Elastic Cross Section Empty Run Acceptance Study and Scale Factor for E08-027

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Directly applying the solid angle acceptance for the purposes of computing elastic cross sections in the E08-027 experiment seems to yield large a systematic error in excess of 25%. By examining the ‘empty’ runs containing only liquid He as a target, and comparing the total elastic cross section without any acceptance applied to the well known Rutherford cross section, it is possible to compute a scale factor that may be used as a substitute for the acceptance in other runs, for other elastic cross sections. This scale factor has a large systematic error of its own, due largely to the uncertainty in the scattering angle, but reduces the error slightly from direct application of the acceptance to the raw cross section.

1. ‘Empty’ Run Cross Sections

In general, it is possible to compute the raw experimental cross section by normalizing the momentum yields, using the formula[1]:

$$\frac{d\sigma^{raw}}{d\Omega dE'} = \frac{ps_1 N}{\frac{Q_{tot}}{e}\rho(LT)\varepsilon_{det}} \frac{1}{\Delta\Omega\Delta E'\Delta Z} \quad (1)$$

Where ps_1 is the prescale factor, Q_{tot} is the total incident charge, e is the electron charge, LT is the livetime, ε_{det} is the product of all detector efficiencies, ρ is the number density of ^4He, and N is the raw number of events. ΔΩ, ΔE’, and ΔZ are the acceptances for the solid angle, the energy resolution, and the target length, respectively. The first quantities were all taken from the MySQL for runs 5650 and 5947, which represent the runs with only liquid helium as a target, 5650 with the longitudinal setting, and 5947 with the transverse setting. ΔZ was obtained from Ryan Zielinski’s tech note on radiation lengths[2]. ΔE’ represents the bin size used, which was 1 MeV. For the solid angle, a cut was used of:

$$-0.005 < \Phi < 0.005 \qquad -0.01 < \theta < 0.01$$

However, it is our goal to try and determine a scale factor to obtain the raw elastic cross sections without directly invoking the acceptance for Theta and Phi. Therefore, we will actually use a modified version of the cross section formula without the solid angle acceptance:

$$\frac{d\sigma^{unscaled}}{d\Omega dE'} = \frac{ps_1 N}{\frac{Q_{tot}}{e}\rho(LT)\varepsilon_{det}} \frac{1}{\Delta E'\Delta Z} \quad (2)$$

First, the number of counts was plotted versus ν = E – E’, the energy transfer, and this plot was then normalized to the unscaled cross section of equation (2). This was done for both elastic empty runs mentioned above, run #5650 for the longitudinal setting, and run #5947 for the transverse setting. The resulting peaks were fit using Toby Badman's fit method: a Gaussian landau convolution was fit to the elastic peak to attempt to model its tail, and a regular Gaussian was fit to the quasielastic peak visible next to it. See Figure 1. This was done to remove the quasielastic contamination in the elastic peak. Since the ultimate goal is to compare the elastic peak for an unscaled cross section to a well known elastic cross section to determine the factor by which the solid angle acceptance scales the elastic peak, it is important to fully separate the data which belongs to the elastic peak from that which belongs to the quasielastic peak.

