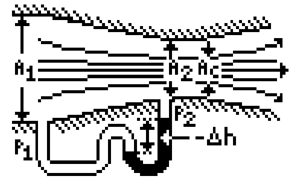
Lower Display

Solution – Select **all** of the **equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. The entries and results are shown in the screen displays above.

Given	**Solution**
da = 12 cm	hfa = 2.16008 m
db = 20 cm	hfb = 2.16008 m
fa = .039	Qa = .028874 m^3/s
fb = .019	Qb = .121126 m^3/s
La = 20 m	va = 2.55302 m/s
Lb = 30 m	vb = 3.85556 m/s
Qt = .15 m^3/s	

21.5.7 Venturi Meter

21.5.7.1 Incompressible Flow



The following equations are used to compute flow rates of an incompressible liquid through a venturi meter, or a constricting orifice. The **first equation** computes the conservation of energy for fluid flow between an inlet, having cross-sectional area, **A1** (m^2), a mean flow velocity, **v1** (m/s), an inlet pressure, **p1** (Pa), a nozzle (constriction) area, **A2** (m^2), nozzle flow velocity, **v2** (m/s), and a nozzle pressure, **p2** (Pa) and head loss, **hf** (m). The conservation of energy equation assumes no head loss occurs due to friction. **Equation 2** computes the velocity in the constriction zone, **v2** (m/s), from the approach velocity, **Fva**, and the pressure difference of the inlet and nozzle (**p1-p2**). The **third equation** computes the volume flow rate, **Qv** (m^3/s), from the flow coefficient **fc**, the area of the nozzle, **A2**, and the pressure drop, **p1-p2**. The **fourth equation** computes the approach velocity, **Fva**, from the area ratio of the inlet, **A1**, nozzle, **A2**, and the contraction coefficient, **cc**. **Equation 5** computes the discharge coefficient, **dc**, from the contraction coefficient, **cc**, and the coefficient of velocity, **vc**. **Table 21.5** lists values of the discharge coefficient for different Reynolds values. The coefficient of velocity, **vc**, accounts for viscous forces at low Reynolds numbers (typically close to 0.98 for Reynolds values greater than 10^5 and decreases at lower Reynolds values). The contraction coefficient, **cc**, in the nozzle is computed in the **sixth equation** as the ratio of the minimum flow area, **Ac** (m^2), to **A2**, the area of the orifice or nozzle in the venturi meter. The contracted flow area, **Ac**, is generally smaller than the contracted flow area, **A2**, for orifices, but usually unity for venturi meters. **Equation 7** calculates the flow coefficient, **fc**, from the approach velocity, **Fva**, and the discharge coefficient, **dc**. **Equation 8** computes the Reynolds number, **Nre**, from the velocity at the inlet, **v1**, and the inlet diameter, **d1** (m). **Equation 9** calculates the initial flow velocity in the inlet, **v1**, from the inlet area, **A1**, and the volume flow rate, **Qv**. **Equation 10** and **11** compute the static pressure head inside the inlet, **hp1** (Pa) and the nozzle, **hp2** (Pa). **Equation 12** and **13** compute the total head, **ht1** (Pa) and **ht2** (Pa). **Equation 14** computes the head difference, **Δh**, of the fluid between the inlet and nozzle from the density of the fluid, ρ (kg/m^3). **Equation 15** computes the head difference measured by a manometer when the fluid inside the manometer (density=**ρm**) has a different density than the flowing fluid (**ρm ≠ ρ).**

Table 21.5: Discharge Coefficient (dc) values for Venturi Meters (2<A1/A2<3)

dc	Nre
.94	6,000
.95	10,000
.96	20,000
.97	50,000
.98	200,000
.99	2,000,000

$$\frac{p1}{\rho \cdot grav}+\frac{v1^2}{2 \cdot grav}=\frac{p2}{\rho \cdot grav}+\frac{v2^2}{2 \cdot grav}+hf$$ **Eq. 1**

$$v2 = Fva \cdot \sqrt{\frac{2 \cdot (p1 - p2)}{\rho}}$$ **Eq. 2**

$$Qv = fc \cdot A2 \sqrt{\frac{2 \cdot (p1 - p2)}{\rho}}$$ **Eq. 3**

$$Fva = \frac{1}{\sqrt{1 - cc^2 \cdot \left(\frac{A2}{A1}\right)^2}}$$ **Eq. 4**

$$dc = vc \cdot cc$$ **Eq. 5**

$$cc = \frac{Ac}{A2}$$ **Eq. 6**

$$fc = Fva \cdot dc$$ **Eq. 7**

$$Nre = \frac{v1 \cdot d1}{\mu k}$$ **Eq. 8**

$$v1 = \frac{Qv}{A1}$$ **Eq. 9**

$$hp1 = \frac{p1}{\rho \cdot grav}$$ **Eq. 10**

$$hp2 = \frac{p2}{\rho \cdot grav}$$ **Eq. 11**

$$ht1 = hp1 + \frac{v1^2}{2 \cdot grav}$$ **Eq. 12**

$$ht2 = hp2 + \frac{v2^2}{2 \cdot grav}$$ **Eq. 13**

when ρ = ρm, the following equation applies

$$\Delta h = \frac{p1 - p2}{\rho \cdot grav}$$ **Eq. 14**

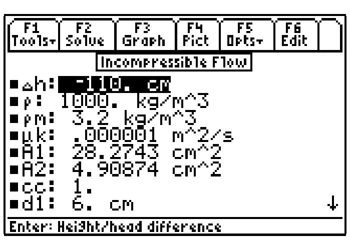
when ρ ≠ ρm, the following equation applies

$$\Delta h = \frac{\rho \cdot \left(\frac{Qv}{fc \cdot A2} \right)^2}{2 \cdot grav \cdot (\rho m - \rho)}$$ **Eq. 15**

Variable	Description	Units
Δh	Head difference	m
ρ	Density	kg/m^3
ρm	Density of fluid in manometer	kg/m^3
μk	Kinematic viscosity	m^2/s
Ac	Contracted flow area	m^2
A1	Area of 1 (inlet)	m^2
A2	Area of 2 (nozzle/orifice)	m^2
cc	Flow area contraction coefficient	m^2
d1	Diameter of inlet	m
dc	Discharge coefficient (see Table 21.5)	unitless
fc	Flow coefficient	unitless
Fva	Approach velocity	unitless
grav	Gravitational acceleration	$9.80665\ m/s^2$
hp1	Pressure head at 1 (inlet)	m
hp2	Pressure head at 2 (nozzle/orifice)	m
ht1	Total head at 1 (inlet)	m
ht2	Total head at 2 (nozzle/orifice)	m
Nre	Reynolds number	unitless
p1	Pressure at 1 (inlet)	Pa
p2	Pressure at 2 (nozzle/orifice)	Pa
Qv	Rate of volume discharge	m^3/s
v1	Velocity at 1-initial	m/s
v2	Velocity at 2-final	m/s
vc	Velocity coefficient	m/s

Example 21.5.7.1:

Water (ρ=1000 kg/m^3, μk = 1x 10^{-6} m^2/s) flows through a circular venturi meter having a circular inlet diameter of 6 cm and a nozzle diameter of 2.5 cm. A tube containing air (3.2 kg/m^3) is connected to the venturi meter and displays a fluid height difference of 110 cm. Compute the volume flow rate. Assume a value of **cc** = 1 and verify that **dc** = 0.97 is an appropriate value of the discharge coefficient using a calculated value of the Reynolds number and **Table 21.5**.



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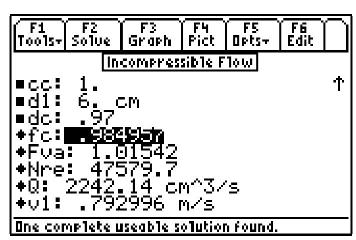


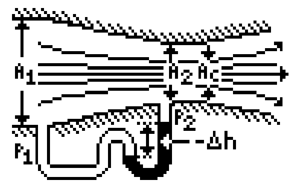
Lower Display

Solution – Select **equations 4, 7, 8, 9** and **15** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. The height difference is negative (**Δh**=-110 cm). The formula for converting diameter, **d** (m) to area, **A** (m^2), is $\mathbf{A=\pi \cdot d^2/4}$. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
Δh = -110 cm
ρ = 1000 kg/m^3
ρm = 3.2 kg/m^3
μk= 1 E –6 m^2/s
A1 = 28.2743 cm^2 ($\pi\cdot 6^2/4$) cm^2
A2 = 4.90874 cm^2 ($\pi\cdot 2.5^2/4$) cm^2
cc = 1
d1 = 6 cm
dc = .97

Solution
fc = .984957
Fva = 1.01542
Nre = 47579.7
Q = 2242.14 cm^3/s
v1 = .792996 m/s

21.5.7.2 Compressible Flow



The following equations compute the flow properties of a compressible fluid, of molar mass **MWT** (kg/mol), in a venturi meter having an inlet cross-section area, **A1** (m^2), and a nozzle diameter, **A2** (m^2). The expansion/compression of the fluid is adiabatic. The **first equation** computes the conservation of energy for a compressible fluid having a specific heat ratio, **k**. The velocity of the fluid at the inlet is **v1** (m/s) and **v2** (m/s) at the nozzle. The conservation of energy equation assumes no head loss occurs due to friction. **Equations 2** and **3** compute the velocities, **v1** (m/s), and **v2** (m/s), the fluid densities, **ρf1** (kg/m^3) and **ρf2** (kg/m^3), and the areas, **A1** and **A2**, at the inlet and nozzle. The **fourth equation** relates the adiabatic change in density of a compressible fluid due to the change in pressure inside the venturi meter, **p1** to **p2**. **Equation 5** computes the mass flow rate, **Qm** (kg/s), from the velocity, **v2**, density, **ρf2**, and area at the venturi thoat (nozzle). The discharge coefficient, **dc** is assumed to be unity, **dc=1**, for high Reynolds numbers, which are characteristic for compressible flow. **Equations 6** and **7** compute the fluid densities **ρf1** and **ρf2**, at the inlet and nozzle for a compressible fluid having ideal gas behavior. The temperature, **T1** (K) and **T2** (K), are the temperatures at the inlet and nozzle of the venturi meter. **Equation 8** and **9** compute the inlet and nozzle static pressure heads, **hp1** (Pa) and **hp2** (Pa). **Equations 10** and **11** compute the total head at the inlet and nozzle. **Equation 12** computes the head difference, **Δh** (m), of the fluid between the inlet and nozzle from the density of the fluid in the manometer, **ρm** (kg/m^3). **Equation 13** computes the Reynolds number, **Nre**, from the velocity at the inlet, **v1** and the inlet diameter, **d1** (m). **Equations 14** and **15** compute the mass flow rate, **Qm**, through a square orifice, having a cross-section area, **Ao** (m^2).

$$\frac{v1^2}{2}+\frac{k\cdot Rm\cdot T1}{k-1}=\frac{v2^2}{2}+\frac{k\cdot Rm\cdot T2}{k-1}$$ **Eq. 1**

$$v1=\frac{\rho f\,2\cdot A2\cdot v2}{\rho f\,1\cdot A1}$$ **Eq. 2**

$$v2=\left(\frac{\frac{2\cdot k}{k-1}\cdot\left(\frac{p1}{\rho f\,1}-\frac{p2}{\rho f\,2}\right)}{1-\left(\frac{\rho f\,2\cdot A2}{\rho f\,1\cdot A1}\right)^2}\right)^{1/2}$$ **Eq. 3**

$$\frac{p1}{p2} = \left(\frac{\rho f\,1}{\rho f\,2}\right)^k$$ **Eq. 4**

$$Qm = dc \cdot \rho f\,2 \cdot A2 \cdot v2$$ **Eq. 5**

$$\rho f\,1 = \frac{p1}{\frac{Rm}{MWT} \cdot T1}$$ **Eq. 6**

$$\rho f\,2 = \frac{p2}{\frac{Rm}{MWT} \cdot T2}$$ **Eq. 7**

$$hp1 = \frac{p1}{\rho f\,1 \cdot grav}$$ **Eq. 8**

$$hp2 = \frac{p2}{\rho f\,2 \cdot grav}$$ **Eq. 9**

$$ht1 = hp1 + \frac{v1^2}{2 \cdot grav}$$ **Eq. 10**

$$ht2 = hp2 + \frac{v2^2}{2 \cdot grav}$$ **Eq. 11**

$$\Delta h = \frac{p1 - p2}{\rho m \cdot grav}$$ **Eq. 12**

$$Nre = \frac{v1 \cdot d1}{\mu k}$$ **Eq. 13**

The following equations are valid for compressible fluid flow through SQUARE orifices

$$Qm = Yc \cdot Ao \cdot fc \cdot \sqrt{2 \cdot \rho f\,1 \cdot (p1 - p2)}$$ **Eq. 14**

$$Yc = 1 - \frac{1}{k} \cdot \left(1 - \frac{p2}{p1}\right) \cdot \left(.41 + .35 \cdot \left(\frac{Ao}{A1}\right)^2\right)$$ **Eq. 15**

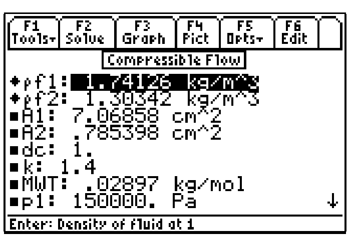
Variable	Description	Units
Δh	Head difference	m
ρf1	Density at 1	kg/m^3
ρf2	Density at 2	kg/m^3

Variable	Description	Units
ρm	Density of fluid in Manometer	kg/m^3
A1	Area of 1 (inlet)	m^2
A2	Area of 2 (nozzle/orifice)	m^2
Ao	Square orifice area	m^2
dc	Discharge Coefficient (see Table 21.5)	unitless
fc	Flow Coefficient	unitless
grav	Gravitational Acceleration	$9.80665\ m/s^2$
hp1	Pressure head at 1 (inlet)	m
hp2	Pressure head at 2 (nozzle/orifice)	m
ht1	Total head at 1 (inlet)	m
ht2	Total head at 2 (nozzle/orifice)	m
k	Specific Heat ratio	unitless
MWT	Molar Mass	kg/mol
p1	Pressure at 1 (inlet)	Pa
p2	Pressure at 2 (nozzle/orifice)	Pa
Qm	Mass flow rate	kg/s
Rm	Ideal Gas constant	8.31451 J/(kg·K)
T1	Temperature at 1	K
T2	Temperature at 2	K
v1	Velocity at 1-initial	m/s
v2	Velocity at 2-final	m/s
Yc	Compressibility factor	unitless

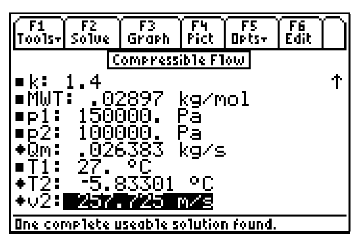
Caution: Because the equations represent a set where several subtopics are covered, the user has to select each equation to be included in the multiple equation solver. Pressing [F2] will not select all the equations and start the solver.

Example 21.5.7.2:

A venturi meter, having a circular throat diameter of 1 cm, is connected to a 3 cm diameter pipe carrying air. The static pressure in the pipe is 150 kPa and the pressure at the throat is 100, kPa. The static temperature of the air in the pipe is 27^oC. The specific heat ratio and the molar mass of air are 1.4 (see *mwa in Reference/Engineering Constants*) and 0.02897 kg/mol. Compute the mass flow rate.



Upper Display



Lower Display

Solution – Select **equations 3, 4, 5, 6, 7** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Use a value of **dc=1** for the discharge coefficient. The formula for converting diameter, **d** (m) to area, **A** (m^2), is $\mathbf{A=\pi \cdot d^2/4}$. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

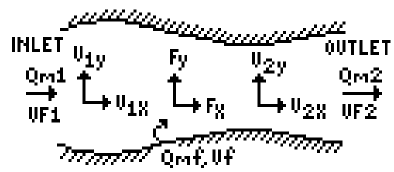
Given	**Solution**
$A1 = 7.06858\ cm^2\ \ (\pi \cdot 3^2/4)\ cm^2$	$\rho f1 = 1.74126\ kg/m^3$
$A2 = .785398\ cm^2\ \ (\pi \cdot 1^2/4)\ cm^2$	$\rho f2 = 1.30342\ kg/m^3$

Given
dc = 1
k = 1.4
MWT = .02897 kg/mol
p1 = 150000 Pa
p2 = 100000 Pa
T1 = 27 ^oC

Solution
Qm = .026383 kg/s
T2 = -5.83301 ^oC
v2 = 257.725 m/s

21.6 Impulse/Momentum

21.6.1 Jet Propulsion



The following equations compute the conservation of momentum properties for fluid flow in a propulsion system. The **first equation** computes the mass flow rate of air through the inlet, **Qm1** (kg/s), from the volumetric flow rate, **VF1** (m^3/s) and the air density, **ρ1** (kg/m^3). **Equation 2** calculates the fuel flow rate, **Qmf** (kg/s). **Equation 3** computes the mass flow of air/fuel mixture leaving the propulsion system through the nozzle, **Qm2** (kg/s). **Equations 4** and **5,** calculate the thrust in the horizontal, **Fx** (N), and vertical, **Fy** (N), directions from the x, y, components of the incoming air, **v1x** (m/s) and **v1y** (m/s), and the outflow of the air/fuel mixture, **v2x** (m/s) and **v2y** (m/s).

$$Qm1 = \rho 1 \cdot VF1 \quad \textbf{Eq. 1}$$

$$Qmf = \rho f \cdot Vf \quad \textbf{Eq. 2}$$

$$Qm2 = Qm1 + Qmf \quad \textbf{Eq. 3}$$

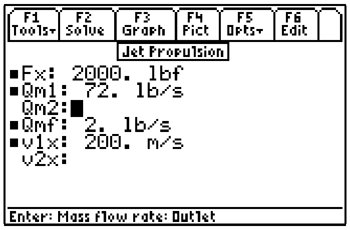
$$Fx = Qm2 \cdot v2x - Qm1 \cdot v1x \quad \textbf{Eq. 4}$$

$$Fy = Qm2 \cdot v2y - Qm1 \cdot v1y \quad \textbf{Eq. 5}$$

Variable	**Description**	**Units**
ρ1	Air density	kg/m^3
ρf	Fuel density	kg/m^3
Fx	Force due to flow – x	N
Fy	Force due to flow – y	N
Qm1	Mass flow rate – inlet	kg/s
Qm2	Mass flow rate – outlet	kg/s
Qmf	Mass flow rate – fuel	kg/s
Vf	Volume flow rate – fuel	m^3/s
VF1	Volume flow rate: Inlet	m^3/s
v1x	Inlet velocity – x	m/s
v1y	Inlet velocity – y	m/s
v2x	Outlet velocity – x	m/s
v2y	Outlet velocity – y	m/s

Example 21.6.1:

A turbo jet is required to exert a thrust of 2000 lbf. If the rate of air flow into the jet is 72 lb/s, the fuel flow rate is 2 lbs and the air velocity is 200 m/s, what is the velocity of the gas leaving the nozzle? Assume all flow occurs along the same coordinate.



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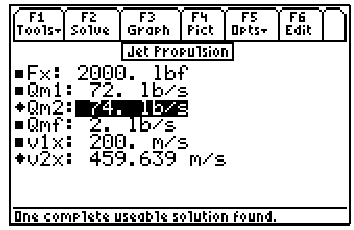


Computed results

Solution – Select the **third** and **fourth equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

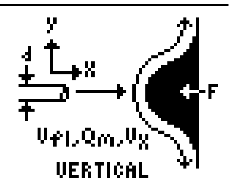
Given
Fx = 2000 lbf
Qm1 = 72 lb/s
Qmf = 2 lb/s
v1x = 200 m/s

Solution
Qm2 = 74 lb/s
v2x = 459.639 m/s

21.6.2 Open Jet

The following sets of equations compute the force exerted by an open jet on a flat plate.

21.6.2.1 Vertical Plate



The **first equation** computes the horizontal force, **Fx** (N), exerted by a jet on a vertical plate parallel to the gravitational field. The jet discharges a mass flow rate of **Qm** (kg/s), with a fluid velocity, **vx** (m/s) perpendicular to the plate. The **second equation** relates the mass flow rate, **Qm** (kg/s), to the volumetric flow rate, **Vf1** (m^3/s). **Equation 3** computes the volumetric flow rate, **Vfl** (m^3/s), from the initial velocity of the stream, **vx**, and the cross sectional area of the jet, **A** (m^2). The **last equation** computes the cross sectional area of the jet, **A**, from the circular jet diameter **d** (m).

$$Fx = Qm \cdot vx \qquad \textbf{Eq. 1}$$

$$Qm = \rho \cdot Vfl \qquad \textbf{Eq. 2}$$

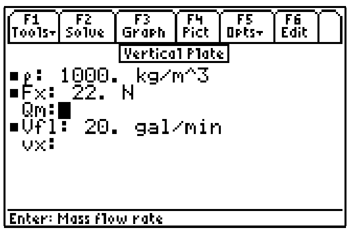
$$Vfl = vx \cdot A \qquad \textbf{Eq. 3}$$

$$A = \frac{\pi \cdot d^2}{4} \qquad \textbf{Eq. 4}$$

Variable	Description	Units
ρ	Density	kg/m^3
A	Area	m^2
d	Diameter	m
Fx	Force due to flow – x	N
Qm	Mass flow rate	kg/s
Vfl	Volume flow rate	m^3/s
vx	Velocity – x	m

Example 21.6.2.1:

An open horizontal jet discharges water onto a wall at a rate of 20 gallons per minute. The wall experiences a force of 22 N. What is the horizontal velocity of the water before it reaches the wall? The density of water is 1000 kg/m^3.



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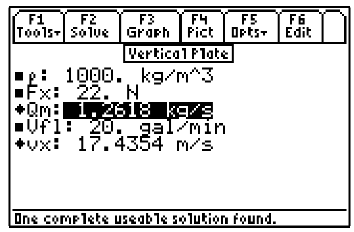
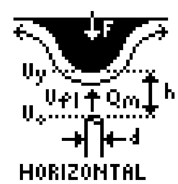


Computed results

Solution – Select the **first two equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
$\rho = 1000\ kg/m^3$	Qm = 1.2618 kg/s
Fx = 22 N	vx = 17.4354 m/s
Vfl = 20 gal/min	

21.6.2.2 Horizontal Plate



These equations compute the force exerted by vertical jet on a horizontal plate at a height, **h** (m), above the jet. The fluid leaves the jet at velocity, **vo** (m/s). The **first equation** computes the vertical velocity of the fluid at the height of the plate, **vy** (m/s), from the initial velocity, **vo**, and the change in kinetic energy due to gravitation acceleration, **grav** (9.80665 m/s). **Equation 2** computes the vertical force, **Fy** (N), exerted by the jet, from the mass flow rate, **Qm** (kg/s). The **third equation** relates the mass flow rate, **Qm**, to the volumetric flow rate, **Vf1** (m^3/s). **Equation 4** computes the volumetric flow rate, **Vfl**, from the initial velocity of the stream, **vy**, and the cross sectional area of the jet, **A** (m^2). The **last equation** computes the cross sectional area of the jet, **A**, from the circular jet diameter, **d** (m).

$$vy = \sqrt{vo^2 - 2 \cdot grav \cdot h} \quad \textbf{Eq. 1}$$

$$Fy = Qm \cdot vy \quad \textbf{Eq. 2}$$

$$Qm = \rho \cdot Vf1 \quad \textbf{Eq. 3}$$

$$Vfl = vo \cdot A \qquad \textbf{Eq. 4}$$

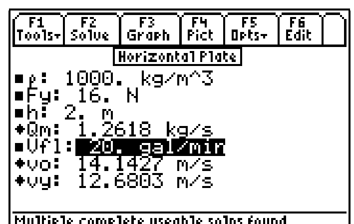
$$A = \frac{\pi \cdot d^2}{4} \qquad \textbf{Eq. 5}$$

Variable	**Description**	**Units**
ρ	Density	kg/m^3
A	Area	m^2
d	Diameter	m
Fy	Force due to flow – y	N
grav	Gravitational Acceleration	$9.80665\ m/s^2$
h	Height	m
Qm	Mass flow rate	kg/s
Vfl	Volume flow rate	m^3/s
vo	Initial velocity	m/s
vy	Velocity - y	m

Example 21.6.2.2:

An open vertical jet discharges water at a rate of 20 gallons per minute. A horizontal plate, 2 m above the nozzle experiences a force of 16 N. What is the vertical velocity of the water at the nozzle and the height of the wall? Assume the density of water is 1000 kg/m^3.

