Photo Multiplier Tube Analysis Report

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Abstract

Photo multiplier tubes (PMTs) are to be used for the coordinate detector (CDET) in the near future for Hall A. The problem arises when each of the 16 pixels within a single PMT is not consistent with the others, as only a single voltage can run through each PMT. Using data from a previous undergraduate student Nathan Murtha. whereby using a constant voltage, was able to detect the current of each PMT. Using these data it is possible to exclude 2 outlier pixels from each PMT, show trends and use statistical methods to determine the optimal selection for the CDET. Of the 186 PMT's, the best 160 have RMSE's below 16.5% when normalized with the mean current of each PMT.

Defining the Problem

The current CDET requires 160 PMT's in order to function correctly. The data that has been acquired shows the output current of 186 individual PMT's given a constant voltage. While this does not directly show the quantum efficiency of each pixel, the current is reasonable enough to determine optimal PMT choices.
Each PMT contains 16 pixels, 14 of which need to be energized for the CDET to be functional. This allows for 2 outlier pixels to be removed before performing analysis.

Method of Analysis

In order to analyze 14 pixels of 16, one must determine a method of removing the 2 outlier pixels. The average of all 16 pixels was calculated, and the difference of each pixel was stored. The largest 2 of these differences were the pixels to be excluded.
The average of the remaining 14 pixels is calculated, and used to calculate the root mean square error (RMSE) which is defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}} \tag{1}$$

This quantity is used in conjunction with the range which is defined as:

$$Range = x_{max} - x_{min} \tag{2}$$

Both of these quantities must be normalized. In this case, with the mean:

$$\frac{RMSE}{\bar{x}} \tag{3}$$

$$\frac{Range}{\bar{x}} \tag{4}$$

Another quantity of interest is the maximum scaling factor (MSF). where the scaling factor is defined as the ratio of a given pixel current and the minimum pixel current in each respective PMT:

$$s_i = \frac{x_i}{x_{min}} \tag{5}$$

This quantity allows us to determine the maximum attenuation required in each PMT.

Results

In all analysis the data is listed by both PMT serial number, and sorted PMT efficiency (by standard of RMSE or maximum scaling factor).
Initially the RMSE is calculated, when normalized with the average the units cancel and the quantity is unitless.

Figure 1: RMSE and Range vs Type 1 PMT # (By Serial Number)

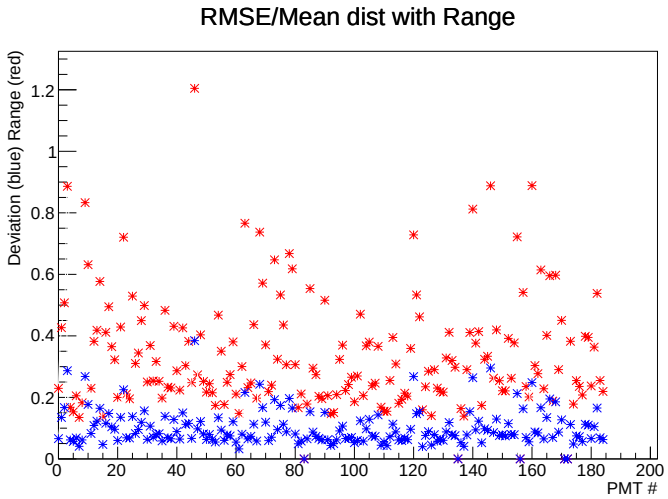
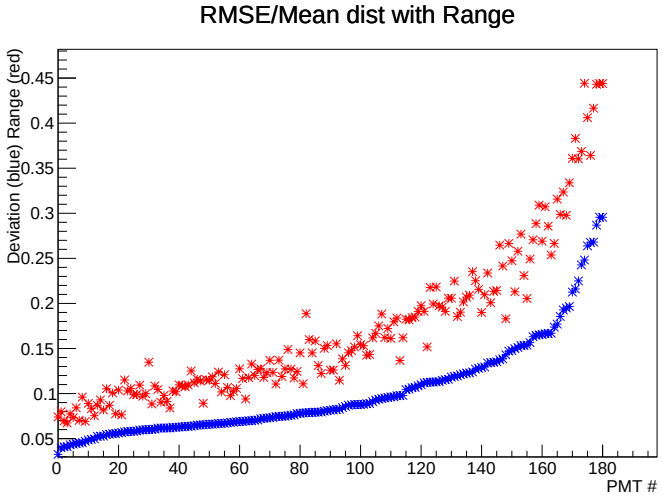


