Neutron Matter as a Composite Bose-Fermi Superfluid

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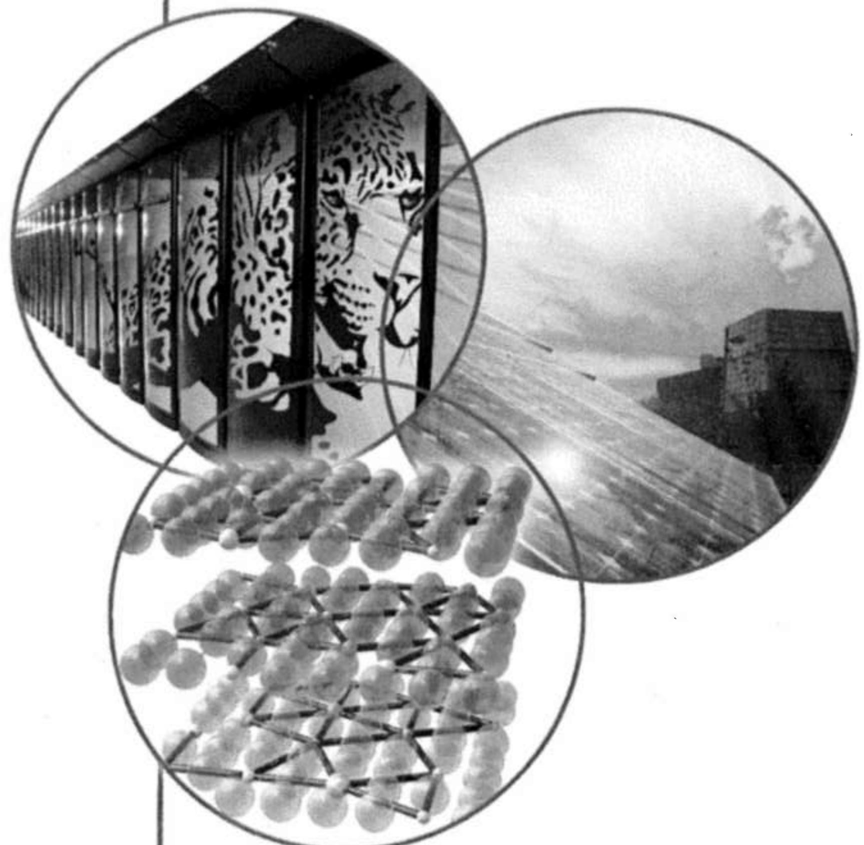
APS DNP Fall Meeting
East Lansing, MI
October 26-29, 2011



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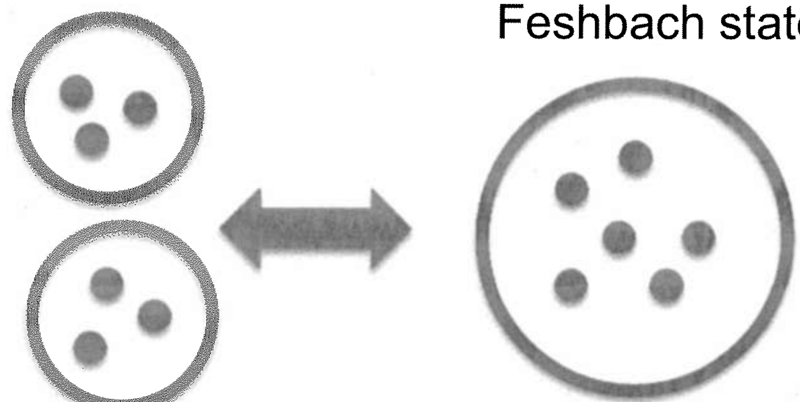
Overview

- **The proposed Hamiltonian model combines:**

  1. **An NN interaction based on three- and six- quark clusters**
     - Nucleon = cluster of three quarks,
     - Two nucleons couple to a cluster of six-quarks
     - The six-quark cluster short-lived non-resonant Feshbach state(s)
     - Feshbach state energy and form factor are found from the NN phase shifts
     - A single Feshbach state may be sufficient to account for $E_{lab} < 350$ MeV