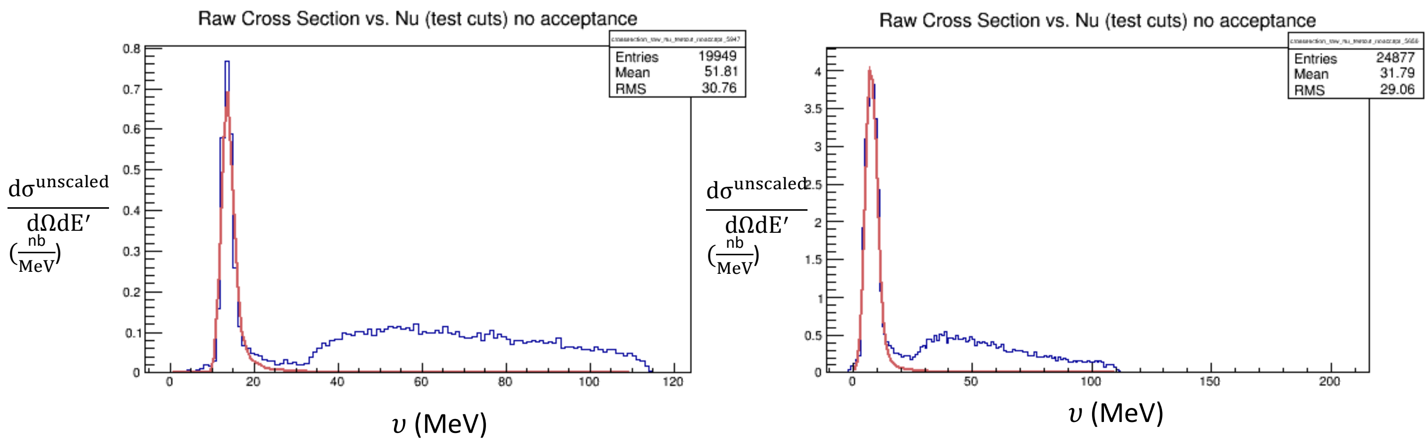

Figure 1: Unscaled cross sections for empty runs with elastic peak fits. Longitudinal setting on run 5650 (left) and transverse setting on run 5947 (right)

It is also relevant to note that the quasielastic peak can only be fitted with a Gaussian on one side, but it is not important for the fit to be good for the whole quasielastic peak, only for the part with significant overlap of the elastic peak. To ensure that the quasielastic peak is fitted over an appropriate range, the Gaussian is actually fit to a modified version of the quasielastic data, with fake zero valued data points at 15 MeV to force the fit to go to zero, before it is subtracted from the elastic peak. See Figure 2.

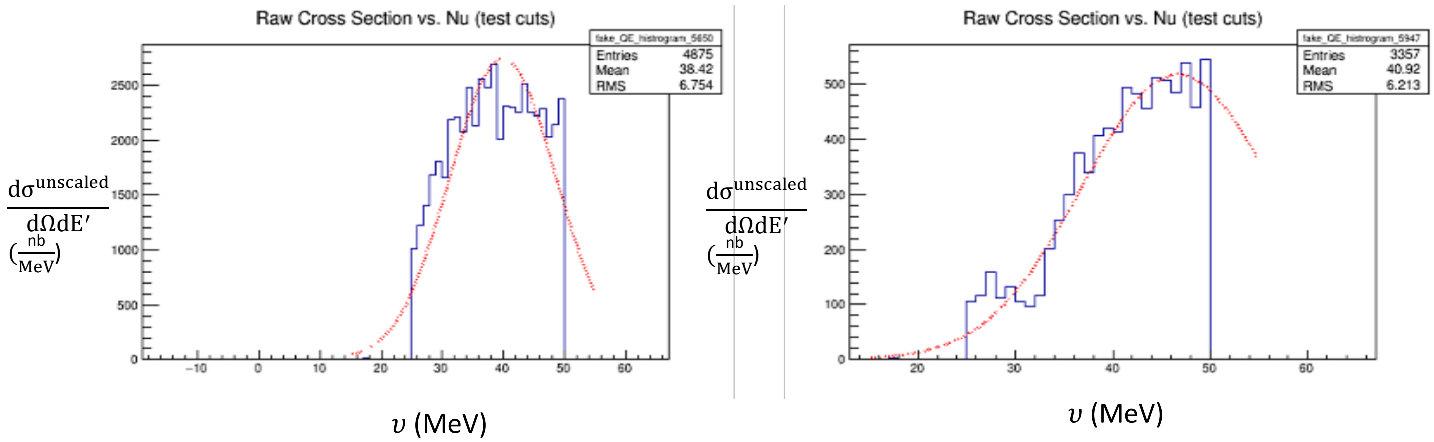

Figure 2: Cross sections for the empty runs with only quasielastic data, with fake data at 15 MeV to force the Gaussian fit to go to zero. Longitudinal setting on run 5650 (left) and transverse setting on run 5947 (right)

The resulting fit for the elastic peak was then integrated over E' to obtain the total unscaled differential cross section. Finally, this peak was radiatively corrected using the method of Mo and Tsai[3]. The formula for the Mo and Tsai radiative correction is too lengthy to reasonably transcribe here but can be found in the original paper referenced below. Most of the radiative effects here are off of the target itself. It is relevant to note that this formula includes a dependence on a "ΔE" term, not present elsewhere, defined in a separate paper[4] as the distance between the center of the peak and the end of the landau tail on the elastic peak. Since there is some element of guesswork involved in quantifying an appropriate cutoff, this unfortunately introduces an additional source of error.