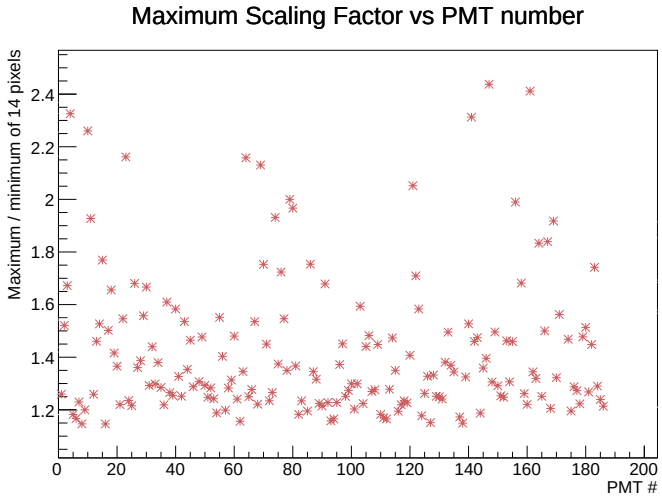
Figure 2: RMSE and Range/2 vs Type 1 PMT # (Sorted by RMSE)



The shape of both the range and RMSE are consistent as one would expect. Note that a the PMT's of interest here are below 160. in terms of the RMSE they are below 16.5%

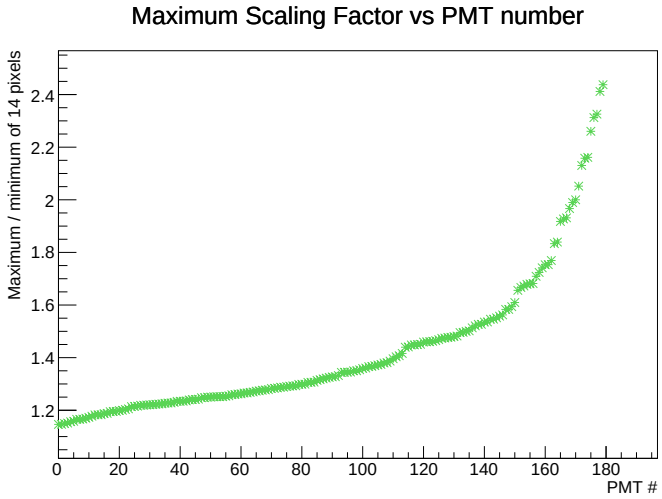
Next the MSF is evaluated for the best 160 PMT's, Note that the MSF is unitless.

Figure 3: MSF vs Type 1 PMT # (By Serial Number)



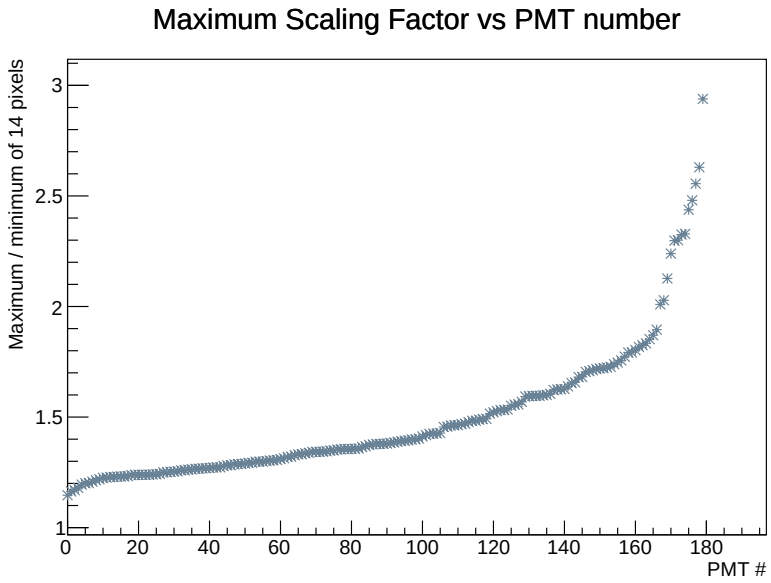
It is important to compare the structure of these data to the unsorted RMSE (Figure 1: Blue). Noticing the sporadic structure of outliers in the RMSE vs the much more consistent and less dense outlier "Field" of the MSF. This would lead one to believe that the RMSE does not give a sufficient description of the data for the purpose of the CDET.