Feshbach state (six quark cluster)

  2. **Many-body model of composite Bose-Fermi superfluid leverages the atomic physics model for the same.**
     - Two-body interaction by coupling nucleons to Feshbach state(s)

1) NN interaction via Feshbach State(s)

- **General Feshbach projection operator method yields:**

$$\tan\delta(k) = -kK_{PP}(k)/(4\pi)$$

$$K_{PP}(k) = H_{PQ}\frac{1}{E - H_{QQ} - \mathrm{P.V.}H_{QP}\frac{1}{E-H_{PP}}H_{QP}}H_{QP}$$

- **Allows for any number of Feshbach States in *Q*-space**
- **For a single Feshbach state:** $H_{QQ} = k_1^2$ **in units** $\hbar^2/m$
  - **Phase shift changes sign at *k_0***

$$H_{PQ} = \sqrt{4\pi}f(k)$$

F. Tabakin, Phys. Rev. C **174**, 1208 (1968);
Kukulin et al., Phys. of Atom. Nuclei, **64**, 1748, (2001)

$$f(q) = (q^2 - k_0^2)h(q)$$

$$h(q) = ge^{-q^2/s^2}$$

Phase-shift for a single Feshbach state

- **Phase-shift changes sign k_0:**

$$\tan\delta(k) = -k\frac{f^2(k)}{k^2 - k_1^2 + \frac{2}{\pi}\int_0^\infty \frac{f^2(q)}{q^2-k^2}q^2dq},$$

$$\tan\delta(k) = -k\frac{(k^2 - k_0^2)h^2(k)}{1 + \frac{2}{\pi}\int_0^\infty \frac{q^2-k_0^2}{q^2-k^2}h^2(q)q^2dq},$$

$$k_1^2 = k_0^2 + \frac{2}{\pi}\int_0^\infty (q^2 - k_0^2)h^2(q)q^2dq$$

- **The energy of the Feshbach state $(k_1)^2$ is uniquely determined by k_0 and $h(q)$ in this model.**

Fitting Low-Energy $^1S_0$ *nn* Data

- **Effective range expansion of tan(δ) yields expressions for the scattering length and the effective range that are equated to experimental values**

$$
\begin{aligned}
a &= -18.9 \text{ fm} \\
r_0 &= 2.75 \text{ fm}
\end{aligned}
$$

Form-factor parameters are found using Mathematica:

$$
\begin{aligned}
k_1 &= 1.62099 \text{ fm}^{-1} \\
g &= 0.812091 \text{ fm}^{3/2} \\
s &= 1.44185 \text{ fm}^{-1}
\end{aligned}
$$

