EPR Analysis

Polarization Gradient

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Outline

1. Polarization Gradient
   - Reduced Diffusion Constant
   - Depolarization Rates
     - Cell Live-Time
     - Beam Depolarization
     - AFP Loss
     - Field Gradients
     - Total Depolarization Rate
   - Diffusion Constant
   - Target Polarization
2. To Do

Reduced Diffusion Constant

Part 1:

Reduced Diffusion Constant

$$P_T = \frac{1}{1 + \frac{\Gamma_T}{d_T}} P_P$$

Transfer Tube Geometry

- Height: 93.98 mm
- Width: 12.8 mm
- Wall Thickness: 1.5 mm
- r = Width/2 - Wall Thickness
- Area = $\pi \times r^2$ = 75.43 mm^2
- Length = 93.98 mm

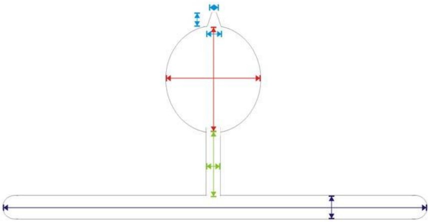

Figure: Samantha Cell Geometry

Reduced Diffusion Constant: Longitudinal Results

Table: List of longitudinal parameters used to calculate d_T

Parameter	Value	Units	Uncertainty [%]
A_{tt}	0.7543	cm^2	5.0
L_{tt}	9.398	cm	1.0
V_T	75.514	mL	5.0
T_T	348.98	K	1.44
T_P	536.0	K	0.93
$D(T_0)$	2.76	cm^2/s	0.3
n_0	0.7733	amg	Neg.
n_T	10.63	amg	3.69
m	1.7	-	Neg.
K	1.170	-	0.01
D_T	0.200	cm^2/s	3.84
d_T	0.892	h^{-1}	19.14

Reduced Diffusion Constant: Transverse Results

Table: List of transverse parameters used to calculate d_T

Parameter	Value	Units	Uncertainty [%]
A_{tt}	0.7543	cm^2	5.0
L_{tt}	9.398	cm	1.0
V_T	75.514	mL	5.0
T_T	349.87	K	1.43
T_P	543.3	K	0.92
$D(T_0)$	2.76	cm^2/s	0.3
n_0	0.7733	amg	Neg.
n_T	10.72	amg	3.76
m	1.7	-	Neg.
K	1.170	-	0.01
D_T	0.199	cm^2/s	3.90
d_T	0.889	h^{-1}	19.14

Depolarization Rate

Part 2:

Depolarization Rate in Target Chamber

$$P_T = \frac{1}{1 + \frac{\Gamma_T}{d_T}} P_P$$

Depolarization Effects

$$\Gamma_T = \Gamma^{He} + \Gamma^{wall} + \Gamma^{beam} + \Gamma^{AFP} + \Gamma^{\nabla B}$$

- Γ^{He}: Nuclear dipolar interaction
- Γ^{wall}: Wall relaxation
- Γ^{beam}: Beam depolarization
- Γ^{AFP}: AFP loss
- $\Gamma^{\nabla B}$: Magnetic field gradient loss

Depolarization Effects

Part 2-1:

Nuclear Dipolar and Wall Relaxations

$$\Gamma_T = \Gamma^{He} + \Gamma^{wall} + \Gamma^{beam} + \Gamma^{AFP} + \Gamma^{\nabla B}$$

Nuclear Dipolar and Wall Relaxation Times

$$\Gamma^{He} + \Gamma^{wall} = \frac{1}{\tau}$$

τ = cell live time

UVa has two live time measurements for our cell:

- Raw: 19 hrs.
- AFP: 22 hrs.

There is no error, so assume 100% error

d2n Cells Pumped at UVa

Long 42-deg UVa cells in blue and W&M cells in cyan

Cell Name	Max Pol	Masing Thresh*	Lasers Power	Spinup (τ_{up})	Lifetime ($\tau_{\mathrm{raw}}/\tau_{\mathrm{afp}}$)
Samantha	70%	56%	3C/60W	4.6 hrs	19/22 hrs
Boris	39%	none	3F/40W	tba	tba
Moss	63%	56%	1C1F/40W	5.4 hrs	33 hrs
Tigger	51%	tba	1C1F/40W	4.9 hrs	12/13 hrs
Alex	59%	59%	2C1F/60W	4.8 hrs	33 hrs

*with gradient coil OFF, all other measurements with gradient ON

PRELIMINARY: May 5, 2009

Figure: Samantha cell performances

Beam Depolarization

Part 2-2:

Depolarization Rate in Target Chamber

$$\Gamma_T = \Gamma^{He} + \Gamma^{wall} + \Gamma^{beam} + \Gamma^{AFP} + \Gamma^{\nabla B}$$

Polarization Loss Due to Beam

- Beam depolarization (ionizing radiation) increases nuclear spin relaxation
- Two step process:
  1. Beam ionizes ^{3}He atom which results in a free electron and atomic ion ^{3}He^{+}. Also possible for the atomic ion to bond with a neutral ^{3}He atom to form molecular ion ^{3}He$_2^{+}$.
  2. Interactions with ^{3}He ions induce ^{3}He spin flips.