Figure 4: MSF vs Type 1 PMT # (Sorted by MSF)



Noting ordered PMT 160 having a MSF of 1.74.

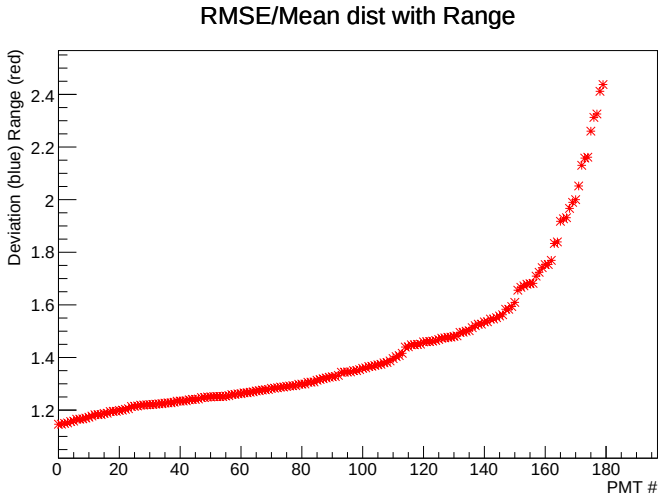
Figure 5: MSF with Minimum Removals vs Type 1 PMT # (Sorted by MSF)



As an aside, suppose that the minimum 2 pixels were to be removed in each PMT, such that the remaining 14 pixels were the maximum remaining.
With ordered PMT 160 having a MSF of 1.79, 0.05 greater than when removing the pixels with the largest deviations.

Included in the analysis were "Type 2" PMT's which were later chosen not to be used.

Figure 6: MSF vs Type 2 PMT # (Sorted by MSF)



Conclusions

Using the RMSE normalized with respect to the average, the 160th PMT deviates 16.5% while the least deviation is 3.3%.
When determining the MSF it was found that the maximum required (i.e. PMT 160) would be 1.74, while the least required would be 1.15.
It was noted that while the RMSE and range showed similar consistent shapes, they did not match well with the MSF when comparing on an individual PMT basis. These data an analysis do not directly reflect the quantum efficiency of the system. However the selection of PMT's will benefit from choosing the PMT's not above the 160 that were determined to be the most efficient by the MSF.

Table 1: Type 1 PMT Selection Table (Ranked by MSF)