for a form factor: $f(q) = (q^2 - k_0^2)h(q)$

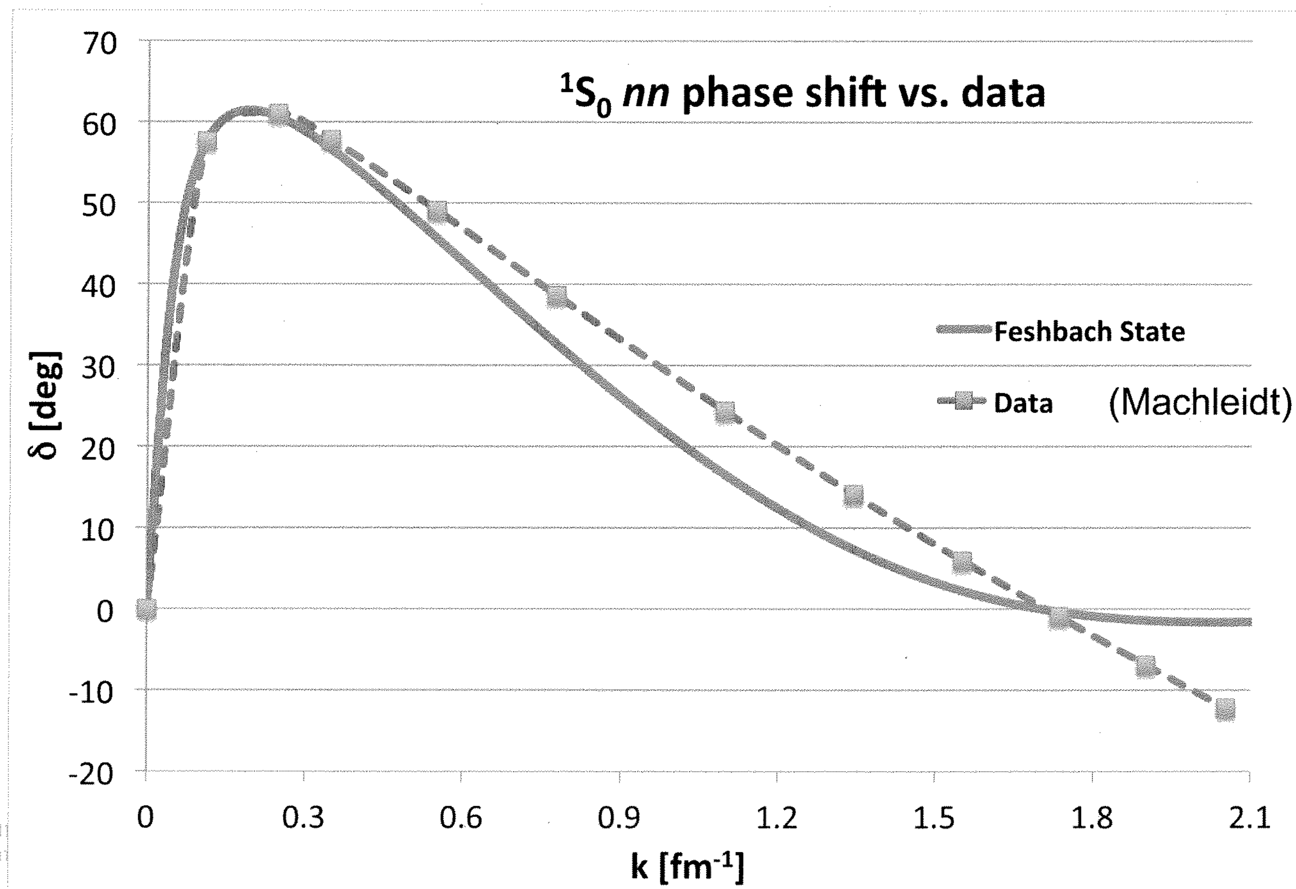
$$h(q) = g e^{-q^2/s^2}$$

(The same method was used to find parameters that yield infinite scattering length...)

Comparison to $^1S_0$ *nn* phase shift data

- Good fit at low energies *and* it crosses zero at E_{lab} = 240 MeV
  - An improvement over fitting just (a, r_0)
    - suitable for higher densities in a many-body problem
  - Additional parameters could improve the fit at other energies

Schwenk and Pethick, PRL 95, 160401 (2005)



2) Neutron matter Grand Canon. Hamilton.

- **Two neutrons couple to Feshbach state(s)**
  - **Simplifies the computation of the interaction (quartic $\longrightarrow$ cubic)**

$$
\begin{aligned}
\hat{K} &= \hat{H} - \mu \hat{N} \\
&= \sum_\sigma \int_{\mathbf{q}} (k^2 - 2\mu) \hat{\psi}^\dagger_\sigma(\mathbf{q}) \hat{\psi}_\sigma(\mathbf{q}) + 2(\epsilon - 2\mu) \int_{\mathbf{q}} \hat{\chi}^\dagger(\mathbf{q}) \hat{\chi}(\mathbf{q}) \\
&\quad + \frac{1}{\sqrt{2}} \int_{\mathbf{q}''} \int_{\mathbf{q}} \int_{\mathbf{q}'} \hat{\chi}^\dagger(\mathbf{q}'') F(\mathbf{q}''; \mathbf{q}, \mathbf{q}') [\hat{\psi}_\uparrow(\mathbf{q}) \hat{\psi}_\downarrow(-\mathbf{q}') - \hat{\psi}_\downarrow(-\mathbf{q}) \hat{\psi}_\uparrow(\mathbf{q}')] + \text{H.c.},
\end{aligned}
$$