Entered Values: 2nd Solution

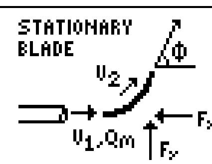


Computed results: 2nd Solution

Solution – Select the **first three equations** to solve this problem. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The gravitational constant, **grav** (9.80665 m/s^2), is automatically inserted into the calculation and does not appear in the list of variables. Select the second solution. The entries and results are shown in the screen displays above.

Given	**Solution**
ρ = 1000 kg/m^3	Qm = 1.2618 kg/s
Fy = 16 N	vo = 14.1427 m/s
h = 2 m	vy = 12.6803 m/s
Vfl = 20 gal/min	

21.6.2.3 Stationary Blade



These equations compute the vertical (y) and horizontal (x) force components of an open jet on a stationary blade having an incline angle, ϕ (rad). The jet has a mass flow rate of **Qm** (kg/s). **Equation 1** calculates the change in the horizontal component of velocity, **Δvx** (m/s), from the initial velocity of the stream before it reaches the plane, **v1** (m/s), and the velocity of the stream following deflection by the blade, **v2** (m/s). **Equation 2** calculates the change in the vertical component of the stream velocity due to blade deflection, **Δvy** (m/s). **Equations 3** and **4** compute the horizontal force, **Fx** (N), and the vertical force, **Fy** (N), exerted by the blade on the stream. **Equation 5** relates the mass flow rate, **Qm,** to the volume flow rate, **Vfl** (m^3/s). **Equation 6** computes the volumetric flow rate, **Vfl**, from the initial velocity of the stream, **v1**, and the area of the jet, **A** (m^2). **Equation 7** computes the cross sectional area of the jet, **A**, from the jet diameter **d** (m). The **last equation** relates the initial velocity, **v1**, and final velocity, **v2**, when the flow on the blade is frictionless.

$$\Delta vx = v2 \cdot \cos(\phi) - v1 \qquad \textbf{Eq. 1}$$

$$\Delta vy = v2 \cdot \sin(\phi) \qquad \textbf{Eq. 2}$$

$$Fx = Qm \cdot \Delta vx \qquad \textbf{Eq. 3}$$

$$Fy = Qm \cdot \Delta vy \qquad \textbf{Eq. 4}$$

$$Qm = \rho \cdot Vf1 \qquad \textbf{Eq. 5}$$

$$Vfl = v1 \cdot A \qquad \textbf{Eq. 6}$$

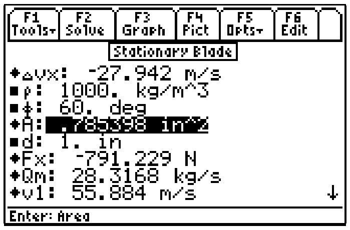
$$A = \frac{\pi \cdot d^2}{4} \qquad \textbf{Eq. 7}$$

$$v1 = v2 \qquad \textbf{Eq. 8}$$

Variable	**Description**	**Units**
Δvx	Change in velocity - x	m/s
Δvy	Change in velocity - y	m/s
ρ	Density	kg/m^3
ϕ	Deflection angle	rad
A	Area	m^2
d	Diameter	m
Fx	Force due to flow - x	N
Fy	Force due to flow - y	N
Qm	Mass flow rate	kg/s
v1	Velocity 1 (Initial)	m/s
v2	Velocity 2 (Final)	m/s
Vfl	Volume flow rate	m^3/s

Example 21.6.2.3:

A stationary frictionless vane having an angle of 60^o deflects a water jet, having a volumetric flow rate of 1 cubic foot per second and a diameter of 1 in. Compute the horizontal force exerted by the vane. The density of water is 1000 kg/m^3.



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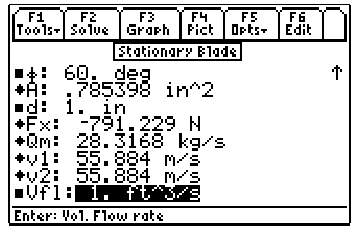


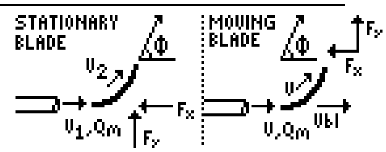
Lower Display

Solution – Select the **Equations 1, 3, 5, 6, 7** and **8** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
$\rho = 1000$ kg/m^3
$\phi = 60$ deg
d = 1 in
Vfl = 1 ft^3/s

Solution
$\Delta vx = -27.942$ m/s
A = .785398 in^2
Fx = -791.229 N
Qm = 28.3168 kg/s
v1 = 55.884 m/s
v2 = 55.884 m/s

21.6.2.4 Moving Blade



These equations compute the vertical (y) and horizontal (x) force components of an open jet, with a mass flow rate of **Qm** (kg/s) incident on a moving blade. The blade is inclined at an angle, **φ** (rad), and moving with a velocity, **vbl** (m/s), in the direction parallel to fluid flow. **Equation 1** calculates the change in the horizontal component of velocity, **Δvx** (m/s), from the initial velocity of the stream before it reaches the blade, **v1** (m/s) and the velocity of the stream following deflection by the blade, **v** (m/s). **Equation 2** calculates the change in the vertical component of the stream velocity due to blade deflection, **Δvy** (m/s). **Equation 3** computes the horizontal force exerted by the blade in deflecting the stream, **Fx** (N). **Equation 4** calculates the vertical force exerted by the blade in deflecting the stream, **Fy** (N). **Equation 5** relates the mass flow rate, **Qm**, to the volume flow rate **VF** (m^3/s). **Equation 6** computes the volumetric flow rate, **Vfl**, from the initial velocity of the stream, **v1**, and the cross sectional area of the jet, **A** (m^2). **Equation 7** computes the cross sectional area of a circular jet, **A**, having a diameter, **d** (m). The **last equation** relates the initial and final velocities, **v1** and **v**, when the flow on the blade is frictionless.

$$\Delta vx = (v - vb1)\cdot(\cos(\phi) - 1) \quad \textbf{Eq. 1}$$

$$\Delta vy = (v - vb1)\cdot\sin(\phi) \quad \textbf{Eq. 2}$$

$$Fx = Qm\cdot\Delta vx \quad \textbf{Eq. 3}$$

$$Fy = Qm\cdot\Delta vy \quad \textbf{Eq. 4}$$

$$Qm = \rho\cdot Vf1 \quad \textbf{Eq. 5}$$

$$Vfl = v1\cdot A \quad \textbf{Eq. 6}$$

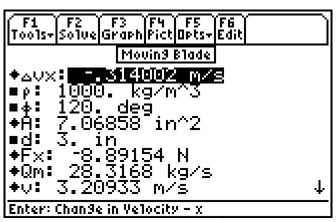
$$A = \frac{\pi \cdot d^2}{4} \quad \textbf{Eq. 7}$$

$$v = v1 - vbl \quad \textbf{Eq. 8}$$

Variable	Description	Units
Δvx	Change in velocity - x	m/s
Δvy	Change in velocity - y	m/s
ρ	Density	kg/m^3
ϕ	Deflection angle	rad
A	Area	m^2
d	Diameter	m
Fx	Force due to flow - x	N
Fy	Force due to flow - y	N
Qm	Mass flow rate	kg/s
v	Velocity (following deflection)	m/s
v1	Velocity 1 (Initial)	m/s
vbl	Velocity of the blade (in direction of flow)	m/s
Vfl	Volume flow rate	m^3/s

Example 21.6.2.4:

A moving frictionless blade, having a deflection angle of 120 deg, channels a water jet, having a volumetric flow rate of 1 ft^3/s and a diameter of 3 in. The vane has a constant velocity of 3 m/s in the direction of the jet flow. Compute the horizontal force of the jet exerted by the vane. Assume the density of water is 1000 kg/m^3.



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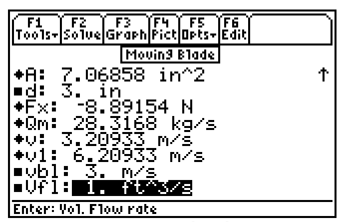

Lower Display

Solution – Select Equations **1, 3, 5, 6, 7, 8** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
ρ = 1000 kg/m^3	Δvx = -.314002 m/s
ϕ = 120 deg	A = 7.06858 in^2
d = 3 in	Fx = -8.89154 N
vbl = 3 m/s	Qm = 28.3168 kg/s
Vfl = 1 ft^3/s	v = 3.20933 m/s
	v1 = 6.20933 m/s

References:

1. John A. Roberson and Clayton T. Crowe, Engineering Fluid Mechanics, 5th Edition, Houghton-Mifflin Company, Boston, MA 1993

2. Ranald V. Giles, Fluid Mechanics & Hydraulics, 2nd Edition, Schaum's outline series, McGraw-Hill Publishing company, New York, NY 1962
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5. Eugene A. Avallone & Theodore Baumeister III, Mark's Handbook for Mechanical Engineers, McGraw-Hill Book Company, New York, NY 1984
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Chapter 22 Dynamics and Statics

This section contains general equations related to the topic of dynamics and statics. The gravitational acceleration constant, **grav** ($g = 9.80665 m/s^2$), and the universal gravitation constant, **G** ($Gc = 6.67259 \times 10^{-11} N\cdot m^2/kg^2$), are predefined in ME•*Pro* and are automatically inserted into calculations. They do not appear in the list of variables for entry screen.

- Laws of Motion
- Angular Motion
- Collisions
- Friction
- Constant Acceleration
- Projectile Motion
- Gravitational Effects
- Statics

22.1 Laws of Motion

The basic laws of force, velocity, acceleration, energy, and momentum are introduced in this topic. The **first equation** represents Newton's second law: an acceleration, **a** (m/s^2), will result from a force, **F** (N), acting on a mass, **m** (kg). The **second equation** calculates momentum, **p** (Pa), of mass, **m**, from the velocity, **v** (m/s). The **third equation** computes kinetic energy, **KE** (J), for a mass with a velocity, **v.** The **fourth equation** calculates the potential energy, **PE** (J), stored or released due to a change in height, **Δh** (m), in the earth's gravitational field. The total energy, **TE** (J), is the sum of **KE** and **PE** and is computed in the **fifth equation**. The instantaneous power, **Pwr** (W), of the system is given in the **last equation**.

$$F = m \cdot a \quad \textbf{Eq. 1}$$

$$p = m \cdot v \quad \textbf{Eq. 2}$$

$$KE = \frac{1}{2} \cdot m \cdot v^2 \quad \textbf{Eq. 3}$$

$$PE = m \cdot grav \cdot \Delta h \quad \textbf{Eq. 4}$$

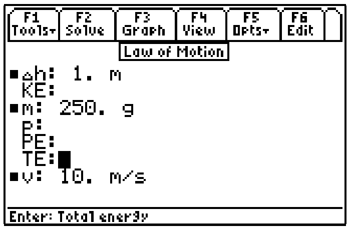
$$TE = PE + KE \quad \textbf{Eq. 5}$$

$$Pwr = F \cdot v \quad \textbf{Eq. 6}$$

Variable	**Description**	**Units**
Δh	Change in height	m
a	Acceleration	m/s^2
F	Force	N
grav	Acceleration due to gravitation	$9.80665 m/s^2$
KE	Kinetic energy	J
m	Mass	kg
p	Linear momentum	kg·m/s
PE	Potential energy	J
Pwr	Power	W
TE	Total energy	J
v	Velocity	m/s

Example 22.1: Part 1

A ball weighing 250 g is thrown vertically from a height of 1 m at 10 m/s. What is the ball's initial momentum and kinetic, potential and total energy?



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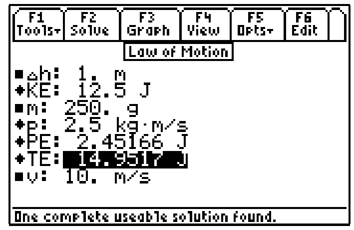


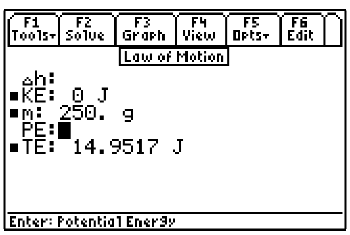
Computed results

Solution – Select the **second, third, fourth, and fifth equations.** Enter the known values and solve for the unknown variables. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the variables. The entries and results are shown in the screen displays above.

Given	Solution
Δh=1 m	KE = 12.5 J
m=250 g	TE = 14.9517 J
v=10 m/s	p = 2.5 kg m/s
	PE = 2.45166 J

Example 22.1: Part 2

Suppose that the total energy calculated in the preceding problem is converted to potential energy when the ball ceases to travel upwards. What height above the ground will the ball reach?



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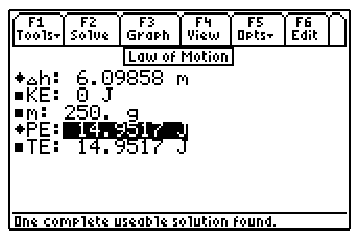


Computed results

Solution - Calculate results for the above problem. Deselect the **second and third equations** and press [F2] to solve. Highlight the calculated value of **TE** and press [F5]**:Opts**, [2]: **Know**. This changes the value to a known parameter used in computation. Enter zero for **KE** and **delete** the values for **Δh** and **PE**.

Given	Solution
KE= 0 J	Δh = 6.0986 m
M = 250 g	PE = 14.9517 J
TE = 14.9517 J	

22.2 Constant Acceleration

22.2.1 Linear Motion

The five equations in this section describe the position and velocity of a moving object, subject to acceleration. The **first equation** computes a change in velocity, **v-vo** (m/s), due to constant acceleration, **a** (m/s^2), over time, **t** (s). The **equations 2** and **3** calculate the final position, **s** (m), of the object from its initial position, **so** (m), from the initial velocity, **vo** (m/s), final velocity, **v** (m/s), time traversed, **t**, and average acceleration, **a**. **Equation 4** describes a method of computing the distance traveled given the initial and final velocity and time spent traveling. The **final equation** computes the velocity, **v**, from the acceleration, **a**, and traversed distance, **s-so** (m).

$$v = vo + a \cdot t \qquad \textbf{Eq. 1}$$

$$s = so + vo \cdot t + \frac{1}{2} \cdot a \cdot t^2 \qquad \textbf{Eq. 2}$$

$$v^2 = vo^2 + 2 \cdot a \cdot (s - so) \qquad \textbf{Eq. 3}$$

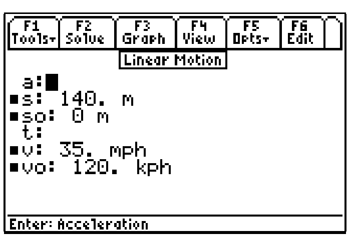
$$s = so + \frac{v + vo}{2} \cdot t \qquad \textbf{Eq. 4}$$

$$s = so + v \cdot t - \frac{1}{2} \cdot a \cdot t^2 \qquad \textbf{Eq. 5}$$

Variable	**Description**	**Units**
a	Acceleration	m/s^2
s	Final position	m
so	Initial position	m
t	Time	s
v	Velocity	m/s
vo	Initial velocity	m/s

Example 22.2.1:

Spotting a police car, Jodi Ulsoor needs to reduce her velocity from 120 km/h to 35 mph and accomplish this in a span of 140 m. Compute the time needed for this task and the deceleration.



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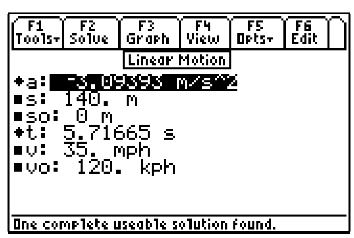


Computed results

Solution - Select the **third** and **fifth equations**. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
s = 140 m	t = 5.71665 s
so = 0 m	a = -3.09383 m/s^2
v = 35 mph	
vo = 120 km/h	

22.2.2 Free Fall

The following equations calculate the vertical velocity, **v** (m/s), at position, **y** (m), of a free falling object in the earth's gravitational field after time, **t** (s), has elapsed since an initial vertical velocity, **vo** (m), and initial vertical position, **yo** (m). Friction effect is ignored.

$$y - yo = vo \cdot t - \frac{1}{2} \cdot grav \cdot t^2$$ **Eq. 1**

$$v^2 = vo^2 - 2 \cdot grav \cdot (y - yo)$$ **Eq. 2**

$$y - yo = \frac{1}{2} \cdot (vo + v) \cdot t$$ **Eq. 3**

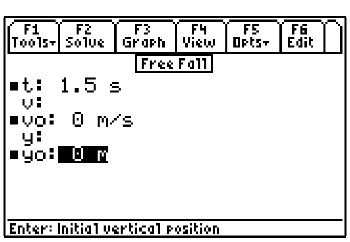
$$y - yo = v \cdot t + \frac{1}{2} \cdot grav \cdot t^2$$ **Eq. 4**

$$v = vo - grav \cdot t$$ **Eq. 5**

Variable	**Description**	**Units**
grav	Acceleration due to gravitation	9.80665 m/s^2
t	Time	s
v	Velocity	m/s
vo	Initial velocity	m/s
y	Displacement	m
yo	Initial vertical position	m

Example 22.2.2:

A construction worker, working on the top floor of a ten-story building, drops her hammer. How far will the hammer travel in 1.5 seconds and what will its velocity be?



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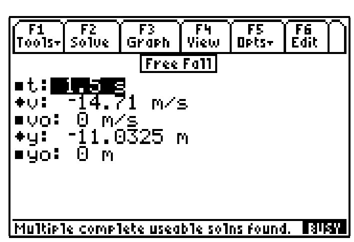


Computed results

Solution – Use the worker's vertical position as a zero reference height. Choose the **first** and **second equations**. Select these by highlighting the equations and pressing [ENTER]. This problem requires the first two equations. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. There are two possible solutions, depending on whether velocity scale is pointing in the upward or downward directions. In this case (1st solution), a positive velocity is assumed to be pointing in the positive direction of height. The entries and results are shown in the screen displays above.

Given	**Solution**
t = 1.5 s	v = -14.71 m/s
vo = 0 m/s	y = -11.0325 m
yo = 0 m	

22.2.3 Circular Motion

These 5 equations describe change in angular position, **θ** (rad), and angular velocity, **ω** (rad/s), from an initial angle position, **θo** (rad), and initial angular velocity, **ωo** (rad/s), due to time, **t** (s), and angular acceleration, **α** (rad/s^2).

$$\omega = \omega o + \alpha \cdot t$$ **Eq. 1**

$$\theta - \theta o = \omega o \cdot t + \frac{1}{2} \cdot \alpha \cdot t^2$$ **Eq. 2**

$$\omega^2 = \omega o^2 + 2 \cdot \alpha \cdot (\theta - \theta o)$$ **Eq. 3**

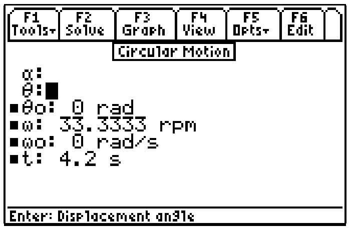
$$\theta - \theta o = \frac{1}{2} \cdot (\omega + \omega o) \cdot t$$ **Eq. 4**

$$\theta - \theta o = \omega \cdot t - \frac{1}{2} \cdot \alpha \cdot t^2$$ **Eq. 5**

Variable	**Description**	**Units**
α	Angular acceleration	rad/s^2
θ	Displacement angle	rad
θo	Initial displacement angle	rad
ω	Angular velocity	rad/s
ωo	Initial angular velocity	rad/s
t	Time	s

Example 22.2.3:

A turntable at rest, is able to accelerate to an angular velocity of 33 $^1/_3$ rpm in 4.2 seconds. What is the average angular acceleration and what is the angle of displacement from the record's initial position?



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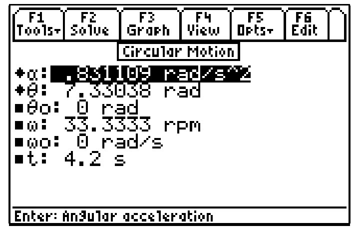


Computed results

Solution - Since two unknowns are being calculated, two equations are needed which, as a set, contain all of the variables. A number of possible equation choices exist. **Equations 1** and **2** were selected for the example above. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
θo=0 rad	θ=7.3308 rad
ω=33.3333 rpm	α=.8311091 rad/s^2
ωo=0 rad/s	
t=4.2 s	

22.3 Angular Motion

22.3.1 Rolling/Rotation

These equations describe the location and rotational properties of a circular object, rolling at a constant speed. The **first equation** describes the circumferential arc length, **sa** (m), between an initial angle, **θo** (rad), and displaced angle, **θ** (rad), for a round object having a radius, **rw** (m). The translation velocity of the center of mass, **vcm** (m/s), of the rotating object is described in the **second equation**. The **third equation** calculates, **KE** (J), the rotational kinetic energy of a rolling object having a total moment of inertia, **Ip** (kg·m^2), and an angular velocity, **ω** (rad/s). The angular momentum, **Lm** (kg·m^2/s), is computed in the **fourth equation**. The **fifth equation** accounts for the adjustment of a rotational moment inertia, **Ip**, for the special case of a mass, **m** (kg), attached at distance, **rm** (m), from the center of rotating object with moment of inertia, **Icm** (kg·m^2). The **next equation** calculates the tangential velocity of the top edge of the rolling object, **vtop** (m/s). The **seventh equation** computes the sum of the rotational and linear kinetic energies of the rolling object, **KEr** (J). The **second-to-last equation** computes the total mass of the rolling object, **mr** (kg). The **last equation** computes the power, **Pwr** (W), exerted by a torque, **τ** (N·m), to achieve an angular velocity, **ω**.

$$sa = rw \cdot (\theta - \theta o)$$ **Eq. 1**

$$vcm = \omega \cdot rw$$ **Eq. 2**

$$KE = \frac{1}{2} \cdot Ip \cdot \omega^2$$ **Eq. 3**

$$Lm = Ip \cdot \omega$$ **Eq. 4**

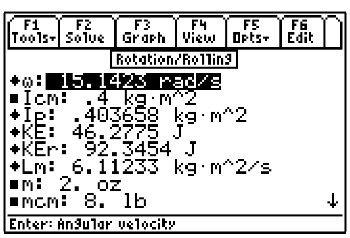
$$Ip = Icm + m \cdot rm^2$$ **Eq. 5**

$vtop = 2 \cdot vcm$	**Eq. 6**
$KEr = KE + \frac{1}{2} \cdot mr \cdot vcm^2$	**Eq. 7**
$mr = mcm + m$	**Eq. 8**
$Pwr = \tau \cdot \omega$	**Eq. 9**

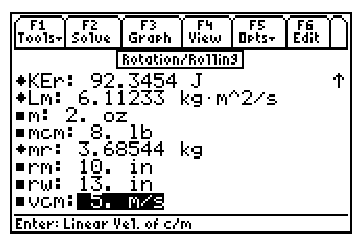
Variable	**Description**	**Units**
θ	Displacement angle	rad
θo	Initial displacement angle	rad
τ	Torque	N·m
ω	Angular velocity	rad/s
Icm	Moment of inertia - c/m	$kg{\cdot}m^2$
Ip	Moment of inertia	kg·m·2
KE	Kinetic energy	J
KEr	Kinetic energy of rolling object	J
Lm	Angular momentum	$kg{\cdot}m^2/s$
m	Mass	kg
mr	Mass of rolling object	kg
mcm	Mass of Icm	kg
Pwr	Power	W
rm	Radius of m to center of mass	m
rw	Radius	m
sa	Arc length	m
vcm	Linear velocity of c/m	m/s
vtop	Linear velocity of top of rolling object	m/s

Example 22.3.1:

A reflector weighing 2 oz is attached 10" from the center of a rolling bicycle wheel weighing 8 lb, having a radius of 13", and a rotational moment of inertia of 0.4 kg/m^2. If the wheel is rolling with a linear velocity of 5 m/s, find the angular velocity, angular momentum and the total kinetic energy.