Rank	Serial #	MSF	RMSE	Range	Rank	Serial #	MSF	RMSE	Range
1	16	1.146	0.047	0.138	46	131	1.241	0.061	0.217
2	8	1.147	0.041	0.135	47	53	1.242	0.064	0.215
3	138	1.149	0.041	0.139	48	51	1.248	0.063	0.217
4	127	1.151	0.045	0.141	49	152	1.248	0.075	0.221
5	62	1.156	0.033	0.148	50	65	1.250	0.065	0.232
6	93	1.158	0.044	0.146	51	130	1.250	0.064	0.219
7	112	1.165	0.043	0.155	52	98	1.251	0.066	0.222
8	94	1.166	0.050	0.151	53	165	1.251	0.065	0.228
9	6	1.167	0.057	0.153	54	129	1.251	0.058	0.230
10	111	1.170	0.056	0.155	55	42	1.252	0.064	0.224
11	137	1.173	0.053	0.164	56	151	1.254	0.078	0.222
12	124	1.177	0.040	0.160	57	39	1.255	0.065	0.231
13	5	1.182	0.062	0.168	58	12	1.259	0.083	0.230
14	82	1.183	0.049	0.166	59	1	1.260	0.066	0.229
15	110	1.183	0.045	0.168	60	159	1.262	0.070	0.235
16	144	1.188	0.053	0.174	61	125	1.262	0.076	0.235
17	54	1.188	0.055	0.174	62	38	1.266	0.069	0.233
18	116	1.195	0.062	0.181	63	73	1.266	0.070	0.240
19	85	1.196	0.065	0.178	64	181	1.268	0.066	0.237
20	175	1.196	0.048	0.178	65	107	1.270	0.075	0.238
21	57	1.199	0.060	0.177	66	177	1.273	0.073	0.236
22	9	1.200	0.062	0.183	67	99	1.274	0.067	0.242
23	101	1.203	0.053	0.186	68	108	1.276	0.073	0.246
24	168	1.205	0.069	0.188	69	66	1.277	0.074	0.247
25	186	1.214	0.045	0.192	70	113	1.278	0.072	0.255
26	90	1.215	0.062	0.196	71	58	1.283	0.078	0.249
27	25	1.216	0.068	0.195	72	52	1.284	0.080	0.245
28	36	1.219	0.059	0.198	73	35	1.285	0.082	0.252
29	117	1.219	0.060	0.194	74	46	1.288	0.067	0.248
30	21	1.220	0.060	0.200	75	176	1.288	0.075	0.255
31	160	1.221	0.056	0.201	76	184	1.290	0.069	0.255
32	68	1.221	0.058	0.198	77	150	1.292	0.076	0.254
33	178	1.223	0.056	0.208	78	31	1.293	0.064	0.250
34	104	1.224	0.058	0.206	79	50	1.293	0.081	0.252
35	89	1.224	0.067	0.204	80	33	1.298	0.071	0.254
36	119	1.227	0.063	0.204	81	102	1.299	0.060	0.270
37	92	1.228	0.063	0.206	82	100	1.299	0.070	0.266
38	95	1.228	0.062	0.209	83	148	1.306	0.086	0.263
39	7	1.230	0.068	0.205	84	154	1.306	0.079	0.262
40	83	1.234	0.058	0.211	85	48	1.307	0.098	0.273
41	72	1.234	0.059	0.219	86	59	1.314	0.075	0.274
42	118	1.235	0.068	0.214	87	88	1.316	0.073	0.274
43	24	1.235	0.069	0.211	88	163	1.319	0.084	0.277
44	185	1.239	0.063	0.219	89	170	1.322	0.087	0.290
45	61	1.241	0.055	0.211	90	139	1.325	0.078	0.290

Type 1 PMT Selection Table (Ranked my MSF)

