Project: E12-14-012 Argon/Ti

Tittle: Beam energy loss in the targets

Document Number: TGT-CALC-16-001

Revision: Rev 0

Author: Dave Meekins

Reference:
Leo: Techniques for Nuclear and Particle Physics
Icropera and DeWitt: Fundamentals of Heat and Mass Transfer

Description:
General calculations for beam energy loss in the Ar target cell and Ti target foil. Calc follows Bete-Bloch with simplified parameterization of density effect instead of full Sternheimer's. Only electron collision energy loss is considered.

Loss is considered for aluminum windows and Ar gas for the target cell. Heat is generated in the target fluid and the aluminum windows. Heat is removed by interaction with the cell wall. Beam is assumed to be rastered 2x2mm.

Target cell block is assumed to be held at fixed T=130K
Initial fill pressure of cell is 600 psia at 300K.
Target cell operating pressure (no beam) 216.85K

Beam on titanium is assumed to be 100 microA with 2x2 mm raster.

Reference Drawing(s):
TGT-103-1000-0012 Cell assy

Units and constants:

$eV := 1.602 \cdot 10^{-19} \cdot J$ def of electron volt

$MeV := 10^{6} \cdot eV$ def of mega eV

$r_e := 2.817 \cdot 10^{-13} \cdot cm$ classical radius of electron

Collision energy loss of elecron beam on target material. The method is from Leo. The shell correction is neglected.

Beam properties:

$I_{beam} := 25 \cdot \mu A$ beam current

$m_e := 0.511 \cdot \frac{MeV}{c^2}$ electron rest mass

$E_e := 2200 \cdot MeV$ Beam energy

$\tau := \frac{E_e - m_e \cdot c^2}{m_e \cdot c^2} = 4.304 \cdot 10^3$ kinetic enervy of e- in units of $m_e c^2$

$P_e := \frac{1}{c} \cdot \sqrt{E_e^{\,2} - \left(m_e \cdot c^2\right)^2}$ momentum of e-

$v_e := \sqrt{\frac{P_e^{\,2}}{m_e^{\,2} + \frac{P_e^{\,2}}{c^2}}}$ velocity of e-

$\beta := \frac{v_e}{c}$ beta

$\gamma := \frac{1}{\sqrt{1 - \beta^2}}$ gamma

$X := \log(\beta \cdot \gamma) = 3.634$

$\eta := \beta \cdot \gamma = 4.305 \cdot 10^3$

$dd1 := \left(0.422 \cdot \eta^{-2} + 0.304 \cdot \eta^{-4} - 0.00038 \cdot \eta^{-6}\right) \cdot 10^{-6}$ dummy for shell correction

$dd2 := \left(3.85 \cdot \eta^{-2} - 0.167 \cdot \eta^{-4} + 0.0016 \cdot \eta^{-6}\right) \cdot 10^{-9}$ dummy for shell correction

Aluminum entrance window
7075 aluminum is treated as pure aluminum for this calculation with the higer density of 7075.

$L_x := 0.010 \cdot in = 0.254\ mm$ length of absorber material

$Z := 13$ Atomic number

$A := 26.98 \cdot \frac{gm}{mol}$ Atomic weight

$\rho := 2.69 \cdot \frac{gm}{cm^3}$ density

$$I := \text{if } Z < 13 \quad \| \ (12 \cdot Z + 7) \cdot eV \quad \text{else} \quad \| \ (9.76 \cdot Z + 58.8 \cdot Z^{-0.19}) \cdot eV$$

Ionization potential

$hv_p := 28.816 \cdot \sqrt{\frac{\rho \cdot Z}{A}} \cdot eV \cdot \sqrt{\frac{cm^3}{mol}} = 32.807\ eV$ plasma energy

$$C_1 := -\left(2 \cdot \ln\left(\frac{I}{hv_p}\right) + 1\right) = -4.206$$

$$\delta := \text{if } X < 3 \quad \| \ 4.6052 \cdot X + C_1 + \frac{C_1}{27} \cdot (3 - X)^3 \quad \text{else} \quad \| \ 4.6052 \cdot X + C_1$$

$C := dd1 \cdot \left(\frac{I}{eV}\right)^2 + dd2 \cdot \left(\frac{I}{eV}\right)^3 = 1.504 \cdot 10^{-9}$ shell correction

$$F_t := 1 - \beta^2 + \frac{\frac{\tau^2}{8} - (2 \cdot \tau + 1) \cdot \ln(2)}{(\tau + 1)^2}$$

first function of τ

$$dE := 2 \cdot \pi \cdot N_A \cdot {r_e}^2 \cdot m_e \cdot c^2 \cdot \rho \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \cdot \left(\ln\left(\tau^2 \cdot \frac{(\tau + 2)}{2 \left(\frac{I}{m_e \cdot c^2} \right)^2} \right) + F_t - \delta + \frac{2\ C}{Z} \right)$$

$$Q_{ent} := dE \cdot L_x \cdot \frac{I_{beam}}{e_c} = 3.549\ W$$

Aluminum exit window

7075 aluminum is treated as pure aluminum for this calculation with the higer density of 7075.