Upper Display



Lower Display

Solution - The **second, third, fourth, fifth, seventh** and **eighth** equations are needed to solve the problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
Icm= 0.4 kg m^2	ω=15.1423 rad/s
m=2 oz	Ip=.403658 kg m^2
mcm=8 lb	KE=46.2775 J
Rm=10 in	KEr=92.3454 J
rw=13 in	Lm=6.11233 kg m^2/s
vcm=5 m/s	mr=3.68544 kg

22.3.2 Forces in Angular Motion

These four equations describe Newton's first law of motion in angular and linear forms. These equations are useful for converting force, **Ft** (N), to torque, **τ** (rad), and linear acceleration, **a** (m/s^2), to angular acceleration, **α** (m/s^2). **I** ($kg{\cdot}m^2$) is the rotational moment of inertia of a rotating body and **m** (kg), is the mass of the rotating object acting as a torque at a distance, **r** (m), from the center of the rotating body.

$$Ft = m \cdot at \qquad \textbf{Eq. 1}$$

$$\tau = Ft \cdot r \qquad \textbf{Eq. 2}$$

$$\tau = m \cdot \alpha \cdot r^2 \qquad \textbf{Eq. 3}$$

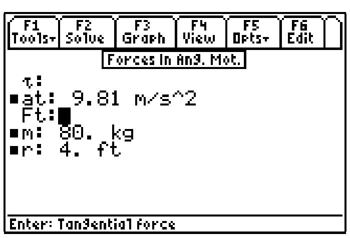
$$\tau = I \cdot \alpha \qquad \textbf{Eq. 4}$$

$$Pwr = \tau \cdot \omega \qquad \textbf{Eq. 5}$$

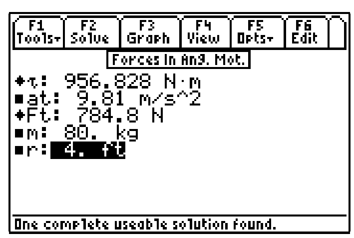
Variable	Description	Units
α	Angular acceleration	rad/s^2
τ	Torque	N·m
ω	Angular velocity	rad/s
at	Tangential acceleration	m/s^2
Ft	Tangential force	N
I	Moment of inertia	kg·m^2
m	Mass	kg
Pwr	Power	W
r	Radius	m

Example 22.3.2:

John McClaine (mass of 80 kg) jumps off the top of Nakatomi plaza with a fire hose wrapped around his waist. The fire hose is attached to the edge of a rotating spool, which is 8 ft in diameter. Assuming the force acting on the spool is solely due to Mr. McClaine's total weight in the gravitational field (9.81 m/s), what is the initial torque on the fire hose spool?



Entered Values



Computed results

Solution - Select the **first** and **second equations** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
at=9.81 m/s^2	τ=956.828 N·m
m=80 kg	Ft=784.8 N
r=4 ft	

22.3.3 Gyroscope Motion

These two equations describe the precession rate, **Ω** (rad/s), and angular momentum, **Lm** (kg·m^2/s), of a gyroscope with a mass, **m** (kg), moment of inertia, **I** (kg·m^2), angular velocity, **ω** (rad/s), and a radius from the fixed point, **r** (m), on an axis perpendicular to the earth's gravitational field.

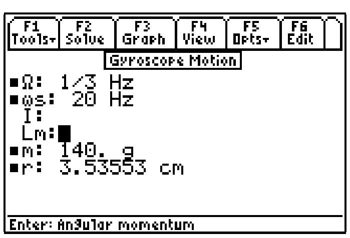
$$\Omega = \frac{m \cdot grav \cdot r}{I \cdot \omega s} \qquad \textbf{Eq. 1}$$

$$Lm = I \cdot \omega s \qquad \textbf{Eq. 2}$$

Variable	**Description**	**Units**
grav	Acceleration due to gravitation	9.80665 m/s^2
Ω	Precession rate	rad/s
ωs	Angular velocity	rad/s
I	Moment of inertia	kg·m^2
Lm	Angular momentum	kg·m^2/s
m	Mass	kg
r	Radius	m

Example 22.3.3:

A gyroscope precesses with its rotational axis 45 degrees from the direction of the gravitational field. The gyroscope's rotational speed is 20 revolutions per second. Calculate the moment of inertia and angular momentum of a gyroscope having a precession rate of one revolution every three seconds, a mass of 140 g, and a longitudinal axle, of negligible mass, 10 cm in length.



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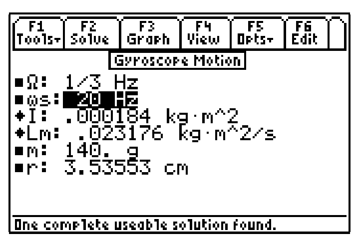


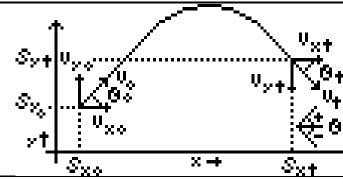
Computed results

Solution - Both equations are needed to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Check the MODE settings of your calculator to determine whether trigonometric functions accept radian or degree entries. Enter the values for the known parameters. The distance from the fixed point to the gyroscope center is half the length of the longitudinal axle. For the radius, **r**, enter 5*cos (π/4) cm if the MODE is set for radian entries, or, 5*cos (45^o). Press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. To convert the result to different units, press [F5]: **Opts**, [4]: **Conv**, and select the desired units from the toolbar.

Given	**Solution**
Ω = 0.33333 Hz	I = 0.018443 kg m^2
ωs = 20 Hz	Lm = 2.31764 kg $m^{2/s}$
m = 140 g	
r = 5*cos(π/4 or 45^o)cm = 3.53553 cm	

22.4 Projectile Motion

The following set of equations describes the horizontal, **x**, and vertical, **y**, components of position, velocity and acceleration of a launched projectile in the earth's gravitational field. Friction is ignored.



$$vxt = vxo + ax \cdot t \quad \textbf{Eq. 1}$$

$$sxt = sxo + vxo \cdot t + \frac{1}{2} \cdot ax \cdot t^2 \quad \textbf{Eq. 2}$$

$$vxt^2 = vxo^2 + 2 \cdot ax \cdot (sxt - sxo) \quad \textbf{Eq. 3}$$

$$vyt = vyo - \frac{1}{2} \cdot grav \cdot t \quad \textbf{Eq. 4}$$

$$syt = syo + vyo \cdot t - \frac{1}{2} \cdot grav \cdot t^2 \quad \textbf{Eq. 5}$$

$$vyt^2 = vyo^2 - 2 \cdot grav \cdot (syt - syo) \quad \textbf{Eq. 6}$$

$$vxo = vo \cdot \cos(\theta o) \quad \textbf{Eq. 7}$$

$$vyo = vo \cdot \sin(\theta o) \quad \textbf{Eq. 8}$$

$$vxt = vt \cdot \cos(\theta t) \quad \textbf{Eq. 9}$$

$$vyt = vt \cdot \sin(\theta t) \quad \textbf{Eq. 10}$$

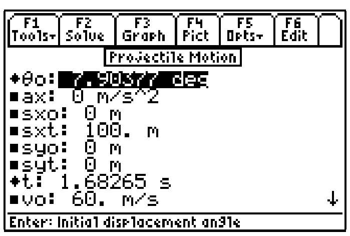
$$vo = \sqrt{vxo^2 + vyo^2} \quad \textbf{Eq. 11}$$

$$vt = \sqrt{vxt^2 + vyt^2}$$ **Eq. 12**

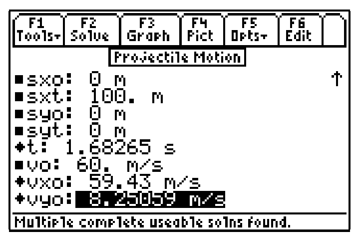
Variable	**Description**	**Units**
θo	Initial displacement angle	rad
θt	Projectile angle at time=t	rad
ax	Acceleration along x axis	m/s^2
grav	Acceleration due to gravitation	$9.80665\ m/s^2$
sxo	x position at t=0	m
sxt	x position at time t	m
syo	y position at t=0	m
syt	y position at time t	m
t	Time	s
vo	Initial velocity	m/s
vt	Velocity at time=t	m/s
vxo	Initial velocity along x axis at t=0	m/s
vxt	Velocity at time=t along x axis	m/s
vyo	Initial velocity along y axis at t=0	m/s
vyt	Velocity at time=t along y axis	m/s

Example 22.4:

An arrow is launched from a bow towards a target at the same height, 100m away. The velocity of the arrow is 60 m/s. What angle above the center of the target must the archer aim? How long does it take the arrow to reach the target?



First Solution: Upper Display



First Solution: Lower Display

Solution - The **second, fifth, seventh,** and **eighth** equations are needed to solve this problem. Select these equations using the highlight bar and pressing [ENTER], press [F2] to display the unknowns. Enter the values using appropriate units, and press [F2] to solve for the unknown variables. The gravitational constant, **grav** ($9.80665\ m/s^2$), is automatically inserted into the calculation and does not appear in the list of variables. Two parabolic trajectories are possible.

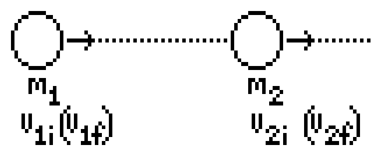
Given	**Solution**
$ax=0\ m/s^2$	θ=7.90377 deg (82.0962 deg)
sxo = 0 m	t = 1.68265 s (12.1204 s)
sxt = 100 m	vxo = 59.43 m/s (8.25059 m/s)
syo=0 m	vyo = 8.25059 m/s (59.43 m/s)
syt = 0 m	
vo = 60 m/s	

22.5 Collisions

22.5.1 Elastic Collisions

22.5.1.1 1D Collision

Elastic collisions are characterized by conservation of energy and momentum. The **first equation** states the law of the conservation of momentum and the **second equation** states the conservation of energy. **Equations 3** and **4** are combinations of the first two equations. We use the convention that velocity is positive when the mass moves to the right.



$$m1 \cdot v1i + m2 \cdot v2i = m1 \cdot v1f + m2 \cdot v2f$$ **Eq. 1**

$$m1 \cdot v1i^2 + m2 \cdot v2i^2 = m1 \cdot v1f^2 + m2 \cdot v2f^2$$ **Eq. 2**

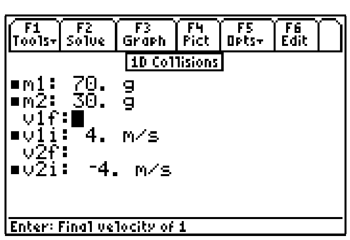
$$v1f = \frac{m1 - m2}{m1 + m2} \cdot v1i + \frac{2 \cdot m2}{m1 + m2} \cdot v2i$$ **Eq. 3**

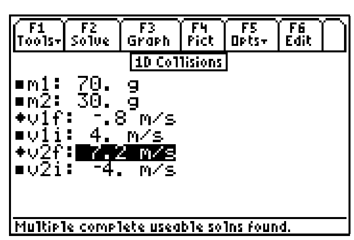
$$v2f = \frac{2 \cdot m1}{m1 + m2} \cdot v1i + \frac{m2 - m1}{m1 + m2} \cdot v2i$$ **Eq. 4**

Variable	**Description**	**Units**
m1	Mass 1	kg
m2	Mass 2	kg
v1f	Final velocity of 1	m/s
v1i	Initial velocity of 1	m/s
v2f	Final velocity of 2	m/s
v2i	Initial velocity of 2	m/s

Example 22.5.1.1:

Tia and Ryan throw two rubber balls toward each other. The balls have masses of 70 g and 30 g and are each traveling at 4 m/s. Assuming a perfectly elastic collision occurs on the axis of flight, what are the velocities and directions of each ball immediately after they collide?





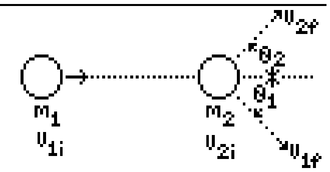
Entered Values *Computed results:*

Solution - Use the **third and fourth** equations to determine the solution, since both include the conservation of energy and momentum along a single axis. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
m1=70 g	vlf= -0.8 m/s (ball rebounds)
m2=30 g	v2f= 7.2 m/s (ball rebounds)
v1i= 4 m/s	
v2i=-4 m/s	

22.5.1.2 2D Collisions

These equations describe an elastic collision between two objects in a two-dimensional coordinate system. One mass, **m2** (kg), is initially at rest and the other, **m1** (kg), has an initial velocity, **v1i** (m/s). Following an elastic collision, the objects have final velocities of, **v1f** (m/s) and **v2f** (m/s). The deflection angles **θ1** (rad) and **θ2** (rad) are relative to the velocity axis for **m1** before the collision.



$$m1 \cdot v1i = m1 \cdot v1f \cdot \cos(\theta 1) + m2 \cdot v2f \cdot \cos(\theta 2) \qquad \textbf{Eq. 1}$$

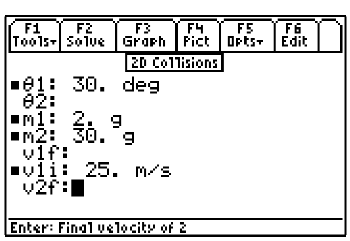
$$m1 \cdot v1f \cdot \sin(\theta 1) = m2 \cdot v2f \cdot \sin(\theta 2) \qquad \textbf{Eq. 2}$$

$$m1 \cdot v1i^2 = m1 \cdot v1f^2 + m2 \cdot v2f^2 \qquad \textbf{Eq. 3}$$

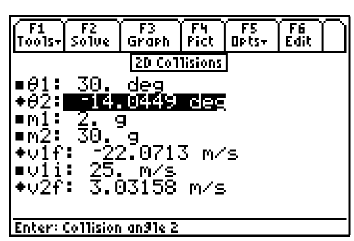
Variable	**Description**	**Units**
θ1	Collision angle 1	rad
θ2	Collision angle 2	rad
m1	Mass 1	kg
m2	Mass 2	kg
v1f	Final velocity of 1	m/s
v1i	Initial velocity of 1	m/s
v2f	Final velocity of 2	m/s

Example 22.5.1.2:

A ping-pong ball is (2 g) is fired at a tennis ball (30 g). Before colliding, the ping-pong ball has a velocity of 25 m/s and rebounds 30 degrees relative to its original axis in the opposite direction. What are the velocities of the ping-pong ball and tennis ball? What is the angle of tennis ball's trajectory relative to the ping-pong ball's original flight path?



Entered Values



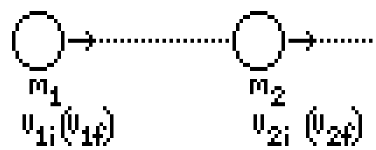
Computed results

Solution - Select all three equations to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. Enter zero when the dialogue box appears to calculate the principal angle solution for θ2. The entries and results are shown in the screen displays above.

Given	**Solution**
θ1=30 deg	θ2=-14.0449 deg
m1= 2g	v1f=-22.0713 m/s
m2= 30 g	v2f=3.03158 m/s
v1i=25 m/s	

22.5.2 Inelastic Collisions

22.5.2.1 1D Collisions



Most collisions are not purely elastic, and some of the kinetic energy is converted to heat or used in deformation of the colliding objects (such as two cars in a head-on collision). The **first equation** represents the law of conservation of momentum; the **second equation** is Newton's collision rule for inelastic collisions. The coefficient of restitution, **cr**, in the second equation is a factor to account for loss of kinetic energy in an inelastic collision. In a perfectly elastic collision, **cr** is equal to **1** (**cr=1,** kinetic energy is completely conserved) and **cr** is equal to **0** in a purely inelastic collision (**cr=0**, all kinetic energy is converted to heat or deformation). The **third** and **fourth** equations are combinations of the first two equations.

$$m1\cdot v1i + m2\cdot v2i = m1\cdot v1f + m2\cdot v2f \quad \textbf{Eq. 1}$$

$$v1f - v2f = -cr\cdot(v1i - v2i) \quad \textbf{Eq. 2}$$

$$v1f = \frac{(m1 - cr\cdot m2)\cdot v1i + m2\cdot(1+cr)\cdot v2i}{m1+m2} \quad \textbf{Eq. 3}$$

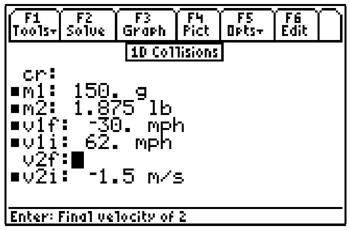
$$v2f = \frac{m1\cdot(1+cr)\cdot v1i + (m2 - cr\cdot m1)\cdot v2i}{m1+m2} \quad \textbf{Eq. 4}$$

Variable	**Description**	**Units**
cr	Coefficient of restitution	unitless
m1	Mass 1	kg
m2	Mass 2	kg
v1f	Final velocity of 1	m/s
v1i	Initial velocity of 1	m/s
v2f	Final velocity of 2	m/s
v2i	Initial velocity of 2	m/s

Example 22.5.2.1:

A baseball, weighing 150 g and traveling at 62 mph, is bunted with a bat, weighing 1.875 lb, moving towards the ball at a velocity of 1.5 m/s. The ball rebounds in the in the opposite direction at 30 mph.

What is the coefficient of restitution for the bat, ball collision? If the bat is released the moment it collides with the ball, what would be the bat's velocity towards the person who was holding it?



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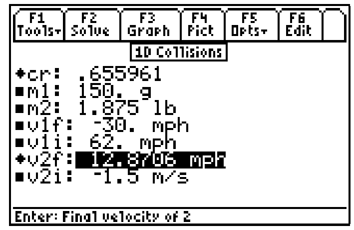
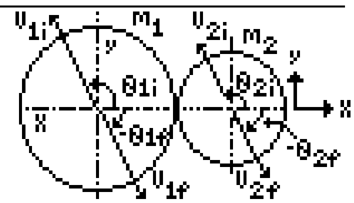


Computed results

Solution – Select **third** and **fourth equations**. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
m1=150 g	cr=0.655961
m2= 1.875 lb	v2f=12.8706 mph
vli=62 mph	
v1f=-30 mph	
v2i= -1.5 m/s	

22.5.2.2 Oblique Collisions



The following equations describe an inelastic collision between two objects in two dimensions. The trajectory angles of the two objects before colliding, **θ1i** & **θ2i** (rad), and after collision, **θ1f** & **θ2f** (rad), are relative to an axis which passes through the center of mass of the two colliding objects and their point of contact. The **first eight equations** convert velocities from polar to rectangular coordinates. The **ninth** and **tenth equations** compute the conservation of tangential velocities (component of velocity perpendicular to the axis of contact) for the two objects. The **last two equations** combine the conservation of momentum principle and Newton's collision rule to determine the velocity components parallel to the axis of contact. The velocity values **v1i**, **v1f**, **v2i**, and **v2f** (m/s) are vector values (i.e. positive when pointing away from the axis, negative when they are pointing towards the axis).

$$v1ix = v1i \cdot \cos(\theta 1i)$$ **Eq. 1**

$$v1iy = v1i \cdot \sin(\theta 1i)$$ **Eq. 2**

$$v2ix = v2i \cdot \cos(\theta 2i)$$ **Eq. 3**

$$v2iy = v2i \cdot \sin(\theta 2i)$$ **Eq. 4**

$$v1fx = v1\mathrm{f} \cdot \cos(\theta 1f)$$ **Eq. 5**

$$v1fy = v1\mathrm{f} \cdot \sin(\theta 1\mathrm{f})$$ **Eq. 6**

$$v2\mathrm{fx} = v2\mathrm{f} \cdot \cos(\theta 2\mathrm{f})$$ **Eq. 7**

$$v2fy = v2f \cdot \sin(\theta 2f)$$ **Eq. 8**

$$v1\mathrm{i} \cdot \sin(\theta 1\mathrm{i}) = v1\mathrm{f} \cdot \sin(\theta 1\mathrm{f})$$ **Eq. 9**

$$v2\mathrm{i} \cdot \sin(\theta 2\mathrm{i}) = v2\mathrm{f} \cdot \sin(\theta 2\mathrm{f})$$ **Eq. 10**

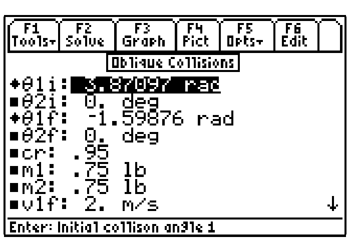
$$v1f \cdot \cos(\theta 1f) = \frac{(m1 - cr \cdot m2) \cdot \cos(\theta 1i) \cdot v1i + m2 \cdot (1 + cr) \cdot v2i \cdot \cos(\theta 2i)}{m1 + m2}$$ **Eq. 11**

$$v2f \cdot \cos(\theta 2f) = \frac{m1 \cdot v1i \cdot \cos(\theta 1i) \cdot (1 + cr) + v2i \cdot \cos(\theta 2i) \cdot (m2 - cr \cdot m1)}{m1 + m2}$$ **Eq. 12**

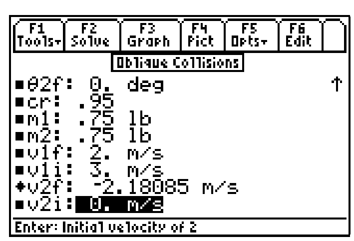
Variable	**Description**	**Units**
θ1i	Initial collision angle 1	rad
θ2i	Initial collision angle 2	rad
θ1f	Final collision angle 1	rad
θ2f	Final collision angle 2	rad
cr	Coefficient of restitution	unitless
m1	Mass 1	kg
m2	Mass 2	kg
v1f	Final velocity of 1	m/s
v1fx	Final velocity of 1 along x-axis	m/s
v1fy	Final velocity of 1 along y-axis	m/s
v1i	Initial velocity of 1	m/s
v1ix	Initial velocity of 1 along x-axis	m/s
v1iy	Initial velocity of 1 along y-axis	m/s
v2f	Final velocity of 2	m/s
v2fx	Final velocity of 2 along x-axis	m/s
v2fy	Final velocity of 2 along y-axis	m/s
v2i	Initial velocity of 2	m/s
v2ix	Initial velocity of 2 along x-axis	m/s
v2iy	Initial velocity of 2 along y-axis	m/s

Example 22.5.2.2:

A pool player hits a cue ball (0.75 lb) towards an eight ball of the same mass at 3 m/s. The player wants to determine a collision angle for the first ball relative to an axis that runs through the center of the two balls at contact so that the cue ball leaves the point of collision at 2 m/s. If the coefficient of restitution for the two balls is 0.95, what is the velocity of the eight ball, what angle does the cue ball leave the collision? (Assume that the balls are sliding before and after the collision instead of rolling).



Upper Display



Lower Display

Solution – Solving angular values from trigonometric expressions often requires the user to choose or convert a solution from many possible answers. Drawing a diagram, similar to the figure in this section, and with vectors for each object before and after collision often helps. In this case, mass 1 is selected as the cue ball and mass 2, the eight ball. **Select the last four equations**, enter the values as stated below, press [F2], enter the known values and press [F2] to solve the unknown variables. The initial and final angles of the eight ball are known to be zero since momentum is only transferred along the axis of collision. In this case, the solution displays the cue ball (1) coming from the *upper right* of the eight ball (2) (a mirror image of the diagram).