Total Relaxation Rate

$$\begin{aligned}
\Gamma^{beam} &= \left[\frac{\text{ionization rate}}{\text{target chamber atom}}\right] \times \left[\frac{\text{mean number of nuclear spin flips}}{\text{atomic ion}}\right] \\
&= \left[\left(\frac{\text{electron}}{\text{unit time}}\right) \times \left(\frac{\text{atomic ions created}}{\text{electron}}\right) \times (\text{atoms in tc})^{-1}\right] \times (n_a + n_m) \\
&= \left[\left(\frac{I}{e}\right) \times \left(\frac{\text{total energy lost}}{\text{mean energy per ion}}\right) \times \left(\frac{1}{V_{tc} n_{tc}}\right)\right] \times (n_a + n_m) \\
&= \left[\left(\frac{I}{e}\right) \times \left(\frac{\left[\frac{1}{\rho}\frac{dE}{dx}\right] L_{tc} n_{tc}}{E_i}\right) \times \left(\frac{1}{V_{tc} n_{tc}}\right)\right] \times (n_a + n_m) \\
&= \left(\frac{I}{e}\frac{1}{E_i}\left[\frac{1}{\rho}\frac{dE}{dx}\right]\frac{1}{A_{tc}}\right) \times (n_a + n_m) \\
&= \Gamma^{ion} \times (n_a + n_m)
\end{aligned}$$

Where I is beam current, E_i is mean energy for ion-electron pair creation, A_{tc} is mean cross sectional area of the target chamber, Γ^{ion} is ionization rate per ^{3}He atom in the target chamber, and n_a (n_m) are average number of spins lost per atomic ion created due to interaction with atomic (molecular) ions.

Beam Depolarization Model

- We did not take data to measure polarization loss due to the electron beam
- Γ^{ion} parameterized by Jaideep Singh, Adian M. Kelleher, and Patricia H. Solvignon
- parameterization within 5% over all JLab energies

$$\Gamma^{ion} = \left(0.0095\frac{cm^2}{\mu A \cdot hr}\right)\frac{I}{A_{tc}} = \left(\frac{1}{21hr}\right)\cdot\left(\frac{I}{10\mu A}\right)\cdot\left(\frac{2cm^2}{A_{tc}}\right) \qquad (1)$$

Beam Depolarization Approximation

$$\Gamma^{beam} = \Gamma^{ion} \cdot (n_a + n_m)$$

- Due to the N_2 in the target cell, n_m is suppressed
- n_a is constrained by the fact that an atomic ion depolarizes no more than one atomic nucleus ($n_a \leq 1$)

$$\Gamma^{beam} = \Gamma^{ion} \cdot (n_a + n_m) \leq \Gamma_{ion} \quad (2)$$

Target Chamber Geometry

- Width: 18.95 $\pm$ 0.537 mm
- Wall Thickness: 1.66 $\pm$ 0.06 mm
- r = Width/2 - Wall Thickness
- $\delta r = \sqrt{\left(\frac{\delta Width}{Width}\right)^2 + \left(\frac{Wall}{Wall}\right)^2}$
- Area = $\pi \times r^2$ = 7.815 mm^2
- $\delta Area = 2\pi r \delta r$ = 9.19%

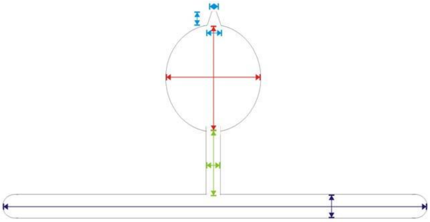

Figure: Samantha Cell Geometry

Beam Depolarization: Results

Table: List of parameters used to calculate Γ^{beam}

Parameter	Value	Units	Uncertainty [%]
A_{tc}	0.7815	cm^2	9.19
I	16.0	μA	-
Γ^{beam}	0.0794	h^{-1}	10.45

AFP Loss

Part 2-3:

Depolarization Rate in Target Chamber

$$\Gamma_T = \Gamma^{He} + \Gamma^{wall} + \Gamma^{beam} + \Gamma^{AFP} + \Gamma^{\nabla B}$$

AFP Loss

- We did not take data to measure depolarization due to AFP
- We could get an upper limit on this by using the depolarization due to AFP from transversity
- Transversity HALOG posts show AFP loss of $\sim 0.5\%$
- Expected not to contribute much
- Still need to do this

Field Gradients

Part 2-4:

Depolarization Due to Magnetic Field Gradients

$$\Gamma_T = \Gamma^{He} + \Gamma^{wall} + \Gamma^{beam} + \Gamma^{AFP} + \Gamma^{\nabla B}$$

Inhomogeneity of Magnetic Field

- Inhomogeneity in holding field B_z induced transverse field gradients ∇B_x and ∇B_y

$$\Gamma^{\nabla B} = 2D\frac{|\nabla B_x|^2 + |\nabla B_y|^2}{B_z^2}$$

- $D \approx 0.28$ cm^2 is ^{3}He self-diffusion coefficient
- Seen values elsewhere on the order of 1/1000 h^{-1}, which is small
- Need to find gradient filed measurements for our data to verify
- Found compass data, but BigBite was at 30 deg.

