HRS First Order Optics and Raster Corrections (G. Franklin, Aug. 18, 2010)

In the standard first order HRS transport matrix, the x-coordinates (vertical bend plane) are decoupled from the y-coordinates. The matrix equation that gives the position and angle of a trajectory at the focal plane (relative to the "central ray") in terms of the ray position, angle, and momentum at the target has the form:

$$\begin{pmatrix} x \\ \theta \end{pmatrix}_{fp} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} x_0 \\ \theta_0 \\ d \end{pmatrix} .$$

Here d gives the ray momentum as a fractional deviation from the spectrometer's central ray; $d = (P - Pcentral)/Pcentral$.

We measure x and θ at the focal plane. We determine x_0 by assuming the particle trajectory intersects the electron beam, so we either set $x_0=0$ or make a raster-dependent adjustment. We want to determine θ_0 and d. The transport equation can be rewritten as:

$$\begin{pmatrix} x \\ \theta \end{pmatrix}_{fp} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} x_0 \\ \theta_0 \\ d \end{pmatrix} = \begin{pmatrix} b & c \\ e & f \end{pmatrix} \begin{pmatrix} \theta_0 \\ d \end{pmatrix} + \begin{pmatrix} a \\ d \end{pmatrix} x_0 = T_1 \begin{pmatrix} \theta_0 \\ d \end{pmatrix} + T_2 x_0$$

Multiplying both sides by the inverse of T_1 gives:

$$\begin{pmatrix} \theta_0 \\ d \end{pmatrix} = T_1^{-1} \begin{pmatrix} x \\ \theta \end{pmatrix}_{fp} - T_1^{-1} T_2 x_0$$

The standard HRS transport matrix is listed at hallaweb.jlab.org/news/minutes/fo_matrix.html:

$$\begin{pmatrix} x \\ \theta \end{pmatrix}_{fp} = \begin{pmatrix} -2.18 & -0.198 & 11.9 \\ -0.10 & -0.469 & 1.967 \end{pmatrix} \begin{pmatrix} x_0 \\ \theta_0 \\ d \end{pmatrix}$$ where distances are in meters, angles in radians.

In my notation, the solution for the ray's vertical angle and momentum at the target is

$$\begin{pmatrix} \theta_0 \\ d \end{pmatrix} = \begin{pmatrix} -0.198 & 11.9 \\ -0.469 & 1.967 \end{pmatrix}^{-1} \begin{pmatrix} x \\ \theta \end{pmatrix}_{fp} - \begin{pmatrix} -0.198 & 11.9 \\ -0.469 & 1.967 \end{pmatrix}^{-1} \begin{pmatrix} -2.18 \\ -0.10 \end{pmatrix} x_0$$

or

$$\begin{pmatrix} \theta_0 \\ d \end{pmatrix} = \begin{pmatrix} 0.379 & -2.229 \\ 0.090 & -0.038 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix}_{fp} - \begin{pmatrix} -1.055 \\ -0.201 \end{pmatrix} x_0$$

To first order, the momentum is thus

$$P\text{-}P_{central} = P_{central}\, d = P_{central}\{0.090\ x\ -0.038\ \theta\ -0.201\ x_0\}$$

Neglecting the non-zero value of x_0 would introduce an error in the calculated momentum of P equal to $P_{central}(-.201\ x_0)$. For the runs studied by Diana, the beam was rastered over about 4 mm and the HRS was set to 0.87 GeV/c, so the error introduced by neglecting this effect sweeps over a range of (0.87 GeV/c)(.201/m)(4mm)= 0.7 MeV/c. (i.e. +- 0.35 MeV/c). This is consistent with Diana's empirical findings.