Given	Solution
θ2f=0 deg	θ1i=221.79 deg
θ2i=0 deg	θ1f= -91.6022 deg
cr=0.95	v2f=-2.18085 m/s
m1=0.75 lb	
m2=0.75 lb	
v1f= 2m/s	
vli=3 m/s	
v2i=0 m/s	

22.6 Gravitational Effects

22.6.1 Law of Gravitation

The following equations compute the orbital properties an object with mass, **m2** (kg), in a circular orbit around a stationary mass, **m1** (kg). **Equation 1** computes the force, **F** (N), between the orbiting mass, **m2**, and central mass, **m1**, due to gravity. **Equation 2** calculates the gravitational acceleration, **ag** (m/s^2), at distance **rd** (m), from the center of mass of **m1**. **Equation 3** calculates the escape velocity, **vesc** (m/s)**,** required for a launched object to overcome the gravitational field of **m1**. The centripetal acceleration, **ag**, of an object orbiting at distance, **rd**, can be calculated from the tangential velocity, **v** (m/s), in **equation 4**, or angular velocity, **ω** (rad/s), in the **equation 8**. The force, **F** (N), acting on **m2** due to centripetal acceleration **ag**, can be calculated in **equation 5**. The period of rotation, **Tp** (s), for an object in circular orbit at distance, **rd**, from a central body, **m1**, is computed in **equation 6**. The tangential velocity, **v**, of an object in orbit at distance **rd** with angular velocity **ω** is calculated in **equation 7**. **Equation 9** calculates the orbital angular momentum, **Lm** ($kg\cdot m^2/s$), of **m2**. **Equations 10** through **12** calculate the potential **PE** (J), kinetic, **KE** (J), and total, **TE** (J), energies of **m2** orbiting at distance **rd** from **m1**.

$$F = \frac{G \cdot m1 \cdot m2}{rd^2}$$ **Eq. 1**

$$ag = \frac{G \cdot m1}{rd^2}$$ **Eq. 2**

$$vesc = \sqrt{\frac{2 \cdot G \cdot m1}{rd}}$$ **Eq. 3**

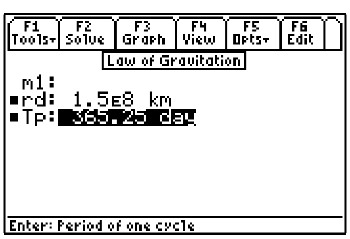
$$ag = \frac{v^2}{rd}$$ **Eq. 4**

Equation	
$F = m2 \cdot ag$	**Eq. 5**
$Tp^2 = \dfrac{4 \cdot \pi^2}{G \cdot m1} \cdot rd^3$	**Eq. 6**
$v = \omega \cdot rd$	**Eq. 7**
$ag = \omega^2 \cdot rd$	**Eq. 8**
$Lm = m2 \cdot rd^2 \cdot \omega$	**Eq. 9**
$PE = \dfrac{-G \cdot m1 \cdot m2}{rd}$	**Eq. 10**
$KE = \dfrac{G \cdot m1 \cdot m2}{2 \cdot rd}$	**Eq. 11**
$TE = KE + PE$	**Eq. 12**

Variable	**Description**	**Units**
ω	Angular Acceleration	rad/s
ag	Gravitational acceleration	m/s^2
F	Force	N
G	Gravitational constant	$6.67x10^{-11}$ $N \cdot m^2/kg^2$
KE	Kinetic energy	J
Lm	Angular momentum	$kg \cdot m^2/s$
m1	Mass 1	kg
m2	Mass 2	kg
PE	Potential Energy	J
rd	Radius between c/m	m
TE	Total energy	J
Tp	Period of one cycle	s
v	Velocity	m/s
vesc	Escape velocity	m/s

Example 22.6.1: Part 1

Calculate a rough estimate of the mass of the sun given the earth orbits the sun once every 365.25 days and the mean distance from the earth to the sun is approximately 150 million kilometers.



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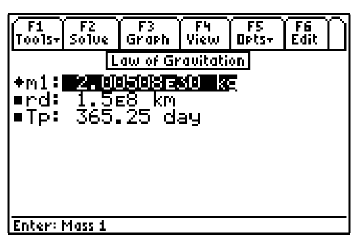


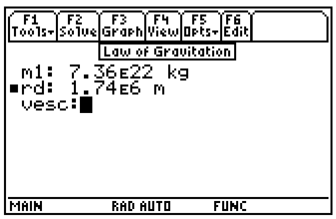
Computed results

Solution **-** Select the **sixth equation** to solve this problem. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
Rd=1.5E8 km	m1 = 2.00508 E 30 kg (actual value is 1.99 E 30 kg)
Tp=365.25 days	

Example 22.6.1: Part 2

An astronaut on the surface of the moon wants to know the minimum velocity needed to send a golf ball beyond the moon's gravitational field. In this case, neglect the added speed of the moon's rotation. The mass and mean radius of the moon are 7.36 x 10^{22} kg and 1.74 x 10^{6} m respectively.



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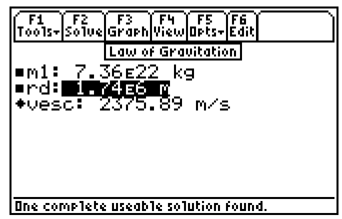
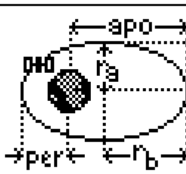


Computed results

Solution - Select the **third equation** to solve this problem. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. The value of **G** the gravitational constant (6.67-11 N m^2/kg) is automatically retrieved from the TI operating system and is not displayed as a variable.

Given	**Solution**
m1=7.36 E22 kg	vesc=2375.89 m/s
rd=1.74E6 m	

22.6.2 Kepler's Laws



Johannes Kepler from Tycho Brahe's recorded observations of planetary orbits , we use these principles to derive the following equations were developed by. Kepler's statements are as follows: 1) Planetary orbits are elliptical in shape. 2) The elliptical area traversed by an orbiting planet over time is constant. 3) The third observation, which is summarized in the first two equations, is that the square of orbital period, **Tp** (s), is proportional to the cube of the minimum distance between the two objects **per** (perigee)**.**

The **first two equations** compute the elliptical area of the orbit, **Area** (m^2), from the minimum, **per**, and maximum, **apo** (m), distances between the two bodies. The **third equation** calculates the distance, **rd** (m), between the central mass, **m** (kg), and the orbiting object at angular position, **θ** (rad), from the perigee location of the orbit. The **fourth equation** computes the period of rotation for the satellite, **Tp** (s). **Equation 5** defines the constant, **hs** (m^2/s), which represents Kepler's second principle, and is used in the second and fifth equations. The **sixth equation** calculates the orbital tangential velocity **v** (m/s) at orbital distance, **rd** (m), from the central mass, **m** (kg). The **seventh equation** relates **apo** (m), and, **per** (m), between the sun (mass **m**) and the planet given a planet's tangential velocity, **vp** (m/s), at the perigee. The **eighth** and **ninth equations** relate the apogee, **apo**, and perigee, **per**, distances to the major and minor elliptical axis. The **last equation** calculates the eccentricity, **ε**, of the elliptical orbit.

$$Area = \frac{\pi}{2} \cdot (per + apo) \cdot \sqrt{apo \cdot per}$$ **Eq. 1**

$$Area = \frac{hs \cdot Tp}{2}$$ **Eq. 2**

$$\frac{1}{rd} = \frac{1}{per} \cdot \left(1 - \frac{G \cdot m}{per \cdot vp^2}\right) \cdot \cos(\theta) + \frac{G \cdot m}{per^2 \cdot vp^2}$$ **Eq. 3**

$$Tp = \frac{\pi}{hs} \cdot (apo + per) \cdot \sqrt{apo \cdot per}$$ **Eq. 4**

$$hs = vp \cdot per$$ **Eq. 5**

$$v = \frac{hs}{rd}$$ **Eq. 6**

$$apo = \frac{per}{\frac{2 \cdot G \cdot m}{per \cdot vp^2} - 1}$$ **Eq. 7**

$$ra = \frac{per + apo}{2}$$ **Eq. 8**

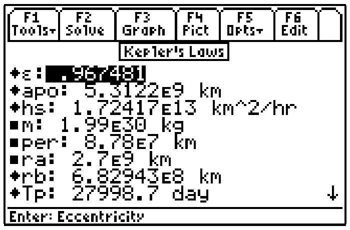
$$rb = \sqrt{per \cdot apo}$$ **Eq. 9**

$$\varepsilon = \frac{1}{per} \cdot \left(1 - \frac{G \cdot m}{per \cdot vp^2}\right) \cdot \frac{hs^2}{G \cdot m}$$ **Eq. 10**

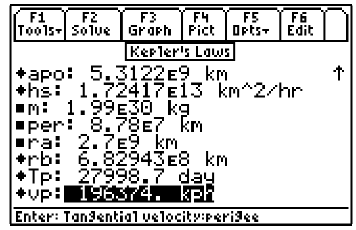
Variable	**Description**	**Units**
ε	Eccentricity	unitless
θ	Displacement angle	rad
apo	Apogee distance – maximum distance to center of mass	m
Area	Area	m^2
G	Gravitational constant	$6.67 \times 10^{-11}\ N \cdot m^2/kg^2$
hs	Constant	m^2/s
m	Mass	kg
per	perigee distance – minimum distance to center of mass	m
ra	Radius: semi-major ellipse axis	m
rb	Radius: minor ellipse axis	m
rd	Radius between center of masses m1 and m2	m
Tp	Period of one cycle	s
v	Tangential velocity	m/s
vp	Tangential velocity at perigee	m/s

Example 22.6.2:

Halley's comet orbits the sun once every 76 years. Verify this information given that the perihelion distance (nearest distance to the sun is 87.8 million kilometers) and the semi-major axis radius is 2.7 billion km. The sun's mass is 1.99 E30 kg. In addition to the orbital period, find the apogee distance, minor axis length and maximum velocity.



Upper Display

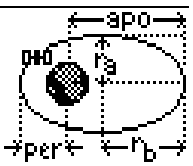


Lower Display

Solution - Choose **the fourth, fifth, seventh, eighth, ninth** and **tenth** equations. The maximum velocity always occurs at the perigee of the orbit.

Given	**Solution**
m=1.99 E30 kg	apo = 5.3122 E 9 km
per=8.78 E7 km	hs = 1.72417 E 13 km^2/hr
ra= 2.7 E9 km	rb = 6.82943 E8 km
	Tp = 27998.7 day (~76.6 years)
	vp = 196374 kph

22.6.3 Satellite Orbit



These equations describe gravitation, launch and elliptical satellite orbit around the earth. **Mea** (kg), the mass of the earth (5.98 E24 kg) is a constant defined in ME•*Pro* that can be viewed in *Reference* under *Engineering constants*. The **first equation** calculates the polar coordinates of a satellite trajectory, **r** (m), and, **θ** (rad), around the earth given initial velocity, **vo** (m/s), and initial release height of the orbiting satellite from the earth's center of gravity, **ro** (m). The **next three equations** are used to calculate the eccentricity **ε** of a conic section of the trajectory. The eccentricity of the conic section will determine whether the free-flight trajectory is a circle (**ε=0**), parabola (**ε=1**), ellipse (**ε<1**), or hyperbola (**ε>1**). The **fifth equation** calculates the minimum escape (launch) velocity, **vesc** (m/s), for a satellite to flee the earth's gravitational field. The **sixth equation** calculates **vc** (m/s), the critical (minimum) velocity required to launch a satellite into a circular orbit at initial height, **ro** (m). The **next five equations** calculate the relationships between **apo** (m), the apogee (maximum distance of a satellite to the center of orbit), **per** (m), the perigee (the minimum distance from the center of orbit), **ra** (m) the minimum distance of a satellite to the center of the earth, **rb** (m), the maximum distance of an orbiting satellite to the center of the earth and **Tp** (s), the time period for a single orbit. The **final equation** computes the tangential velocity of the orbiting satellite, **v** (m/s), at radius, **r** (m), given the product of the tangential velocity **vo** and the perigee (initial) distance, **per**.

$$\frac{1}{r} = \frac{1}{ro} \cdot \left(1 - \frac{G \cdot Mea}{ro \cdot vo^2}\right) \cdot \cos(\theta) + \frac{G \cdot Mea}{ro^2 \cdot vo^2}$$ **Eq. 1**

$$\varepsilon = \frac{Cs \cdot hs^2}{G \cdot Mea}$$ **Eq. 2**

$$hs = vo \cdot ro$$ **Eq. 3**

$$Cs = \frac{1}{ro} \cdot \left(1 - \frac{G \cdot Mea}{ro \cdot vo^2}\right)$$ **Eq. 4**

$$vesc = \sqrt{\frac{2 \cdot G \cdot Mea}{ro}}$$ **Eq. 5**

$$vc = \sqrt{\frac{G \cdot Mea}{ro}}$$ **Eq. 6**

$$per = ro$$ **Eq. 7**

$$apo = \frac{ro}{\frac{2 \cdot G \cdot Mea}{ro \cdot vo^2} - 1}$$ **Eq. 8**

$$Tp = \frac{\pi}{hs} \cdot (apo + per) \cdot \sqrt{apo \cdot per}$$ **Eq. 9**

$$ra = \frac{per + apo}{2}$$ **Eq. 10**

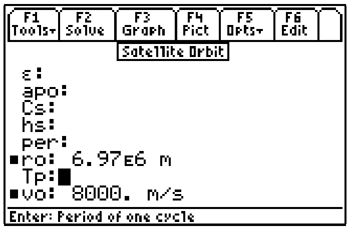
$$rb = \sqrt{per \cdot apo}$$ **Eq. 11**

$$v = \frac{hs}{rd}$$ **Eq. 12**

Variable	**Description**	**Units**
ε	Eccentricity	unitless
θ	Displacement angle	rad
apo	Apogee-maximum distance to center of mass	m
Cs	Constant	1/m
G	Gravitational constant	$6.67\text{x}10^{-11}$ N·m^2/kg^2
hs	Constant	m^2/s
per	Perigee – min. distance to center of mass	m
Pwr	Power	W
r	Radius	m
ra	Radius or semi-major axis of ellipse	m
rb	Radius or minor axis of ellipse	m
ro	Initial height/distance from planet center	m
Tp	Period of one cycle	s
v	Tangential velocity	m/s
vc	Critical velocity	m/s
vesc	Escape velocity	m/s
vo	Initial tangential velocity at ro/per	m/s

Example 22.6.3:

A satellite is launched 600 km above the earth's surface and has an initial velocity of 8000 m/s. Calculate the features of the satellite trajectory including its minimum maximum distance from the earth's center, period of orbit, and the shape of the trajectory.



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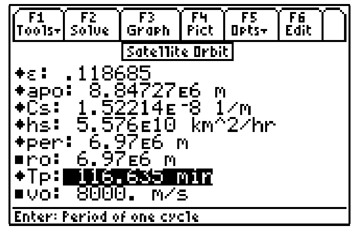


Computed results

Solution - The value for the mean radius of the earth **rea**=6.37E6 m is located in the **Engineering Constants** section of **Reference**. Use **equations 2, 3, 4, 7, 8** and **9** to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given
ro=6 E5 m (surface to satellite) + 6.37 E6 m(surface to earth center)=6.97E6 m.
vo=8000 m/s

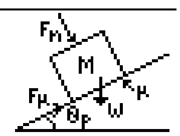
Solution
ε = .118685 (ellipse)

apo =8.84727 E6 m
Cs=1.52214 E-8 1/m
hs=5.576E10 km^2/hr
per=6.97E6 m
T=116.635 min

22.7 Friction

22.7.1 Frictional Force

These equations compute static friction for a mass on an inclined plane. **θp** (rad), is the minimum angle of incline required for gravity to overcome the frictional forces, which maintain the object from sliding. **Fn** (N), is the component of the object weight, **W** (N), normal to the plane of the inclined surface, **μ** is the coefficient of static friction for the mass/plane system and, **Fμ** (N), is the force exerted by friction in the static system.



$$F\mu = \mu \cdot Fn \quad \textbf{Eq. 1}$$

$$Fn = W \cdot \cos(\theta p) \quad \textbf{Eq. 2}$$

$$\mu \cdot W \cdot \cos(\theta p) = W \cdot \sin(\theta p) \quad \textbf{Eq. 3}$$

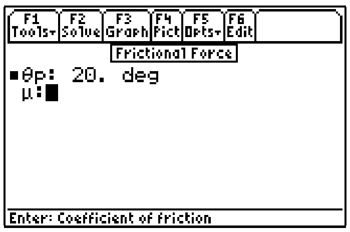
$$\mu = \tan(\theta p) \quad \textbf{Eq. 4}$$

$$W = m \cdot grav \quad \textbf{Eq. 5}$$

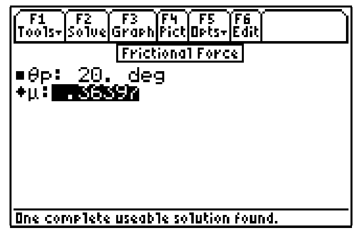
Variable	Description	Units
θp	Angle	rad
μ	Coefficient of friction	unitless
Fn	Normal force	N
grav	Acceleration due to gravitation	9.80665 m/s^2
m	Mass	kg
W	Weight	N

Example 22.7.1:

A calculator, weighing 7.1 oz, rests on an inclined textbook. The angle of incline required for the calculator to slide is 20 degrees. What is the coefficient of friction? If the textbook is laid flat, what force, parallel to the surface, must be exerted on the calculator to achieve the same effect?



Entered Values: Step 1

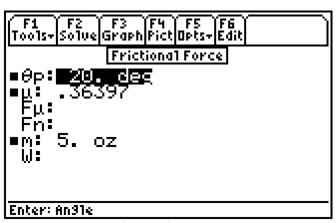


Computed results: Step 1

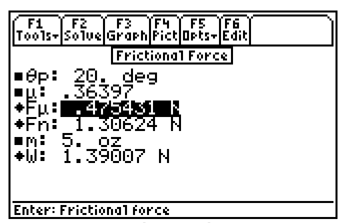
Given (Step 1)
θp = 20 deg

Solution (Step 1)
μ = 0.36397

Solution (Step 1) – This problem can be solved in two steps. First, select the **fourth equation**. Select this by highlighting the equation and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables.



Entered Values: Step 2



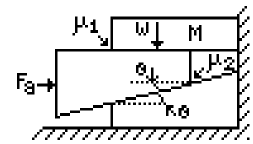
Computed results: Step 2

Given (Step 2)
θp = 20 deg
μ = 0.36397
m = 5 oz

Solution (Step 2)
Fμ = .475431 N
Fn = 1.30624 N
W = 1.39007 N

Solution (Step 2) Next, select the **first, second** and **last** equations. Enter the computed value of **μ**, the mass of the calculator, and a zero angle of incline. The entries and results are shown in the screen displays above.

22.7.2 Wedge



The following equations calculate forces on a wedge lodged between two blocks. There are five forces, which act on the wedge shown in the diagram. These forces add to zero when these forces are in equilibrium. **W** (N), is the weight of the upper block, **μ1** is the friction coefficient between the upper block and wedge, **μ2**, is the coefficient of friction on the inclined surface of the wedge and the lower block and **θ** (rad) is the angle of incline of the wedge. **Fa** (N), is the force required to overcome the forces of friction, which maintain the wedge-block system in equilibrium. The coefficient of friction between the upper block and the vertical surface to its right is assumed to be negligible.

$$Fa = \mu 1 \cdot W + \frac{W \cdot (\mu 2 \cdot \cos(\theta) + \sin(\theta))}{\cos(\theta) - \mu 2 \cdot \sin(\theta)}$$ **Eq. 1**

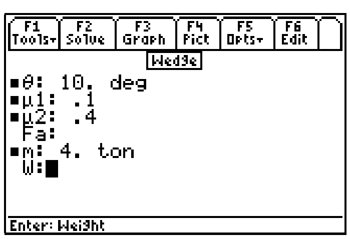
$$Fa = W \cdot \left(\mu 1 + \frac{\mu 2 + \tan(\theta)}{1 - \mu 2 \cdot \tan(\theta)} \right)$$ **Eq. 2**

$$W = m \cdot grav$$ **Eq. 3**

Variable	Description	Units
θ	Displacement angle	rad
μ1	Coefficient of friction	unitless
μ2	Coefficient of friction	unitless
Fa	Horizontal force	N
grav	Acceleration due to gravitation	9.80665 m/s^2
m	Mass	kg
W	Weight	N

Example 22.7.2:

What force must be exerted on the wedge with a 10-degree incline to lift a 4-ton load. The friction coefficients for **μ2** and **μ1** in the above diagram, are 0.1 and 0.4, respectively.



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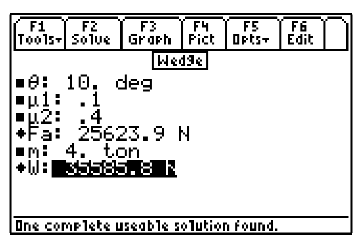
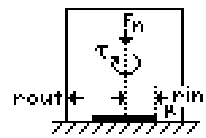


Computed results

Solution – The **first and third, or second and third equations** are needed to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
θ = 10 deg	Fa=25623.9 N
$\mu 1$ = 0.1	W=35585.8 N
$\mu 2$ = 0.4	
m=4 ton	

22.7.3 Rotating Cylinder



The following equations describe a rotating cylinder having an annulus-shaped contact with a stationary surface. The inner radius of the contact area is **rin** (m), and an outer radius of contact is **rout** (m). The **first equation** calculates the **Area** (m^2) of the annulus-shaped contact between the cylinder and the surface. The **second equation** estimates the pressure, **σ** (Pa), over the area of the annulus-shaped contact surface, **Area**. The **third equation** computes the torque, **τ** (N·m), required to overcome the static forces of friction between the surface and the cylinder.

$$Area = \pi \cdot \left(rout^2 - rin^2\right) \qquad \textbf{Eq. 1}$$

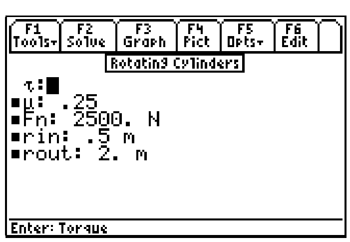
$$\sigma = \frac{Fn}{\pi \cdot \left(rout^2 - rin^2\right)} \qquad \textbf{Eq. 2}$$

$$\tau = \frac{2 \cdot \mu \cdot Fn \cdot \left(rout^3 - rin^3\right)}{3 \cdot \left(rout^2 - rin^2\right)} \qquad \textbf{Eq. 3}$$

Variable	**Description**	**Units**
σ	Stress	Pa
τ	Torque	N·m
μ	Coefficient of static friction	unitless
Area	Cross-sectional area	m^2
Fn	Normal force	N
grav	Acceleration due to gravitation	9.80665 m/s^2
rin	Inner radius	m
rout	Outer radius	m

Example 22.7.3:

A rotating cylinder, at rest, exerts a force of 2500 N on a surface. The cylinder has inner and outer contact radii of 0.5 m and 2 m, respectively. The coefficient of static friction is 0.25. What is the torque required to initiate rotation of the cylinder?



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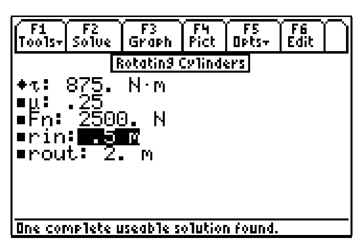


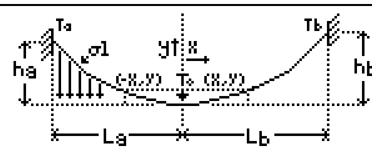
Computed results

Solution – Select the **last equation**. Highlight the equation and press [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	**Solution**
μ = 0.25	τ = 875 N m
Fn = 2500 N	
rin = 0.5 m	
rout = 2 m	

22.8 Statics

22.8.1 Parabolic cable



A cable between two fixed anchor points (a, b) assumes the shape of a parabola when there is a constant load per *horizontal distance*, **σ1** (N/m), *between the two stationary ends*. In such a case, the weight of the cable is typically insignificant in reference to the load. The **first equation** calculates **y** (m), the vertical distance from the lowest point of the cable at distance, **x** (m), from **To** (N), the tension of the cable at the lowest point. The **second equation** calculates the total distance, **L** (m), between the two anchors from horizontal distances to the lowest point, **La** (m) and **Lb** (m), from the location of **To**. The **third** and **fourth** equations compute **To** from the vertical, **ha** (m) and **hb** (m), and horizontal distances, **La**, and **Lb**, of the anchors to **To**. The **fifth** and **sixth** equations estimate the tension at the two fixed points, **Ta** (N) and **Tb** (N).

$$y = \frac{\sigma 1 \cdot x^2}{2 \cdot To}$$ **Eq. 1**

$$L = La + Lb$$ **Eq. 2**

$$To = \frac{\sigma 1 \cdot La^2}{2 \cdot ha}$$ **Eq. 3**

$$To = \frac{\sigma 1 \cdot Lb^2}{2 \cdot hb}$$ **Eq. 4**

$$Ta = \sigma 1 \cdot La \cdot \sqrt{1 + \frac{La^2}{4 \cdot ha^2}}$$ **Eq. 5**

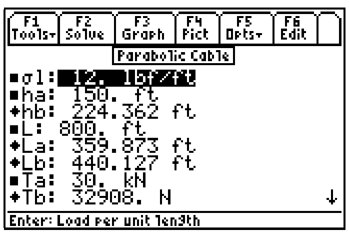
$$Tb = \sigma 1 \cdot Lb \cdot \sqrt{1 + \frac{Lb^2}{4 \cdot hb^2}}$$ **Eq. 6**

Variable	**Description**	**Units**
σl	Load per horizontal unit length	N/m
ha	Height from low point to left support	m
hb	Height from low point to right support	m
L	Total length	m

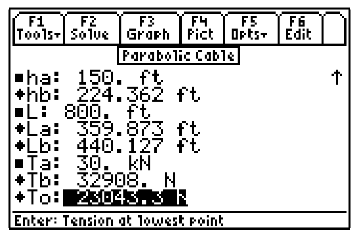
Variable	Description	Units
La	Distance from low point to left support	m
Lb	Distance from low point to right support	m
Ta	Tension at left anchor	N
Tb	Tension at right anchor	N
To	Tension at lowest point	N
x	Distance from origin along x axis	m
y	Displacement	m

Example 22.8.1:

A parabolic cable supports a uniform horizontal load of 12-lb/ft distances across a river gorge 800 ft wide. If an anchor on one side is able to support a load of 30kN at a height of 150 ft above the river, what is the minimum height and tension of the anchor on the opposite side of the gorge? What is the minimum distance of one side to the sag point (location of **To**)? What is the tension at the lowest point of the cable?