Rank	Serial #	MSF	RMSE	Range	Rank	Serial #	MSF	RMSE	Range
91	41	1.327	0.089	0.287	136	17	1.502	0.116	0.412
92	126	1.328	0.089	0.285	137	180	1.513	0.110	0.395
93	128	1.332	0.079	0.290	138	2	1.521	0.135	0.427
94	135	1.344	0.076	0.298	139	14	1.527	0.123	0.418
95	162	1.344	0.088	0.303	140	140	1.527	0.154	0.411
96	87	1.345	0.087	0.295	141	67	1.535	0.113	0.436
97	63	1.345	0.080	0.300	142	43	1.536	0.150	0.426
98	115	1.350	0.088	0.309	143	22	1.546	0.135	0.429
99	78	1.350	0.088	0.306	144	77	1.546	0.113	0.435
100	44	1.354	0.112	0.303	145	55	1.551	0.134	0.467
101	145	1.359	0.098	0.324	146	29	1.558	0.119	0.450
102	27	1.361	0.082	0.310	147	171	1.562	0.127	0.450
103	20	1.366	0.096	0.322	148	123	1.583	0.154	0.462
104	81	1.367	0.081	0.307	149	40	1.583	0.128	0.431
105	134	1.370	0.079	0.320	150	103	1.594	0.124	0.471
106	96	1.372	0.091	0.324	151	37	1.610	0.138	0.483
107	75	1.374	0.095	0.324	152	18	1.656	0.148	0.495
108	34	1.380	0.079	0.317	153	30	1.667	0.157	0.499
109	132	1.381	0.088	0.328	154	3	1.672	0.167	0.507
110	28	1.387	0.096	0.344	155	91	1.679	0.151	0.516
111	146	1.396	0.093	0.334	156	26	1.681	0.138	0.529
112	56	1.403	0.094	0.350	157	158	1.682	0.163	0.541
113	120	1.407	0.096	0.359	158	122	1.709	0.147	0.533
114	19	1.416	0.104	0.365	159	76	1.724	0.174	0.533
115	32	1.440	0.107	0.369	160	183	1.742	0.165	0.538
116	105	1.441	0.098	0.368	161	70	1.753	0.167	0.571
117	182	1.448	0.105	0.363	162	86	1.754	0.153	0.554
118	109	1.448	0.143	0.366	163	15	1.769	0.165	0.577
119	71	1.450	0.119	0.371	164	164	1.833	0.166	0.614
120	97	1.451	0.107	0.370	165	167	1.839	0.195	0.595
121	155	1.459	0.079	0.377	166	169	1.918	0.186	0.597
122	142	1.460	0.094	0.376	167	11	1.927	0.177	0.631
123	13	1.461	0.109	0.382	168	74	1.931	0.193	0.647
124	153	1.462	0.115	0.392	169	80	1.967	0.165	0.618
125	45	1.465	0.115	0.382	170	156	1.990	0.213	0.722
126	174	1.468	0.112	0.382	171	79	2.000	0.197	0.668
127	114	1.473	0.113	0.395	172	121	2.052	0.268	0.728
128	143	1.475	0.123	0.414	173	69	2.130	0.243	0.737
129	49	1.477	0.122	0.403	174	64	2.158	0.216	0.766
130	179	1.477	0.113	0.399	175	23	2.161	0.225	0.721
131	60	1.480	0.121	0.381	176	10	2.260	0.268	0.833
132	106	1.482	0.129	0.380	177	141	2.312	0.264	0.812
133	133	1.495	0.118	0.412	178	4	2.326	0.287	0.886
134	149	1.496	0.130	0.419	179	161	2.412	0.248	0.888
135	166	1.500	0.135	0.402	180	147	2.438	0.296	0.888

Table 2: Type 2 PMT Selection Table (Ranked by MSF)