Total Depolarization Rate: Results

Table: List of parameters used to calculate Γ^T

Parameter	Value	Units	Uncertainty [%]
$\frac{1}{\tau^{raw}}$	0.0526	h^{-1}	100
$\frac{1}{\tau^{afp}}$	0.0455	h^{-1}	100
Γ^{beam}	0.0794	h^{-1}	10.45
Γ^{AFP}	-	h^{-1}	-
$\Gamma^{\nabla B}$	0.001	h^{-1}	-
Γ_T^{raw}	0.1330	h^{-1}	100
Γ_T^{afp}	0.1249	h^{-1}	100

Diffusion Constant

$$C_{Diff} = \frac{1}{1 + \frac{\Gamma_T}{d_T}}$$

Table: List of parameters used to calculate C_{Diff}

Parameter	Target Spin	Live Time	Value	Units	Uncertainty [%]
d_T	Long.	-	0.892	h^{-1}	19.14
d_T	Trans.	-	0.889	h^{-1}	19.14
Γ_T	-	Raw	0.1330	h^{-1}	100
Γ_T	-	AFP	0.1249	h^{-1}	100
C_{Diff}	Long.	Raw	0.8702	h^{-1}	13.18
C_{Diff}	Long.	AFP	0.8771	h^{-1}	12.58
C_{Diff}	Trans.	Raw	0.8699	h^{-1}	13.22
C_{Diff}	Trans.	AFP	0.8769	h^{-1}	12.61

Target Chamber Polarization

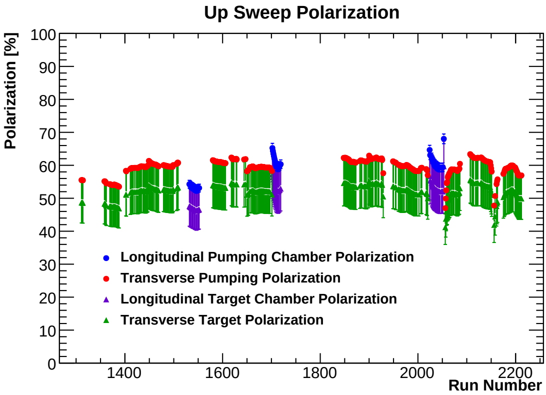

Figure: Target chamber polarization calculated from EPR calibration constant, C_{EPR} and polarization diffusion constant, C_{Diff}. These constants are applied to interpolated NMR production run amplitudes.

Beam Depolarization: Results

Table: Target polarization uncertainties form C_{EPR} and C_{Diff} in both chambers using up-sweep measurements and τ_{afp}

Chamber	Target Spin Direction	Relative Uncertainty [%]
Pumping	Long.	2.17
Pumping	Trans.	1.46
Target	Long.	12.76
Target	Trans.	12.69

TO Do

- Use Dave's cell fluxes to try and get water calibration
- Start looking into PolRad/RadCor

Polarization Gradient Model

- EPR measures the polarization in the pumping chamber
- In order to get polarization in the target chamber a model of the polarization gradient between the two chambers is used [Romalis]

$$\frac{dP_T}{dt} = d_P \left(P_T - P_P\right) + \gamma_{SE} \left(P_{Rb} - P_P\right) - \Gamma_P P_P$$
$$\frac{dP_P}{dt} = d_T \left(P_P - P_T\right) + \Gamma_T P_T$$

- $P_{T,P,Rb}$: target,pumping and Rubidium polarizations
- Γ_T : total relaxation rate during running conditions
- γ_{SE}: spin exchange rate
- $d_{P,T}$: reduced diffusion constants

Reduced Diffusion Constants

$$
\begin{aligned}
d_P &= \frac{A_{tr} D_T}{V T L tr} K \\
d_T &= \frac{A_{tr} D_T n_T}{V_P L_{tr} n_P} K
\end{aligned}
$$

- A_{tr}: cross sectional area of the transfer tube
- $L_t r$: length of the transfer tube
- $V_{P,T}$: volume of the pumping and target chambers
- n_T: density of the target chamber
- K: dimensionless constant
- D_T: diffusion constant along the length of the transfer tube

K Constant

$$K = \frac{(m-2)\,(T_T - T_P)\,T_T}{(T_T/T_P)^m\,T_P^2 - T_T^2}$$

- m = 1.7
- See Romalis,1997 and Zheng, 2002 for more details

Diffuse Constant D_T

$$D_T = D\left(T_0\right)\frac{n_0}{n_T}\left(\frac{T_T}{T_0}\right)^{m-1}$$

- m = 1.7
- $D\left(T_0\right) = 2.76$ cm^2 at $T_0 = 80°C$
- $n_0 = 0.773$ amg
- See Romalis,1997 and Zheng, 2002 for more details

The Equilibrium Solution (1)

- For the equilibrium solution $dP_T/dt = 0$. This leads to

$$P_T = \frac{1}{1 + \frac{\Gamma_T}{d_T}} P_P$$

- Γ_T: Depolarization rate in the target