Upper Display

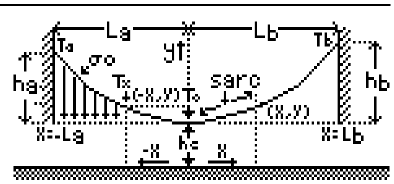


Lower Display

Solution – The **second through sixth equations** are needed to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above.

Given	Solution
σl = 12 lbf/ft	hb = 224.362 ft
ha=150 ft	La = 359.873 ft
L = 800 ft	Lb = 440.127 ft
Ta=30 kN	Tb = 32908 N
	To = 23043.3 N

22.8.2 Catenary cable



A catenary cable supports a uniform load *per length of* cable, **σo** (N/m). The cable forms a hyperbolic curve when the load is solely due to the weight of the cable. The **first equation** calculates the vertical displacement, **y** (m), at position, **x** (m), from the point of sag, above a plane located at a distance, **hc** (m), below the sag point. **Equation 2** computes the total horizontal distance between the two anchors, **L** (m), as the sum of the horizontal distances, **La** and **Lb**, to the sag point from each end. The variable, **hc**, in the **third equation**, is defined as the height of the cable where, **x=0**. The **fourth equation** computes the arc length of the cable, **sarc** (m), at a distance, **x** (m), from the low point. The **fifth** and **sixth equations** calculate the heights, **ha** (m) and **hb** (m), of the fixed ends above the sag point (location of **To**). **Equation 7** calculates the tension of the cable, **Tx** (N), at distance, **x**, from the sag point. In the diagram above, the variable, **x**, is positive to the right of the cable and negative to the left. The **eighth** and **ninth** equations estimate the tension at the two fixed points, **Ta** (N) and **Tb** (N). The **last equation** computes the length of the cable, **cl** (m), given cable parameters, **hc**, **La** and **Lb**.

$$y = hc \cdot \cosh\left(\frac{x}{hc}\right)$$ **Eq. 1**

$$L = La + Lb$$ **Eq. 2**

$$hc = \frac{To}{\sigma o}$$ **Eq. 3**

$$sarc = hc \cdot \sinh\left(\frac{x}{hc}\right)$$ **Eq. 4**

$$ha = hc \cdot \left(\cosh\left(\frac{La}{hc}\right) - 1\right)$$ **Eq. 5**

$$hb = hc \cdot \left(\cosh\left(\frac{Lb}{hc}\right) - 1\right)$$ **Eq. 6**

$$Tx = \sigma o \cdot hc \cdot \cosh\left(\frac{x}{hc}\right)$$ **Eq. 7**

$$Ta = \sigma o \cdot hc \cdot \cosh\left(\frac{La}{hc}\right)$$ **Eq. 8**

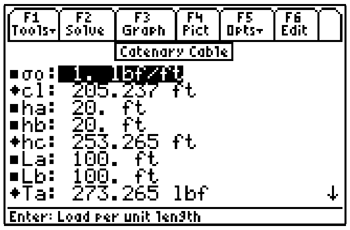
$$Tb = \sigma o \cdot hc \cdot \cosh\left(\frac{Lb}{hc}\right)$$ **Eq. 9**

$$cl = hc\left(\sinh\left(\frac{La}{hc}\right) + \sinh\left(\frac{Lb}{hc}\right)\right)$$ **Eq. 10**

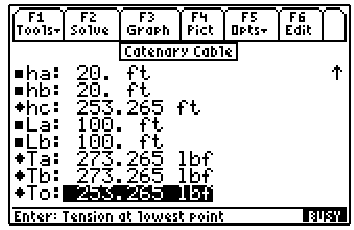
Variable	**Description**	**Units**
σo	Load per cable length	N/m
cl	Length of cable	m
ha	Height from low point to left support	m
hb	Height from low point to right support	m
hc	Characteristic length	m
L	Distance between a and b	m
La	Dist. from low point to left support	m
Lb	Dist. from low point to right support	m
sarc	Length of cable from lowest point	m
Ta	Tension at left support	N
Tb	Tension at right support	N
Tx	Tension at a location x	N
To	Tension at lowest point	N
x	Distance from origin along x axis	m
y	Displacement	m

Example 22.8.2:

A cable weighing 1 lb per foot length is strung between two towers 200 ft apart. The sag is 20 ft below each tower. Calculate the tension at each tower and the length of cable required.



Upper Display



Lower Display

Solution - The **third, fifth, sixth, eighth, ninth,** and **tenth equations** are needed to solve this problem. Select these by highlighting the equations and pressing [ENTER]. Press [F2] to display the variables. Enter the values for the known parameters and press [F2] to solve for the unknown variables. The entries and results are shown in the screen displays above. Since the towers are equal of equal height, the sag point should be halfway in between the two towers (100 ft). Enter the known values and press [F2] to solve for the unknown variables. This problem takes about 5 minutes to solve.

Given	**Solution**
σo = 1 lbf/ft	cl = 205.237 ft
ha = 20 ft	hc = 253.265 ft
hb = 20 ft	Ta = 273.265 lbf
La = 100 ft	Tb = 273.265 lbf
Lb = 100 ft	To = 253.265 lbf

References:

1. Engineering Mechanics, Vol. 1, Statics, Edition 2, J. L. Meriam and L. G. Kraige, John Wiley and Sons, New York, 1986
2. Engineering Mechanics, Vol. 2, Dynamics, Edition 2, J. L. Meriam and L. G. Kraige, John Wiley and Sons, New York, 1986
3. Engineering Mechanics, 7th Edition, R. C. Hibbeler, Prentice-Hall, Englewood Cliffs, NJ, 1995
4. Fundamentals of Physics, D. Halliday, R. Resnick and J. Walker, 4th Edition, John Wiley and sons, Inc. New York, 1993
5. Michael R. Lindeburg, Mechanical Engineering Reference Manual, Professional Publications, Belmont, CA 1990
6. Murray R. Spiegel, Theoretical Mechanics, Schaum's Outline Series, McGraw-Hill Book Company, New York, NY, 1967
7. Lane K. Bronson, Engineering Mechanics, Statics and Dynamics, Simon and Schuster technical outlines series, Simon and Schuster, New York, NY 1970

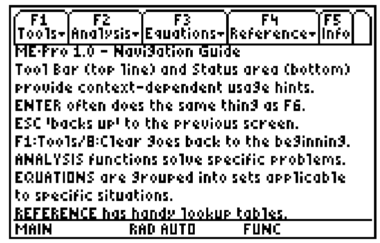
Part III: Reference

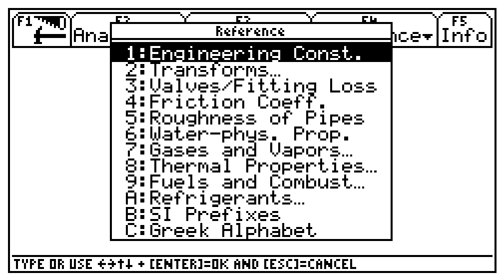
Chapter 23: Introduction to Reference

This chapter guides the user through the *Reference* Section of ME•*Pro*. The information in the *Reference* Section of the software is organized in a fashion similar to that in *Analysis* and *Equations*, except that it is generally non-interactive.

23.1 Introduction

The Reference Part is organized in twelve sections that include the following topics: Engineering Constants; Fourier and Laplace Transforms; Valves/Fitting Loss; Friction Coefficients; Roughness of Pipes; Water Physical Properties; Gases and Vapors; Thermal Properties; Fuels and Combustion; Refrigerants; SI prefixes; and, the Greek Alphabet.





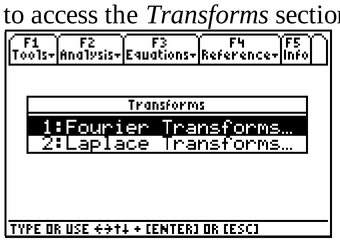
Pull down menu on TI - 89 and TI - 92 Plus

Unlike *Analysis* and *Equations*, the screen formats for the topics in the *Reference* section can differ significantly, depending on the information presented.

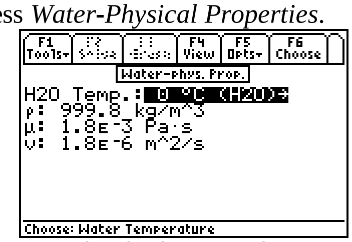
23.2 Finding Reference

Starting ME•Pro you can access the Reference Part.

1. Start ME•*Pro*:
   - TI-89 and TI-92 Plus: Press [APPS] key to display the pull down menu. Press [ENTER] or press [1] to view flash applications. Use ▼ key to move the highlight bar to ME•*Pro* and press [ENTER].
   - Pressing [F4] accesses the menu for the *Reference* section listing the topics. ME•*Pro* is structured with a hierarchy of screens for choosing a specific topic.
2. Select a topic of *Reference* by moving the highlight bar to the desired section using the ▼ key and pressing [ENTER]. Alternatively, type in the number corresponding to the section desired. For example, press [2] to access the *Transforms* section or press [6] to access *Water-Physical Properties*.



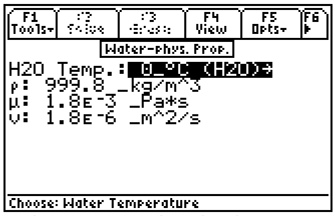
Transforms Menu



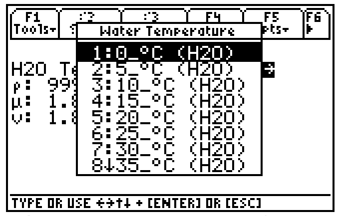
Water-Physical Properties Screen

23.3 Reference Screens

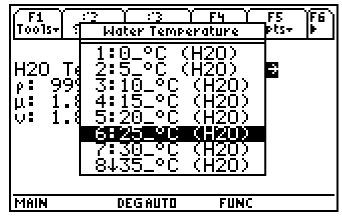
The *Water-Physical Properties* section has been chosen to illustrate how to navigate within a topic of the *Reference* section. When accessing the *Water-Physical Properties* section the properties of H_2O should be displayed.



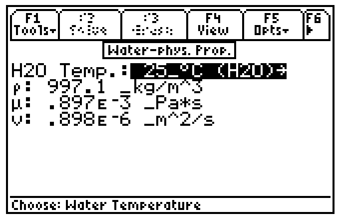
Main Water-Physical Properties Screen



Other Temperatures of H_2O



Highlight bar at 25°C (H_2O)

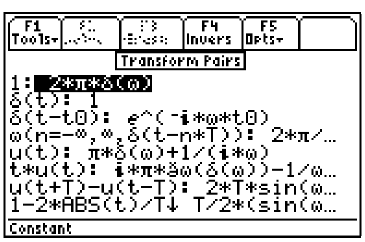


Properties of H_2O at 25°C

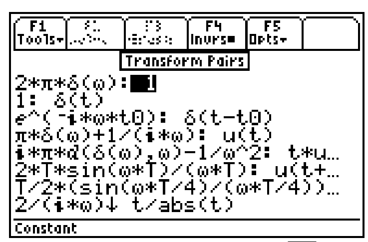
- Note that H_2O Temp has an arrow → to its right indicating that there are other temperatures of H_20 whose properties are also listed. Move the highlight bar to H_20 Temp., press [ENTER] to view the other temperatures of H_20. The list of other temperatures include 0 °C to 100 °C in five degree increments. To display the properties of H_2O at 25 °C, use the ⊖ key to move the high light bar to 25°C (H_2O) and press [ENTER]. Alternatively, type in [6] when the pull down menu appears.
- The data displayed automatically updates to list the properties of water at 25 °C as shown above.

23.4 Using Reference Tables

The *Transform* section is used as an example of viewing reference tables. *Transforms* allows the user to inspect *Fourier and Laplace Transforms*. Each of these topics is divided into three sub-topics: *Definitions*, *Properties* and *Transform Pairs*. For example, navigate from *Transforms* → *Fourier Transforms* → *Transform Pairs*. A screen display below lists the equations displaying the fundamental properties of *Fourier* transform pairs. Note that the name of the selected transform equation appears in the status line (at the bottom of the screen). For example, if the highlight bar is moved to the sixth equation, the status line displays "Rectangular Pulse" as the description of the property.

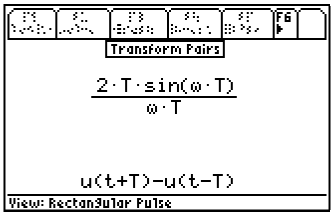


Normal View

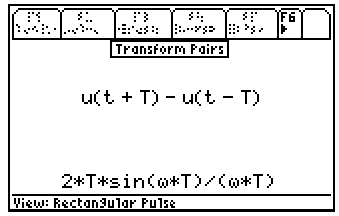


Inverse View (press [F4])

To view an equation in *Pretty Print* format, press ⊙. The contents on the right side of the colon (:) are displayed in *Pretty Print*, while the contents to the left of the colon are displayed in regular type above the status line. To reverse this display (display the inverse of the property), press [ESC] to exit *Pretty Print* mode; press [F4] to display the *inverse* form of the transform property; and, ⊙ to view the inverse transform in *Pretty Print.* This function is only available in the Transforms Section of the ME-Pro Reference.



Normal View of Rectangular Pulse



Inverse View of Rectangular Pulse

Chapter 24: Engineering Constants

This *Constants Reference Table* section lists the values and units for 46 commonly used universal constants. These constants are embedded in equations in the *Equations* section of ME•*Pro* and are automatically inserted during computations, when required.

24.1 Using Constants

The *Constants* section in *Reference* is designed to give a quick glance for commonly used constants. It lists values of accuracy available by the standards of measurement established by appropriate international agencies. This section does not include any information about the uncertainty in measurement, if any.

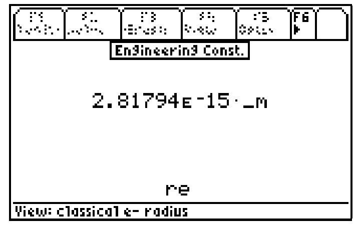
Table 24.1 Constants Reference Table

Display	***Description***	***Display***	***Description***
π	Circle ratio	ϕ0	Magnetic flux quantum
e	Napier constant	µB	Bohr magneton
γ	Euler constant	µe	e- magnetic moment
ϕ	Golden ratio	µN	Nuclear magneton
α	Fine structure	µp	p+ magnetic moment
c	Speed of light in vacuum	µµ	µ magnetic moment
ε0	Permittivity of a vacuum	a0	Bohr radius
F	Faraday constant	R∞	Rydberg constant
G	Newtonian constant of gravitation	brc1	1st radiation constant
grav	Acceleration of gravity	brc2	2nd radiation constant
h	Planck constant	Brc3	Wien displacement
hb	Dirac constant	σ	Stefan-Boltzmann constant
k	Boltzmann constant	λc	e- Compton wavelength
µ0	Permeability of vacuum	λn	n Compton wavelength
q	e- charge	λp	p+ Compton wavelength
em	e- charge / mass	SP	Standard pressure
me	e- rest mass	ST	Standard temperature
mn	n rest mass	Vm	Molar volume at STP
mp	p+ rest mass	mwa	Molecular mass of dry air
mµ	muon rest mass	NA	Avogadro constant
pe	Mass p+ / mass e-	Rm	Molar gas constant
re	Classical e- radius	Mea	Mass of the earth
µq	Mass µ / mass e-	rea	Mean radius of earth

These constants were arranged in the following order: universal mathematical constants lead the list, followed by universal physical constants; atomic and quantum mechanical constants; radiation constants; standard temperature and pressure; universal gas constant; and, molar constants. To view a constant, use the arrow key ⊙ key to move the highlight bar to the value and press the *View* key [F4]. The status line at the bottom of the screen gives a verbal description of the constant.

Example 24.1:

Look up the classical radius of an electron.



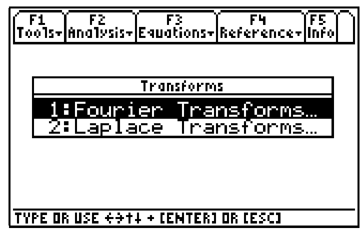
1. **Re** value is located about half way down the list. Make sure it is selected by the highlight bar using the arrow keys.
2. Access the *View* function by pressing key [F4].
3. Press any key to return to the constants screen.

Chapter 25: Transforms

This section accesses a series of tables containing transforms of common interest to mechanical engineers. The transforms are listed are – *Fourier Transforms*, and *Laplace Transforms*. Each topic contains information categorized under three subtopics; Definitions, Properties and Transform Pairs. All formulae can be viewed in *Pretty Print* equation-display format. These sub-topics are not interactive; i.e., one cannot specify an arbitrary expression and expect to compute a transformed result.

25.1 Using Transforms

When the **Transform** section is selected, a dialog box is appears. To choose a topic, move the highlight bar using ▼ or ▲ keys and press [ENTER]**;** alternatively, press the number associated with the topic.



1. Select Fourier, or Laplace Transforms.
2. Select Definitions, Properties or Transform Pairs
3. Use the ▼ or ▲ keys to move to the transform line desired.
4. The forward transforms are displayed by default.
5. Pressing [F4] toggles between the forward and inverse formats.
6. Press ▶ to view the transform property in *Pretty Print.*
   - Information is presented on either side of the colon as shown below. The term on the left side of the colon, F(ω), represents the function in the frequency domain. The right side of the colon represents the exact definition for F(ω) in terms of the time domain function, f(t), integrated over all time modulated by $e^{-i\cdot\omega\cdot t}$.

$$F(\omega):\int_{-\infty}^{+\infty}\left(f(t)\cdot e^{-i\cdot\omega\cdot t}\right)dt$$ ***Forward Fourier Transform***

$$F(t):\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(F(\omega)\cdot e^{i\cdot\omega\cdot t}\right)d\omega$$ ***Inverse Fourier Transform***

Forward and Inverse Formats

- The information can be displayed in the inverse (as opposed to forward) form. This means that the information on either side of the colon changes positions when the Inverse key [F4] is pressed. A ‘▪‘ symbol appears in the [F4] tool bar to indicate the inverse form of the transform function is being displayed.

Status Line Message

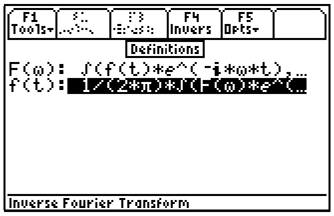
- The status line gives a description of the equation highlighted. The descriptions use standard terminology such as *Modulation, Convolution, Frequency Integration*, etc.

Example 25.1:

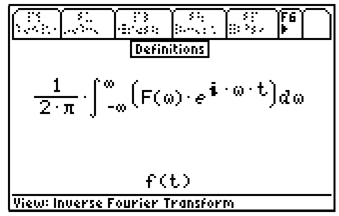
What is the definition of the *Inverse Fourier transform*?

Normally, Fourier Transform is a representation of a periodic time domain function f(t) in terms of an integral involving the frequency domain function F(ω). An inverse Fourier Transform refers to a representation F(ω) in terms of f(t).

1. In the main **Transforms** screen, move the highlight bar to **Fourier Transforms** and press [ENTER].
2. Press [ENTER] a second time to access **Definitions**.
3. Move the highlight bar to the second definition and press ▶ to display the equation in *Pretty Print* format.
4. Press ◀ or ▶ to scroll. Press any key to return to the previous screen, if the Pretty Print display is larger than the screen.



Fourier Definitions

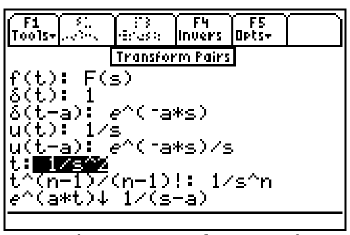


Pretty Print of Fourier (Inverse Fourier Transform)

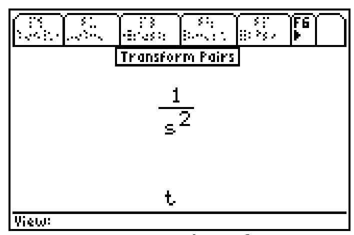
Example 25.2:

View the **Laplace** transform of the time function **f(t)=t**, in terms of a universal frequency variable Laplace s.

1. In the initial **Transforms** screen, move the highlight bar to **Laplace Transforms** and press [ENTER].
2. Move the highlight bar to **Transform Pairs** and press [ENTER].
3. Scroll down to t: 1/s^2 and press ▶ to view the equation in *Pretty Print* format.



Laplace Transform Pairs



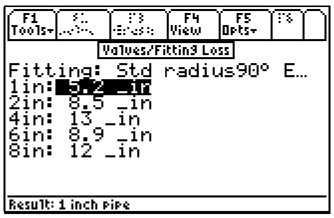
*Pretty Print of '**t**'*

Chapter 26: Valves and Fitting Loss

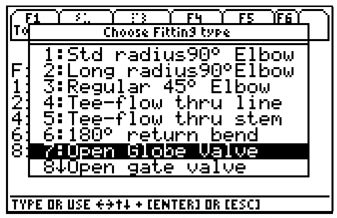
This Section covers the head loss of 11 different valves and fittings.

26.1 Valves and Fitting Loss Screens

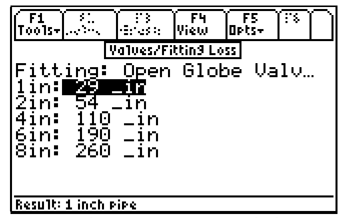
When accessing the *Valves and Fitting Loss* section, the next screen will show the properties of the Std radius 90° Elbow. The Valve/Fitting Loss for each type of fitting is listed for a 1in., 2in., 4in., 6in., and 8in. radius.



Properties Std radius 90° Elbow



Materials available in Fitting type



Properties of Open Globe Valve

- Note that *Fitting:* has an arrow → to its right indicating that there are additional cases of materials whose properties are also listed as shown above. Move the highlight bar to *Std radius 90°*; press [ENTER] to view the other materials. The list of other materials includes Long radius 90° elbow, Regular 45° elbow, Tee-flow thru line, Tee-flow thru stem, 180° return bend, Open globe valve, Open gate valve, Open angle valve, Swing check valve, Coupling or union. To display the properties of *Open Globe Valve*, use the ⊝ key to move the high light bar to *Open Globe Valve* and press [ENTER]. Alternatively, type in [7] when the pull down menu appears.
- The data displayed automatically updates to list the properties of Open Globe Valve as shown above.

Chapter 27: Friction Coefficients

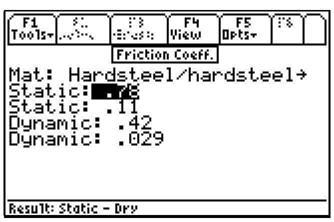
This Section covers the friction coefficients of 34 materials. Four types of friction coefficient values are displayed when known. Static or dynamic friction coefficients under dry or greasy conditions are displayed. The friction coefficients are unitless quantities, when the display shows "-" character it implies there is no reliable data available.

Table 27.1 Friction Coefficients Reference

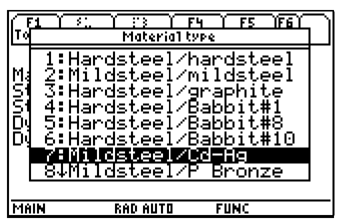
Display	***Status Line***	***Display***	***Status Line***
Static	Static – Dry	Dynamic	Dynamic – Dry
Static	Static – Greasy	Dynamic	Dynamic – Greasy

27.1 Friction Coefficients Screens

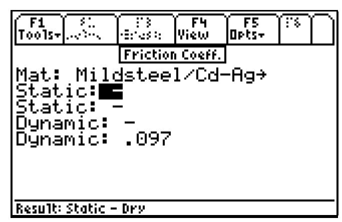
When accessing the *Friction Coefficients* section, the next screen will show the properties of the *Hardsteel/hardsteel*.