Rank	Serial #	MSF	RMSE	Range	Rank	Serial #	MSF	RMSE	Range
1	373	1.151	0.039	0.140	46	352	1.325	0.095	0.277
2	228	1.156	0.049	0.144	47	497	1.326	0.094	0.286
3	338	1.160	0.055	0.149	48	266	1.327	0.072	0.278
4	529	1.185	0.050	0.171	49	379	1.330	0.098	0.288
5	320	1.186	0.054	0.170	50	312	1.335	0.073	0.278
6	218	1.191	0.049	0.175	51	278	1.336	0.095	0.294
7	558	1.202	0.061	0.182	52	541	1.337	0.081	0.282
8	294	1.203	0.064	0.184	53	393	1.338	0.083	0.299
9	355	1.203	0.050	0.184	54	289	1.338	0.090	0.295
10	469	1.209	0.052	0.192	55	567	1.339	0.082	0.292
11	248	1.223	0.056	0.201	56	556	1.341	0.094	0.293
12	234	1.228	0.061	0.206	57	237	1.343	0.089	0.289
13	253	1.228	0.064	0.205	58	498	1.343	0.095	0.291
14	201	1.229	0.056	0.209	59	335	1.349	0.087	0.288
15	332	1.234	0.071	0.212	60	276	1.350	0.096	0.289
16	486	1.238	0.061	0.214	61	291	1.354	0.096	0.291
17	521	1.242	0.058	0.216	62	254	1.355	0.089	0.302
18	193	1.242	0.070	0.220	63	380	1.359	0.105	0.319
19	550	1.244	0.073	0.223	64	576	1.371	0.099	0.318
20	425	1.250	0.060	0.230	65	522	1.378	0.095	0.313
21	478	1.251	0.069	0.222	66	370	1.385	0.097	0.324
22	555	1.255	0.073	0.230	67	238	1.388	0.093	0.322
23	268	1.260	0.077	0.230	68	466	1.391	0.126	0.339
24	514	1.261	0.075	0.236	69	210	1.391	0.102	0.334
25	431	1.265	0.063	0.239	70	499	1.393	0.102	0.340
26	594	1.266	0.073	0.229	71	451	1.400	0.095	0.361
27	357	1.266	0.069	0.235	72	573	1.401	0.111	0.345
28	336	1.268	0.091	0.238	73	387	1.403	0.114	0.353
29	490	1.278	0.069	0.249	74	240	1.403	0.087	0.343
30	563	1.278	0.061	0.235	75	565	1.403	0.129	0.337
31	315	1.279	0.070	0.244	76	376	1.405	0.093	0.340
32	433	1.290	0.073	0.253	77	592	1.406	0.088	0.326
33	517	1.291	0.084	0.253	78	561	1.406	0.088	0.326
34	303	1.292	0.083	0.261	79	261	1.407	0.097	0.347
35	477	1.293	0.084	0.253	80	313	1.409	0.095	0.336
36	562	1.302	0.099	0.268	81	440	1.410	0.107	0.351
37	409	1.304	0.085	0.271	82	582	1.411	0.083	0.332
38	520	1.306	0.078	0.270	83	334	1.419	0.127	0.354
39	255	1.307	0.093	0.270	84	493	1.419	0.100	0.334
40	384	1.308	0.077	0.274	85	188	1.419	0.099	0.344
41	585	1.322	0.072	0.270	86	270	1.425	0.092	0.339
42	471	1.322	0.099	0.278	87	476	1.426	0.099	0.347
43	203	1.322	0.089	0.287	88	197	1.426	0.113	0.339
44	367	1.323	0.098	0.284	89	526	1.426	0.110	0.364
45	421	1.324	0.081	0.291	90	559	1.426	0.115	0.351

PMT Type 2 Selection Table (Ranked my MSF)