A two-body form-factor

where

$$
F(\mathbf{q}''; \mathbf{q}, \mathbf{q}') = 2\sqrt{4\pi} f\left(\tfrac{\mathbf{q}+\mathbf{q}'}{2}\right) \delta(\mathbf{q}'' - (\mathbf{q} - \mathbf{q}'))
$$

- **A mean molecular field approx.** $\langle 0 | \hat{\chi}(\mathbf{q}) | 0 \rangle = \delta(\mathbf{q}) \frac{\chi}{\sqrt{2}}$ **(cubic $\longrightarrow$ quadratic):**

$$
\begin{aligned}
\hat{K} &= \sum_\sigma \int_{\mathbf{q}} (k^2 - \mu) \hat{\psi}^\dagger_\sigma(\mathbf{q}) \hat{\psi}_\sigma(\mathbf{q}) - (2k_1^2 - 2\mu) \chi^2 \\
&\quad + 2\sqrt{4\pi} \chi \int_{\mathbf{q}} f(\mathbf{q}) \hat{\psi}_\uparrow(\mathbf{q}) \hat{\psi}_\downarrow(-\mathbf{q}) + \text{H.c.}.
\end{aligned}
$$

The First Law of Thermodynamics

- **Use it to relate pressure, energy density, and particle density**

$$dE = -PdV + \mu dN$$

$$P = -\mathcal{K}/V = -u + \mu\rho$$

$$e = \frac{u}{\rho} = \mu - \frac{P}{\rho}$$

Neutron matter gap function and EOS

- Application of the Bogoliubov transform technique and the First Law of thermodynamics yields:

$$P = 2\int_{\mathbf{q}} E(q)v^2(q) - 2(k_1^2-\mu)\chi^2$$

$$\rho = 2\int_{\mathbf{q}} v^2(q) + 2\chi^2$$

- Two fermions per boson
- Energy units in the lab frame $\hbar^2/2m$

where

$$E(q) = \sqrt{(q^2-\mu)^2 + 16\pi\chi^2 f^2(q)}$$

$$v^2(q) = \frac{1}{2}\left[1 - \frac{q^2-\mu}{E(q)}\right],$$

- Extremizing pressure w.r.t. mean field yields a gap equation

Equation of State

- Extremizing pressure w.r.t. the mean field yields a gap eq.:

$$\frac{\partial P}{\partial \chi} = 0 \Rightarrow \qquad \mu - k_1^2 + \frac{2}{\pi}\int_0^\infty \frac{f^2(q)}{E(q)} q^2 dq = 0$$

- For a given μ this integral equation is solved for χ
- the energy per particle is $\quad e = u/\rho = \mu - P/\rho$