$L_x := 0.011 \cdot in = 0.279\ mm$ length of absorber material

$Z := 13$ Atomic number

$A := 26.98 \cdot \frac{gm}{mol}$ Atomic weight

$\rho := 2.69 \cdot \frac{gm}{cm^3}$ density

$$I := \text{if } Z < 13 \;\; \| (12 \cdot Z + 7) \cdot eV \;\; \text{else} \;\; \| (9.76 \cdot Z + 58.8 \cdot Z^{-0.19}) \cdot eV$$

Ionization potential

$hv_p := 28.816 \cdot \sqrt{\frac{\rho \cdot Z}{A}} \cdot eV \cdot \sqrt{\frac{cm^3}{mol}} = 32.807\ eV$ plasma energy

$C_1 := -\left(2 \cdot \ln\left(\frac{I}{hv_p}\right) + 1\right) = -4.206$ constant

$$\delta := \text{if } X < 3 \;\; \left\| 4.6052 \cdot X + C_1 + \frac{C_1}{27} \cdot (3 - X)^3 \right. \;\; \text{else} \;\; \| 4.6052 \cdot X + C_1$$

$C := dd1 \cdot \left(\frac{I}{eV}\right)^2 + dd2 \cdot \left(\frac{I}{eV}\right)^3 = 1.504 \cdot 10^{-9}$ shell correction

$$F_t := 1-\beta^2+\frac{\frac{\tau^2}{8}-(2\cdot\tau+1)\cdot\ln(2)}{(\tau+1)^2}$$ first function of τ

$$dE := 2\cdot\pi\cdot N_A\cdot r_e^2\cdot m_e\cdot c^2\cdot\rho\cdot\frac{Z}{A}\cdot\frac{1}{\beta^2}\cdot\left(\ln\left(\tau^2\cdot\frac{(\tau+2)}{2\left(\frac{I}{m_e\cdot c^2}\right)^2}\right)+F_t-\delta+2\cdot\frac{C}{Z}\right)$$

$$dE\cdot L_x=0.156\ MeV$$

$$Q_{exit} := dE\cdot L_x\cdot\frac{I_{beam}}{e_c}=3.904\ W$$

Energy loss in the Argon gas. Average density of the gas is considered. Local variations along path are ignored.

$L_x := 250 \cdot mm$ length of absorber material

$Z := 18$ Atomic number

$A := 40 \cdot \frac{gm}{mol}$ Atomic weight

$\rho_{Ar} := 0.068 \cdot \frac{gm}{cm^3}$ density

$$I := \text{if } Z < 13 \;\; \| (12 \cdot Z + 7) \cdot eV \;\; \text{else} \;\; \| \left(9.76 \cdot Z + 58.8 \cdot Z^{-0.19}\right) \cdot eV$$

Ionization potential

$hv_p := 28.816 \cdot \sqrt{\frac{\rho \cdot Z}{A}} \cdot eV \cdot \sqrt{\frac{cm^3}{mol}} = 31.704\ eV$ plasma energy

$C_1 := -\left(2 \cdot \ln\left(\frac{I}{hv_p}\right) + 1\right) = -4.778$ constant

$$\delta := \text{if } X < 3 \;\; \| 4.6052 \cdot X + C_1 + \frac{C_1}{27} \cdot (3 - X)^3 \;\; \text{else} \;\; \| 4.6052 \cdot X + C_1$$

$C := dd1 \cdot \left(\frac{I}{eV}\right)^2 + dd2 \cdot \left(\frac{I}{eV}\right)^3 = 2.914 \cdot 10^{-9}$ shell correction

$$F_t := 1 - \beta^2 + \frac{\frac{\tau^2}{8} - (2 \cdot \tau + 1) \cdot \ln(2)}{(\tau + 1)^2}$$

first function of τ

$$dE := 2 \cdot \pi \cdot N_A \cdot {r_e}^2 \cdot m_e \cdot c^2 \cdot \rho_{Ar} \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \cdot \left(\ln \left(\tau^2 \cdot \frac{(\tau + 2)}{2 \left(\frac{I}{m_e \cdot c^2} \right)^2} \right) + F_t - \delta + 2 \cdot \frac{C}{Z} \right)$$

$$dE \cdot L_x = 3.307 \ \mathbf{MeV}$$

$$Q_{Ar} := dE \cdot L_x \cdot \frac{I_{beam}}{e_c} = 82.666 \ \mathbf{W}$$

A more detailed CFD analysis is given in TGT-CALC-16-003. This analysis accounts for the significant change of density with beam power and beam power dissipation with density. The power loss as a function of desity is therefore required.

$$dQ_{Ar} := \frac{Q_{Ar}}{\rho_{Ar}} = 1.216 \ \frac{\mathbf{W} \cdot \mathbf{m}^3}{\mathbf{kg}}$$

Total target energy loss

Total energy lost in the argon target is then:

$Q_{tot} := Q_{Ar} + Q_{ent} + Q_{exit} = 90.119\ W$ total power dissipation

Energy Density:

For a 2x2 raster with 25 μA beam current the power density in the aluminum is

$d_{raster} := 2 \cdot mm$ raster leg size

$A := d_{raster}^{2}$ raster area

$t := 0.01 \cdot in$ thickness of entrance window

$V := t \cdot A = \left(1.016 \cdot 10^{-9}\right)\ m^{3}$ Volume of material impacted by the beam

$q := \frac{Q_{ent}}{V} = \left(3.493 \cdot 10^{9}\right)\ \frac{W}{m^{3}}$ volumetric heat density from beam in entrance window

$q_{ent} := \frac{Q_{ent}}{A} = \left(8.872 \cdot 10^{5}\right)\ \frac{1}{m^{2}} \cdot W$

This result is applicable to the exit window as well. It shall be used in the thermal FEA.

$q_{exit} := \frac{Q_{exit}}{A} = \left(9.76 \cdot 10^{5}\right)\ \frac{W}{m^{2}}$

In the target gas

$d_{raster} := 2 \cdot mm$ raster leg size

$A := d_{raster}^{2}$ raster area

$t := 25 \cdot cm$ thickness of gas target

$V := t \cdot A = \left(1 \cdot 10^{-6}\right)\ m^{3}$ Volume of material impacted by the beam

$q := \frac{Q_{Ar}}{V} = \left(8.267 \cdot 10^{7}\right)\ \frac{W}{m^{3}}$ volumetric heat density from beam

Heat flux to wall of cell:

Heat flux from heated fluid to wall of cell from beam heat

$d_{cell} := 12.7 \cdot mm$

$L_{cell} := 225 \cdot mm$

$A_{cap} := \pi \cdot {d_{cell}}^2$

$A_{cell} := A_{cap} + L_{cell} \cdot \pi \cdot d_{cell}$

$q_{cell} := \frac{Q_{Ar}}{A_{cell}} = \left(8.717 \cdot 10^3\right) \frac{W}{m^2}$ flux to wall of cell

This heat flux is assumed to obe uniform to all of the cell surfaces. This is a reasonable assumption given the dimensions of the cell tube.

The density varies with heat load and this in turn varies the density. The density indepentent flux is:

$dq_{cell} := \frac{q_{cell}}{\rho_{Ar}} = 128.185 \frac{W}{m^2} \cdot \frac{m^3}{kg}$ density ind flux to wall

Titanium Foil:

Beam current assumed is 100 microA with a 2x2 mm square raster.

$L_x := 1.5 \cdot mm$ length of absorber material

$Z := 22$ Atomic number

$A := 48 \cdot \frac{gm}{mol}$ Atomic weight

$\rho := 4.5 \cdot \frac{gm}{cm^3}$ density

$$I := \text{if } Z < 13 \;\; \left\| (12 \cdot Z + 7) \cdot eV \right. \;\; \text{else} \;\; \left\| \left(9.76 \cdot Z + 58.8 \cdot Z^{-0.19}\right) \cdot eV \right.$$

Ionization potential

$$hv_p := 28.816 \cdot \sqrt{\frac{\rho \cdot Z}{A}} \cdot eV \cdot \sqrt{\frac{cm^3}{mol}} = 41.384 \; eV$$

plasma energy

$$C_1 := -\left(2 \cdot \ln\left(\frac{I}{hv_p}\right) + 1\right) = -4.576$$

$$\delta := \text{if } X < 3 \;\; \left\| 4.6052 \cdot X + C_1 + \frac{C_1}{27} \cdot (3 - X)^3 \right. \;\; \text{else} \;\; \left\| 4.6052 \cdot X + C_1 \right.$$

$$C := dd1 \cdot \left(\frac{I}{eV}\right)^2 + dd2 \cdot \left(\frac{I}{eV}\right)^3 = 4.539 \cdot 10^{-9}$$

shell correction

$$F_t := 1 - \beta^2 + \frac{\frac{\tau^2}{8} - (2 \cdot \tau + 1) \cdot \ln(2)}{(\tau + 1)^2}$$ first function of τ

$$dE := 2 \cdot \pi \cdot N_A \cdot r_e^{\ 2} \cdot m_e \cdot c^2 \cdot \rho \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \cdot \left(\ln\left(\tau^2 \cdot \frac{(\tau + 2)}{2 \left(\frac{I}{m_e \cdot c^2} \right)^2} \right) + F_t - \delta + 2 \cdot \frac{C}{Z} \right)$$

$I_{beam} := 100 \cdot \mu A$ max current for Ti target foil

$Q_{Ti} := dE \cdot L_x \cdot \frac{I_{beam}}{e_c} = 131.196\ W$ power deposited in Ti foil

Power density deposited:

$d_{raster} := 2 \cdot mm$ raster leg size

$A := d_{raster}^{\ 2}$ raster area

$t := 1.5 \cdot cm$ thickness of gas target

$V := t \cdot A = (6 \cdot 10^{-8})\ m^3$ Volume of material impacted by the beam

$q := \frac{Q_{Ti}}{V} = (2.187 \cdot 10^9)\ \frac{W}{m^3}$ volumetric heat density from beam