Correction for contamination from ^{3}H$\rightarrow$^{3}He decay in tritium target

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Initial state of ^{3}H target

From TGT-RPT-17-007 and TGT-CALC-17-020:

- The tritium cell was filled on October 23, 2017 ($t = 0$ for decay)
- The cell contained the following masses of tritium and helium:

$$m_{^3\mathrm{H}} = 0.102 \pm 0.001 \text{ g}$$
$$m_{^3\mathrm{He}} = 3.01 \times 10^{-5} \pm 3.01 \times 10^{-5} \text{ g}^*$$

*This assumes 100% uncertainty, as none was provided

- Taking into account cell parameters and corresponding uncertainties, this leads to initial target thicknesses (*in the tritium cell*) of:

$$\eta^0_{^3\mathrm{H}} = 0.077 \pm 0.001 \text{ g cm}^{-2}$$
$$\eta^0_{^3\mathrm{He}} = 2.26 \times 10^{-5} \pm 2.26 \times 10^{-5} \text{ g cm}^{-2}$$

^{3}H decay

Review of measurements[1] gives a tritium half-life of

$$\tau_{^3\mathrm{H}} \equiv \tau = 4500 \pm 8 \text{ days}$$

Measurement of decay final states[2] found branching ratios of:

$$^3\mathrm{H}_2 \to (^3\mathrm{H}^3\mathrm{He})^+ \qquad (94.5 \pm 0.6\%)$$
$$^3\mathrm{H}_2 \to (^3\mathrm{H})^+ + (^3\mathrm{He})^+ \qquad (5.5 \pm 0.6\%)$$

although molecular effects are irrelevant for DIS.

[1] L.L. Lucas & M.P. Unterweger, J. Res. Natl. Inst. Stand. Technol. 105, 541 (2000)
[2] S. Wexler, J. Inorg. Nucl. Chem. 10, 8 (1959)

Evolution of target composition

Tritium thickness:

$$\eta_{^3\mathrm{H}} \equiv \eta_{^3\mathrm{H}}(t) = \eta^0_{^3\mathrm{H}}(e^{-t/\tau})$$

Helium thickness *in tritium cell* (approximating equal nuclear masses):

$$\eta_{^3\mathrm{He}} \equiv \eta_{^3\mathrm{He}}(t) = \eta^0_{^3\mathrm{He}} + \eta^0_{^3\mathrm{H}}(1 - e^{-t/\tau})$$

Total target thickness (constant in time):

$$\begin{aligned} \eta_{tot} &= \eta_{^3\mathrm{H}}(t) + \eta_{^3\mathrm{He}}(t) \\ &= \eta^0_{^3\mathrm{H}} + \eta^0_{^3\mathrm{He}} \end{aligned}$$

Helium contamination factor:

$$c \equiv c(t) = \frac{\eta_{^3\mathrm{He}}(t)}{\eta_{tot}}$$

Uncertainties

Uncertainty in target thickness:

$$\delta\eta = \sqrt{\sum_i \left(\frac{\partial\eta}{\partial v_i}\delta v_i\right)^2} \quad , \quad v_i = \eta^0_{^3\mathrm{H}},\ \eta^0_{^3\mathrm{He}},\ \tau$$

However, $(\partial\eta/\partial\tau)\delta\tau \propto (\delta\tau/\tau^2)t$, which takes 7 years to reach 0.1%
$\rightarrow$ ignore uncertainty in half-life.

Tritium thickness uncertainty:

$$\delta\eta_{^3\mathrm{H}} = \delta\eta^0_{^3\mathrm{H}}\, e^{-t/\tau}$$

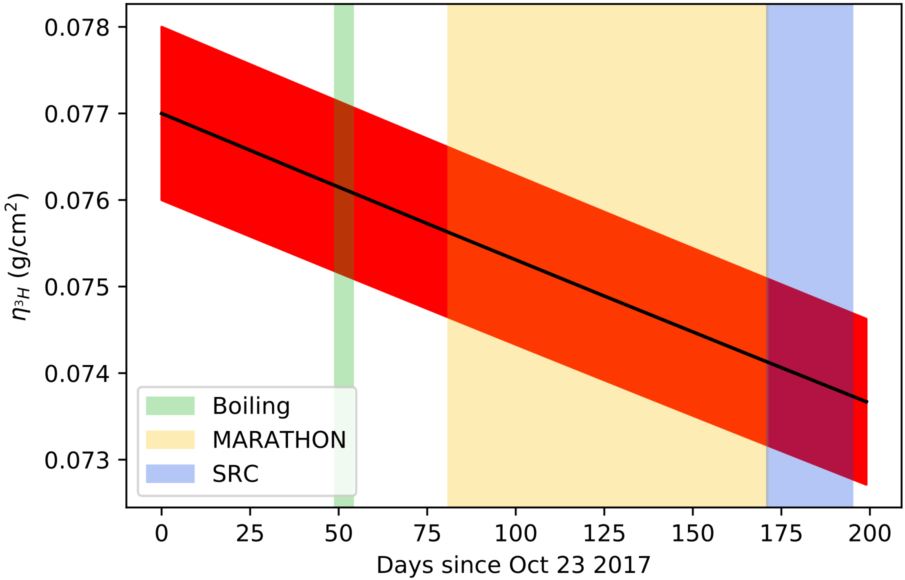
Helium thickness uncertainty:

$$\delta\eta_{^3\mathrm{He}} = \sqrt{(\delta\eta^0_{^3\mathrm{He}})^2 + ((1 - e^{-t/\tau})\delta\eta^0_{^3\mathrm{H}})^2}$$

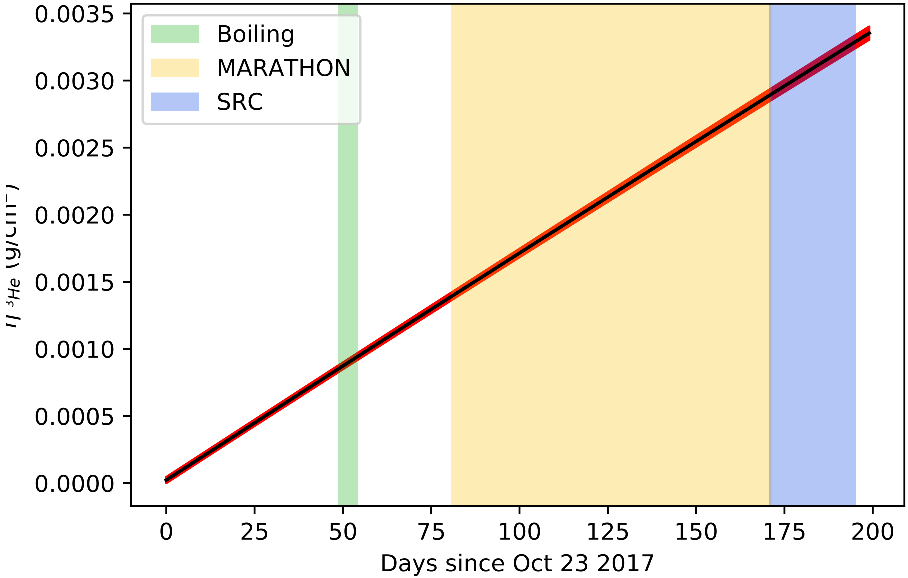
Contamination factor uncertainty:

$$\frac{\delta c}{c} = \sqrt{\left(\frac{\delta\eta^0_{^3\mathrm{He}}}{\eta^0_{^3\mathrm{He}}}\right)^2 + \left(\frac{\delta\eta_{tot}}{\eta_{tot}}\right)^2}$$