Rank	Serial #	MSF	RMSE	Range	Rank	Serial #	MSF	RMSE	Range
91	539	1.428	0.108	0.356	136	381	1.513	0.117	0.422
92	552	1.429	0.098	0.344	137	213	1.515	0.129	0.424
93	309	1.430	0.122	0.360	138	386	1.517	0.126	0.426
94	489	1.431	0.121	0.368	139	366	1.518	0.141	0.421
95	259	1.432	0.112	0.362	140	536	1.519	0.127	0.423
96	575	1.432	0.105	0.370	141	437	1.523	0.125	0.409
97	595	1.434	0.120	0.354	142	584	1.524	0.138	0.411
98	329	1.435	0.124	0.358	143	411	1.524	0.132	0.434
99	401	1.437	0.095	0.364	144	282	1.525	0.127	0.443
100	547	1.437	0.123	0.359	145	191	1.527	0.104	0.404
101	337	1.437	0.138	0.352	146	516	1.529	0.116	0.414
102	417	1.441	0.096	0.368	147	419	1.531	0.138	0.433
103	435	1.441	0.111	0.353	148	524	1.534	0.110	0.406
104	453	1.455	0.114	0.380	149	484	1.542	0.147	0.438
105	533	1.459	0.099	0.390	150	580	1.543	0.113	0.415
106	501	1.460	0.108	0.377	151	374	1.544	0.148	0.445
107	491	1.462	0.116	0.384	152	546	1.544	0.126	0.452
108	424	1.464	0.118	0.379	153	317	1.545	0.125	0.424
109	531	1.466	0.119	0.402	154	239	1.547	0.119	0.432
110	564	1.468	0.104	0.377	155	422	1.551	0.160	0.439
111	318	1.469	0.114	0.381	156	211	1.554	0.136	0.436
112	342	1.471	0.099	0.383	157	340	1.556	0.130	0.441
113	444	1.472	0.098	0.376	158	509	1.556	0.135	0.438
114	488	1.473	0.105	0.391	159	364	1.557	0.156	0.442
115	280	1.475	0.125	0.387	160	244	1.559	0.149	0.438
116	391	1.476	0.133	0.407	161	262	1.560	0.137	0.441
117	256	1.477	0.134	0.403	162	507	1.561	0.125	0.437
118	360	1.480	0.136	0.401	163	290	1.565	0.137	0.429
119	599	1.480	0.128	0.392	164	363	1.573	0.130	0.445
120	369	1.482	0.111	0.383	165	581	1.574	0.127	0.460
121	532	1.483	0.111	0.399	166	574	1.575	0.159	0.449
122	583	1.483	0.126	0.387	167	274	1.578	0.128	0.452
123	399	1.484	0.106	0.399	168	458	1.583	0.133	0.493
124	284	1.488	0.094	0.383	169	480	1.588	0.150	0.456
125	596	1.489	0.107	0.387	170	600	1.595	0.120	0.455
126	220	1.491	0.125	0.387	171	513	1.605	0.139	0.447
127	341	1.492	0.116	0.380	172	389	1.606	0.133	0.458
128	405	1.493	0.127	0.419	173	479	1.610	0.153	0.475
129	365	1.494	0.125	0.399	174	459	1.611	0.132	0.470
130	395	1.495	0.108	0.387	175	416	1.611	0.131	0.443
131	302	1.497	0.134	0.409	176	410	1.613	0.144	0.482
132	347	1.500	0.110	0.408	177	407	1.614	0.130	0.469
133	293	1.500	0.111	0.399	178	322	1.614	0.153	0.462
134	412	1.500	0.112	0.402	179	388	1.617	0.131	0.478
135	408	1.506	0.127	0.413	180	400	1.617	0.140	0.448

PMT Type 2 Selection Table (Ranked my MSF)