Properties Friction Coefficients



Materials available in Material type



Properties of Open Mildsteel/Cd-Ag

- Note that *Hardsteel/hardsteel* has an arrow → to its right indicating that there are other materials whose properties are also listed. Move the highlight bar to *Hardsteel/hardsteel*, press [ENTER] to view the other materials. The list of other materials includes Mildsteel/mildsteel, Hardsteel/graphite, Hardsteel/Babbit#1, Hardsteel/Babbit#8, Hardsteel/Babbit#10, Mildsteel/Cd-Ag, Mildsteel/P Bronze, Mildsteel/Cu Pb, Mildsteel/cast Iron, Mildsteel/Pb, Ni/Mildsteel, Al/mildsteel, Mg/mildsteel, Teflon/steel, Wcarbide/Wcarbide, Wcarbide/steel, Wcarbide/Copper, Wcarbide/Iron, Bonded Carbide/Cu, Bonded carbide/Iron, Cd/Mildsteel, Cu/Mildsteel, Brass/Mildsteel, Brass/cast Iron, Zinc/cast Iron, Cu/cast iron, Tin/castIron, Lead/cast Iron, Aluminum/Aluminum, Glass/glass, Glass/Nickel, Copper/glass, Cast Iron/Cast Iron. To display the properties of Mildsteel/Cd-Ag, use the ⊖ key to move the high light bar to Mildsteel/Cd-Ag and press [ENTER]. Alternatively, type in [7] when the pull down menu appears.
- The data displayed automatically updates to list the properties of Mildsteel/Cd-Ag as shown above.
- To return to display listing material types, press [F6]. To return to screen with Reference sections, press [ESC].

Chapter 28: Relative Roughness of Pipes

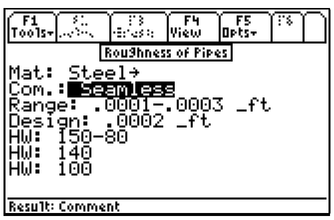
This Section covers 23 different pipe materials and their relative roughness.

Table 28.1 Relative Roughness Reference

Display	***Status Line***	***Display***	***Status Line***
Com.	Comment	HW	HW const – range
Range	Roughness range	HW	HW const – clean
Design	Roughness design	HW	HW const - design

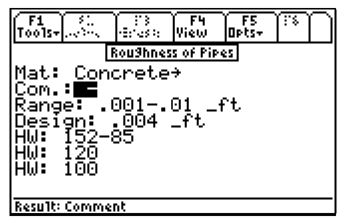
28.1 Relative Roughness Screens

When accessing the *Roughness of Pipes* section, the next screen will show the properties of *Steel*.



Properties Steel

Materials available in Material type



Properties of Concrete

- Note that *Steel has* an arrow → to its right indicating that there are other materi als whose properties are also listed. Move the highlight bar to *Steel*, press [ENTER] to view the other materials. The list of other materials includes Steel-rivets, Steel-girth, Steel-horiz. rivets, Spiral rivets, vitrified, Concrete, Cement-asbestos, vitrified Clay, Brick sewer, Cast Iron, Cast Iron coated, Cast Iron lined, Cast Iron Bituminous, Cast Iron spun, Cast Iron galvanized, Wrought Iron, Fiber, Copper and Brass, Wood stave, Transite, lead, tin, glass, and Plastic. To display the properties of Concrete, use the ⊙ key to move the high light bar to Concrete and press [ENTER]. Alternatively, type in [7] when the pull down menu appears.
- The data displayed automatically updates to list the properties of Concrete as shown above. You may return to Pipe material type screen by highlighting Mat: Concrete→ and press ⊙.

Chapter 29: Water-Physical Properties

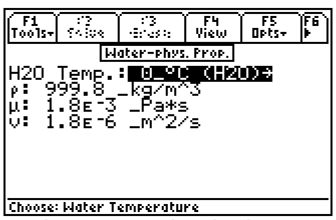
This section covers the properties of water from 0 °C – 100 °C. Properties tabulated are; density, dynamic, and kinematic viscosity.

Table 29.1 Properties of Water

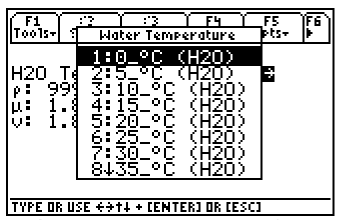
Water-Physical Properties	
Display	***Status Line***
ρ	Density
μ	Dynamic Viscosity
ν	Kinematic Viscosity

29.1 Water-Physical Properties Screens

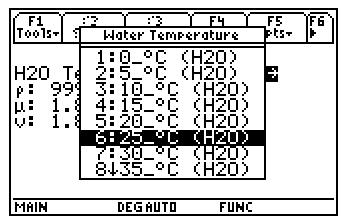
When accessing the *Water-Physical Properties* section the properties of H_2O should be displayed.



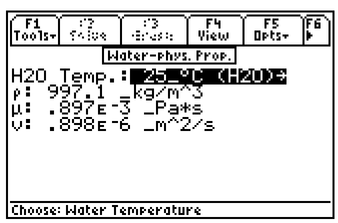
Main Water-Physical Properties Screen



Other Temperatures of H_2O



Highlight bar at 25°C (H_2O)



Properties of H_2O at 25 °C

- Note that H_2O Temp has an arrow → to its right indicating that there are other temperatures of H_2O whose properties are also listed. Move the highlight bar to H20 Temp., press [ENTER] to view the other temperatures of H_2O. The list of other temperatures include 0 °C to 100 °C in five degree increments. To display the properties of H_2O at 25 °C, use the ⊙ key to move the high light bar to 25°C (H_2O) and press [ENTER]. Alternatively, type in [6] when the pull down menu appears.
- The data displayed automatically updates to list the properties of water at 25 °C as shown above.

Chapter 30: Gases and Vapors

This section covers the reference data for six topics relating to the behavior of gases and vapors.

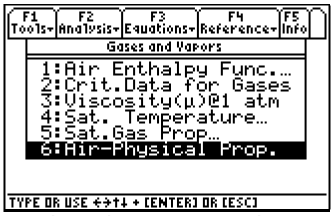
- Air Enthalpy and Psi Function
  - 200K – 780K
  - 800K – 1480K
  - 1500K – 2200K
- Critical Data for Various Gases
  - Gases, general
  - Gases, organic
- Viscosity (μ) at One Atmosphere
- Saturation Temperature
  - <1 bar
  - ≥1 bar
- Saturated Gas Properties
  - Ammonia
  - Carbon Dioxide
  - Ethane (R170)
  - Hydrogen (n)
  - Methane
  - Refrigerant 11
- Air Physical Properties

Table 30.1 Gases and Vapors Reference Table

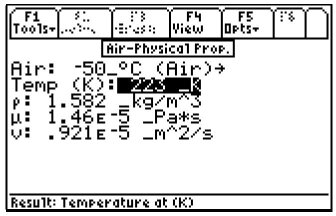
Display	***Status Line***	***Display***	***Status Line***
h	Specific enthalpy	vg	Specific volume, gas
Psi	Psi function	hf	Specific enthalpy, liquid
Gas	Gas type	hg	Specific enthalpy, gas
Temp	Boiling temperature	sf	Specific entropy, liquid
Temp	Critical temperature	sg	Specific entropy, gas
Temp	Temperature at K	cpf	Specific heat at constant pressure
Press.	Critical pressure	ρ	Density
Vol.	Critical Volume	μ	Dynamic viscosity
P	Vapor Pressure	v	Kinematic viscosity
vf	Specific volume, liquid		

30.1 Gases and Vapors Screens

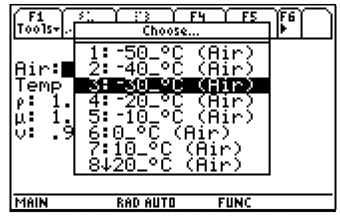
When accessing the *Gases and Vapors* section, a dialog box appears listing the available topics. Use ⊝ key to move the highlight bar to *Air – physical properties* and press [ENTER]. This will display the properties of air at -50°C.



Sections in Gases and Vapors



Properties of air at -50°C



Materials available in ***Air – Physical Properties***

- Note that -50°C has an arrow → to its right indicating that there are other Air Temperatures whose properties are also listed. Move the highlight bar to -50°C, press [ENTER] to view the other materials. The list of other materials includes -40°C (Air), -30°C (Air), -20°C (Air), -10°C (Air), 0°C (Air), 10°C (Air), 20°C (Air), 30°C (Air), 40°C (Air), 50°C (Air), 60°C (Air), 70°C (Air), 80°C (Air), 90°C (Air), 100°C (Air), 150°C (Air), 200°C (Air), 250°C (Air), and 300°C (Air). To display the properties of -30°C (Air), use the ⊝ key to move the high light bar to -30°C (Air) and press [ENTER]. Alternatively, type in [3] when the pull down menu appears.
- The data displayed automatically updates to list the properties of Air at -30°C.

Chapter 31: Thermal Properties

This section covers the Thermal Conductivity of metals, liquids, gases and other specified materials; and the Specific Heat of liquids and gases.

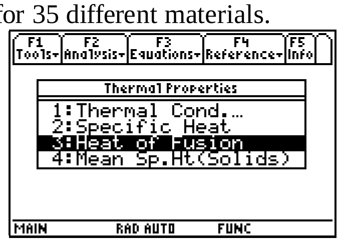
- ♦ Thermal Conditions
  - Metals
    - Elemental
    - Alloys
- ♦ Specific Heat
  - Cp of Liquids and gases
  - Cp/Cv of liquids and gases at 1atm
- ♦ Heat of Fusion (Various elements)
- ♦ Mean Specific Heat (Solids)

Table 31.1 Thermal Properties Reference Table

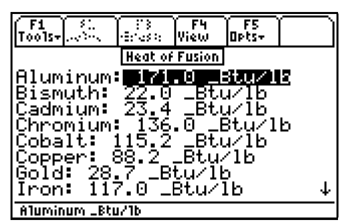
Display	***Status Line***	***Display***	***Status Line***
Temp	Temperature range	cp	Specific heat at constant pressure
Temp	Temperature	μ	Viscosity at Bulk Temperature
kto	Thermal Conductivity	Melt temp	Melting Temperature
k	Thermal Conductivity	Max temp	Maximum temperature
a	Empirical constant	Density	Bulk Density
ρ	Density		

31.1 Thermal Properties Screens

When accessing the *Thermal Properties* section, a dialog box appears listing the available topics. Use ⊖ key to move the highlight bar to *Heat of Fusion* and press [ENTER]. This will display the heat of fusion properties for 35 different materials.



Sections in Thermal Properties



Heat of fusion Properties for *34 different materials*

- To select any of the materials on the list use the ⊖ or ⊖ keys. Now you can view the heat of fusion values for the materials listed.

Chapter 32: Fuels and Combustion

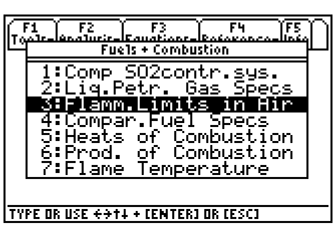
This section covers mechanical engineering reference data for the below mentioned seven topics:

Table 32.1 Fuels and Combustion Reference Table

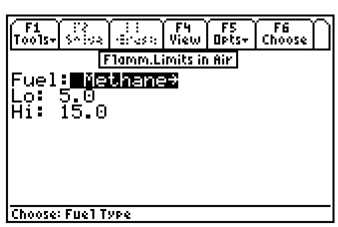
Display	*Status Line*	*Display*	*Status Line*
	Compare SO2 Control Systems		**Heats of Combustion**
Req.	Requirements	Sym.	Chemical symbol
Rec.	Recoverable material	Qv	Qv (per lb)
Cost	Extra cost power gen%	Qv	Qv (per ft^3)
SO2	SO2 efficiency %	Qp	Qp (per lb)
Inv.	Extra plant investment %	Qp	Qp (per ft^3)
	Liquid Petro Gas Specs		**Products of Combustion**
v.p.	Max v.p. at 100°F	Formula	Chemical formula
Butane	Max % butane v. residue	Mol.	Molecular weight
Pentane	Max % pentane v. residue	Wt.	Specific Weight at STP
Res.	Residue from evap, max	Vol	Volume ration air to fuel
Oil	Oil stain observation	CO_2	CO2 from $1ft^3$ Fuel
Corr	Corrosion, Cu strip, max	H_2O	H2O from $1ft^3$ Fuel
Sulfur	Sulfur, max	N2	N2 from $1ft^3$ Fuel
H2O	Free H20 content	Wt	Weight ratio air to fuel
	Flammable Limits in Air	CO_2	CO2 from 1lb fuel
Lo	Low limit in air vol%	H_2O	H2O from 1lb fuel
Hi	High limit in air vol%	N_2	N2 from 1lb fuel
	Comparative Fuel Specs		**Flame Temperature**
Petrol	Conventional petroleum	Gas	Gas Type
Tar	Tar sand bitumen	80%	80% Theoretical air
Syn.	Synthetic crude oil	90%	90% Theoretical air
		100%	100% Theoretical air
		120%	120% Theoretical air
		140%	140% Theoretical air

32.1 Fuels and Combustion Screens

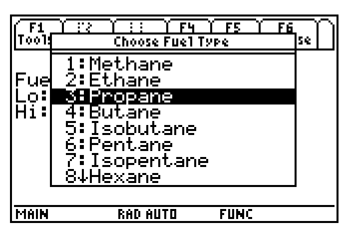
When accessing the *Fuels and Combustion* section, a dialog box appears listing the available topics. Use ⊝ key to move the highlight bar to *Flamm..Limits in Air*, and press [ENTER]. This will display the properties of Methane.



Sections in Fuels and Combustion



Properties of Methane



*Materials available in **Fuels***

- Note that *Methane* has an arrow → to its right indicating that there are other materials whose properties are also listed. Move the highlight bar to *Methane*, press [ENTER] to view the other materials. The list of other materials include: Ethane; Propane; Butane; Isobutane; Pentane; Isopentane; Hexane; Ethylene; Propylene; Butylene; Acetylene; Hydrogen; Carbon monoxide; Ammonia; Hydrogen sulfide; Natural; Producer; Blast-furnace; Water; Carbureted-water; Coal; Coke-oven; and, High-Btu oil. To display the properties of Propane, use the ⊝ key to move the high light bar to Propane and press [ENTER]. Alternatively, type in [3] when the pull down menu appears.

- The data displayed automatically updates to list the properties of Propane.

Chapter 33: Refrigerants

This section covers selected refrigerants and gases in low-temperature applications, transition temperatures and common cryogenic properties

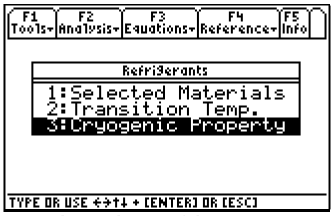
- Selected Materials
- Transition Temperatures
  - **I** Type I superconductor
  - **II** Type II superconductors
- Common Cryogenic Prop.

Table 33.1 Refrigerants Reference Table

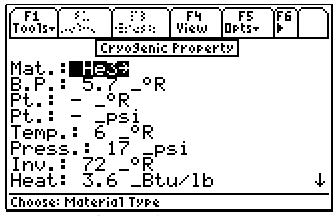
Display	***Status Line***	***Display***	***Status Line***
No.	Refrigerant number	Pt.	Triple pt. pressure
Mol.	Molecular weight	Temp.	Critical temperature
B.P.	Boiling point	Press.	Critical pressure
Temp.	Critical point temperature	Inv.	Upper inversion temperature
Press.	Critical point pressure	Heat	Heat of vapor (wt)
To	Transition temperature	Heat	Heat of vapor (vol)
Ho	Critical magnetic fields	Liq.	Liquid density
Hc2	Upper critical fields	Vap.	Vapor density
T(K)	Temp of upper critical fields	Gas	Gas density
Pt.	Triple pt. temperature		

33.1 Refrigerants Screens

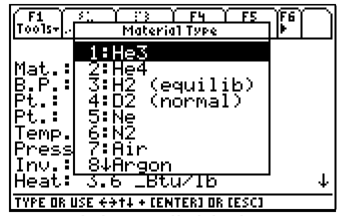
When accessing the *Refrigerants* section, a dialog box appears listing the available topics. Use ⊝ key to move the highlight bar to *Cryogenic Properties* and press [ENTER]. This will display the properties He3 .



Sections in Refrigerants



Properties of He3



Materials available in ***Cryogenic property***

- Note that He3 has an arrow → to its right indicating that there are other materials whose properties are also listed. Move the highlight bar to He3; press [ENTER] to view the other materials. The list of other materials includes He4, H2 (equilib), D2 (normal), Ne, N2, Air, Argon, F2, O2, and CH4. To display the properties of H2 (equilib), use the ⊝ key to move the high light bar to H2 (equilib) and press [ENTER]. Alternatively, type in [3] when the pull down menu appears.
- The data displayed automatically updates to list the properties of H2.

Chapter 34: SI Prefixes

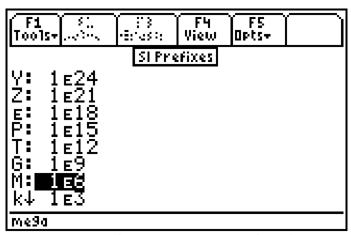
The SI Prefixes section displays the prefixes adapted by the Systeme International [d'Unit[eacute]s] (SI).

34.1 Using SI Prefixes

The prefixes are listed in the order shown in Table 34-1. The ⊝ key is used to move the highlight bar to select a SI prefix multiplier. The name of the prefix is displayed in the status line. The prefix and multiplier can be viewed by pressing the F4 key.

Table 34-1 SI Prefix Table

Prefix	Multiplier	Prefix	Multiplier
Y: (Yotta)	1E24	d: (deci)	1E-1
Z: (Zetta)	1E21	c: (Centi)	1E-2
E: (Exa)	1E18	m: (Milli)	1E-3
P: (Peta)	1E15	μ: (Micro)	1E-6
T: (Tera)	1E12	n: (Nano)	1E-9
G: (Giga)	1E9	p: (Pico)	1E-12
M: (Mega)	1E6	f: (Femto)	1E-15
k: (Kilo)	1E3	a: (Atto)	1E-18
h: (Hecto)	1E2	z: (Zepto)	1E-21
da: (Deka)	1E1	y: (Yocto)	1E-24



SI Prefix Screen Display

Chapter 35: Greek Alphabet

This section displays the Greek Alphabet and their names. There are several Greek letters supported by the TI-89. To enter the Greek letters, the sequential keystrokes are listed in the TI-89 manual. They are repeated here for convenience of the user. Alternatively, [2nd] [+] (or [CHAR]) followed by [1] will access an internal menu listing several Greek characters.

Table 35-1

Key stroke Sequence	Greek Letter	Key stroke Sequence	Greek Letter
[◆] [(] [alpha] [=]	α		
[◆] [(] [alpha] [(]	β		
[◆] [(] [alpha] [,]	δ	[◆] [(] [alpha] ↑ [,]	Δ
[◆] [(] [alpha] [÷]	ε		
[◆] [(] [alpha] [I]	φ		
[◆] [(] [alpha] [7]	γ	[◆] [(] [alpha] ↑ [7]	Γ
[◆] [(] [alpha] [4]	λ		
[◆] [(] [alpha] [5]	μ		
[◆] [(] [alpha] [STO▸]	π	[◆] [(] [alpha] ↑[STO▸]	Π
[◆] [(] [alpha] [2]	ρ		
[◆] [(] [alpha] [3]	σ	[◆] [(] [alpha] ↑[3]	Σ
[◆] [(] [alpha] [T]	τ		
[◆] [(] [alpha] [.]	ω	[◆] [(] [alpha] ↑[.]	Ω
[◆] [(] [alpha] [X]	ξ		
[◆] [(] [alpha] [Y]	ψ		
[◆] [(] [alpha] [Z]	ζ		

Part IV: Appendix and Index

Appendix A Frequently Asked Questions

A.1 Questions and Answers

A list of commonly asked questions about the ME•*Pro* is listed here. Review this list of questions prior to contacting the Technical support either at Texas Instruments or at da Vinci. You might save yourself a phone call! These questions are classified under four general headings.

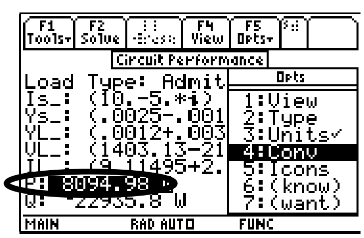
- Equations Questions
- General Questions
- Reference Questions
- Analysis Questions

A.2 General Questions

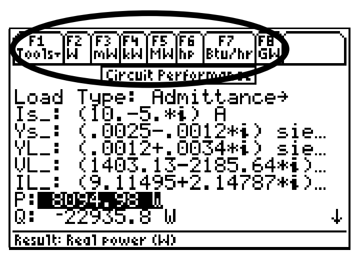
The following is a list of questions about the general features of ME •*Pro*:

Q. Where can I find additional information about a variable?
A. A brief description of a highlighted variable appears in the status line at the bottom of the screen. More information, including its allowable entry parameters (i.e.: whether complex, symbolic or negative values can be entered, etc.) can be accessed by pressing [F5]**/Opts** and **2:Type**.

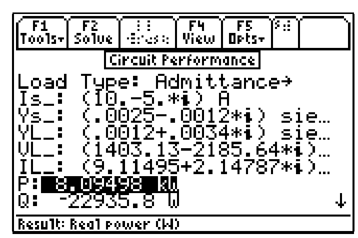
Q. How do I convert the results of a calculation in analysis and equations into different units?
A. Highlight the value which you want to convert; Press [F5]**/Opts**, **4:Conv**. A unit menu for the variable is displayed. Select the desired unit by pressing the function key corresponding to the appropriate units.



*To display a result in different units, highlight the variable (**P**) and press* [F5]**:Opts**, *move the cursor to '***4:Conv***'*



The unit menu for the variable appears in the top bar. Press the function key corresponding to the desired units. ([F4] in this case).



The computed value for Real Power, ***P****, is now displayed in kilowatts (***kW***).*

Q. What does the underscore "_" next to a variable mean?
A. This designates a variable, which allows entry of complex values.

Q. I am in the middle of a computation and it seems to stay busy for longer than I would like. How can I halt this process?
A. Some computations can take a long time, particular if many equations and unknowns are being solved or a complex analysis function has been entered. Notice if the message in the status line at the bottom-right of the screen reads BUSY. This indicates that the TI math engine is attempting to solve the problem. Pressing the [ON] key usually halts a computation and allows the user to regain control of the software. If, for some reason, the calculator locks up and does not allow user intervention, a "cold start" will have to be performed to reset the calculator. This can be done by holding down the keys; [2nd], ◀, and ▶, and pressing [ON] for the TI-89 (for the TI-92 Plus, press [2nd], [hand] and [ON]).

WARNING: This will delete folders containing any defined variables or stored programs. Use it as a last resort. A "cold start" will **not** delete ME•*Pro* from your calculator.

Q. What do three dots (...) mean at the end of an item on the screen?
A. The three dots (an ellipsis) indicate the item is too wide to fit on the available screen area. To view an item in its entirety, select it by moving the highlight bar and press [F4] (or ▶, in some cases) to view the item in *Pretty Print*. Press ◀ or ▶ to scroll the item back and forth across the screen to view the entire object.

Q. How can I recall, or view values of a previously computed problem?
A. ME•*Pro* automatically stores its variables in the current folder specified by the user in [MODE] or the HOME screens. The current folder name is displayed in the lower left corner of the screen (default is "Main"). To create a new folder to store values for a particular session of ME•*Pro*, press [F1]**:/TOOLS,** [3]**:/NEW** and type the name of the new folder (see Chapter 5 of the TI-89 Guidebook for the complete details of creating and managing folders).

There are several ways to display or recall a value:

- The contents of variables in any folder can be displayed using the [VAR-LINK], moving the cursor to the variable name and pressing [F6] to display the contents of a particular variable.
- Variables in a current folder can be recalled in the HOME screen by typing the variable name.
- Finally, values and units can be copied and recalled using the [F1]**/Tools** 5:COPY and **6:PASTE** feature. The COPY/PASTE function will copy the highlighted value and units in *Analysis* and *Equations* but will only copy the *value* (no units) in the *Reference* section.

All inputs and calculated results from *Analysis* and *Equations* section are saved as variable names. Previously calculated, or entered values for variables in a folder are replaced when equations are solved using new values for inputs.