2. Rosenbluth Elastic Cross Section

The ultimate goal is to compare this unscaled cross section to the well known Rosenbluth elastic cross section, for a ^{4}He target[5]:

$$\sigma_{Rosenbluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \times \{\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\} \qquad (3)$$

Where G_E and G_M are the electric and magnetic Sachs form factors for Helium-4, θ will here be the reconstructed scattering angle, and $\tau = \frac{Q^2}{4M^2}$, with Q^2 as the momentum transfer, and M as the mass of Helium. The Mott cross section can be obtained from

$$(\frac{d\sigma}{d\Omega})_{Mott} = \frac{\alpha^2}{4E^2}\frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\frac{E'}{E} \qquad (4)$$

With α as the fine structure constant, E’ as the electron energy after elastic scattering, and E as the incident electron energy. Since the reconstructed scattering angle is a very narrow peak for the very tight cut used here, it is not necessary to weight it by the Mott cross section, but simply to take the mean value for the reconstructed scattering angle and use it as θ.

The form factors were obtained from the McCarthy, Sick and Whitney fit[6], which provides that the magnetic form factor for Helium-4 is zero, as well as a fit to data for the charge form factor with a reduced χ^2 of 0.8. Invoking the relationship that $G_E = 2\ F_{ch}$, this allows us to compute the Rosenbluth cross section from equation (3).

3. Scaling Factor Results

As mentioned before, the ultimate goal of this study is to determine a scaling factor by which it shall be possible to determine the scaled elastic cross section for other, non-empty runs, by multiplying a cross section not yet scaled by the solid angle acceptance with some scale factor S:

$$\sigma_{unscaled} \times S = \sigma_{raw} \tag{5}$$

Hence, we will define this scaling factor as the ratio of our unscaled 'Empty' cross section to the more well known Rosenbluth cross section:

$$S = \frac{(\frac{d\sigma}{d\Omega})_{Rosenbluth}}{(\frac{d\sigma}{d\Omega})_{Unscaled}} \tag{6}$$

Under this definition, using the results of sections 1 and 2, scale factors were computed for both the longitudinal and transverse setting as follows:

$$S_{Longitudinal} = 4759.26\ Sr^{-1} \pm 16.6\%$$

$$S_{Transverse} = 6173.17\ Sr^{-1} \pm 20.1\%$$

The errors are summarized in Table 1 below, but dominated by the systematic uncertainty in the scattering angle of ±2%. This creates a large uncertainty in both the Mott cross section, and the form factors. This error was determined simply by varying the mean reconstructed scattering angle by 2% and observing the change in the final scale factor. The error due to the ΔE term, and the uncertainty in the upper bound on the quasielastic fit, was determined by varying the relevant parameter by ±5 MeV. The error in the MSW form factors was determined by averaging the error in the data used to fit the charge form factors, near the appropriate Q^2 range. Incident charge and detector efficiency errors were suggested by Ryan Zielinski to be less than 1%.

Quantity	Systematic Error in S	
Scattering Angle	13.8% (Longitudinal)	17.9% (Transverse)
Mo & Tsai ΔE	4%	
Fit Upper Bound	3.6%	
MSW Form Factors	7.5%	
Incident Charge	1%	
Detector Efficiencies	1%	
Total in Quadrature	16.6% (Longitudinal)	20.1% (Transverse)

Table 1: Summary of all scaling factor systematic errors

In summary, the scaling factors provided above may provide a method for applying the solid angle acceptance to the elastic cross sections of E08-027 with less systematic error. However, the systematic error is still relatively large due to the uncertainty in the scattering angle producing a large systematic error in the Rosenbluth cross section. More work may need to be done to investigate the possible superelastic contribution visible on the plots in Figure 1, but this is likely a very small contribution to the integrated elastic peak.

References

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