Rank	Serial #	MSF	RMSE	Range	Rank	Serial #	MSF	RMSE	Range
181	306	1.619	0.155	0.485	226	537	1.736	0.143	0.552
182	372	1.619	0.128	0.486	227	394	1.744	0.188	0.553
183	519	1.622	0.157	0.486	228	542	1.745	0.182	0.552
184	230	1.626	0.138	0.455	229	323	1.747	0.163	0.574
185	449	1.627	0.154	0.470	230	591	1.750	0.192	0.524
186	378	1.628	0.169	0.485	231	267	1.750	0.146	0.553
187	528	1.628	0.142	0.481	232	361	1.750	0.165	0.539
188	456	1.629	0.173	0.472	233	461	1.750	0.151	0.538
189	597	1.629	0.146	0.488	234	260	1.750	0.150	0.529
190	215	1.633	0.177	0.485	235	452	1.758	0.156	0.584
191	272	1.634	0.155	0.475	236	307	1.761	0.198	0.561
192	589	1.635	0.130	0.483	237	327	1.764	0.166	0.608
193	446	1.647	0.153	0.486	238	217	1.767	0.133	0.558
194	457	1.647	0.146	0.488	239	246	1.768	0.164	0.583
195	578	1.653	0.145	0.455	240	396	1.771	0.166	0.559
196	377	1.656	0.165	0.500	241	548	1.772	0.179	0.568
197	385	1.660	0.162	0.503	242	353	1.776	0.154	0.512
198	382	1.663	0.144	0.499	243	333	1.783	0.164	0.568
199	570	1.664	0.150	0.486	244	568	1.791	0.167	0.582
200	392	1.667	0.133	0.504	245	590	1.792	0.187	0.597
201	344	1.669	0.144	0.495	246	426	1.794	0.190	0.577
202	420	1.670	0.157	0.521	247	549	1.797	0.173	0.590
203	251	1.675	0.168	0.504	248	383	1.800	0.175	0.571
204	319	1.677	0.145	0.537	249	430	1.808	0.152	0.604
205	545	1.678	0.106	0.448	250	398	1.810	0.163	0.616
206	222	1.680	0.160	0.519	251	349	1.818	0.178	0.607
207	258	1.684	0.176	0.524	252	345	1.819	0.161	0.567
208	375	1.684	0.170	0.491	253	299	1.819	0.210	0.579
209	427	1.685	0.144	0.509	254	418	1.821	0.183	0.550
210	506	1.687	0.185	0.501	255	263	1.823	0.174	0.592
211	481	1.688	0.154	0.555	256	241	1.827	0.163	0.587
212	198	1.695	0.134	0.524	257	242	1.833	0.192	0.593
213	543	1.695	0.166	0.516	258	402	1.842	0.166	0.583
214	455	1.702	0.129	0.503	259	454	1.862	0.177	0.596
215	362	1.704	0.171	0.532	260	298	1.867	0.168	0.569
216	194	1.707	0.161	0.516	261	359	1.877	0.208	0.592
217	285	1.718	0.145	0.506	262	297	1.883	0.200	0.609
218	475	1.719	0.157	0.517	263	525	1.892	0.177	0.633
219	269	1.720	0.158	0.520	264	226	1.895	0.192	0.635
220	496	1.722	0.182	0.560	265	450	1.897	0.177	0.614
221	214	1.727	0.160	0.556	266	523	1.906	0.173	0.617
222	414	1.730	0.165	0.564	267	199	1.909	0.185	0.608
223	277	1.730	0.160	0.530	268	350	1.909	0.205	0.656
224	301	1.731	0.167	0.562	269	404	1.918	0.179	0.617
225	423	1.732	0.167	0.540	270	286	1.921	0.201	0.651

PMT Type 2 Selection Table (Ranked my MSF)

Rank	Serial #	MSF	RMSE	Range
271	557	1.923	0.194	0.646
272	371	1.925	0.213	0.641
273	316	1.935	0.213	0.640
274	192	1.938	0.211	0.731
275	243	1.939	0.159	0.629
276	403	1.944	0.181	0.677
277	579	1.945	0.174	0.643
278	208	1.951	0.226	0.619
279	296	1.951	0.226	0.680
280	288	1.955	0.192	0.629
281	406	1.960	0.188	0.661
282	212	1.964	0.215	0.691
283	287	1.967	0.224	0.655
284	292	1.968	0.193	0.663
285	465	1.973	0.186	0.647
286	205	1.977	0.223	0.652
287	560	1.987	0.200	0.670
288	510	1.987	0.170	0.664
289	443	1.991	0.213	0.644
290	326	2.000	0.168	0.667
291	415	2.000	0.200	0.705
292	527	2.000	0.197	0.651
293	588	2.000	0.182	0.627
294	304	2.045	0.205	0.698
295	273	2.072	0.241	0.677
296	279	2.077	0.232	0.680
297	434	2.091	0.230	0.747
298	252	2.109	0.188	0.734
299	295	2.125	0.212	0.715
300	348	2.126	0.210	0.763
301	346	2.136	0.211	0.725
302	530	2.136	0.187	0.686
303	275	2.160	0.279	0.794
304	200	2.167	0.247	0.747
305	324	2.174	0.177	0.666
306	204	2.193	0.220	0.845
307	390	2.262	0.208	0.708
308	264	2.323	0.225	0.879
309	281	2.333	0.215	0.740
310	343	2.392	0.267	0.804
311	518	2.417	0.273	0.881
312	463	2.467	0.226	0.842
313	356	2.556	0.232	0.789
314	209	2.568	0.284	0.885
315	358	3.667	0.299	1.000
316	413	3.842	0.445	1.499
317	339	976.296	3.548	13.779