Q. I am not able to calculate a value for a particular variable. Why?
A. Check the status-line message to insure that a complete or partial solution was found. Some variables have range restrictions; for example, only positive values are allowed for kinetic energy or satellite distance from the earth. Make sure that the inputs are meaningful in context of the allowable result(s). Sometimes, variable conflicts can occur from other applications. It is recommended that ME•*Pro* be used in a separate folder. Press [F1]**: Tools,** [3]**: New** to create a new folder. As a last resort, open a new folder or clear the variables in the current folder using [VAR-LINK] and repeat the calculation.

Q. An item, which is supposed to be displayed in a menu, doesn't appear.
A. Some menus have more than eight items. If an arrow ↓ appears next to the digit 8, use the arrow key ▼ to scroll the menu and view the remaining topics or press [2nd] ▼ jump to the bottom of the menu.

Q. Can I copy and paste entries from one variable to another?
A. You can copy and paste a highlighted value using [F1]**/Tools** 5:COPY and **6:PASTE** feature**. The COPY/PASTE function will copy the highlighted value and units in *Analysis, Equations and reference sections.***

Q. Is there a help section in the software?
A. There is a short series (slides) of general hints which can be accessed from the main screen of ME•*Pro* under [F5]**/Info**. A different message appears each time [F5] is pressed. We've attempted to keep most of the explanation of certain topics to the manual in an effort to keep the software compact. Consult the chapter corresponding to the appropriate section of the software. A compiled list of the received questions and answers will be posted periodically on the da Vinci website at http://www.dvtg.com/faq/mepro

A.3 Analysis Questions

These are some commonly asked questions about the *Analysis* section of ME•*Pro*.

Q. The screen display of computed results does not look identical to the example in the manual.
A. The [MODE] setting, which controls the number of floating point digits displayed in a value and whether an answer appears in exact or approximate form, may have been set differently on the calculator used to make the screen displays for the example problem. Press the [MODE] key to view or change the mode settings. The first page will display the number of floating-point digits, whether the display is in NORMAL, SCIENTIFIC, OR ENGINEERING exponential formats. Pressing [F2] will display whether computed answers are displayed in APPROXIMATE or EXACT formats. The default mode setting for ME•*Pro* is *Float 6*, *Radian* and *Approximate* calculations.

Q. The calculated angle isn't correct.
A. If the result is greater than 2π or less than -2π ($|\text{result}| \geq 360^o$), the TI *solve (...)* function may be generating a non-principal solution. A principal solution is defined as a value between 0 and 2π (or between 0 and 360^o). A non-principal solution can be converted to a principal solution by adding or subtracting integer multiples of 2π (or 360^o) until the remainder is within the range of 0 and 2π (or 0 and 360^o). The remainder is the principal solution.

Example: When solving the equation $\sin(x) = 0.5$, non-principle solutions include: 30°, 390°, -330°, 750°, etc., but the *principal* solution is 30°.

Q. I am not able to calculate a value for a particular variable. Why?
A. If a computed result displays no value, try clearing the variables in the current folder or opening a new folder to remove any possible variable conflicts and repeat the calculation. To delete variables in the current folder, press [VAR-LINK], [F5]: All, [1]: Select All, [F1]: Manage, [1]: Delete, [ENTER]: Yes. To open a new folder in ME•*Pro*, press [F1]: Tools, [3]: New. Type the name of the new folder and press [ENTER].

Q. What is the multiple graph feature in **Capital Budgeting** and how do I get it to plot sev eral projects simultaneously?
A. Activating the multiple graph features allows successive graphs to be overlaid on the same. To do this, the graphing execution must be repeated each time a new project is plotted.

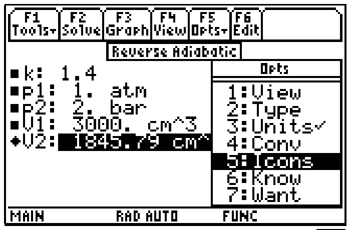
1. Make sure the cash flows for all the named projects have been entered.
2. Enable the **Multiple Graphs** feature by highlighting and pressing the [ENTER] key.
3. Select the name of a project you wish to graph and press [F3].
4. To overlay a second project on the first, select a different project name and press [F3] to graph.
5. Repeat step 4 each time a new project is to be graphed on top of previously plotted functions.

A.4 Equations Questions

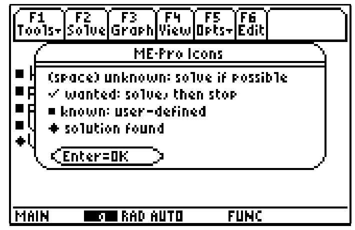
These are some common questions about the Equations section of ME•*Pro*.

Q. What do the icons next the variables and equations mean?
A. The explanation for these icons '♦', '▪', and '√' can be displayed by pressing [F5]:Opts, [5] Icons.

√ **-** This appears when an equation or feature has been selected.
▪ **-** This appears when a value has been accepted by the system.
♦ **-** Designates a value as having been calculated.



To view icon descriptions, press [F5]:*Opts*, [5]:*Icons*

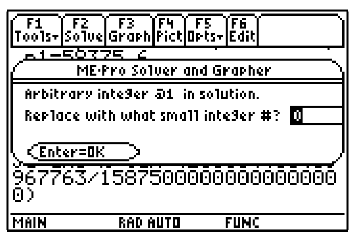


Icon descriptions

Q. There are already values stored in some of my variables. How do I clear or use those values?
A. These values remain from previous solving operations. It is okay to ignore the values. As long as they aren't selected '▪', they will be overwritten by new solutions. If you want to reset the values, clear one or all of the variables. A value can be re-used in a computation by highlighting the displayed value and pressing [ENTER] twice. The values can be accessed via VAR-LINK menu. To delete variables in VAR-LINK, use the file management tool provided (use [F1] key to access file management tools), [F4] check the variables you want deleted and delete the variables.

Q. Why do the values of the entered or calculated results change when the Units feature is deactivated in the [F5] options menu?
A. When the Units feature is on, values can be entered and saved in any unit. When units are off, values can be entered in any unit, but the values will automatically be displayed on the screen in the default SI units. This is necessary so that when a series of equations are solved, all the values are consistent.

Q.: Why does this screen appear when I solve for a value? What do I enter?



A. This happens when angles or radian frequencies are involved in the equation(s) you are solving. The TI 89 may have found a non-principal solution to your equation, or may have displayed the angle in radians. If a non-principal solution is found, it may then be used to solve other equations, leading to strange results. Example: Imagine solving the equation $x + y = 90°$. If x is 30°, then y should be 60°. But if a non-principal solution for x was found, such as 750°, then the value of y will be -660°, which although technically correct, is also not a principal solution. **Select an arbitrary integer of 0** to compute the principal solution (the principal solution, P, in a periodic trigonometric function, trig(…), is P=trig(θ + n·π) and n is the arbitrary integer).

Q. When solving a set of equations "Too many unknowns to finish solving." is displayed. Why?
A. Sometimes the solver doesn't have enough information to solve for all the remaining, unknown variables. In some cases, a *Partial Solution* set will be displayed. If the unknown value(s) is not calculated, more known values (or selected equations) will need to be selected to compute the solution.

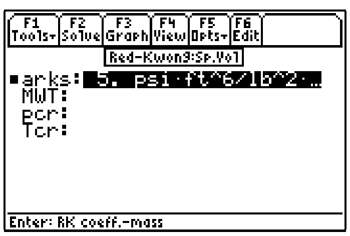
Q. Why can't I select multiple equations under different '**when**' headings?
A. Conditions for each **when (…)** statement is usually exclusive to each other. In *Beams and Columns*, equations for the slope of beam deflection are different depending on whether the deflection at distance x along the beam is calculated to the left side of the load (x<a) or right side of the load (x>a). Since these conditions cannot simultaneously exist, ME•*Pro* restricts selection to equations under a single **when (…)** heading. However, in at least one known case (*Beams and Columns/Simple Beams/Point load*), conditions in more than one **when (…)** statement can occur simultaneously. A work around is to solve the equation set in

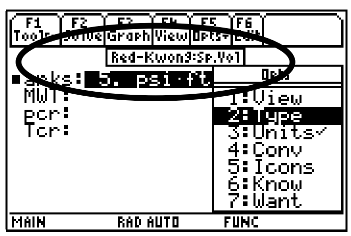
two steps, using equations under a single **when (…)** heading at a time and designating the results from one calculation as the input into the second (to designate a calculated or previously entered value as a known variable for a calculation, move the cursor to the value, press [F5]**:Opts**, [6]**:Know**). A marker '▪' should appear next to the variable.

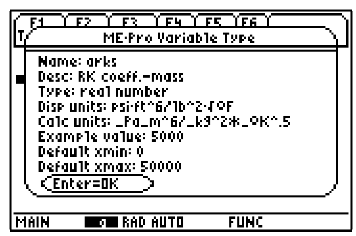
Q. If I view the value of a variable in *Pretty Print*, I notice that the units contain an extra character (such as 'Δ').
A. In a few cases ME•*Pro* and the TI-89 and TI-92 operating systems use slightly different conventions for displaying units. The unit system in ME•*Pro* is designed to conform to the convention established by SI, however, in order to CUT and PASTE a value and units from ME•*Pro* to another area of the TI operating system, ME•*Pro* must insert extra characters in the units to match TI's syntax. This causes extra characters to appear or symbols to appear differently in *Pretty Print.*

Q. The solution to my problem is clearly wrong! An angle might be negative or unreasonably large. Why?
A. This is most likely to happen when angles are involved in the equation(s) you are solving. The TI 89 may have found a non-principal solution to your equation, or may have displayed the angle in radians. If a non-principal solution is found, it may then be used to solve other equations, leading to strange results. Example: Imagine solving the equation $x + y = 90°$. If x is 30°, then y should be 60°. But if a non-principal solution for x was found, such as 750°, then the value of y will be -660°, which although technically correct, is also not a principal solution.

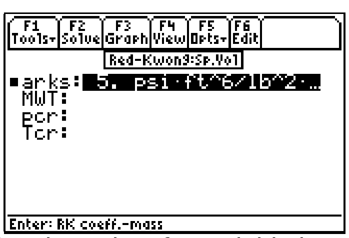
Q. There is not enough room on the screen to display the units for a variable. How can I see the entire unit string?
A. There are two methods to view the units. First, highlight the variable using the cursor bar and press [F5]Opts. Press [2]:Type. This will display the information for the variable including the displayed units.



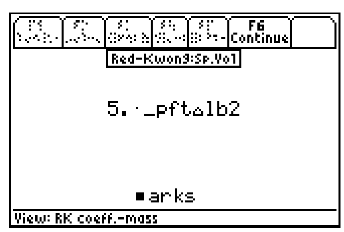




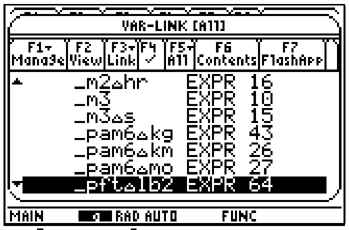
An alternative method that displays the units for the variable in TI syntax is to highlight the variable and press the right arrow key, ⊙. This will display the value with the name of the variable containing the unit string (usually begins with an underscore '_'). Press [VAR-LINK]. Scroll down the list of variables and highlight the name of variable containing the unit string. Press [F6]: Contents. This displays the units in TI syntax.



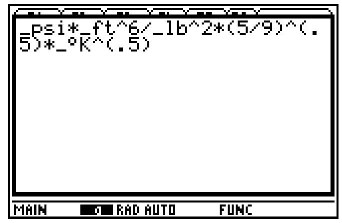
To display the units of a variable in TI syntax, press ⊙ to view the value and unit string.



The unit string usually begins with an underscore '_'.

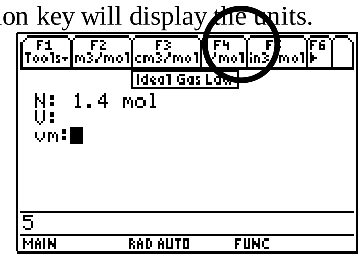


Press [VAR-LINK] and scroll to the unit variable name

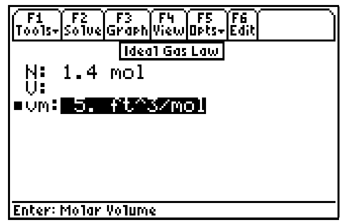


Press [F6] to view the contents.

Q. Why does an arrow '▶' appear on one of the function keys when the unit menu for a variable is being displayed?
A. There are a few unit menus, which are too large to fit on the available space for the unit toolbar. The arrow indicates that there is an additional unit entry, which cannot be shown in th e allotted width. Pressing the function key will display the units.



An arrow ▶ attached to one of the function keys indicates an additional unit for the variable.



Pressing the function key [F6] *appends the units to the entered value.*

Q. I am solving for a variable inside the **erf** or **erfc** functions, a notice appears that the calculation may take a long time?
A. User-defined functions, which are external to the equation set, including the error functions (erf and erfc), use the nsolve function -an iterative solving process. The nsolve function will not generate multiple solutions and the solution which nsolve converges upon may not be unique. It may be possible to find a solution starting from a different initial guess. To specify an initial guess, enter a value for the unknown and then use **F5:Opts/7:Want** to designate it as the variable to solve for.

Q. Why can't I select all of the equations automatically by pressing [F2]?
A. You can however have some equation sets do not form a consistent set, which can be solved together. An example occurs in *Equations/Fluid Mechanics/Fluid Dynamics/Equivalent Diameter* (see **Chapter 24.3.3**), where each equation represents fluid flow through a different-shaped cross-section. In such a case, the equations must be selected explicitly by moving the cursor to each equation and pressing [ENTER]. Special restrictions for a particular equation or group of equations should appear in status line while an equation is being highlighted. In some cases a **'when'** clause preceding the equation(s) state which condition must be fulfilled.

A.5 Graphing

Q. How do I switch between ME•*Pro* and a graph in split screen mode?
A. Press [2nd] [APPS] shift from one screen to the other.

Q. If I have created a graph in full screen mode, how do I return to ME•*Pro*?
A. Press [2nd] [APPS]

Q. How do I clear the split screen after I am finished with graphing?
A. You will need to change the display settings in the MODE screen of the calculator. To do this:

1. Press [MODE]
2. Press [F2]**:Page 2**, move the cursor to **Split Screen**.

3. Press the right arrow key ▶ to display a pop-up menu.
4. Select FULL.
5. Press [ENTER] twice.

A.6 Reference

Q. Can I do any computations in the Reference Section?
A. By the nature of the design of the product, the Reference Section was intended to be non-computing, and table look up only.

Q. Can I convert units in the reference section?
A. The feature of unit conversion has not been implemented in the Reference section.

Q. How can I view an equation or value, which does not fit the width of the screen?
A. Highlight the object and press the right arrow key ▶ to view the object in *Pretty Print.*

Appendix B Warranty, Technical Support

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B.2 How to Contact Customer Support

Customers in the U.S., Canada, Puerto Rico, and the Virgin Islands

For questions that are specific to the purchase, download and installation of ME•*Pro*, or questions regarding the operation of your TI calculator, contact Texas Instruments Customer Support:

phone: **1.800.TI.CARES (1-800-842-2737)**
e-mail: **ti-cares@ti.com**

For questions specific to the use and features of ME•*Pro*, contact da Vinci Technologies Group, Inc.

phone: **1-541-757-8416, 9 AM-3 PM, P.S.T. (Pacific Standard Time), Monday thru Friday (except normal holidays observed in the US).**
e-mail: **support@dvtg.com**

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For questions that are specific to the purchase, download and installation of ME•*Pro*, or questions regarding the operation of your TI calculator, contact Texas Instruments Customer Support:

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Internet: **www.dvtg.com**

Appendix C: TI-89 & TI-92 Plus- Keystroke and Display Differences

C.1 Display Property Differences between the TI-89 and TI-92 Plus

The complete display specifications for both the TI-89 and TI-92 Plus calculators are displayed below.

Table C-1 TI-89 and TI-92 Plus display specifications.

Property	TI-89	TI-92 Plus
Display size		
Pixel	160 x 100	240 x 128
Aspect ratio	1.60	1.88
Full Screen	26 characters/line 10 lines	40 characters/line 13 Lines
Horizontal Split Screen	156 x 39 pixels 25 characters 4 lines	236 x 51 pixels 39 characters, 6 lines
Vertical Split Screen	77 x 80 pixels 12 characters 10 lines	117 x 104 pixels 19 characters, 13 lines
Vertical Split Screen (1/3rd)	Not supported	236 x 33 pixels 39 characters, 4 lines
Vertical Split screen (2/3rd)	Not supported	236 x 69 pixels 39 characters, 8 lines
Horizontal Split Screen (1/3rd)	Not supported	77 x 104 pixels 12 characters, 13 lines
Horizontal Split Screen (2/3rd)	Not supported	157 x 104 pixels 26 characters, 13 lines
Key legends	16 pixel rows	20 pixel row

C.2 Keyboard Differences Between TI-89 and TI-92 Plus

The keystrokes in the manual for ME•*Pro* are written for the TI-89. The equivalent keystrokes for the TI-92 Plus are listed in the following tables.

Table C-2 Keyboard Differences, Representation in Manual

Function	Specific Key	TI-89 Key strokes	TI-92 Plus key strokes	Representation in the manual
Function Keys	F1	[F1]	[F1]	**[F1]**
	F2	[F2]	[F2]	**[F2]**
	F3	[F3]	[F3]	**[F3]**
	F4	[F4]	[F4]	**[F4]**
	F5	[F5]	[F5]	**[F5]**
	F6	[2nd] [F1]	[F6]	**[F6]**
	F7	[2nd] [F2]	[F7]	**[F7]**
	F8	[2nd] [F3]	[F8]	**[F8]**
Trig Functions	Sin	[2nd] [Y]	[SIN]	**[SIN]**
	Cos	[2nd] [Z]	[COS]	**[COS]**
	Tan	[2nd] [T]	[TAN]	**[TAN]**
	Sin^{-1}	[♦] [Y]	[2nd] [SIN]	**[SIN^{-1}]**
	Cos^{-1}	[♦] [Z]	[2nd] [COS]	**[COS^{-1}]**
	Tan^{-1}	[♦] [T]	[2nd] [TAN]	**[TAN^{-1}]**
Alphabet keys	A	[alpha] [=]	[A]	**A**
	B	[alpha] [(]	[B]	**B**
	C	[alpha] [)]	[C]	**C**
	D	[alpha] [,]		**D**
	E	[alpha] [÷]		**E**
	F	[alpha] [|]	[F]	**F**
	G	[alpha] [7]		**G**
	H	[alpha] [8]		**H**
	I	[alpha] [9]		**I**
	J	[alpha] [×]		**J**
	K	[alpha] [EE]		**K**
	L	[alpha] [4]		**L**
	M	[alpha] [5]		**M**
	N	[alpha] [6]	[N]	**N**
	O	[alpha] [-]	[O]	**O**
	P	[alpha] [STO▸]		**P**
	Q	[alpha] [1]		**Q**
	R	[alpha] [2]		**R**
	S	[alpha] [3]	[S]	**S**
	T	[T]		**T**

	U	[alpha] [+]		**U**
	V	[alpha] [0]	[V]	**V**
	W	[alpha] [.]		**W**
	X	[X]	[X]	**X**
	Y	[Y]		**Y**
	Z	[Z]		**Z**
	Space	[alpha] [(-)]		[␣]

Function	Specific Key	TI-89 Key strokes	TI-92 Plus key strokes	Representation in the manual
Log, EXP	LN	[2nd] [X]	[LN]	[LN]
	e^x	[♦] [X]	[2nd] [LN]	[e^x]
Special Characters	π	[2nd] [^]	[2nd] [^]	[π]
	θ	[♦] [^]	[θ]	[θ]
	Negation	[(-)]	[(-)]	[(-)]
	i	[2nd] [CATALOG]	[2nd] I	[*i*]
	∞	[♦] [CATALOG]	[2nd] J	[∞]
Graphing Functions	Y=	[♦] [F1]	[♦] W	[Y=]
	Window	[♦] [F2]	[♦] E	[WINDOW]
	Graph	[♦] [F3]	[♦] R	[GRAPH]
Editing Functions	Cut	[♦] [2nd]		[CUT]
	Copy	[♦] [↑]		[COPY]
	Paste	[♦] [ESC]		[PASTE]
	Delete	[♦] [←]	[♦] [←]	[DEL]
	Quit	[2nd] [ESC]		[QUIT]
	Insert	[2nd] [←]	[2nd] [←]	[INS]
	Recall	[2nd] [STO▸]	[2nd] [STO▸]	[RCL]
	Store	[STO▸]	[STO▸]	[STO▸]
	Backspace	[←]	[←]	[←]
Parenthesis, Brackets	(	[(]	[(]	[(]
	)	[)]	[)]	[)]
	{	[2nd] [(]	[2nd] [(]	[{]
	}	[2nd] [)]	[2nd] [)]	[}]
	[	[2nd] [,]	[2nd] [,]	[[]
	]	[2nd] [÷]	[2nd] [÷]	[]]
Math Operations	Addition	[+]	[+]	[+]
	Subtraction	[−]	[−]	[−]
	Multiplication	[×]	[×]	[×]
	Division	[÷]	[÷]	[÷]
	Raise to power	[^]	[^]	[^]
	Enter	[EE]	[2nd] [1]	[EE]

	Exponent for power of 10			
	Equal	[=]	[=]	[=]
	Integrate	[2nd] [7]	[2nd] [7]	[∫]
	Differentiate	[2nd] [8]	[2nd] [8]	[*d*]

Function	Specific Key	TI-89 Key strokes	TI-92 Plus key strokes	Representation in the manual
Math Operations	Less than	[2nd] [0]	[2nd] [0]	[<]
	Greater than	[2nd] [.]	[2nd] [.]	[>]
cont.	Absolute value	[|]	[2nd] K	[|]
	Angle	[2nd] [EE]		[∠]
	Square root	[2nd] [×]	[2nd] [×]	[√]
	Approximate	[◆] [ENTER]	[◆] [ENTER]	[≈]
Tables	TblSet	[◆] [F4]	[◆] T	[TblSet]
	Table	[◆] [F5]	[◆] Y	[TABLE]
Modifier Keys	2^{nd}	[2nd]	[2nd]	[2nd]
	Diamond	[◆]	[◆]	[◆]
	Shift	↑	[↑]	↑
	Alphabet	[alpha]		[alpha]
	Alphabet lock	[2nd] [alpha]		[a-lock]
Special Areas	Math	[2nd] [5]	[2nd] [5]	[MATH]
	Mem	[2nd] [6]	[2nd] [6]	[MEM]
	Var-Link	[2nd] [−]	[2nd] [−]	[VAR-LINK]
	Units	[2nd] [3]		[UNITS]
	Char	[2nd] [+]	[2nd] [+]	[CHAR]
	Ans	[2nd] [(-)]	[2nd] [(-)]	[ANS]
	Entry	[2nd] [ENTER]	[2nd] [ENTER]	[ENTRY]
Special Characters	Single Quote	[2nd] [=]		[′]
	Double Quote	[2nd] [1]	[2nd] L	["]
	Back slash	[2nd] [2]	[2nd] [=]	[\\]
	Underscore	[◆] [MODE]		[_]
	Colon	[2nd] [4]	[2nd] [θ]	[:]
	Semicolon	[2nd] [9]	[2nd] M	[;]
Number keys	One	[1]	[1]	**1**
	Two	[2]	[2]	**2**
	Three	[3]	[3]	**3**
	Four	[4]	[4]	**4**
	Five	[5]	[5]	**5**
	Six	[6]	[6]	**6**
	Seven	[7]	[7]	**7**
	Eight	[8]	[8]	**8**
	Nine	[9]	[9]	**9**

Function	Specific Key	TI-89 Key strokes	TI-92 Plus key strokes	Representation in the manual
	Zero	[0]	[0]	**0**
	Comma	[,]	[,]	**,**
	Decimal point	[.]	[.]	**.**
Main Functions	Home		[♦] Q	[HOME]
Main Functions cont.	Mode		[MODE]	[MODE]
	Catalog		[2nd] [2]	[CATALOG]
	Clear		[CLEAR]	[CLEAR]
	Custom		[2nd] [3]	[CUSTOM]
	Enter		[ENTER]	[ENTER]
	ON	[ON]	[ON]	[ON]
	OFF	[2nd] [ON]	[2nd] [ON]	[OFF]
	ESCAPE	[ESC]	[ESC]	[ESC]
	Application	[APPS]	[APPS]	[APPS]
Cursor Movement	Top	⊖ (up)	⊙ (up)	⊖ (up)
	Right	⊙ (right)	⊙ (right)	⊙ (right)
	Left	⊙ (left)	⊙ (left)	⊙ (left)
	Bottom	⊖ (down)	⊙ (down)	⊖ (down)

Appendix D Error Messages

- General Error Messages
- Equations Error Messages
- Analysis Error Messages
- Reference Error Messages

D.1 General Error Messages

1. **NOTE:** Make sure the settings in the [MODE] screen **do not** have the following configuration.
   Angle: DEGREE
   Complex Format: POLAR
   ME•*Pro* works best in the default mode settings of your calculator (ie. **Complex Format: REAL**, or **Angle: RADIAN**). If one set of error messages appears which includes **"An error has occurred while converting....", Data Error"**, **"Domain Error"**, and/or **"Internal Error",** check to see if the above settings in the [MODE] screen exists. If it does, change or reset your calculator to the default mode settings ([2nd] [MEM] [F1] [1] [2] [ENTER]).