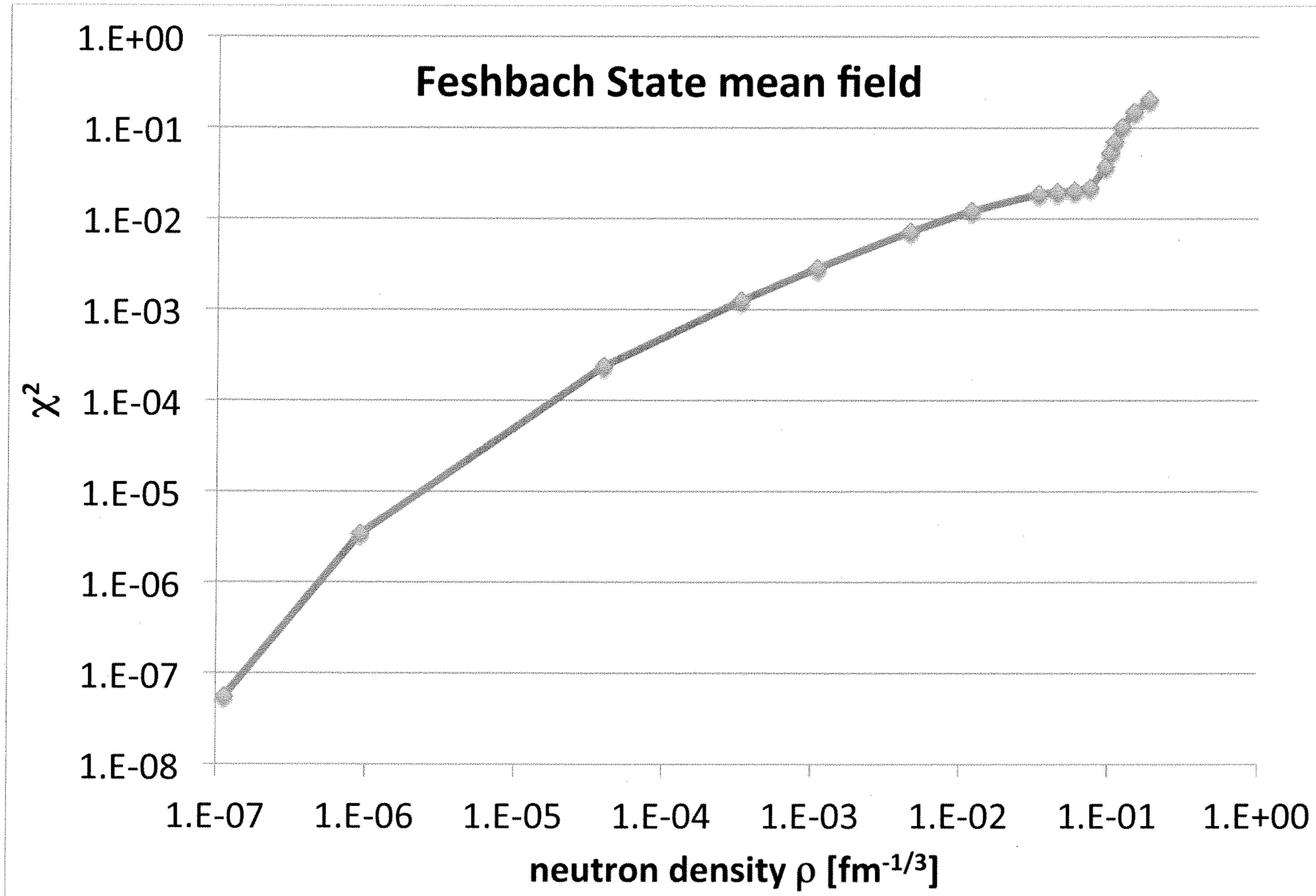
  that is plotted in units of energy per particle of a Fermi gas:

$$k_F^2 = (3\pi^2\rho)^{2/3} \neq \mu$$

$$e_{FG} = \frac{3}{5}k_F^2$$

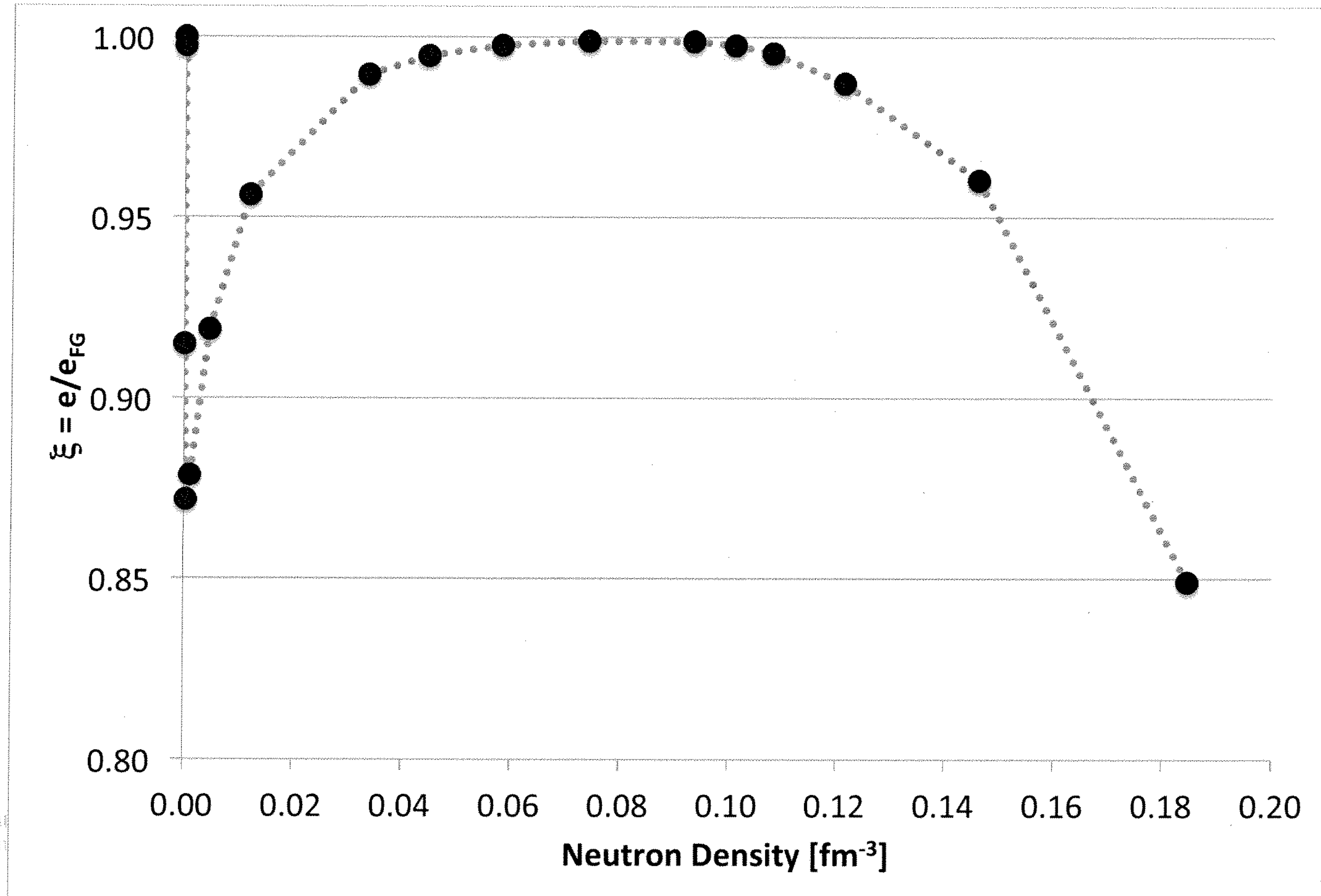
Mean field as a function of neutron density