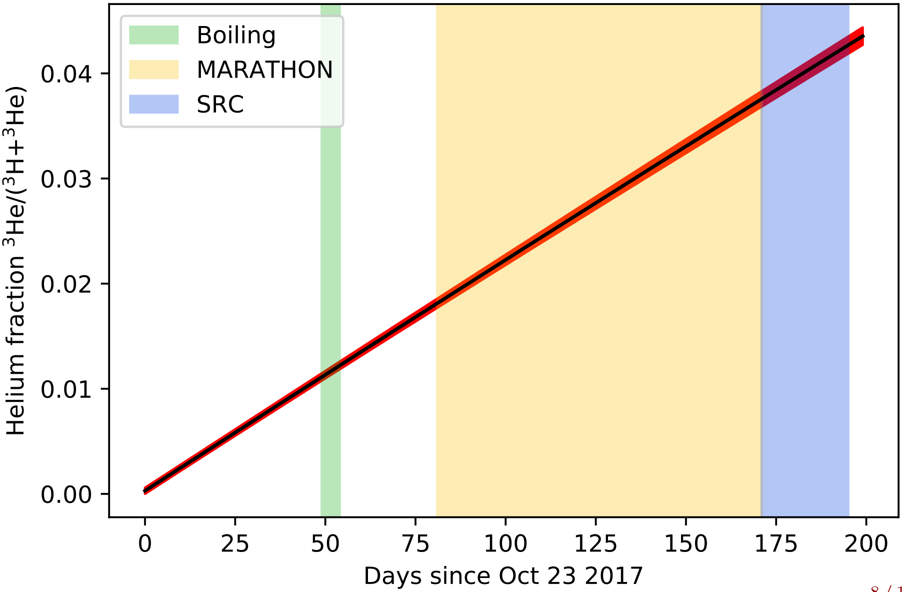
Tritium effective thickness



Helium effective thickness (in tritium cell)



Contamination



Correcting for contamination

We want the tritium cross section,

$$\sigma_{^3\mathrm{H}} = \beta \left(\frac{N_{^3\mathrm{H}}}{\eta_{^3\mathrm{H}}} \right),$$

- $N_{^3\mathrm{H}}$ is the number of detected electrons scattered by tritium
- $\eta_{^3\mathrm{H}}$ is the tritium target thickness in g cm^{-2} and is a function of time
- β includes all decay-independent normalization (charge, efficiencies, etc.)

However, some detected electrons actually scattered from helium. The total number of detected electrons is:

$$N_{tot} = N_{^3\mathrm{H}} + N_{^3\mathrm{He}}$$

Call the uncorrected measurement the total cross section:

$$\sigma_{tot} = \beta \left(\frac{N_{tot}}{\eta_{tot}} \right) = \beta \left(\frac{N_{^3\mathrm{H}} + N_{^3\mathrm{He}}}{\eta_{tot}} \right)$$

Correcting for contamination

Correct using our knowledge of the contamination factor c

$$\sigma_{tot} = \beta\left(\frac{N_{^3\mathrm{H}} + N_{^3\mathrm{He}}}{\eta_{tot}}\right)$$

$$= \beta\left(\frac{\eta_{^3\mathrm{H}}}{\eta_{tot}}\right)\left(\frac{N_{^3\mathrm{H}}}{\eta_{^3\mathrm{H}}}\right) + \beta\left(\frac{\eta_{^3\mathrm{He}}}{\eta_{tot}}\right)\left(\frac{N_{^3\mathrm{He}}}{\eta_{^3\mathrm{He}}}\right)$$

$$= \beta(1-c)\left(\frac{N_{^3\mathrm{H}}}{\eta_{^3\mathrm{H}}}\right) + \beta c\left(\frac{N_{^3\mathrm{He}}}{\eta_{^3\mathrm{He}}}\right)$$

$$\rightarrow \sigma_{tot} = (1-c)\sigma_{^3\mathrm{H}} + c\,\sigma_{^3\mathrm{He}}$$

$$\boxed{\sigma_{^3\mathrm{H}} = \frac{\sigma_{tot} - c\,\sigma_{^3\mathrm{He}}}{1-c} \rightarrow \frac{\sigma_{^3\mathrm{H}}}{\sigma_{^3\mathrm{He}}} = \left(\frac{\sigma_{tot}}{\sigma_{^3\mathrm{He}}}\right)\left(\frac{1}{1-c}\right) - \frac{c}{1-c}}$$

Comments

- ^{3}H$\rightarrow$^{3}He decay results in non-negligible helium content in the tritium target cell
- Fractional helium contamination $c = \eta_{^3\mathrm{He}}/(\eta_{^3\mathrm{H}} + \eta_{^3\mathrm{He}})$ grows from 0% to 4% during spring run
- Measured cross section ratio $\sigma_{tot}/\sigma_{^3\mathrm{He}}$ can be corrected by c to yield $\sigma_{^3\mathrm{H}}/\sigma_{^3\mathrm{He}}$