2. "**Syntax Error**" -- occurs if the entered information does not meet the syntax requirements of the expected entry. Check to make sure extra parenthesis are removed and the entered value meets the legal rules for number entry.

3. **"Invalid Entry…"**: This occurs when the entered value or units do not match the specified format of the variable. Check to make sure the entry can be reduced to a numeric value, which is within the range of the allowed values for the variable (i.e.: greater than zero, between zero and one, real, etc.). If a value with units is being copied from one section and pasted in another, using the [F1]**: Tools,** [5]**:Copy (or** [4]**:Cut)** [6]**:Paste** feature, this can only work if the unit feature is deactivated. To turn off the unit feature, press [F5]**:Opts. /**[3]**:Units**.

4. "**Insufficient Table Space**" or "**Insufficient Memory**" can occur when the system is low on available memory resources. Consult your TI-89 manual on methods of viewing memory status and procedures for deleting variables and folders to make more memory available.

5. The message "**Unable to save ME•Pro data**" will be displayed if ME•*Pro* is unable to save information of its last location in the program before exiting due to low memory availability. Consult your TI-89 manual under the index heading: *Memory-manage*.

6. "**The variable prodata1 was not created by ME•Pro...**" ME•*Pro* uses a variable called "prodata1" to recall its last location in the program when it is re-accessed. If this variable list is changed to a format, which is non-recognizable to ME•*Pro*, it displays this message before overwriting.

7. "**Data length exceeds buffer size. The variable name will be displayed instead. The variable's value may be viewed with VAR-LINK using** [F6] **or recalled to the status line of the HOME screen."**

8. "**An error has occurred while converting this variable's data for display**. (The name of the variable is in the title of this dialog box.) **There may be something stored in the variable that ME•Pro can't make sense of. You may be able to correct the problem by deleting the variable**."

9. **“Storage error...”** This message is set to occur if the user attempts to enter a value into a variable, which is locked or archived, or a memory error has occurred. Check the current status of the variable by pressing [VAR-LINK] and scrolling to the variable name, or check the memory parameters by pressing [MEM].

10. **“Invalid variable reference. Conflict with system variable or reserved name.”** This can occur if a variable name is entered which is reserved by the TI operating system. A list of reserved variable names is included in **Appendix F**.

D.2 Analysis Error Messages

EE for MEs

1. **“No impedances/admittances defined” This** message is displayed if an entry for impedance **ZZ_** or **YY_** admittance is not a real, complex number or a defined variable in the **Voltage Divider** and **Current Divider** sections of **AC Circuits**.

Capital Budgeting

1. When changing the name of a project, the error **"Duplicate project variable name."** will occur if the entered name is already in use by an existing project.
2. The **"Too few cash flows"** message will be displayed if an attempt is made to solve a project which contains too few or no cash flow entries.
3. The message **"Data Error. Reinitializing project."** will occur if the project data has been altered or is inconsistent with current state. ME•*Pro* will restart.

D.3 Equation Messages

If a value is entered that is inconsistent with the expected data type, an error dialog will appear which lists the entry name, the description, and the expected data type(s), and the expected units.

If an error occurs during a computation that involves temperature, **"temp conversion err"** or **"deg/watt conversion err"** will be displayed.

When solving equation sets, several messages can be displayed. These messages include:

- **“One or more equations has no unknowns.....”** This message occurs if one or more of the selected equations in a solution set have all of its variables defined by the user. This can be remedied by pressing [ESC], deselecting the equation(s) where all of the variables are defined and resolving the solution set by pressing [F2] twice. To determine which equation has all of its variables defined, press [ESC] to view the equations, select an equation in question by highlighting the equation and pressing [ENTER], and pressing [F2] to view the list of variables. A ‘▪‘ next to a variable indicates a value has been specified for that variable by the user. If all of the variables in an equation are marked with a ‘▪‘, no unknown variables exist for that equation. This equation should not be included in the solution set. Press [ESC] to view the list of equations. Select the equations to be solved, *excluding the equation with no unknowns*, and press [F2] twice to resolve the set of equations.
- **"Unable to find a solution in the time allowed. Examine variables meinput and meprob to see the exact statement of the problem. ME•*Pro* sets Exact/Approx mode to AUTO during solve.”**
- **“No equations have been selected. Please select either a single equation to solve by itself, or several equations to solve simultaneously.”** This can occur when the set contains equations, which are mutually exclusive (i.e.: cannot be solved altogether) and must be explicitly selected. In such a case, the equations must be selected explicitly by moving the cursor to each equation and pressing [ENTER]. Special restrictions for a particular equation or group of equations should appear in status line while an equation is being highlighted. In some cases a **‘when’** clause preceding the equation(s) states which condition must be fulfilled before using an equation.

- **"Too many unknowns to finish solving"-**generally occurs if the number of equations is less than the number of unknowns
- **"It may take a long time to find a complete solution, if one can be found at all. You may abort the calculation at any time by pressing the ON key."** -this occurs if there are many unknowns or multiple solutions.
- **"No input values provided...."** occurs if none of the variables have values designated when solving an equation set.
- **"The nsolve command will be used. The existing value for the unknown, if any, will be used as an initial guess."** The *nsolve* function is used when a single unknown exists in the equation and the unknown variable is an input in a user-defined function. The nsolve function will not generate multiple solutions and the solution which nsolve converges upon may not be unique. It may be possible to find a solution starting from a different initial guess. To specify an initial guess, enter a value for the unknown and then use [F5]**:Opts/**[7]**: Want** to designate it as the variable to solve for. More information on the differences between *solve*, *nsolve* and *csolve* functions is listed in the TI-89 manual.
- **"One complete useable solution found."** All of the unknown variables can be solved in the selected equations.
- **"One partial useable solution found."** Only some of the variables in the selected equations could be solved.
- **"Multiple complete useable solns found."** One or more variables in the selected equations have two possible values
- **"Multiple partial useable solns found."** One or more variables in the selected equations have two possible values, however not all of the unknown variables could be solved.
- **"No Apparent solution found."** A solution could not be computed with the available inputs. Make sure the known variables compute accepted ranges of the target value (e.g. a negative electrical resistance is not being computed). Press [F5]**:Opts./**[2]**:Type** to view the acceptable ranges of a particular variable. If a large number of equations are being used in a solving routine, you might try reducing the number of equations being solved at one time. Example, instead of solving 10 equations for 10 unknown variables, solve the first five equations and use the computed results and known variables as inputs to compute the second five variables in the remaining five equations.

The following messages can appear when attempting to graph equation set functions:

- **"Independent and dependent variables are the same."**
- **"Unable to define Pro (x)"-**cannot resolve the dependent and independent variables. This may occur when graphing a complex equation or an equation, which contains user-defined functions such as **erf (…)** and **erfc (…)**.
- **"Undefined variable" too** many dependent variables or dependent variable unable to be defined in terms of the independent variable.
- **"Error while graphing."**

D.4 Reference Error Messages

No error messages specific to this section have been documented.

Appendix E: System Variables and Reserved Names

The TI-89 and TI-92 Plus has a number of variable names that are reserved for the Operating System. The table below lists all the reserved names that are not allowed for use as variables or algebraic names.

Graph	y1(x)-y99(x)* xt(t)-xt99(t)* ui1-ui99* tc xfact xmax ymax Δx zsc1 ncontour tmin tplot Estep nmax	y1'(t)-y99'(t)* yt(1)-y99(t)* xc rc yfact xsc1 ysc1 Δy eye0 θmin tmax ncurves fldpic plotStrt	Yi1-yi99* z1(x,y)-z99(x,y) yc θc zfact xrid ygrid zmin eyeϕ θmax tstep difto1 fldres plotStep	r1(θ)-r99(θ)* u1(n)-u99(n)* zc nc xmin ymin xres zmax eyeφ θstep t0 dtime nmin sysMath
Graph Zoom	Zxmin Zymin Zxres Ztmin Ztmaxde Zmax zeyeφ zpltstep	Zxmax Zymax zθmin ztmax ztstepde zzsc1 znmin	Zxscl Zyscl zθmax ztstep ztplotde zeyeθ znmax	Zxgrid Zygrid zθstep zt0de zzmin zeyeϕ zpltstrt
Statistics	$\bar{x}$ Σx^2 Σy^2 medStat medyl minY regCoef Sx	$\bar{y}$ Σxy corr medx1 medy2 nStat regEq(x)* Sy	Σx Σy maxX medx2 medy3 q1 seedl R^2	σx σy maxY medx3 minX q3 seed2
Table	tblStart	Δtbl	tblInput	
Data/Matrix	C1-c99	SysData		
Miscellaneous	Main	Ok	Errornum	
Solver	Eqn*	Exp*		

Index

#

[VAR-LINK] 391

1

1 ϕ Induction Motor 96
1 Fixed End / 1 Free End..............................252
1 Section - Even 46
1 Section - Uneven.. 45
1D Collision 349
1D Collisions...351
1D Heat Transfer 122
1st radiation constant 372

2

2D Collision 350
2nd radiation constant 372

A

About 21
AC Circuits
  circuit performance 33
  impedance calculations 32
AC Motors 92
Add 21
advanced user 63
Affinity Law-Constant Speed........................226
Affinity Law-Variable Speed 225
AGMA Stresses 201
Air Properties 23
Analysis
  finding 18
Analysis Error Messages..............................405
Analysis Screens... 18
Angular Harmonic Oscillation.......................232
Angular Motion 343
apogee 356
Apparent Power 34
area of cross-section...................................... 42
armature resistance 86
ASME Weirs 39
axial 274
Axial Load 270
Axial Loaded 217

B

back emf 88
bars in torsion 285
Basic Definitions 222
Basic Electricity.. 83
Basic Transfer Mechanisms......................... 118
Beams and Columns...................................... 64
Bearing Life 197
Bearings.. 197
Bending 208
Bernoulli Equation 297
BHN 52
blackbody 121
Blackbody Radiation 146
Both Ends Fixed.. 251
Both Ends Free 254
Brinell’s method 52
Buckling 75
buffer size 404

C

C Section 47
Cantilever Beams 70
Capilllary Rise 289
Capital Budgeting 28
Cash 21
Catenary cable 365
center of mass 42
Centrifugal Compressor............................... 228
Centrifugal Pumps 225
centripetal acceleration................................ 354
CF_t 28
Charles’s Law 99
choosing equations 62
Cipolletti 40
Circle 43
circuit performance 33
Circular Cross Section 220
Circular Flat Plate 255
Circular Motion 342
Circular Ring 44
Circular Shafts 284
Circular Straight Bar 215
Clear 21
Clutches 204
Clutches and Brakes 204
coefficient of performance........................... 261
coefficient of resitution 351
Coiled Springs 212
cold start 390
Collisions 349
Columns 75
Compressible Flow 327
Compressible Fluid 292
Compression Cycle 264

compression ratio....263
Compute Hardness Number 52
conditional constraints 58
Conduction....118, 122
Conical Circular Section 217
Conical Vessel.... 37
conservation of energy....349
conservation of momentum....330
Constant Acceleration....340
Constant Liquid Level.... 36
Constant Pressure 99
Constant Surface-Heat Flux 143
Constant Temperature....103
Constant Volume....101
Conv 21
Convection 120
convective coefficient....123
Convective Source....123
Convolution....374
Copy 21
Copy/Paste 59
Coupling or union....376
Crane Hook 192
Critical Damping (ξ=1)....239
Critical magnetic fields....385
critical pressure....109, 112
critical temperature....108, 109
csolve.... 61
Cylinder with Insulation Wrap 137
Cylinder-Critical radius....139
Cylinders....188
Cylindrical Helical-Circular wire 212
Cylindrical-Helical 219

D

da Vinci website 391
Damped Forced Vibration....245
damping ratio 238
Darcy-Weisbach equation 313
DC Generators.... 90
DC Motors 86
DC Series Generator 90
DC Series Motor.... 86
DC Shunt Generator.... 91
DC Shunt Motor 88
Definitions....374
deflection angle 247
Delete.... 21
design in the number of revolutions....197
design life....197
design load 197
Differences of 89 & 92Plus.... 13
Dirac constant....372
discharge coefficient.... 36
discharge rate....225
Discount Rate.... 30
Dynamic Load 272
Dynamic Viscosity.... 379
Dynamics and Statics 338

E

eccentricity 356
eccentricity ratio 77
Eccentricity, Axial Load.... 76
EE for MEs, Analysis.... 32
EE for MEs, Equations.... 83
Efflux 36, 61
Elastic Collisions 349
Elasticity.... 288
Electrical Analogy.... 128
ellipsis 391
emissivity 125
Engineering Constants.... 372
Equation Messages.... 405
Equations Questions.... 392
Equivalent Diameter.... 300
Error Messages 404
Euler constant 372
examples
  capital budgeting.... 29
  Constants reference table.... 373
external periodic force.... 244
external resistance.... 91

F

Faraday constant 372
field coil resistance.... 91
field constant 86
Field current.... 88
finding
  Analysis.... 18
  Reference.... 369
first kind, radiative transfer.... 148
Flame Temperature 383
Flammable Limits in Air 383
FlashApps.... 13
Flat Plates 255
Floating Bodies 294
Flow in Conduits.... 311
Flow pipe Inlet.... 319
Flow Pipe Inlet.... 321
Fluid Dynamics.... 297
Fluid Mass Acceleration.... 302
Fluid Mechanics.... 288
Fluid Properties.... 288
Fluid Statics 291
Fluid vapor pressure.... 222
flux.... 86
Forced Vibrations.... 244
Forces in Angular Motion.... 345
Fourier Transforms 374, 375
Free Fall 341

Free Vibration 236
Frequently asked Questions...........................390
Friction...360
Friction Coefficients 377
Friction Coefficients Screens 377
Frictional Force 360
Frictions Coefficients Reference 377
Fuels and Combustion..................................383
Fuels and Combustion Screens......................383
function 14

G

Gas Laws.. 98
Gases and Vapors 380
Gases and Vapors Screens............................381
Gay-Lussac's Law 101
General Cycle..261
General Questions..390
Graph 21
Graphing a Funtion 59
Gravitational Effects 354
Greek Alphabet..388
guessed value 61
Gyroscope Motion 346

H

Hardness Number 52
harmonic motion..233
Heat of Fusion 382
heat pump..261
Heat Transfer...118
heat transfer coefficient................................259
Heating Load...259
help...390
Hertzian Stresses 193
Hollow Circle 45
Hollow Cylinder 135
Hollow Polygon.. 50
Hollow Rectangle 43
Hollow Sphere..136
homologous pump 226
Hook's law 278
Horizontal Cylinder 38
Horizontal Plate...332
How to Contact Customer Support................398
How to use this Manual 14

I

Icons 21
ideal blackbody..146
Ideal Gas Laws 98
impedance calculations 32
impeller diameter...226
Imperfections in Columns 79
Impulse/Momentum.....................................330
inclined plane 309
Inclined Plane/Surface................................. 296
Incompressible.. 324
incompressible fluid 311
Inelastic Buckling 81
Insufficient Table Space 404
Insulated pipe.. 137
Interfacial temperature 138
Internal Enegry/Ethalpy............................... 104
internal rate of return............................... 17, 28
International Conventions............................... 23
Introduction to Analysis 17
Introduction to Equations 55
Introduction to Reference 369
Introductions to ME•*Pro* 12
Invalid Entry 404
Inverse Fourier transform 374
isentropic 113

J

Jet Propulsion 330

K

Kepler's Laws 356
Kepler's second principle 356
Key Features 12
Keystroke and Display Differences.............. 399
Kinematic Viscosity 379
Kinetic Gas Theory 106

L

Laminar Flow
  Smooth Pipe.. 311
Laminar Flow – Flat Plate 304
Laminar Flow on an Inclined Plane 309
Laplace Transforms............................. 374, 375
Large Rectangular Orifice 38
Law of Gravitation 354
Laws of Motion... 338
Lewis Formula 200
License Agreement 397
Linear Acceleration..................................... 302
Linear Harmonic Oscillation 231
Linear Motion 340
Liquid Petro Gas Spees 383
Load Admittance... 33
Load Problems 270
Loss Coefficients 319

M

Machine Design 188
Machine Elements....................................... 188
magnetic flux 86
Manual Disclaimer 14
Maximum Power.. 34
Maximum Shear Stress................................ 277
Maxwell distribution 106

mean roughness 316
Mean Specific Heat 382
mean velocity 313
mechanical angular speed 92
Mechanical power 93
Memory Requirements 13
minor elliptical axis 356
Modulation 374
Mohr's circle radius 280
Mohr's Circle Stress 280
molar volume 98
Moment Load 68, 73
Moving Blade 335
Multiple Graphs 30

N

Napier constant 372
Natural and Forced Vibrations 236
Natural Frequencies 247
Natural Vibrations 236
net present value 17, 28
New 21
No apparent solution found 406
Non-Blackbody radiation 148
normal force 274
Normal Stress and Strain 267
nsolve 61

O

Oblique Collisions 352
Ohm 128
Ohm's Law and Power 84
One partial useable solution found 406
Open 21
Open angle valve 376
Open gate valve 376
Open globe valve 376
Open Jet 331
Ordering a Manual 13
Overdamped Case (ξ>1) 238

P

Parabolic cable 364
Parallel 32
Parallel Pipe System 322
Parallel Series 132
Partial Solutions 59
paste 21
payback period 17, 28
Pendulums 233
perfectly plastic impact 272
perigee 356
perimeter 259
Periodic solutions 58
period of oscillation 232
Petroff's law 198
Phase voltage 93
Physical Pendulum 235
Pict 21
Planck constant 372
Plane Stress 278
Plane Wall 122
Plate and Two Fluids 127
PocketProfessional 12
Point Load 66, 71
Poisson's ratio 193
polar area moment 42
Polygon 49
polytropic 115
Polytropic Porcess 115
Power Factor 34
Power in rotor 93
Pressure and Shrink Fits 190
Pressure Fed Bearings 199
Pressure Variation 291, 292
principal axis stress 280
principal shear stress 280
principal solution 392
Principal Stresses 276
Printed manual 13
Pro (x) 406
Pro software 12
profitability index 17, 28
Project 30
Projectile Motion 347
Properties 374
Psi Function 380
pump performance 225
Pump Power 223
pump seed 225
Pump Similarity 227
Pumps and Hydraulics 222
Purchasing, Downloading 13
Pure Shear 275, 283
Pure Torsion 281

Q

questions and answers 390

R

Radial Systems 135
Radiation 121, 146
Radiative Source 125
radiative transfer 121
radius of gyration 42, 77
rated life 197
rated number of revolutions 197
rating load 197
Reactive Power 33
Real Gas Laws 108
Real Power 33
Rectangle 42

Rectangular Cross Section 219
Rectangular Flat Plate..................................257
Rectangular Notch 39
Rectangular Plate..208
Rectangular Spiral 213
Rectangular Straight Bar..............................216
Redlich-Kwong
  Molar...112
  Sp. Vol 110
Redlich-Kwong coefficients..........................110
Reference 396
  finding...369
  using reference tables...............................370
Reference Screens..370
Refrigerants...385
Refrigerants Screens 386
Refrigeration 261
Refrigeration and Air Conditioning 259
refrigeration cycle 261
Relative Roughness of Pipes 378
Relative Roughness Screens..........................378
Reserved Variables 22
Resistance Formulas 83
restoring torque..232
Restrictions..397
Restricted Rights 397
Reverse Adiabatic...113
Reverse Brayton 263
Reverse Carnot 262
reversible adiabatic process..........................113
Reynolds Number 299
RMS velocity 107
Rolling/Rotation 343
Rotating Cylinder 363
rotating magnetic field 92
Rotating Rings..189
Rotational Acceleration................................303
rotational moment of inertia 232
rotor 92
Rotor current 93
Rydberg constant 372

S

Satellite Orbit 358
Saturated Steam Properties............................ 23
screens
  Analysis.. 18
Secant Formula.. 77
second kind, heat transfer.............................148
Section Properties... 42
Semi Elliptical 210
Semi-Infinite Solid 142
Series 32
Series Pipe System.......................................321
Session Folders... 22
Shafts 203
Shape factors 119
shear force 274
Shear Stress and Modulus............................ 269
shear stress coefficient................................. 306
SI Prefixes 387
Simple Beams 64
Simple Harmonic Motion 231
Simple Pendulum 233
Simply Supported....................................... 250
slenderness ratio.. 77
solve 61
Solve 21
Solving Equations Sets 55
Sommerfeld 198
Source Admittance 33
Specific Speed 229
Sphere-Critical radius.................................. 141
Split Screen... 395
Spring Design 208
Spring stiffness 231
Starting ME•*Pro* 13
static elongation 272
Statics 364
Stationary Blade.. 333
stator... 92
Statosphere 293
status line.. 390
steady state oscillation................................. 244
Steam Tables 23
Stefan-Boltzmann constant........................... 372
Stefan-Boltzmann law 146
Storing and recalling variable 61
Strength Materials.. 267
Stress Analysis 274
Stress and Strain Basics 267
Stress on an inclined Section 274
Stretched String 247
Summary 15
Superheated Steam Properties......................... 23
Suppressed Weir 40
Surface Convection 145
Surface Resistance 304
Surface Temperature 142
Swing check valve 376
Syntax Error.. 404
System Variables and Reserved Names 407

T

T Section 48
Technical Support 397
Temperature Effect 85, 271
terminal voltage 88
thermal conductivity.................................... 118
thermal expansion coefficient 271
Thermal Properties...................................... 382
Thermal Radiation Shield 149

thermal resistance......................... 122, 128, 259
Thermocouples.. 26
Three ϕ Induction Motor I.............................. 92
Three ϕ Induction Motor II............................. 94
Torsion...281
Torsional Member 285
Torsional Pendulum.....................................236
Torsional Spring..215
Transfoms 374
Transform Pairs...374
Transitions temperature................................385
Trapezoid 48
Triangular Plate...209
Triangular Weir 39
Triple pt. pressure..385
Troposhpere 292
troubleshooting..390
Turbulent Flow
  Rough Pipe...316
  Smooth Pipe 313
Turbulent Flow – Flat Plate..........................306
Two Conductors in Parallel..........................131
Two Conductors in Series 129
Two Cylinders..195
Two Spheres..193

U

undamped oscillating...................................236
Undefined variable.....................................406
Underdamped Case (ξ<1).............................241
Underdamped Forced Vibration 244
underscore...390
Uniform Beams 249
Uniform Fluid..291
Uniform Load...64, 70
Uniform Pressure - Cone Break.....................207
uniform thermal strain..................................271
Uniform Wear - Cone Brake 206
Units 21
Universal Gas Constant................................288
Upper inversion temperature.........................385
User-defined functions.................................. 61
Using Constants...372
Using SI Prefixes 387
Using Transforms....................................... 374

V

Validity Range.. 25
Valves and Fitting Loss 376
Valves and Fitting Loss Screens 376
van der Waals
  Molar form.. 109
  Specific Volume...................................... 108
Variable
  Overwriting... 22
Variable Names 22
Varying Liquid Level.................................... 36
Venturi Meter 324
Vertical Plate 331
VHN.. 52
Vibration Isolation 248
vibration mode 250
Vickers method... 52
View.. 21
Viewing in *Pretty Print*................................. 56
Viewing Multiple Solutions........................... 57
Viewing Results in different units.................. 56
Volume Dilation 268
volumetric flow rate 332

W

Want... 21
Warranty... 397
Water Physical Properties............................ 379
Water-Physical Properties Screens............... 379
Waves and Oscillation................................. 231
Web Store... 13
Wedge 362
when (…).. 393
Wien displacement...................................... 372
Wye ↔ Δ Conversion 34
Wye Imedances... 34

Y

Young’s modulus.. 250

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