- A solution to the gap equation



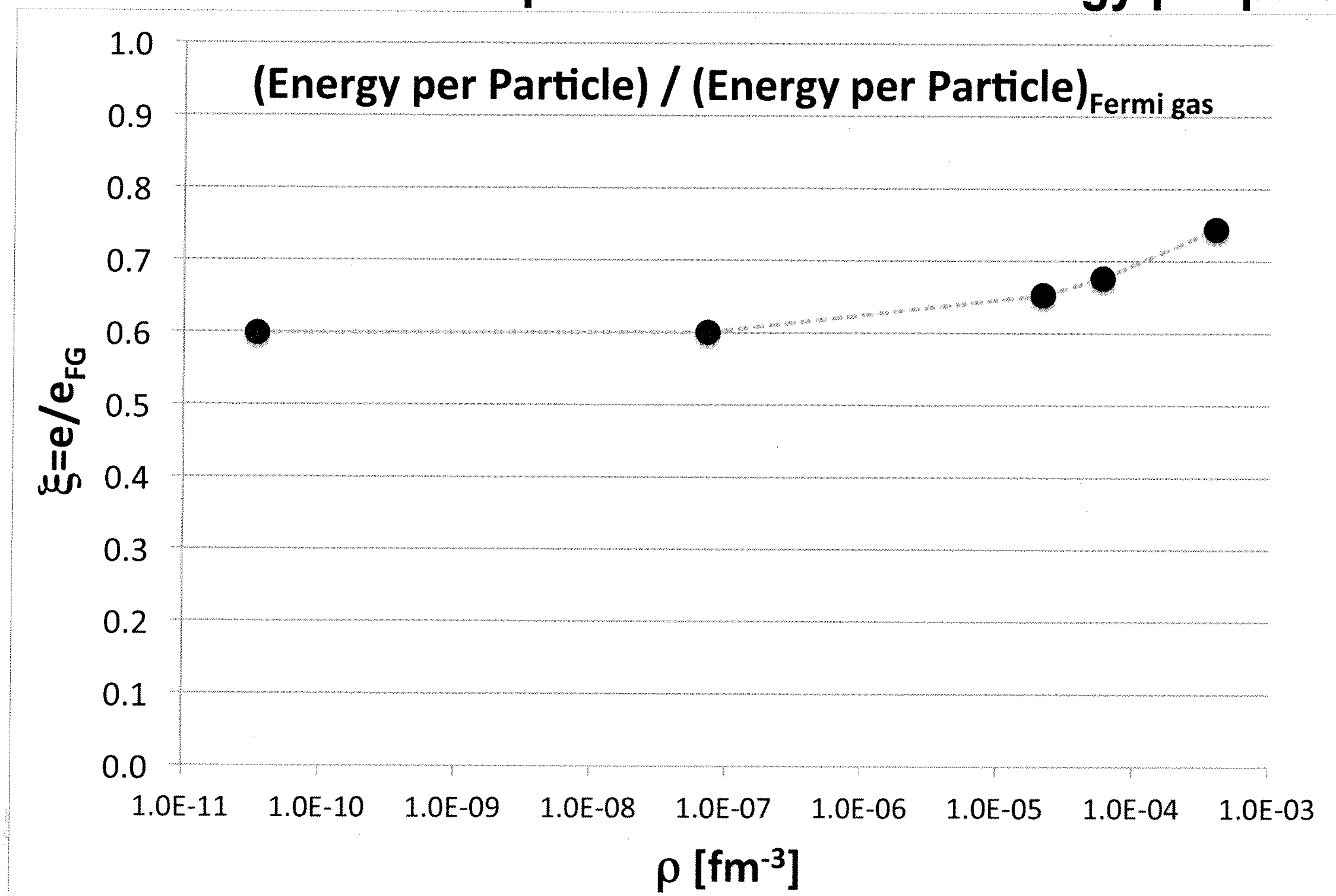
Neutron Matter EOS

- Energy per particle is higher than in extant models
  - However, it is always smaller than that of a free Fermi gas.
- This computation does not include RPA correction
  - RPA correction are expected to lower the energy per particle



Unitary Fermi Gas EOS

- Energy per particle is higher than in extant models
  - However, it is always smaller than that of a free Fermi gas.
- This computation does not include RPA correction
  - RPA correction are expected to lower the energy per particle



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Summary and Outlook

- A novel use of a six-quark cluster NN interaction in a composite bose-fermion field theory at T=0 is suggested
  - Feshbach state parameters were fitted to $^1S_0$ phase shift
  - EOS was solved for neutron matter and a unitary Fermi gas
  - Energy per particle was found to be higher than in extant models
  - RPA correction is needed to decrease energy/particle (in progress)
- Outlook:
  - Extend to symmetric nuclear matter
  - Apply to ground states of nuclei
  - Apply to excited states (T>0)
  - Connect to nuclear reactions?